Femtoscopy as a precision tool to determine hadronic interactions ?

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by NRW-FAIR

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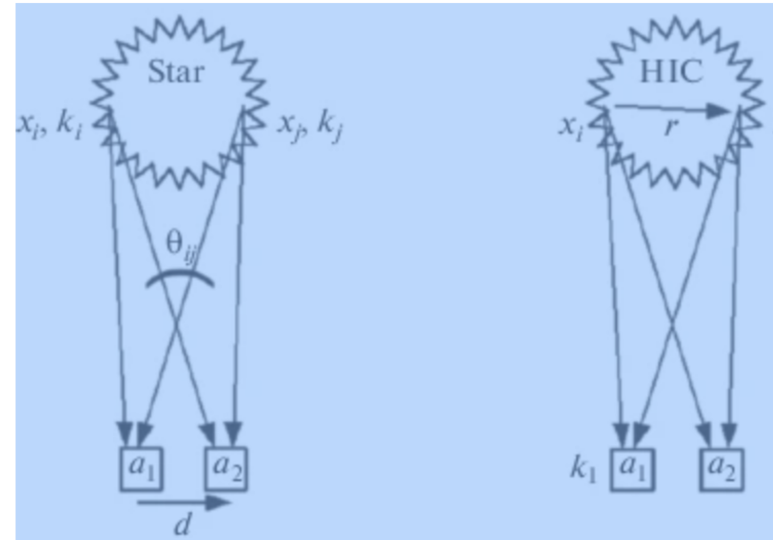
Epelbaum, Heihoff, UGM, Tscherwon, arXiv:2504.08631 [nucl-th]

Introduction: Basic ideas

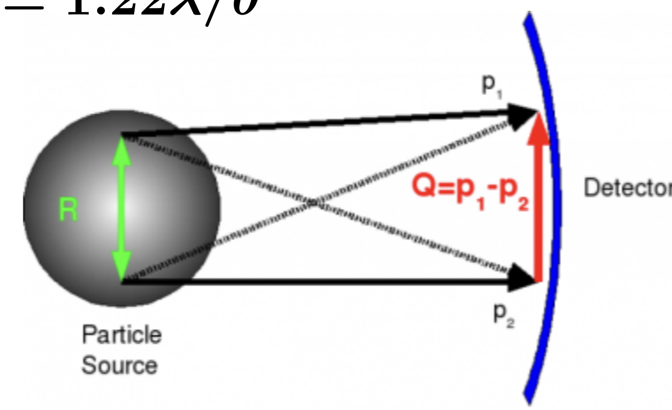
Basics of Femtoscopy I

