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CENTER FOR SCIENCE & THOUGHT

JÜLICH FORSCHUNGSZENTRUM

Recent Progress in Nuclear Lattice EFT

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

supported by DFG, SFB/TR-110 by CAS, PIFI by VolkswagenStftung by ERC, EXOTIC by NRW-FAIR

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VolkswagenStiftung

erc European Research Council Established by the European Commission

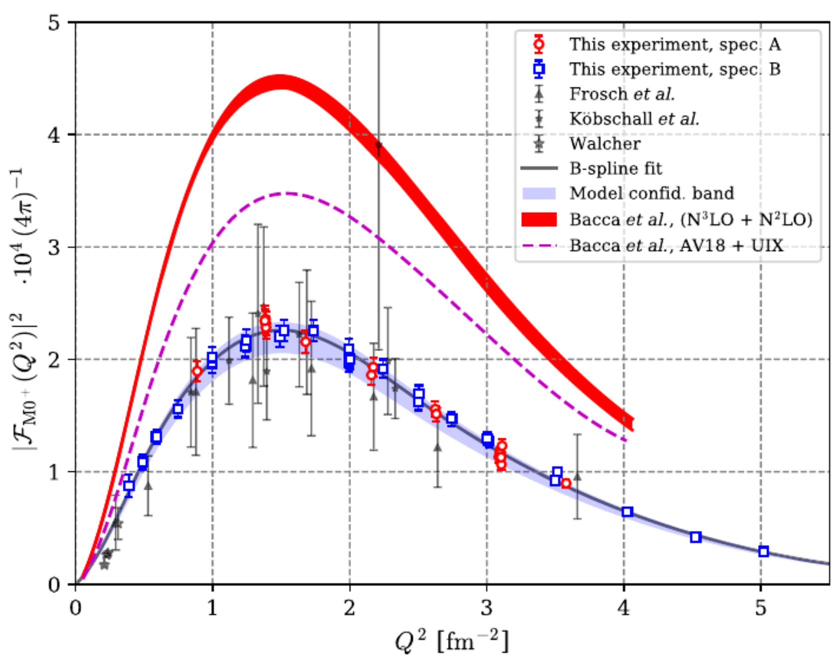
Contents

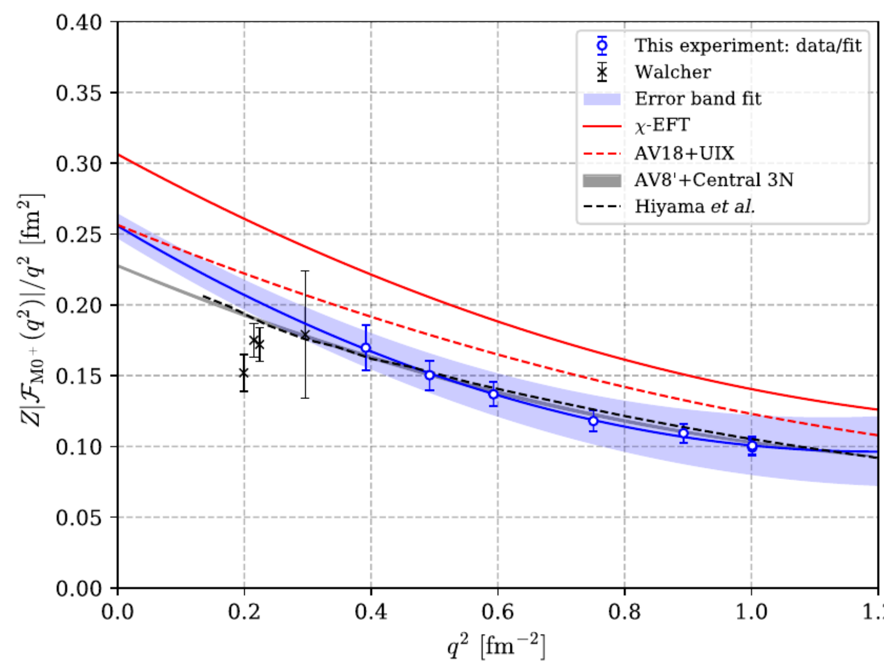
Two motivations

The ^{4}He form factor puzzle

- Recent Mainz measurements of $F_{M0}(0_2^+ \to 0_1^+)$ appear to be in stark disagreement with *ab initio* nuclear theory Kegel et al., Phys. Rev. Lett. **130** (2023) 152502

- Monopole transition ff
- low-momentum expansion





$\Rightarrow$ **A low-energy puzzle for nuclear forces?**

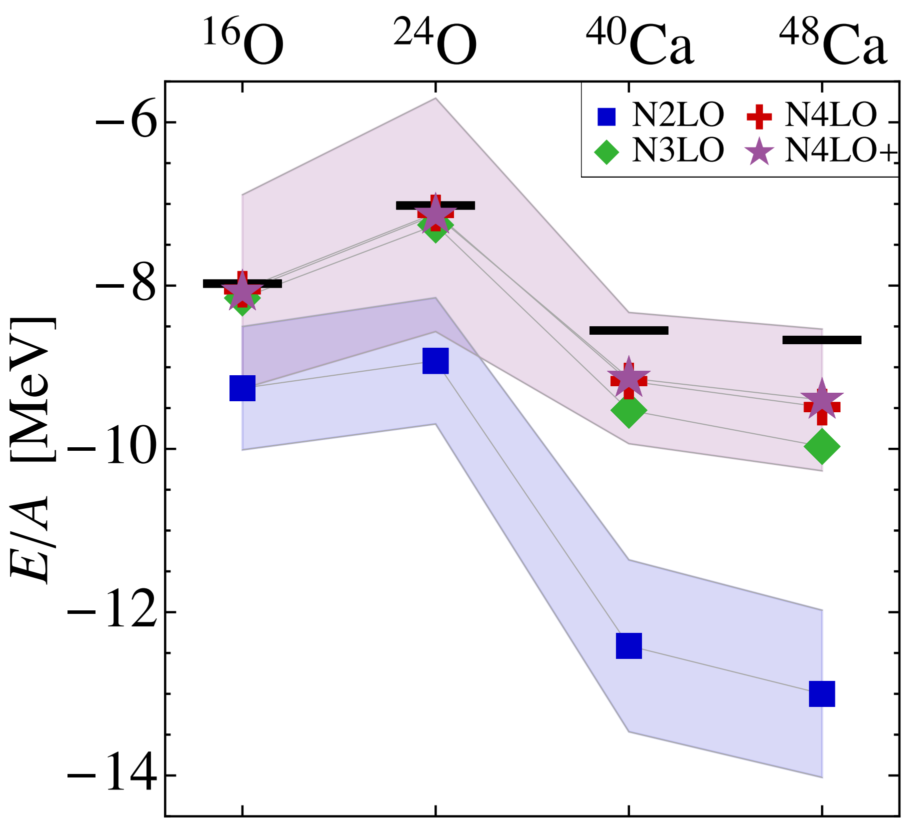
The nuclear radii puzzle

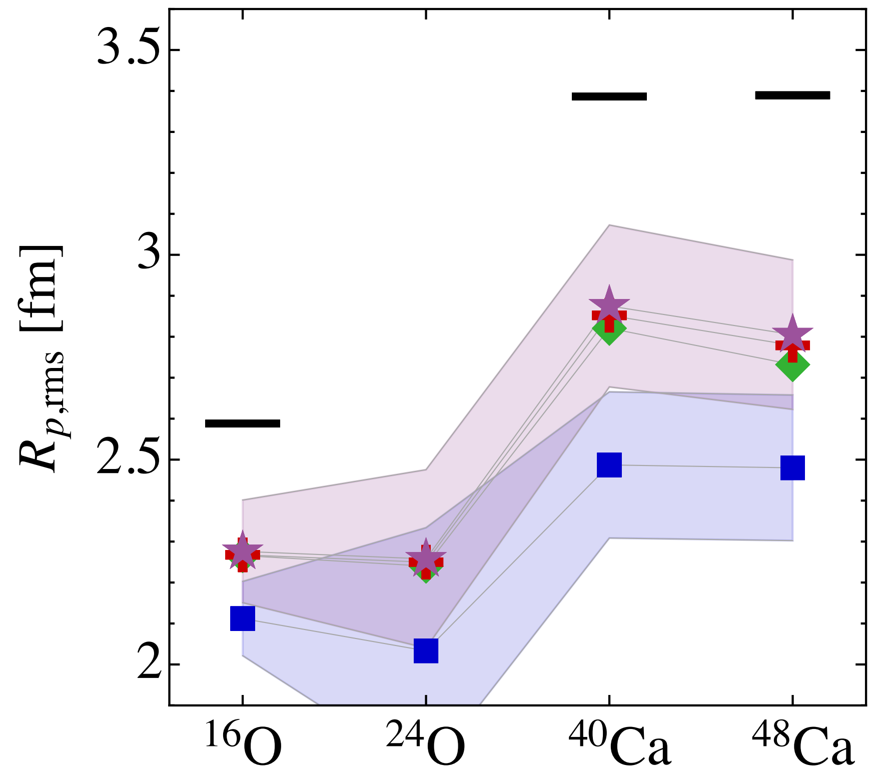
- Modern *ab initio* methods get correct energies, but incorrect radii

Cipollone et al., Phys. Rev. C **92** (2015) 014306, ...

- E.g. shell model with SRG evolved chiral NN and NNN interactions

LENPIC, Phys. Rev. C **106** (2022) 064002



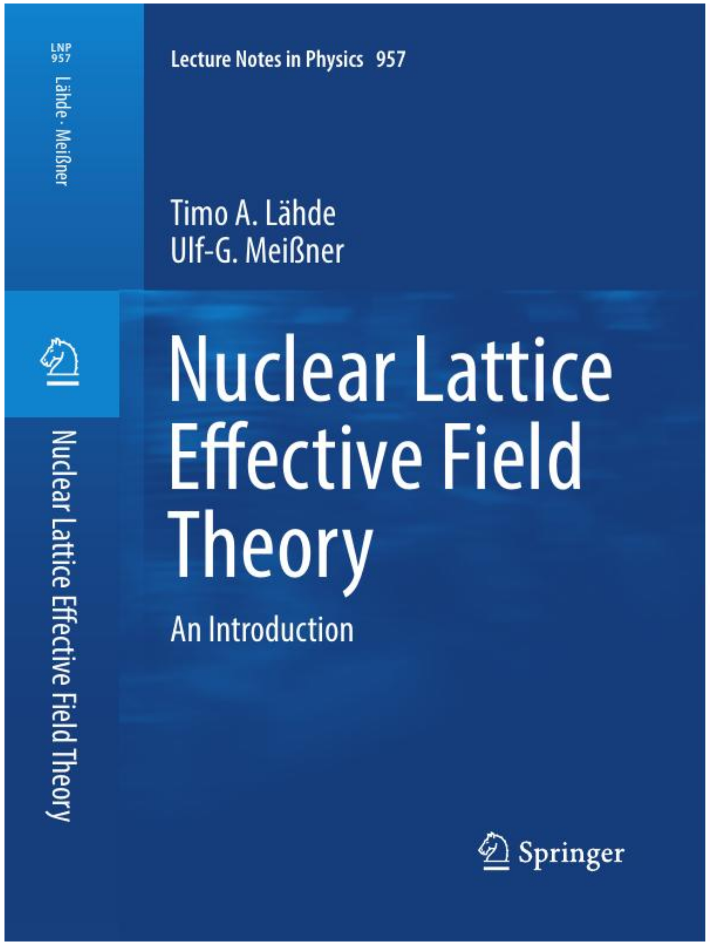


Can we solve these puzzles with NLEFT?

- Work on a discretized Euclidean space-time $L^3 \times L_t$
- Build on successful continuum chiral NN + NNN forces

  $\hookrightarrow$ discretized chiral potential w/ pion exchanges and contact interactions + Coulomb

  see e.g. Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773



- Typical lattice parameters:

  $- \; a = 1 \ldots 2$ fm

  $\rightarrow p_{\rm max} = \dfrac{\pi}{a} \simeq 315 - 630$ MeV [UV cutoff]

  $- \; L = 5 \ldots 15$ in units of a

- Special features:

  $\hookrightarrow$ no continuum limes (EFT)

  $\hookrightarrow$ approximate Wigner SU(4) spin-isopin symmetry suppresses sign oscillations

  E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

Essentials of Nuclear Binding

B. N. Lu, N. Li, S. Elhatisari, D. Lee, E. Epelbaum, UGM,
Phys. Lett. **B 797** (2019) 134863

A minimal nuclear interaction

- Basic idea:

  $\hookrightarrow$ explore the approximate SU(4) spin-isospin symmetry of the nuclear forces

  Wigner (1936)

  $\hookrightarrow$ particular friendly for MC simulations (suppression of sign oscillations)

  Chen, Lee, Schäfer, Phys. Rev. Lett. **93** (2004) 242302

  $\hookrightarrow$ the ^{4}He nucleus is a prime candidate ($I = S = 0$)

- Ingredients:

  $\hookrightarrow$ 2N & 3N forces (contact interactions)

  $\hookrightarrow$ local & non-local smearing (generates range of these forces)

  $\hookrightarrow$ use later as the LO action free of sign problem (simple Hamiltonian)

Short reminder of Wigner SU(4) symmetry

Wigner, Phys. Rev. **C 51** (1937) 106

- If the nuclear Hamiltonian does not depend on spin and isospin, then it is obviously invariant under SU(4) transformations [really $U(4) = U(1) \times SU(4)$]:

$$N \to UN\,, \quad U \in SU(4)\,, \quad N = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$N \to N + \delta N\,, \quad \delta N = i\epsilon_{\mu\nu}\sigma^\mu\tau^\nu\, N\,, \quad \sigma^\mu = (1, \sigma_i)\,, \quad \tau^\mu = (1, \tau_i)$$

- LO pionless EFT: $\mathcal{L}_{\not\pi} = N^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m_N} \right) N - \frac{1}{2} \left(C_S (N^\dagger N)^2 + C_T (N^\dagger \vec{\sigma} N)^2 \right)$

Mehen, Stewart, Wise, Phys. Rev. Lett. **83** (1999) 931

- Partial wave LECs: $C(^1S_0) = C_S - 3C_T\,, \quad C(^3S_1) = C_S + C_T$

⇒ The operator $(N^\dagger N)^2$ is invariant under Wigner SU(4), but $(N^\dagger \vec{\sigma} N)^2$ is not

⇒ In the Wigner SU(4) limit, one finds: $C(^1S_0) = C(^3S_1) \; \to a_{np}^{S=0} = a_{np}^{S=1} \to \infty$

⇒ The exact symmetry limit corresponds to a scale invariant non-relativistic system

Essential elements for nuclear binding I

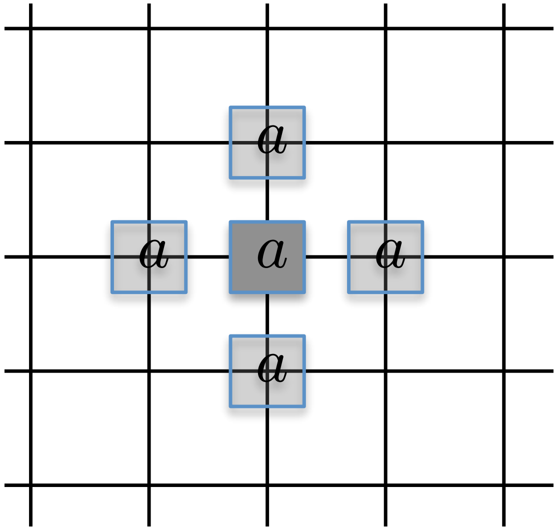
Lu, Li, Elhatisari, Epelbaum, Lee, UGM. Phys. Lett. B **797** (2019) 134863 [arXiv:1812.10928]

- Highly SU(4) symmetric LO action without pions, local and non-local smearing:

$$H_{\mathrm{SU}(4)} = H_{\mathrm{free}} + \frac{1}{2!} C_2 \sum_n \tilde\rho(n)^2 + \frac{1}{3!} C_3 \sum_n \tilde\rho(n)^3$$

$$\tilde\rho(n) = \sum_i \tilde a_i^\dagger(n) \tilde a_i(n) + s_L \sum_{|n'-n|=1} \sum_i \tilde a_i^\dagger(n') \tilde a_i(n')$$

$$\tilde a_i(n) = a_i(n) + s_{NL} \sum_{|n'-n|=1} a_i(n')$$



- Only **four** parameters!

  C_2 and C_3 = strength of the leading two- and three-body interactions

  s_L and s_{NL} = strength of the local and the non-local interaction

Essential elements for nuclear binding II

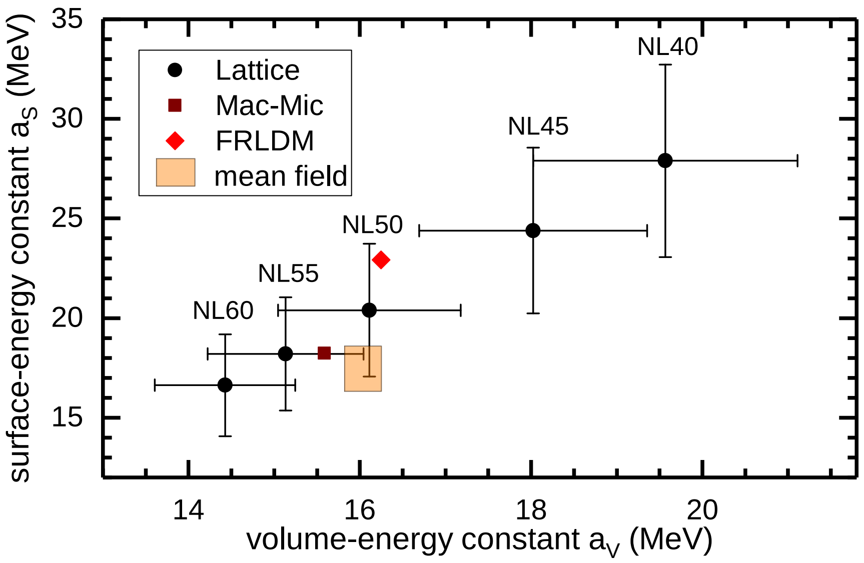
- Fixing the parameters:
  - ⋆ interaction strength C_2 and range s_L from the average S-wave scattering lengths and effective ranges (requires SU(4) breaking later)
  - ⋆ interaction strength C_3 from the ^{3}H binding energy
  - ⋆ interaction range s_{NL} can not be determined in light nuclei
  - ↪ calculate the volume- and surface energy of mid-mass nuclei $16 \leq A \leq 40$
- compare w/ existing calculations:

↪ $\boxed{s_{NL} = 0.5}$

Mac-Mic: Wang et al., Phys. Lett. B **734** (2014) 215

FRLDM: Möller et al., Atom Data Nucl. Data Tabl. **59** (1995) 184

mean field: Bender et al., Rev. Mod. Phys. **75** (2003) 121



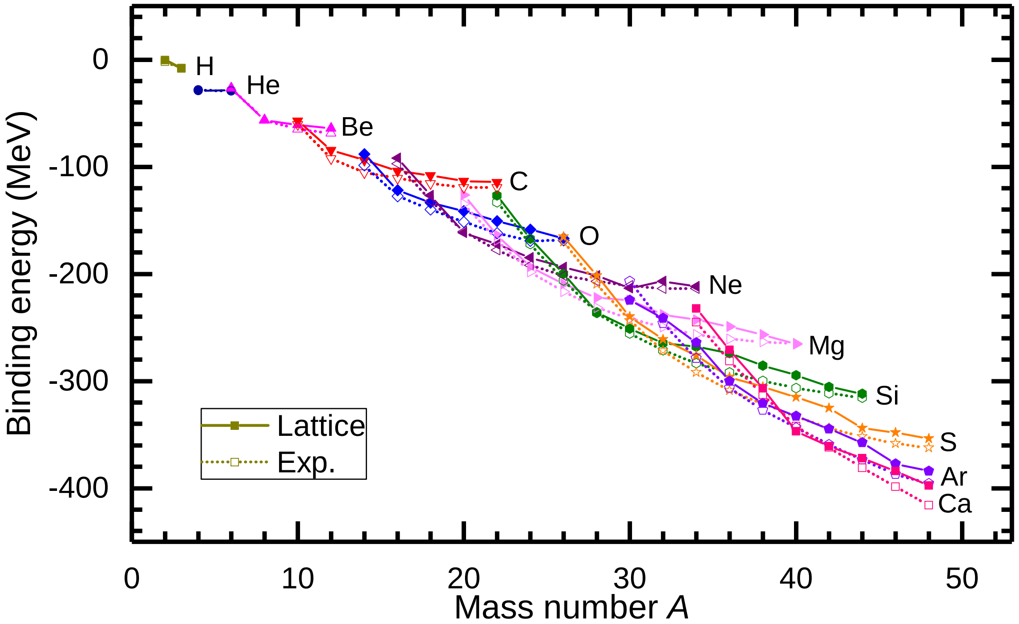
Energies for selected nuclei

- Calculated binding energies for 3N & alpha-type nuclei:

	B [MeV]	Coul. [MeV]	B/Exp.
^{3}H	8.48(2)*	0.0	1.00
^{3}He	7.75(2)	0.73(1)	1.00
^{4}He	28.89(1)	0.80(1)	1.02
^{16}O	121.9(3)	13.9(1)	0.96
^{20}Ne	161.6(1)	20.2(1)	1.01
^{24}Mg	193.5(17)	28.0(2)	0.98
^{28}Si	235.8(17)	37.1(4)	1.00
^{40}Ca	346.8(8)	71.7(6)	1.01

[* = input]

- Binding energies for 86 even-even nuclei



- selected nuclei: amazingly precise, all deviations $\leq 4\%$ (except ^{12}C)
- even-even isotopic chains come out amazingly precise, general trends reproduced

$\hookrightarrow$ on the proton-rich side better than on the neutron-rich one $\rightarrow$ spin-dep. effects

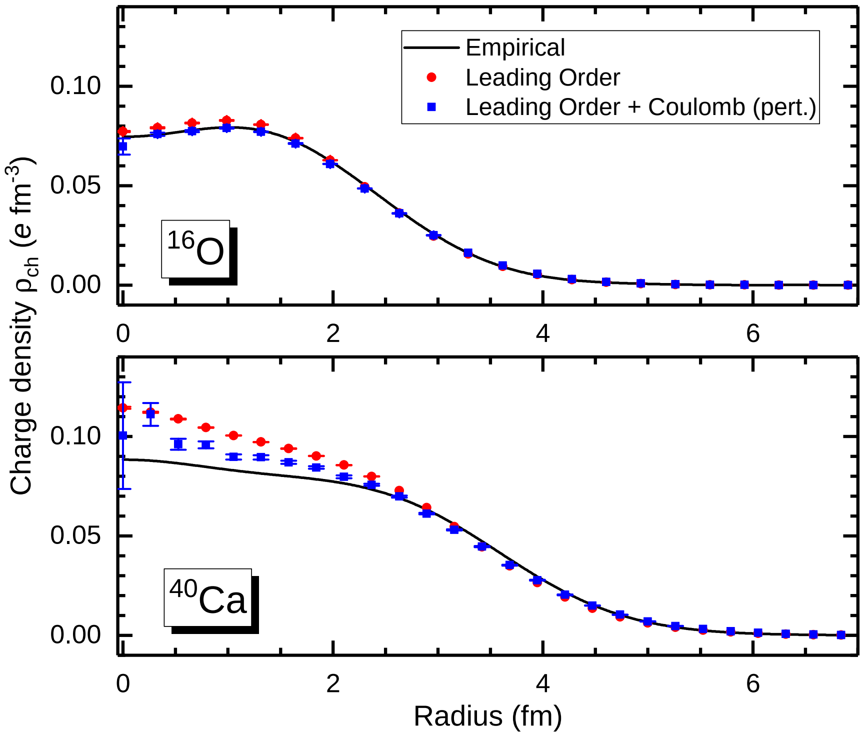
- but remember: this is only leading order!

Radii for selected nuclei

- Calculated charge radii
  for 3N & alpha-type nuclei:

	$R_{\rm ch}$	Exp.	$R_{\rm ch}$/Exp.
^{3}H	1.90(1)	1.76	1.08
^{3}He	1.99(1)	1.97	1.01
^{4}He	1.72(3)	1.68	1.02
^{16}O	2.74(1)	2.70	1.01
^{20}Ne	2.95(1)	3.01	0.98
^{24}Mg	3.13(2)	3.06	1.02
^{28}Si	3.26(1)	3.12	1.04
^{40}Ca	3.42(3)	3.48	0.98

- Charge distributions
  for ^{16}O and ^{40}Ca



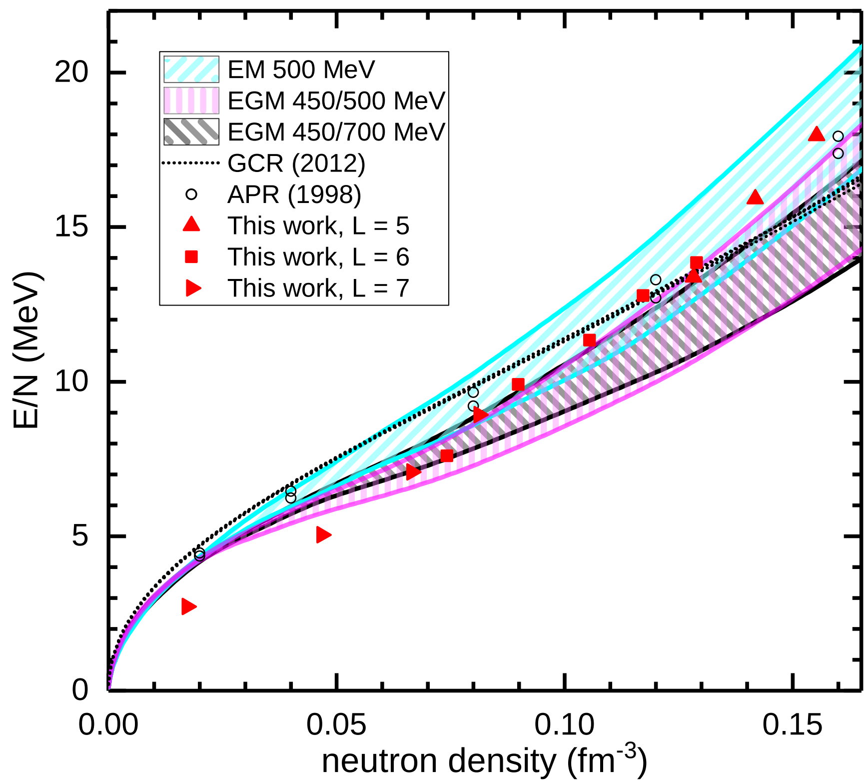
- Radii quite well described (except ^{12}C)

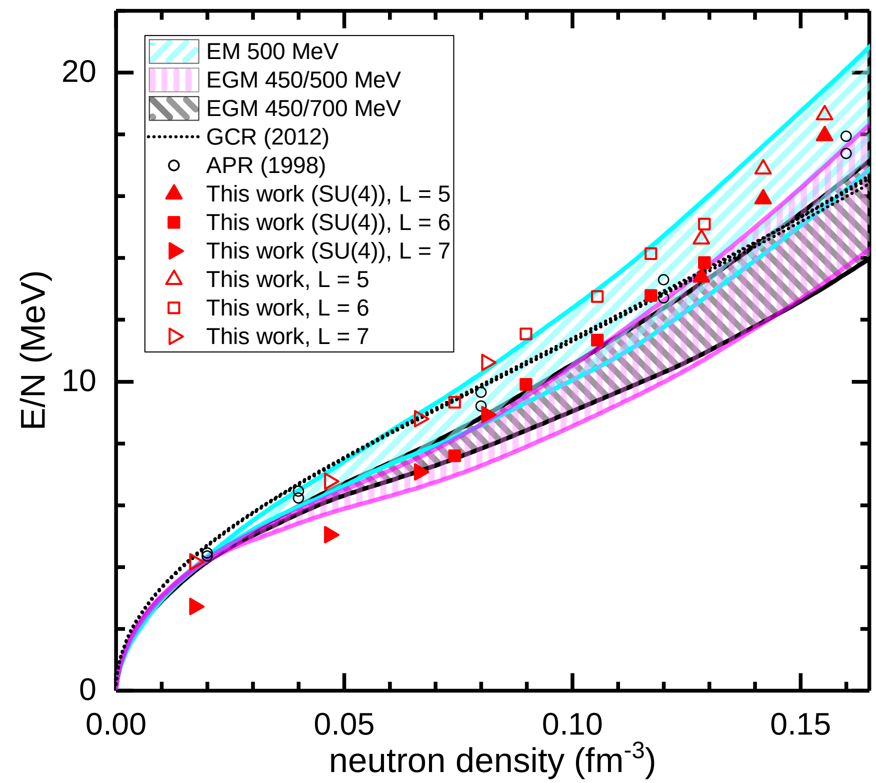
↪ overcomes earlier problems (see PRL 109 (2012) 252501, 112 (2014) 102501)

- Also a fair description of the charge distributions at LO!

Neutron matter

- 14 to 66 neutrons in $L = 5, 6, 7 \rightarrow \rho = 0.02 - 0.15\,\text{fm}^{-3}$





- exact SU(4)

$\hookrightarrow$ deviations at low densities

- SU(4) breaking term $\rightarrow a_{nn}$ ✓

$\hookrightarrow$ good overall description

APR = Akmal, Pandharipande, Ravenhall, Phys. Rev. C **58** (1998) 1804; GCR = Gandolfi, Carlson, Reddy, Phys. Rev. C **85** (2012) 032801; all others in: Tews et al., Phys. Rev. Lett. **110** (2013) 032504.

Ab initio calculation of the ^{4}He transition form factor

UGM, S. Shen, S. Elhatisari, D. Lee, 2309.01558 [nucl-th]

Basic considerations

- Use the essential elements action, **all parameters fixed!**

- Calculate the transition ff and its low-energy expansion form the transition density

$$\rho_{\rm tr}(r) = \langle 0_1^+|\hat\rho(\vec r)|0_2^+\rangle$$

$$F(q) = \frac{4\pi}{Z}\int_0^\infty \rho_{\rm tr}(r) j_0(qr) r^2 dr = \frac{1}{Z}\sum_{\lambda=1}^{\infty}\frac{(-1)^\lambda}{(2\lambda+1)!} q^{2\lambda}\langle r^{2\lambda}\rangle_{\rm tr}$$

$$\frac{Z|F(q^2)|}{q^2} = \frac{1}{6}\langle r^2\rangle_{\rm tr}\left[1 - \frac{q^2}{20}\mathcal{R}_{\rm tr}^2 + \mathcal{O}(q^4)\right]$$

$$\mathcal{R}_{\rm tr}^2 = \langle r^4\rangle_{\rm tr}/\langle r^2\rangle_{\rm tr}$$

- The first excited state sits in the continuum & close to the ^3H-p threshold

$\hookrightarrow$ use large volumes $L = 10, 11, 12$ or $L = 13.2$ fm, 14.5 fm, 15.7 fm

$\hookrightarrow$ the lattice spacing is fixed to $a = 1.32$ fm, corresponding $\Lambda = \pi/a = 465$ MeV

The first excited state

- 3 coupled channels with $\mathbf{0^+}$ q.n's $\rightarrow$ accelerates convergence as $L_t \rightarrow \infty$
- Shell-model wave functions (4 nucleons in $1s_{1/2}$, twice 3 in $1s_{1/2}$ and 1 in $2s_{1/2}$)

L [fm]	$E(0_1^+)$ [MeV]	$E(0_2^+)$ [MeV]	ΔE [MeV]
13.2	$-28.32(3)$	$-8.37(14)$	$0.28(14)$
14.5	$-28.30(3)$	$-8.02(14)$	$0.42(14)$
15.7	$-28.30(3)$	$-7.96(9)$	$0.39(9)$

$\hookrightarrow$ statistical and large-L_t errors

$\hookrightarrow$ agreement w/ experiment: $E(0_1^+) = 28.3$ MeV, $\Delta E = 0.4$ MeV

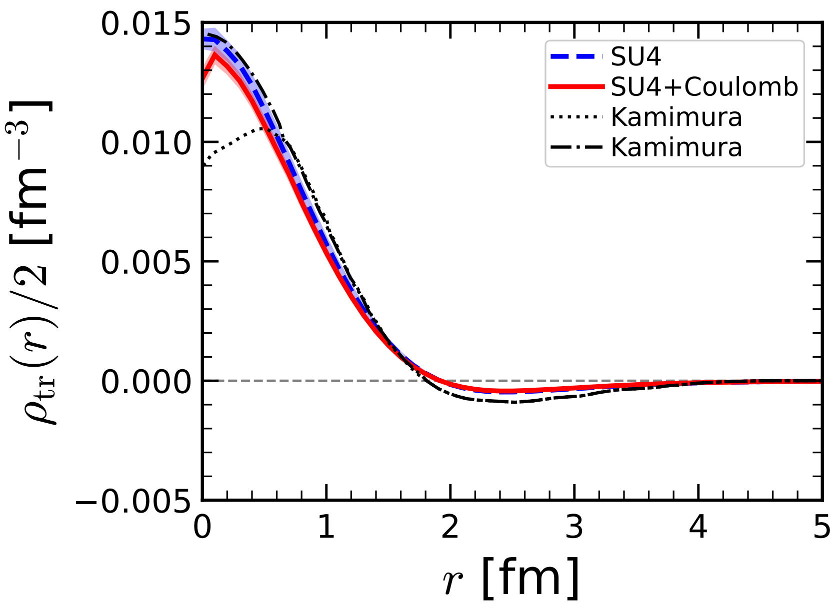
$\hookrightarrow$ ΔE consistent w/ no-core Gamov shell model Michel et al., 2306.05192 [nucl-th]

$\hookrightarrow$ consistent w/ the Efimov tetramer analysis $\Delta E = 0.38(2)$ MeV

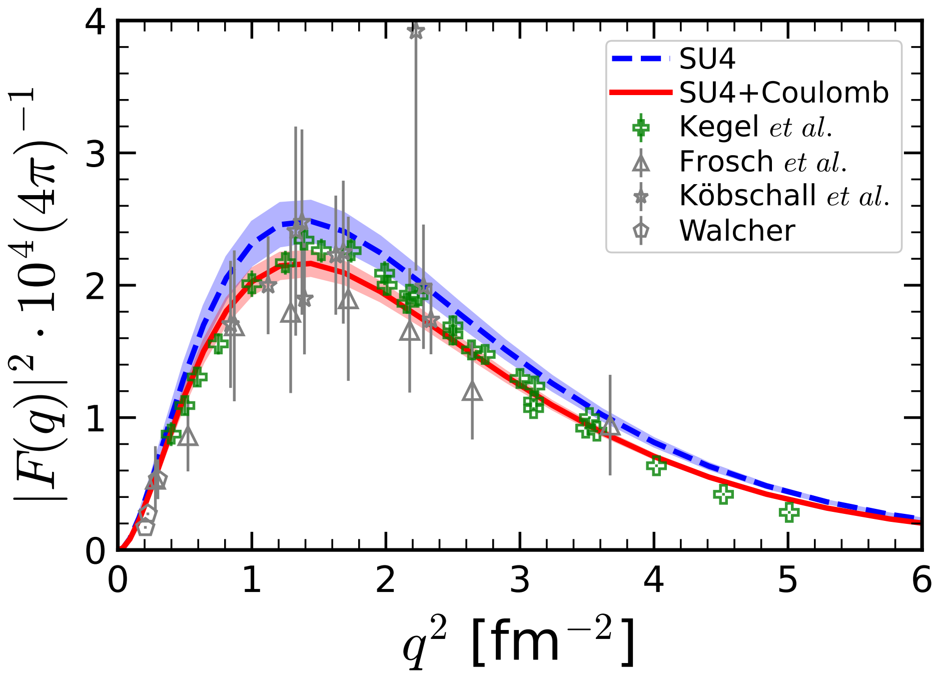
von Stecher, D'Incao, Greene, Nat. Phys. **5** (2009) 417; Hammer, Platter, EPJA **32** (2007) 113

The transition form factor

- Transition charge density



- Transition form factor



$\hookrightarrow$ agrees with the reconstructed one from Kamimura PTEP **2023** (2023) 071D01

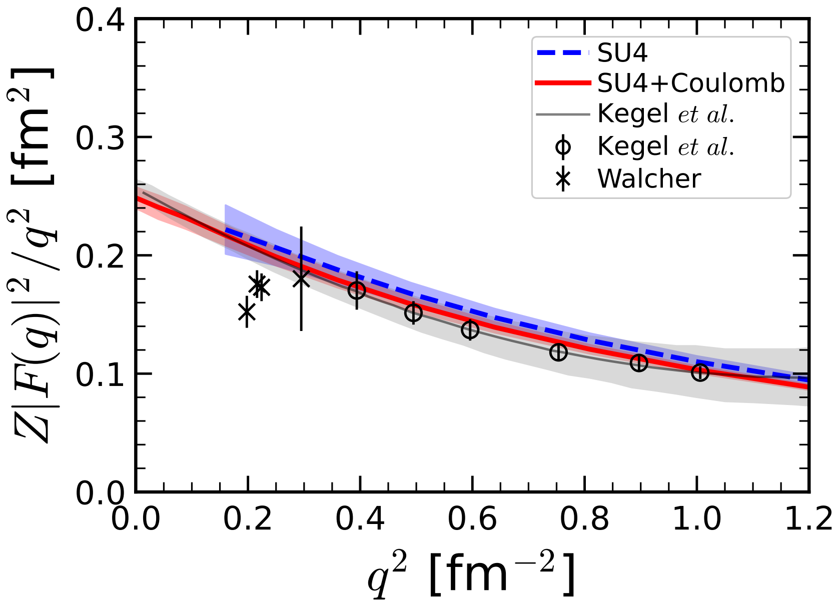
$\hookrightarrow$ very small central depletion (no zero)

$\hookrightarrow$ excellent description of the data

$\hookrightarrow$ Coulomb required plus smaller uncertainty (improved signal)

The transition form factor II

- Small momentum expansion



	$\langle r^2\rangle_{\rm tr}$ [fm^2]	$\mathcal{R}_{\rm tr}$ [fm]
Experiment	1.53 ± 0.05	4.56 ± 0.15
Th (AV8'+ centr. 3N)*	1.36 ± 0.01	4.01 ± 0.05
Th (AV18 + UIX)	1.54 ± 0.01	3.77 ± 0.08
Th (NLEFT)	1.49 ± 0.01	4.00 ± 0.04

*Hiyama, Gibson, Kamimura, PRC **70** (2004) 031001

↪ Also consistent description of the low-energy data

↪ **No puzzle** to the nuclear forces!

↪ Can be improved using N3LO action + wave function matching

Elhatisari et al., 2210.17488 [nucl-th]

Emergent geometry and duality in the carbon nucleus

Wigner's SU(4) symmetry and the carbon spectrum

- Study of the spectrum of ^{12}C Shen, Lähde, Lee, UGM, Eur. Phys.J. A **57** (2021) 276

  $\hookrightarrow$ spin-orbit splittings are known to be weak

  Hayes, Navratil, Vary, Phys. Rev. Lett. **91** (2003) 012502 Johnson, Phys. Rev. C **91** (2015) 034313

  $\hookrightarrow$ start with cluster and shell-model configurations $\longrightarrow$ next slide

- Locally and non-locally smeared SU(4) invariant interaction:

$$V = C_2 \sum_{\mathbf{n}',\mathbf{n},\mathbf{n}''} :\rho_{\rm NL}(\mathbf{n}') f_{s_{\rm L}}(\mathbf{n}'-\mathbf{n}) f_{s_{\rm L}}(\mathbf{n}-\mathbf{n}'')\rho_{\rm NL}(\mathbf{n}''):\,, \quad f_{s_{\rm L}}(\mathbf{n}) = \begin{cases} 1, & |\mathbf{n}| = 0, \\ s_{\rm L}, & |\mathbf{n}| = 1, \\ 0, & \text{otherwise} \end{cases}$$

$$\rho_{\rm NL}(\mathbf{n}) = a^\dagger_{\rm NL}(\mathbf{n}) a_{\rm NL}(\mathbf{n})$$

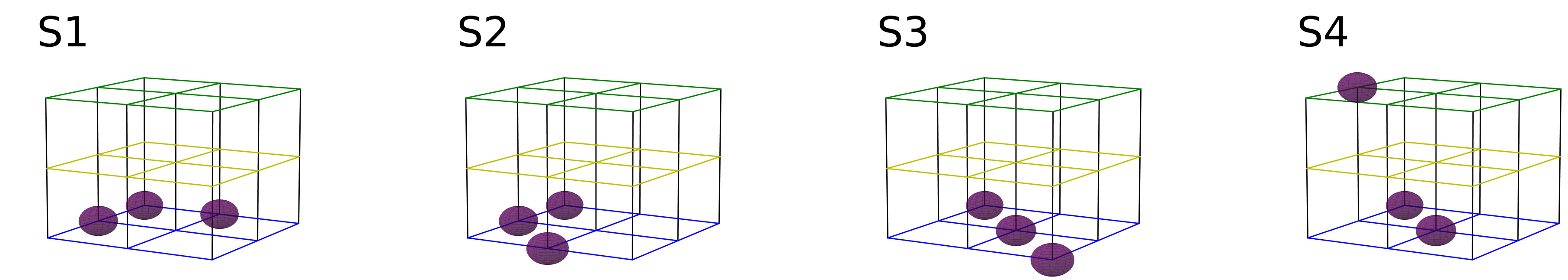
$$a^{(\dagger)}_{\rm NL}(\mathbf{n}) = a^{(\dagger)}(\mathbf{n}) + s_{\rm NL} \sum_{|\mathbf{n}'|=1} a^{(\dagger)}(\mathbf{n}+\mathbf{n}')\,, \quad s_{\rm NL} = 0.2$$

$\hookrightarrow$ only two adjustable parameters (C_2, s_L) fitted to $E_{^4\rm He}$ & $E_{^{12}\rm C}$

$\hookrightarrow$ investigate the spectrum for $a = 1.64$ fm and $a = 1.97$ fm

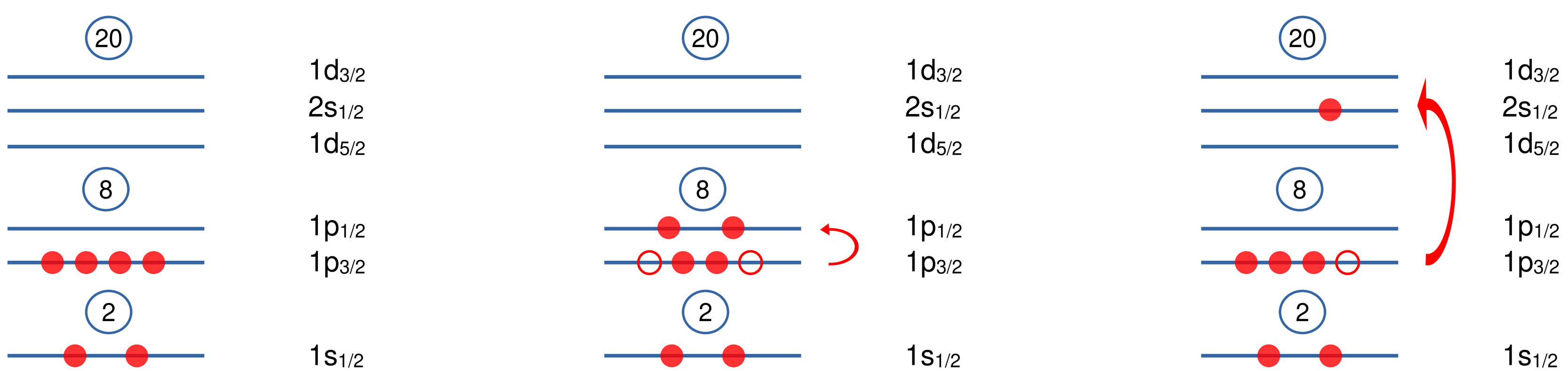
Configurations

- Cluster and shell model configurations



— isoscele right triangle — "bent-arm" shape — linear diagonal chain — acute isoscele triangle

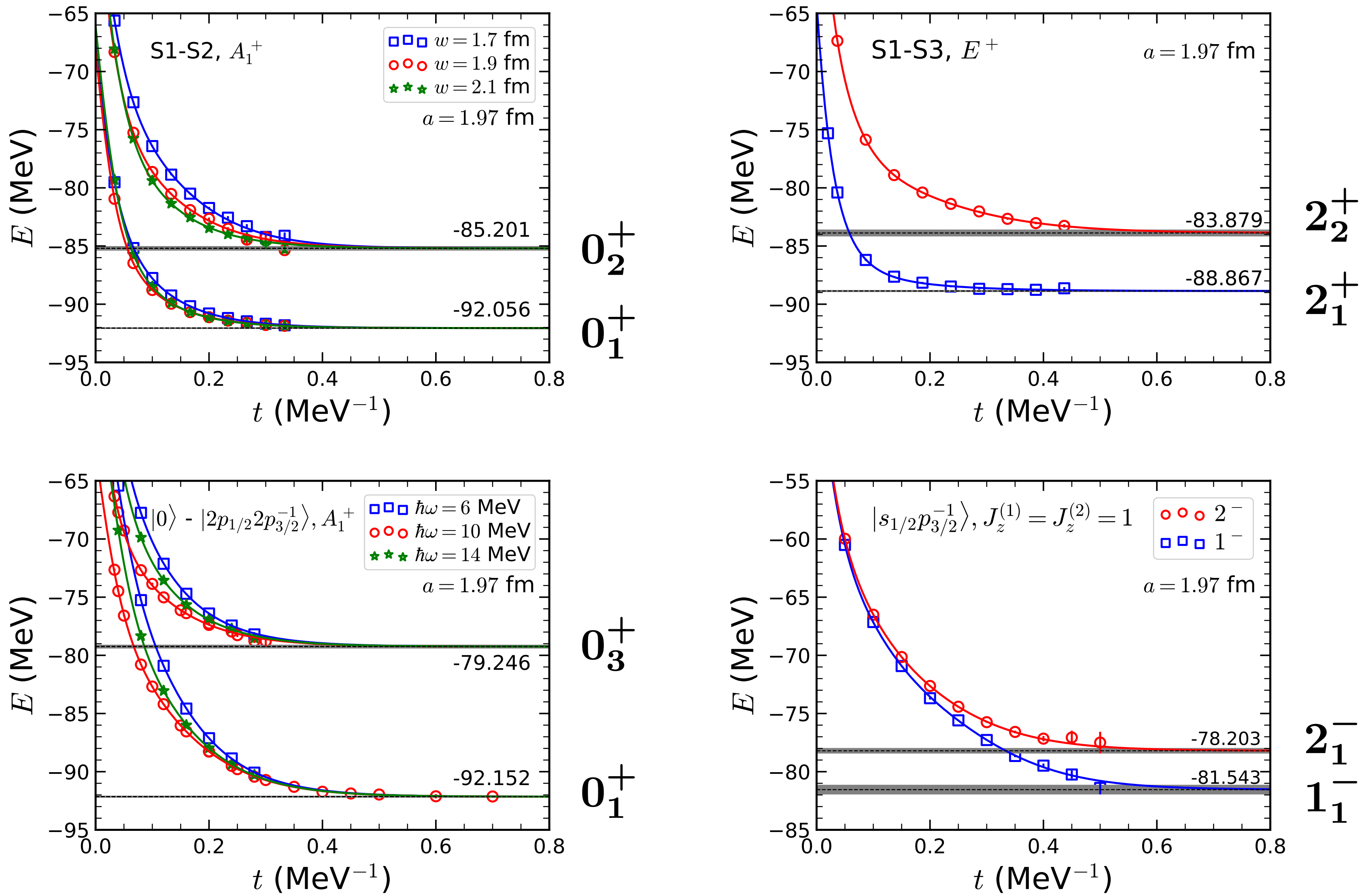
Gaussian wave packets
$w = 1.7 - 2.1$ fm



— ground state $|0\rangle$ — $2p$-$2h$ state, $J_z = 0$ — $1p$-$1h$ state, $J_z^{(1)} = J_z^{(2)} = 1$

Transient energies

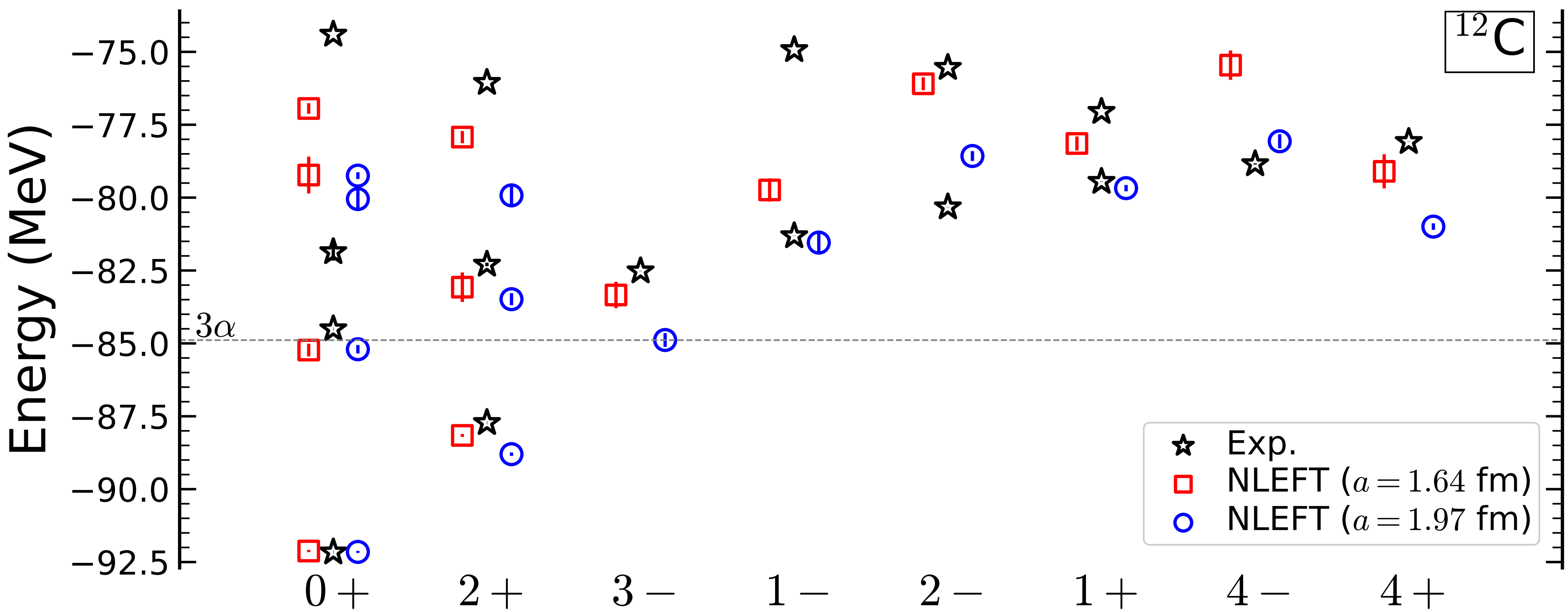
- Transient energies from cluster and shell-model configurations



Spectrum of ^{12}C

Shen, Lähde, Lee, UGM, Eur. Phys.J. A **57** (2021) 276 [arXiv:2106.04834]

- Amazingly precise description $\rightarrow$ great starting point



Gives all states, other than NCSM

$\rightarrow$ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

A closer look at the spectrum of ^{12}C

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777

- Include also 3NFs: $$V = \frac{C_2}{2!}\sum_{\mathbf{n}}\tilde{\rho}(\mathbf{n})^2 + \frac{C_3}{3!}\sum_{\mathbf{n}}\tilde{\rho}(\mathbf{n})^3$$

- Fit the four parameters:

  C_2, C_3 – ground state energies of ^{4}He and ^{12}C

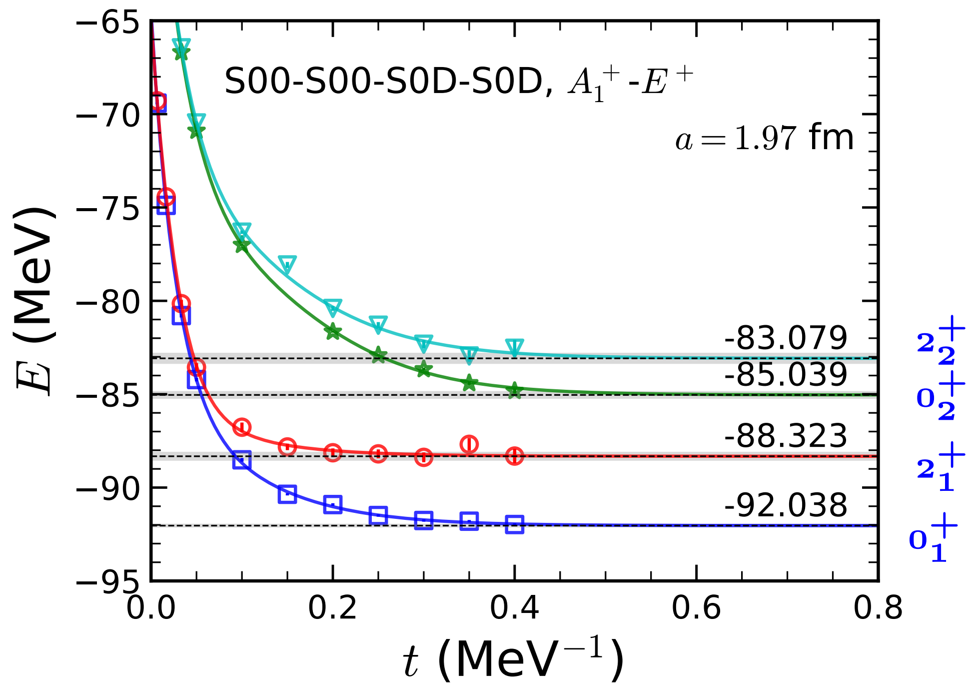
  $s_{\rm L}$ – radius of ^{12}C around 2.4 fm

  $s_{\rm NL}$ – best overall description of the transition rates

- Calculation of em transitions

  requires coupled-channel approach

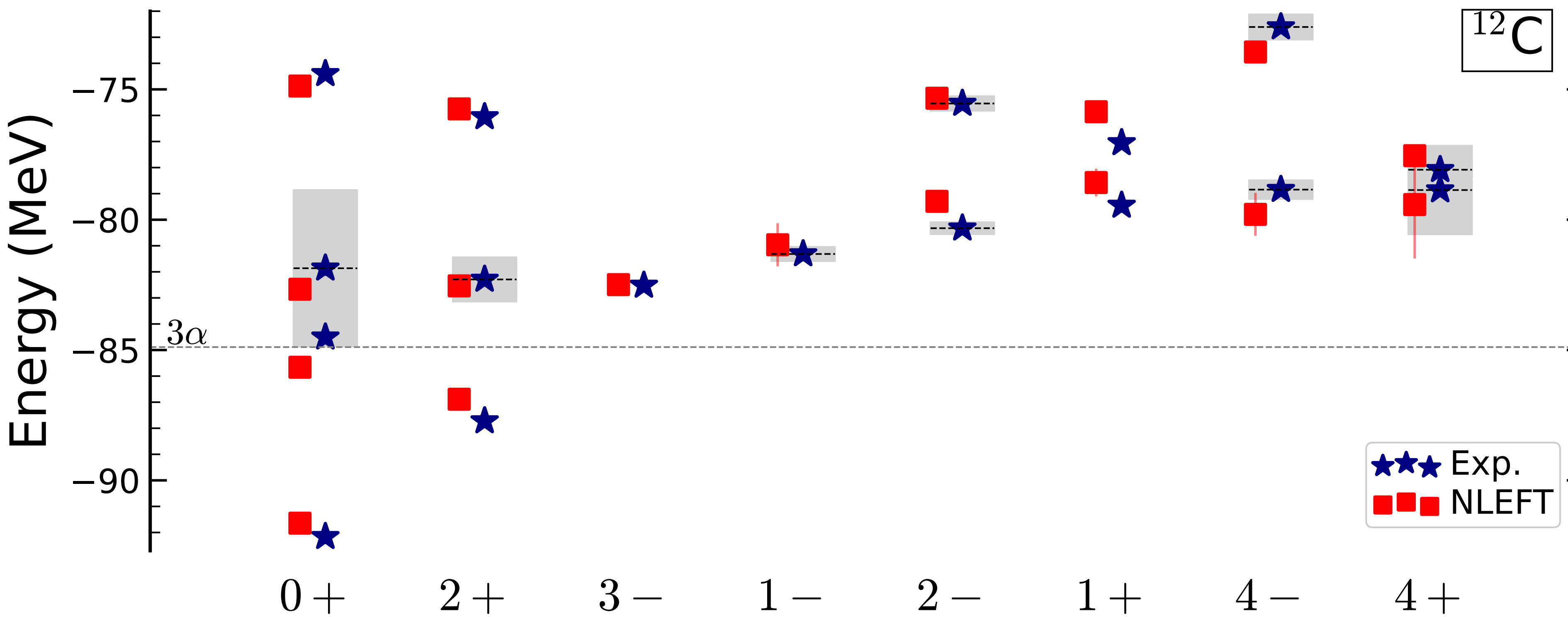
  e.g. 0^+ and 2^+ states



Spectrum of ^{12}C reloaded

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777

- Improved description when 3NFs are included, amazingly good



→ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

Electromagnetic properties

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777

- Radii (be aware of excited states), quadrupole moments & transition rates

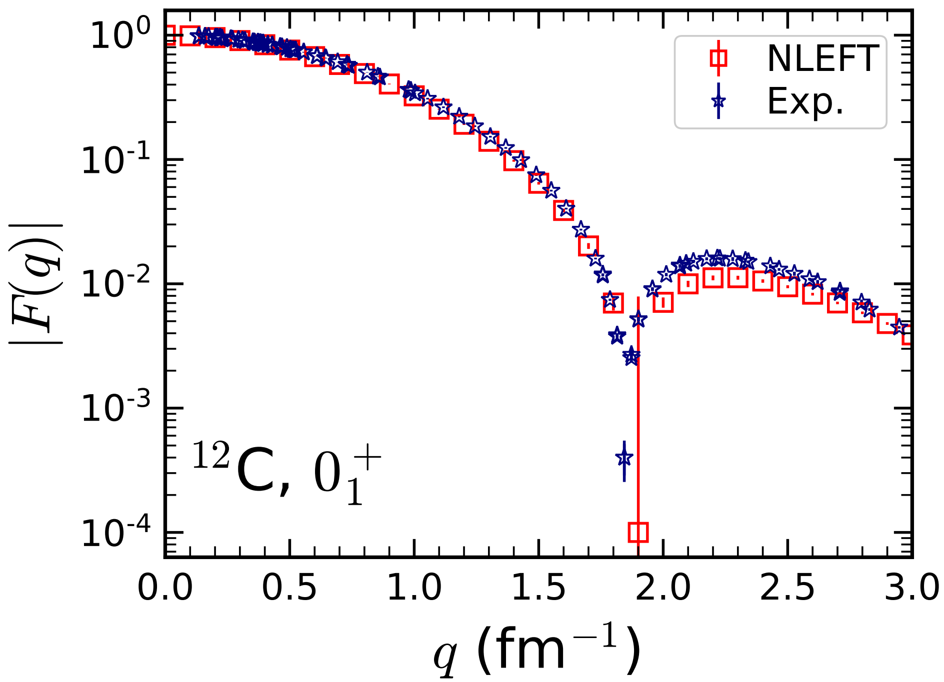
	NLEFT	FMD	α cluster	BEC	RXMC	Exp.
$r_c(0_1^+)$ [fm]	$\mathbf{2.53(1)}$	2.53	2.54	2.53	2.65	$\mathbf{2.47(2)}$
$r(0_2^+)$ [fm]	$\mathbf{3.45(2)}$	3.38	3.71	3.83	4.00	–
$r(0_3^+)$ [fm]	$\mathbf{3.47(1)}$	4.62	4.75	–	4.80	–
$r(2_1^+)$ [fm]	$\mathbf{2.42(1)}$	2.50	2.37	2.38	–	–
$r(2_2^+)$ [fm]	$\mathbf{3.30(1)}$	4.43	4.43	–	–	–

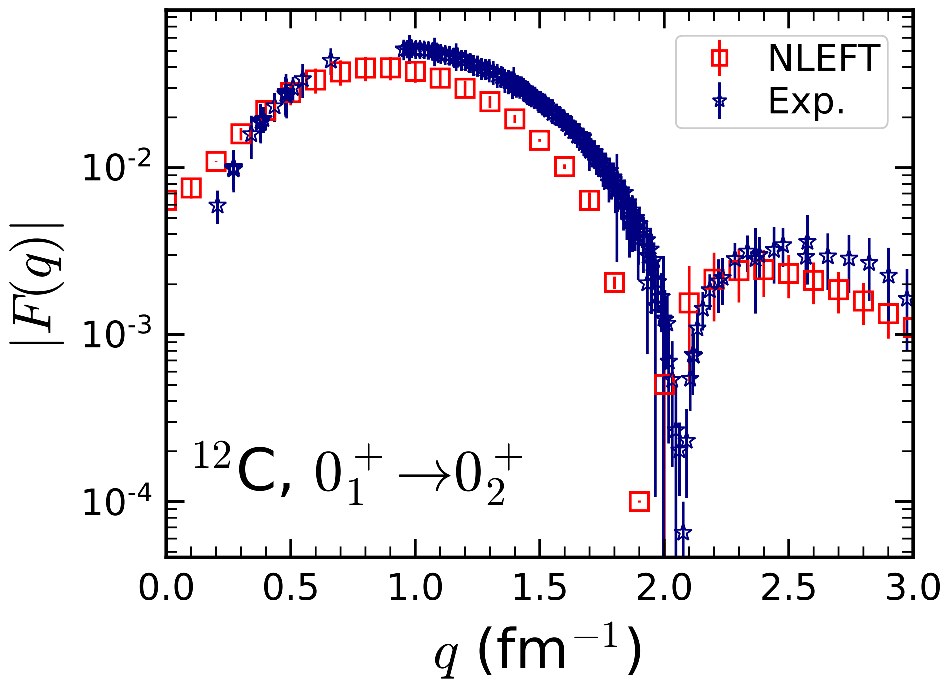
	NLEFT	FMD	α cluster	NCSM	Exp.
$Q(2_1^+)$ [e fm^2]	$\mathbf{6.8(3)}$	–	–	6.3(3)	$\mathbf{8.1(2.3)}$
$Q(2_2^+)$ [e fm^2]	$\mathbf{-35(1)}$	–	–	–	–
$M(E0, 0_1^+ \to 0_2^+)$ [e fm^2]	$\mathbf{4.8(3)}$	6.5	6.5	–	$\mathbf{5.4(2)}$
$M(E0, 0_1^+ \to 0_3^+)$ [e fm^2]	$\mathbf{0.4(3)}$	–	–	–	–
$M(E0, 0_2^+ \to 0_3^+)$ [e fm^2]	$\mathbf{7.4(4)}$	–	–	–	–
$B(E2, 2_1^+ \to 0_1^+)$ [e^2 fm^4]	$\mathbf{11.4(1)}$	8.7	9.2	8.7(9)	$\mathbf{7.9(4)}$
$B(E2, 2_1^+ \to 0_2^+)$ [e^2 fm^4]	$\mathbf{2.5(2)}$	3.8	0.8	–	$\mathbf{2.6(4)}$

Electromagnetic properties

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777

- Form factors and transition ffs [essentially parameter-free]:



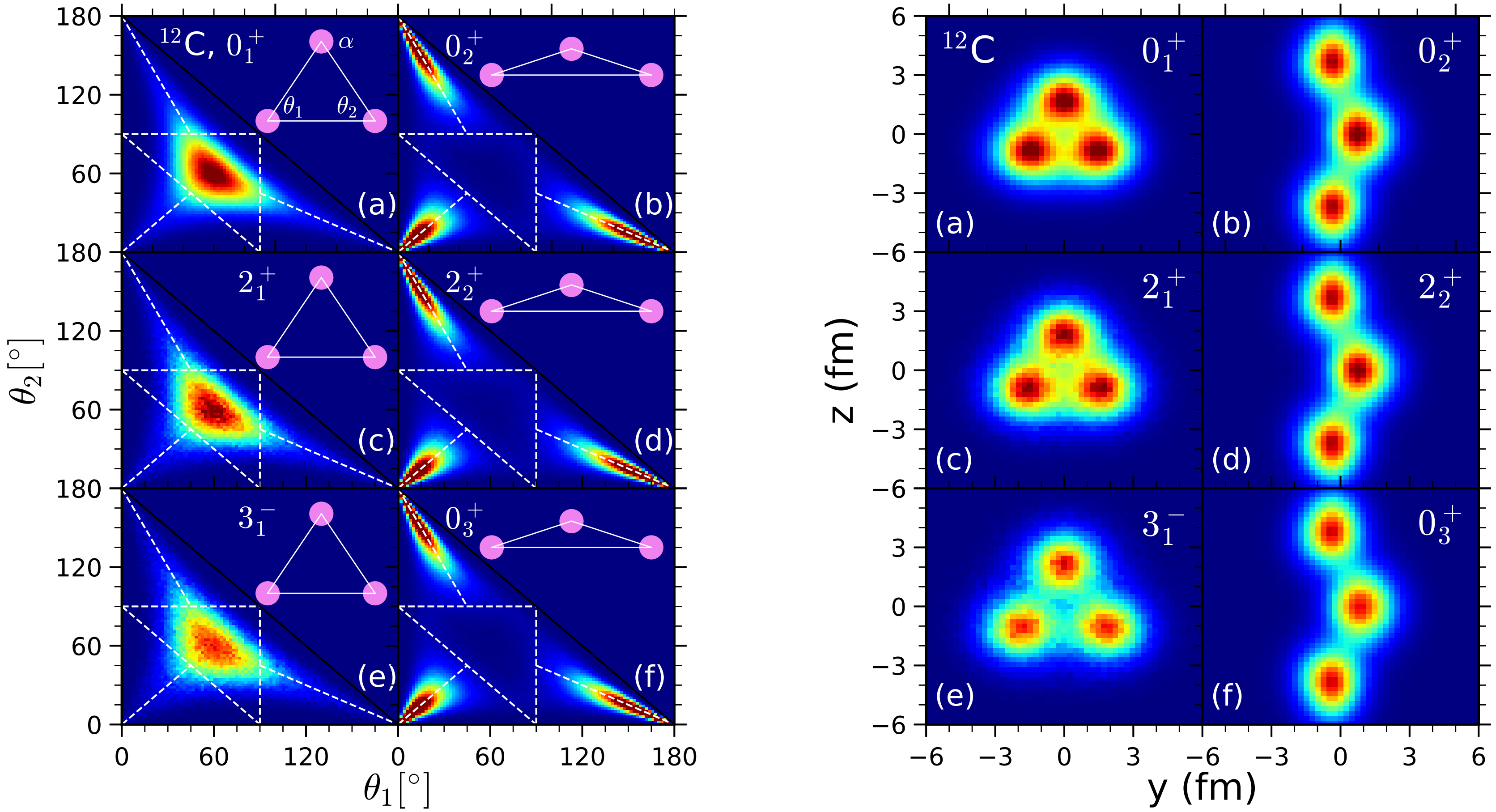


Sick, McCarthy, Nucl. Phys. A 150 (1970) 631
Strehl, Z. Phys. 234 (1970) 416
Crannell et al., Nucl. Phys. A 758 (2005) 399

Chernykh et al., Phys. Rev. Lett. 105 (2010) 022501

Emergence of geometry

- Use the pinhole algorithm to measure the distribution of α-clusters/matter:

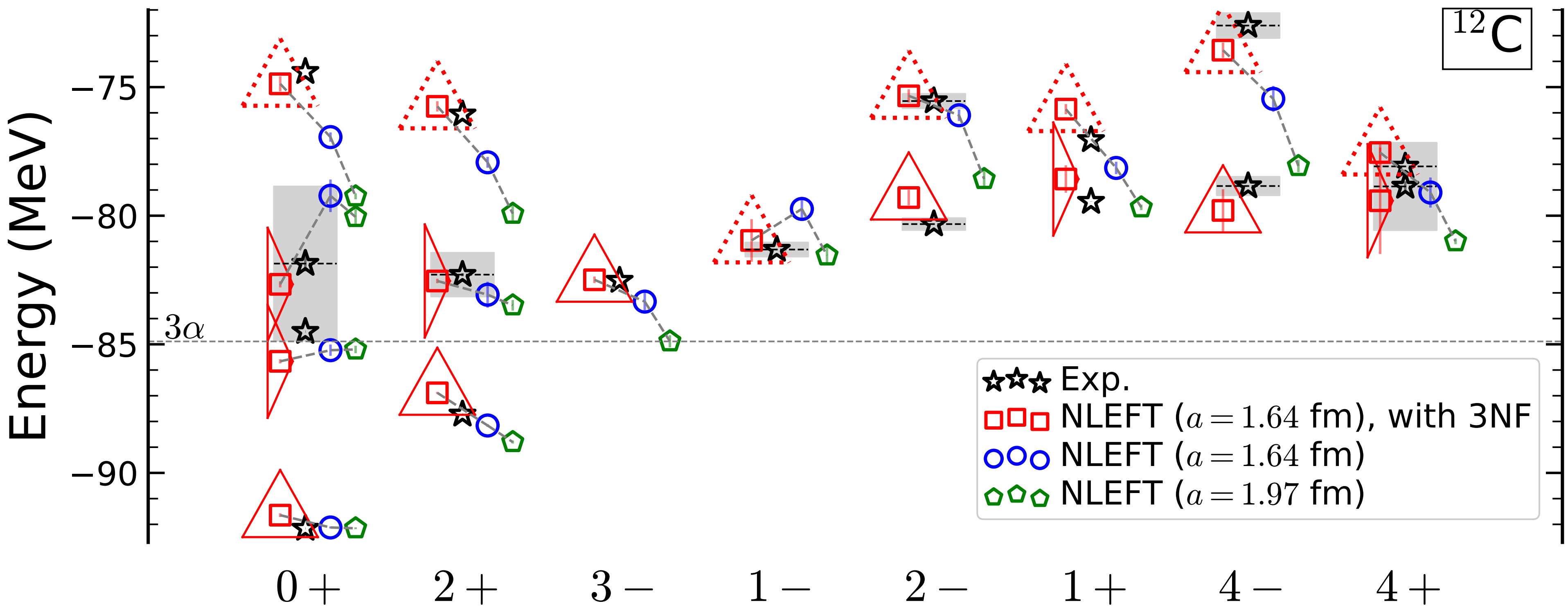


- equilateral & obstuse triangles $\rightarrow$ $\mathbf{2^+}$ states are excitations of the $\mathbf{0^+}$ states

Emergence of duality

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777

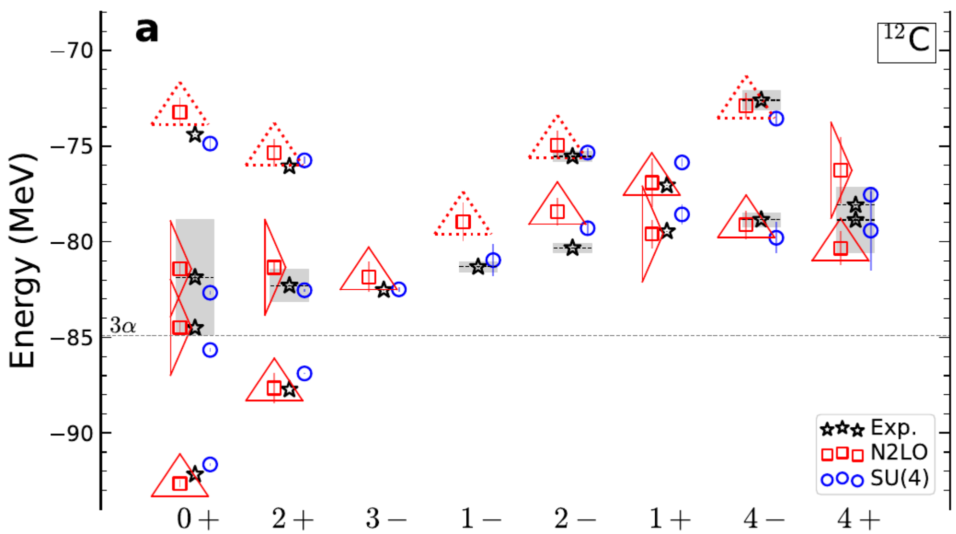
- ^{12}C spectrum shows a cluster/shell-model duality

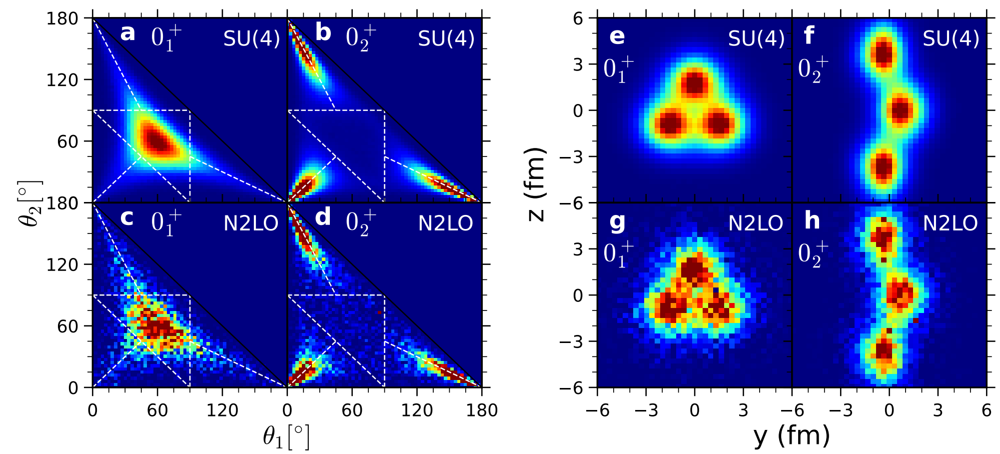


- dashed triangles: strong 1p-1h admixture in the wave function

Sanity check

- Repeat the calculations w/ the time-honored N2LO chiral interaction
  $\hookrightarrow$ better NN phase shifts than the SU(4) interaction
  $\hookrightarrow$ but calculations are much more difficult (sign problem)





- spectrum as before (good agreement w/ data)
- density distributions as before (more noisy, stronger sign problem)

Towards heavy nuclei and nuclear matter: Wave function matching

Wave function matching I

Elhatisari et al., [arXiv:2210.17488 [nucl-th]]

- $H_{\rm soft}$ has tolerable sign oscillations, good for many-body observables
- H_χ has severe sign oscillations, derived from the underlying theory

$\hookrightarrow$ can we find a unitary trafo, that creates a chiral H_χ that is pert. th'y friendly?

$$H'_\chi = U^\dagger \, H_\chi \, U$$

□ Let $|\psi^0_{\rm soft}\rangle$ be the lowest eigenstate of $H_{\rm soft}$

□ Let $|\psi^0_\chi\rangle$ be the lowest eigenstate of H_χ

□ Let $|\phi_{\rm soft}\rangle$ be the projected and normalized lowest eigenstate of $H_{\rm soft}$

$$|\phi_{\rm soft}\rangle = \mathcal{P}\,|\psi^0_{\rm soft}\rangle / ||\psi^0_{\rm soft}\rangle||$$

□ Let $|\phi_\chi\rangle$ be the projected and normalized lowest eigenstate of H_χ

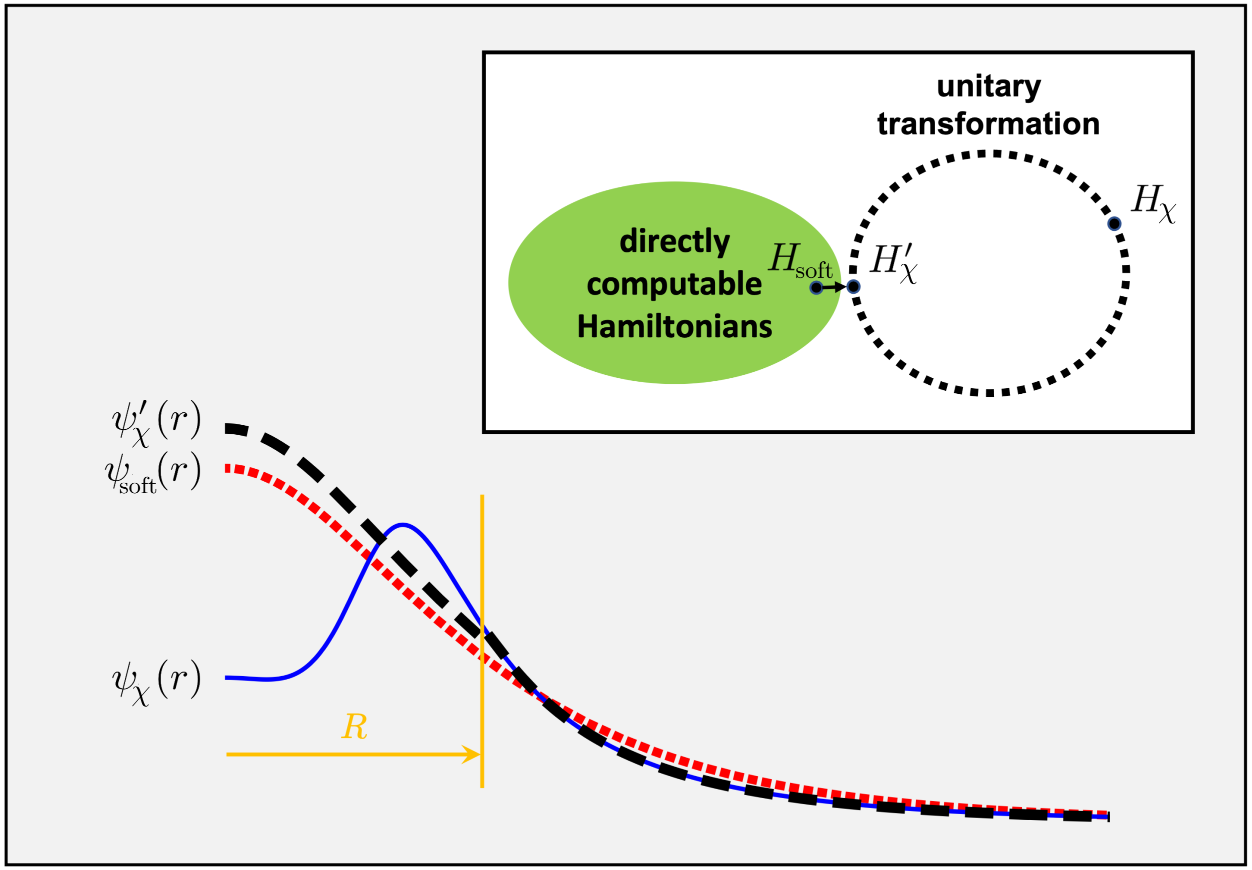
$$|\phi_\chi\rangle = \mathcal{P}\,|\psi^0_\chi\rangle / ||\psi^0_\chi\rangle||$$

$$\hookrightarrow U_{R',R} = \theta(r-R)\delta_{R',R} + \theta(R'-r)\theta(R-r)|\phi^\perp_\chi\rangle\langle\phi^\perp_{\rm soft}|$$

Wave function matching II

Elhatisari et al., [arXiv:2210.17488 [nucl-th]]

- Graphical representation of w.f. matching



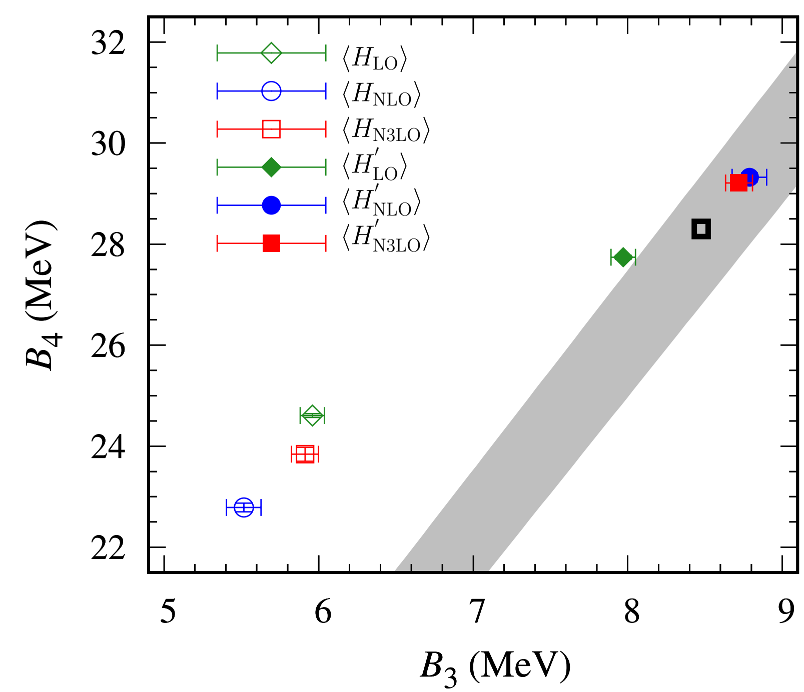
- W.F. matching is a “Hamiltonian translator”:
  eigenenergies from $\boldsymbol{H_1}$ but w.f. from $\boldsymbol{H_2 = U^\dagger H_1 U}$

Wave function matching III

Elhatisari et al., [arXiv:2210.17488 [nucl-th]], L. Bovermann, PhD thesis

- W.F. matching for the light nuclei

Nucleus	$B_{\rm LO}$ [MeV]	$B_{\rm N3LO}$ [MeV]	Exp. [MeV]
$E_{\chi,d}$	1.79	2.21	2.22
$\langle\psi^0_{\rm soft}\,\vert\, H_{\chi,d}\,\vert\psi^0_{\rm soft}\rangle$	0.45	0.62	
$\langle\psi^0_{\rm soft}\,\vert\, H'_{\chi,d}\,\vert\psi^0_{\rm soft}\rangle$	1.65	2.01	
$\langle\psi^0_{\rm soft}\,\vert\, H_{\chi,t}\,\vert\psi^0_{\rm soft}\rangle$	5.96(8)	5.91(9)	8.48
$\langle\psi^0_{\rm soft}\,\vert\, H'_{\chi,t}\,\vert\psi^0_{\rm soft}\rangle$	7.97(8)	8.72(9)	
$\langle\psi^0_{\rm soft}\,\vert\, H_{\chi,\alpha}\,\vert\psi^0_{\rm soft}\rangle$	24.61(4)	23.84(14)	28.30
$\langle\psi^0_{\rm soft}\,\vert\, H'_{\chi,\alpha}\,\vert\psi^0_{\rm soft}\rangle$	27.74(4)	29.21(14)	



- reasonable accuracy for the light nuclei

- Tjon-band recovered with H'_χ

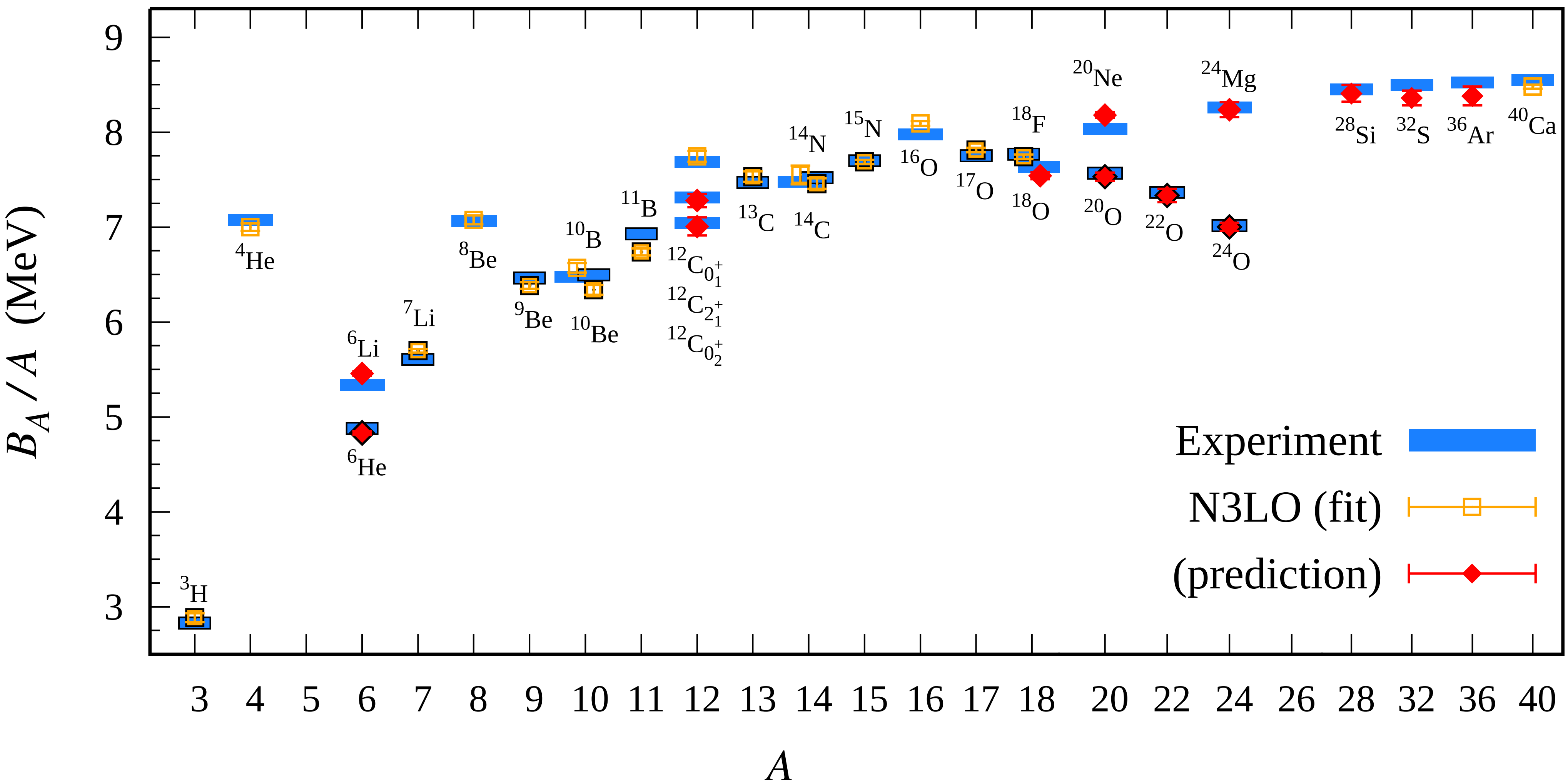
  Platter, Hammer, UGM, Phys. Lett. B **607** (2005) 254

$\hookrightarrow$ now let us go to larger nuclei....

Nuclei at N3LO

- Binding energies of nuclei for $a = 1.32$ fm ($p_{\rm max} = 470$ MeV)

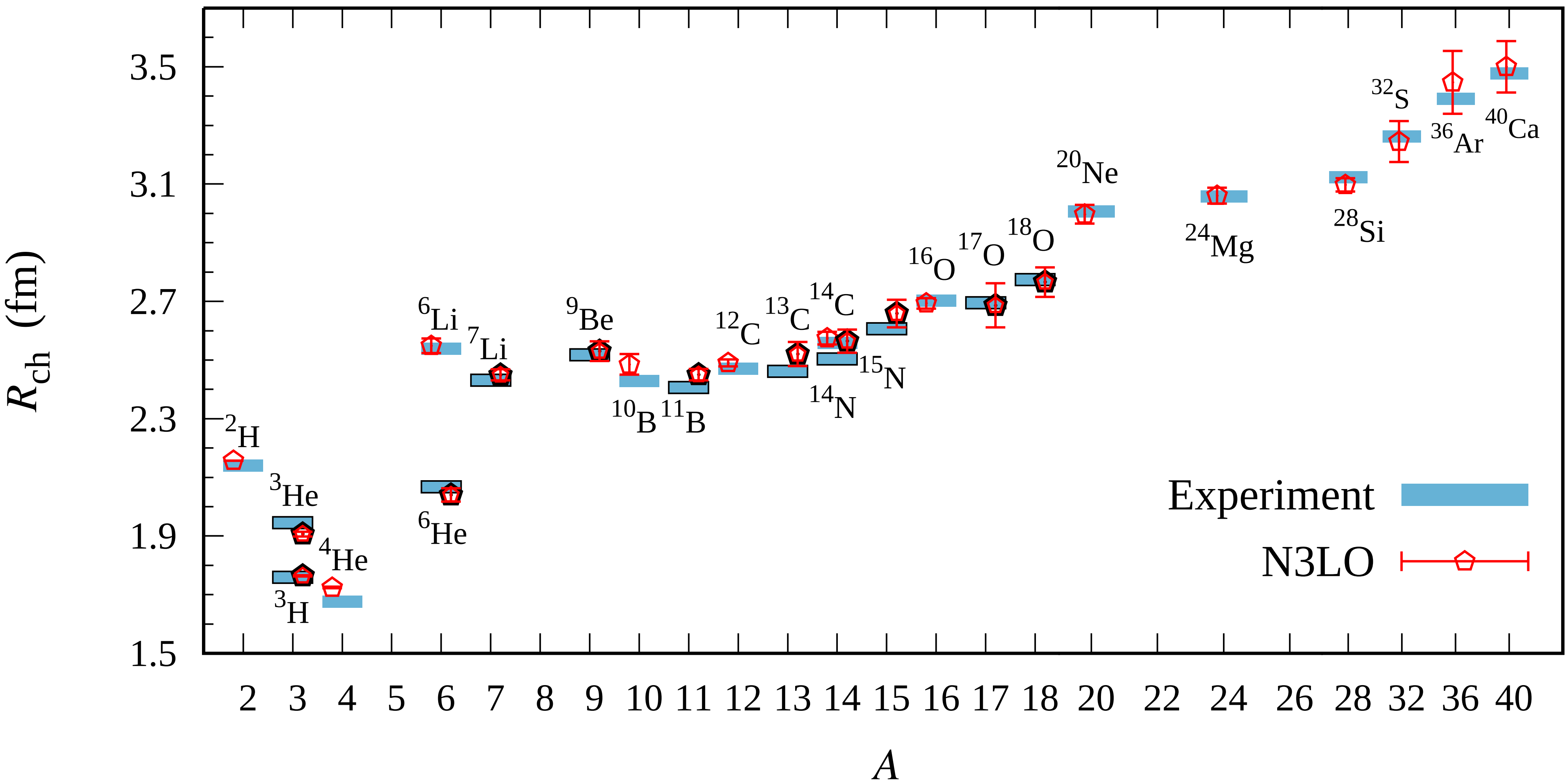
$\rightarrow$ systematic errors via history matching Elhatisari et al., [arXiv:2210.17488 [nucl-th]]



Charge radii at N3LO

- Charge radii ($a = 1.32$ fm, statistical errors can be reduced)

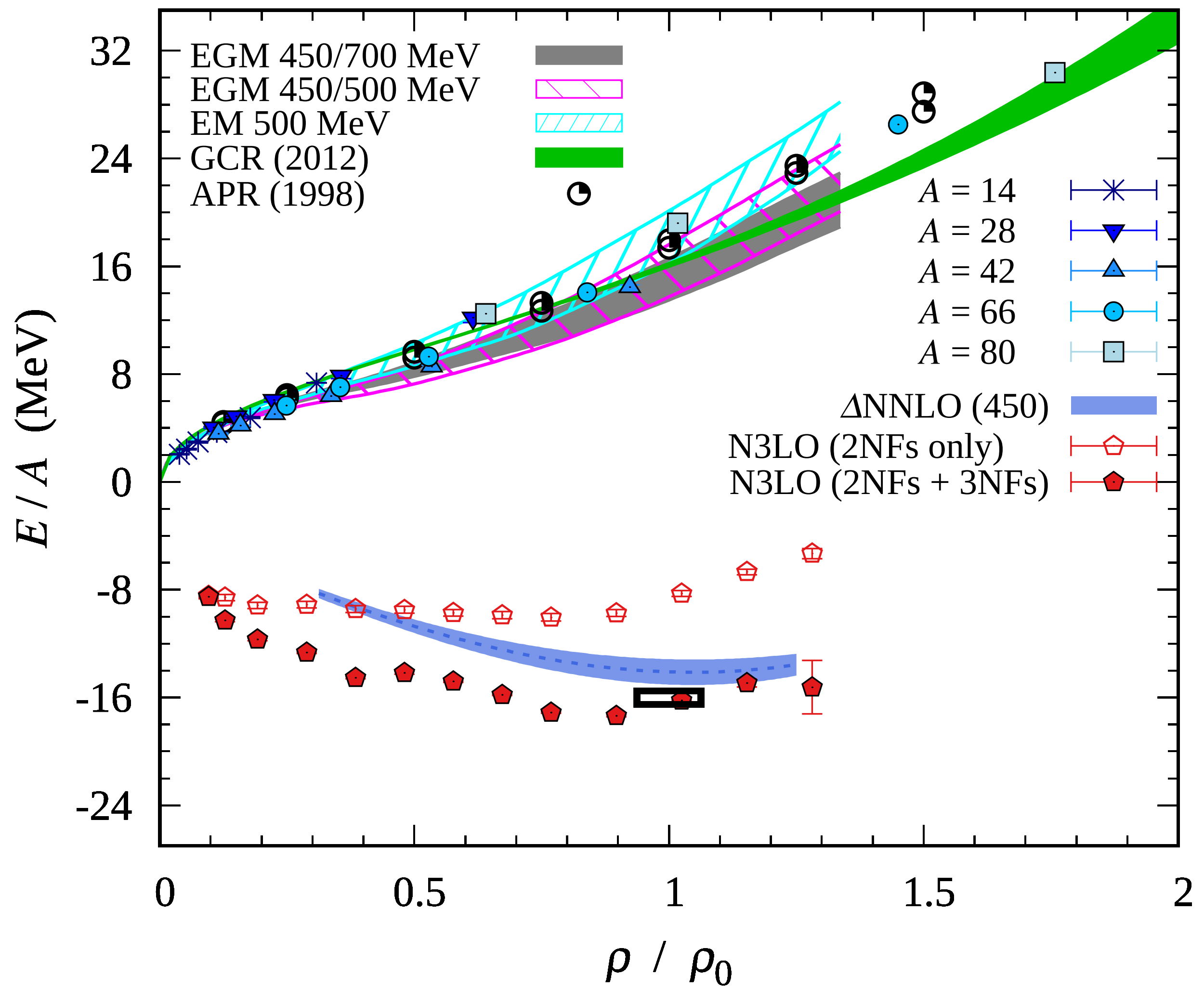
Elhatisari et al., [arXiv:2210.17488 [nucl-th]]



Neutron & nuclear matter at N3LO

- EoS of pure neutron matter & nuclear matter ($a = 1.32$ fm)

Elhatisari et al., [arXiv:2210.17488 [nucl-th]]



Sanity check

- One referee asked us to do calculations outside the history matching interval

$\hookrightarrow$ so let us look at ^{50}Cr and ^{58}Ni:

Nucleus	$E_{\mathbf{N3LO}}$ [MeV]	$E_{\mathbf{exp}}$ [MeV]	$R_{\mathbf{N3LO}}$ [fm]	$R_{\mathbf{exp}}$ [fm]
^{50}Cr	$\mathbf{-425.32(943)}$	$\mathbf{-435.05}$	$\mathbf{3.6469(229)}$	$\mathbf{3.6588}$
^{58}Ni	$\mathbf{-493.13(661)}$	$\mathbf{-506.46}$	$\mathbf{3.7754(202)}$	$\mathbf{3.7752}$

$\hookrightarrow$ Energies within 2-3%, uncertainties on the 1-2% level

$\hookrightarrow$ Radii smack on, uncertainties can be improved

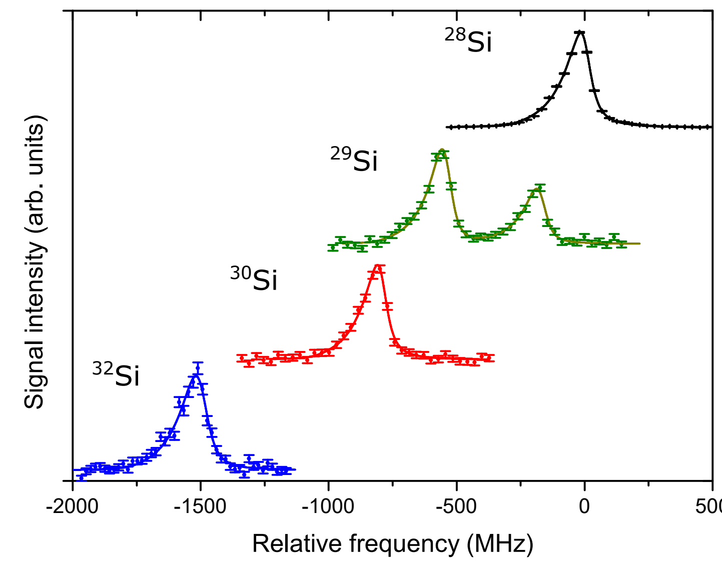
$\hookrightarrow$ Test passed $\surd$

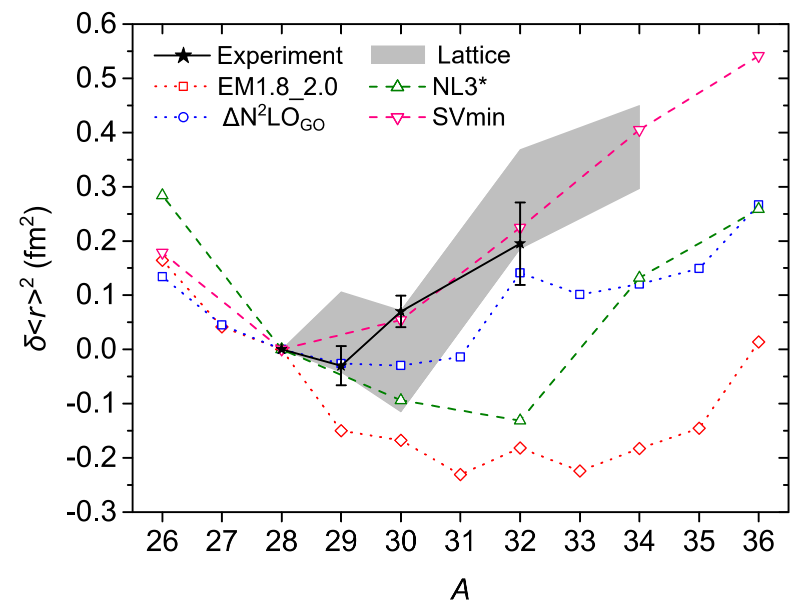
Recent results

Nuclear charge radii of Si isotopes

König et al., 2309.02037 [nucl-th]

- Charge radius of ^{32}Si not known
  - $\hookrightarrow$ mirror nucleus to ^{32}Ar $\rightarrow$ L (slope of symm. en.)
  - $\hookrightarrow$ (dis)appearance of magic numbers
- Use laser spectroscopy at BECOLA/FRIB
  - $\hookrightarrow$ radius related to frequency shift
  - $\hookrightarrow$ $R_{\rm ch}(^{32}{\rm Si}) = 3.153(12)$ fm
  - $\hookrightarrow$ completes the chain from ^{28}Si to ^{32}Si
  - $\hookrightarrow$ combined with ^{32}Ar radius $\rightarrow$ $L \leq 60$ MeV
- NLEFT calculation reproduces the trend of the data well $\surd$
  - $\hookrightarrow$ also: $L_{\rm NLEFT} = 50 \pm 1$ MeV





Study of the Be isotopes

Shen, Elhatisari, UGM, ... *in preparation*

- Be isotopes show many interesting features, e.g.

  $\hookrightarrow$ alpha-cluster nuclei (like ^{8}Be)

  $\hookrightarrow$ large halo nuclei (like ^{11}Be)

  $\hookrightarrow$ parity inversion of the ^{11}Be g.s.

- many calculations of one or a few isotopes (QMC, NCSM + cont., cluster models, ...)

  $\hookrightarrow$ no unified picture

- NLEFT provides a large basis of cluster and shell model states

  $\hookrightarrow$ perform a calculation of $A = 7 - 12$ using the SU(4) minimal interaction

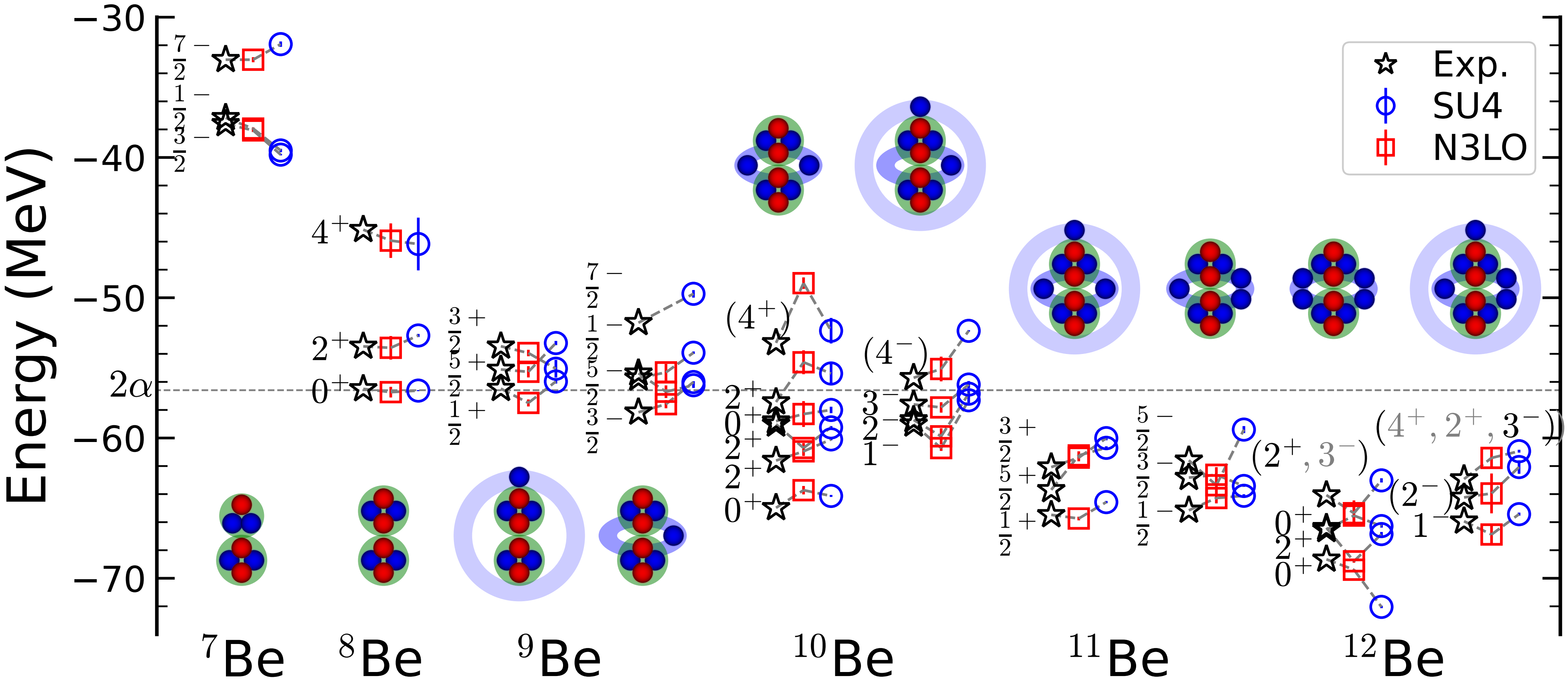
  $\hookrightarrow$ perform a calculation of $A = 7 - 12$ using the N3LO wfm interaction

  $\hookrightarrow$ all parameters fixed $\rightarrow$ **true** predictions

  $\hookrightarrow$ here: spectra, EM observables in the works

Spectra of the BE isotopes from $A = 7 - 12$

Shen, Elhatisari, UGM, ... *in preparation*



- SU(4) works astonishingly well, but some visible deviations
- N3LO gives an overall very good description, all levels correctly ordered $\surd$

Structure factors for hot neutron matter

Ma, Liu, Lu, Elhatisari, Lee, Li, UGM, Steiner, Wang, 2306.04500 [nucl-th]

- Core collapse supernovae: 99% of the gravitational energy escapes via neutrinos
  $\hookrightarrow$ need precise calculations of neutrino-nucleus cross sections
  $\hookrightarrow$ these XS are determined by the *structure factors* (correlation functions):

$$S_V(q) = \int d^3r\, e^{-i\vec{q}\cdot\vec{r}}\, \langle \delta\rho(0,\vec{r})\delta\rho(0,0)\rangle\ , \qquad \delta\rho = \rho - \langle\rho\rangle$$

$$S_A(q) = \int d^3r\, e^{-i\vec{q}\cdot\vec{r}}\, \langle \delta\rho_z(0,\vec{r})\delta\rho_z(0,0)\rangle\ , \quad \delta\rho_z = \rho_z - \langle\rho_z\rangle$$

- Various calculations using different methods exist
  $\hookrightarrow$ HF, RPA, extended virial expansions (model-independent for high T and low ρ)
  $\hookrightarrow$ *ab initio* calculation based on pionless EFT — Alexandru et al., Phys. Rev. Lett. **126** (2021) 132701
  $\hookrightarrow$ perform an *ab initio* calculation within NLEFT at N3LO

Computational scheme

Ma, Liu, Lu, Elhatisari, Lee, Li, UGM, Steiner, Wang, 2306.04500 [nucl-th]

- Consider the grand canonical ensemble (inverse temp. $\beta = 1/T$, N nucleons):

$$\mathcal{Z} = \sum_N e^{\beta\mu_G N}\, Z(\beta, N)\,, \quad Z(\beta, N) = \sum_{c_1,\ldots,c_N} \langle c_1, \ldots, c_N | \exp(-\beta H) | c_1, \ldots, c_N \rangle$$

$c_i = (\vec{n}_i, \sigma_i, \tau_i)$ single-particle basis

- Many-body operators induce exponentially growing contractions

$\hookrightarrow$ rank-one operator (RO) method: $F_\alpha = \sum_{\mathbf{n}} a_{i,j}(\vec{n})\, \underbrace{f_{\alpha,i,j}(\vec{n})}_{s.p.\ orb.\ fct}$

$\hookrightarrow$ rank-one operator: $F^\dagger_{\alpha_1'} F_{\alpha_1} = \lim\limits_{c_1\to\infty} : \exp\left(c_1 F^\dagger_{\alpha_1'} F_{\alpha_1}\right) :$

$\hookrightarrow$ can be easily extended to higher-body rank-one operators

$\hookrightarrow$ reduction to exponentials of one-body operators $\to$ enormous speed-up

Structure factors

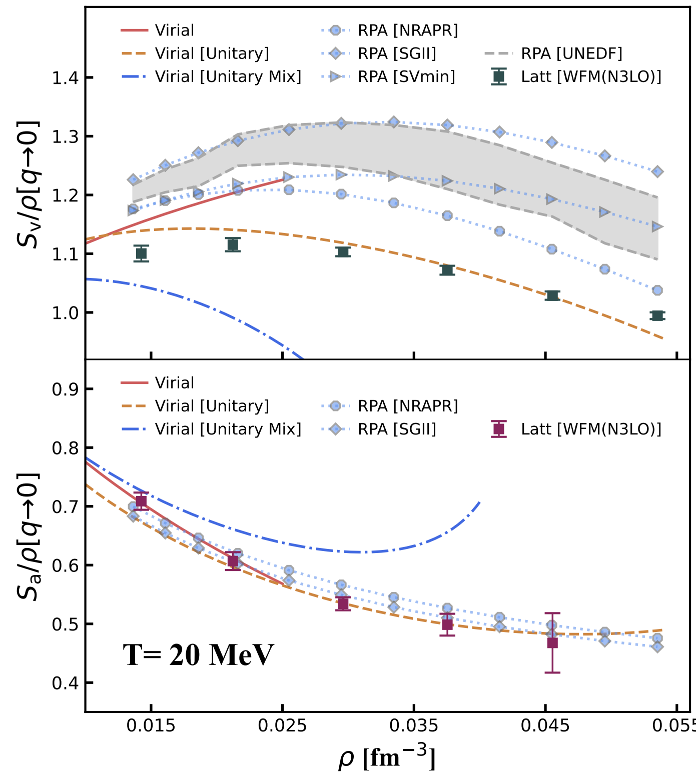
Ma, Liu, Lu, Elhatisari, Lee, Li, UGM, Steiner, Wang, 2306.04500 [nucl-th]

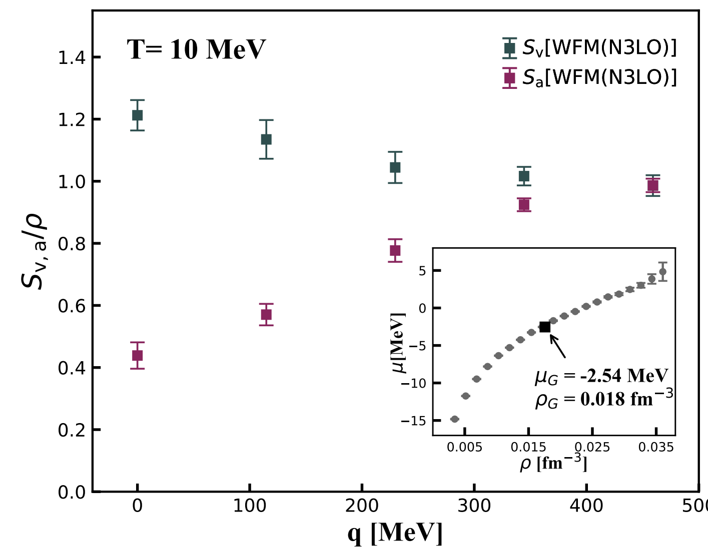
- Simulation details:
  - $\hookrightarrow$ $L^3 = 6^3, 7^3, 8^3$, $a = 1.32$ fm, $a_t = 0.2$ fm
  - $\hookrightarrow$ average over twisted b.c.'s $\longrightarrow$ better t.d. limit

    Lu et al., Phys. Rev. Lett. **125** (2020) 192502

- Results in the long wavelength limit ($q \to 0$)
  - $\hookrightarrow$ can be used to calibrate RPA /Skyrme calc's
  - $\hookrightarrow$ unitary virial expansion works amazingly well

- $S_{V,A}(q)$ at $T = 10$ MeV, $\mu_G = 0.018$ fm^{-3}
  - $\hookrightarrow$ in the high-density limit $S_V = S_A = \rho$
  - $\hookrightarrow$ controlled corrections to the long wavelength limit
  - $\hookrightarrow$ estimated uncertainty: $\sim 5\%$





Summary & outlook

- Investigated the ^{4}He transition form factor using NLEFT w/o tuning any parameter

  $\hookrightarrow$ minimal nuclear interaction gives a good description of the data

  $\hookrightarrow$ no problem to the nuclear forces

- New insights into the emergent geometry and duality in the carbon nucleus

- New computational tool for quantum physics: Wave function matching

  $\hookrightarrow$ allows for precision calculations into the mid-mass region and beyond

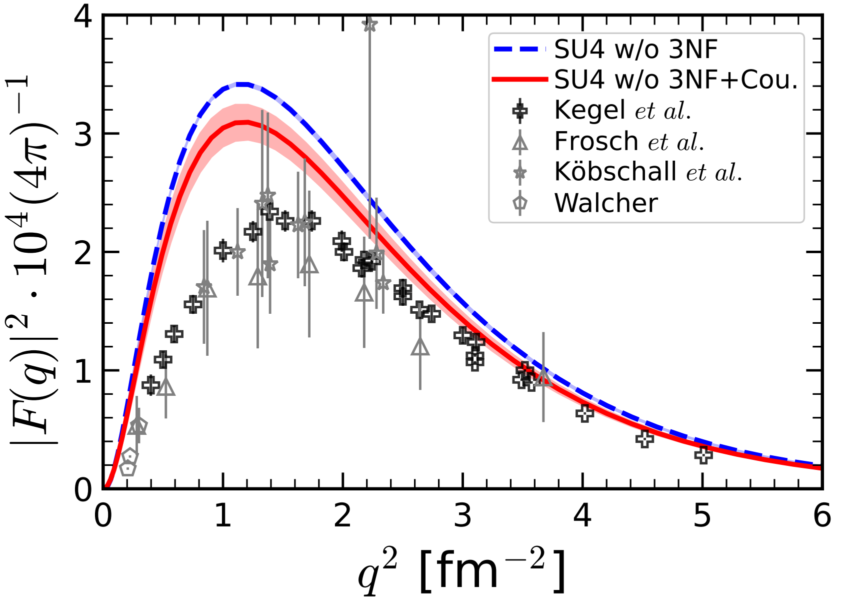
  $\hookrightarrow$ consistent energies and radii into the mid-mass region

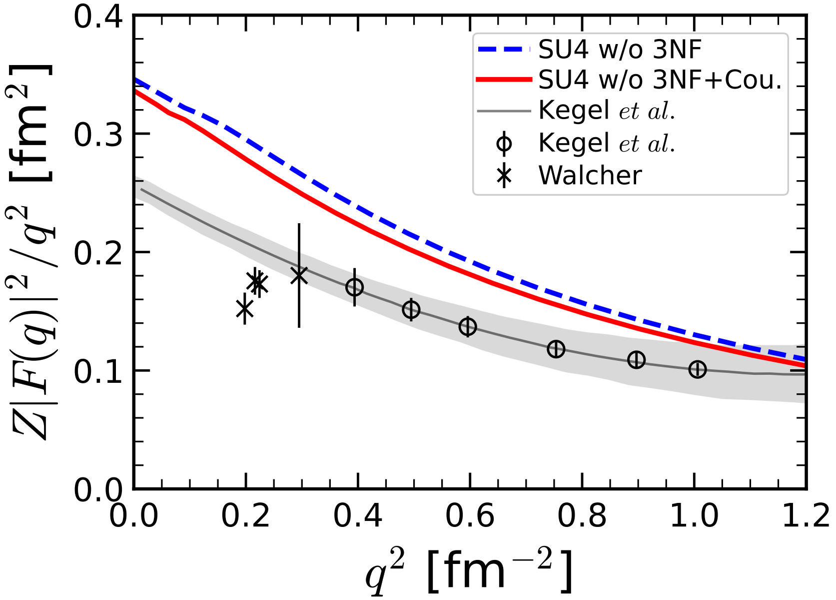
  $\hookrightarrow$ many intriguing results shown & more to come

Spares

The transition form factor w/o 3NFs

- What is the role of the 3NFs? Switch them off!





$\hookrightarrow$ FF goes up, so does $\boldsymbol{\Delta E = 0.50(6)}$ MeV, consistent w/ Michel et al.

$\hookrightarrow$ radius too large ($\mathbf{2.02(0.01)}$ fm$^\mathbf{2}$), fourth moment ok ($\mathbf{4.15(0.05)}$ fm$^\mathbf{2}$)

$\hookrightarrow$ for $\boldsymbol{q^2 \gtrsim 3}$ fm$^{\mathbf{-2}}$, the ff is ok (range of the 3NFs)

Electron scattering off nucleons and nuclei

- Electron scattering is a versatile tool to

  ⇒ reveal the structure of the nucleon

  ⇒ reveal the structure of atomic nuclei

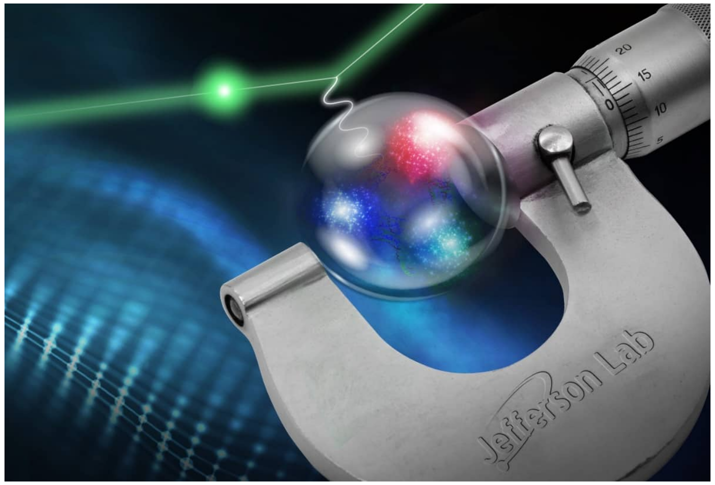
  ⇒ information encoded in **form factors**, ...

- Often complimentary information through
  final-state interactions (FSI) in reactions or decays

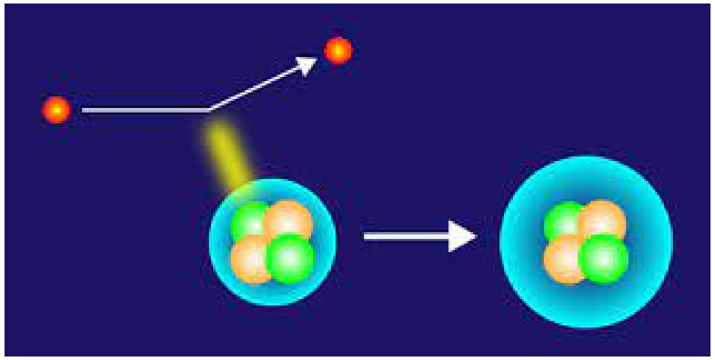
- this talk addresses two topics of high current interest:

  – a new method to measure the proton charge radius

  – an *ab initio* calculation of the ^{4}He transition ff



ⓒPhysics World



ⓒTohoku Univ.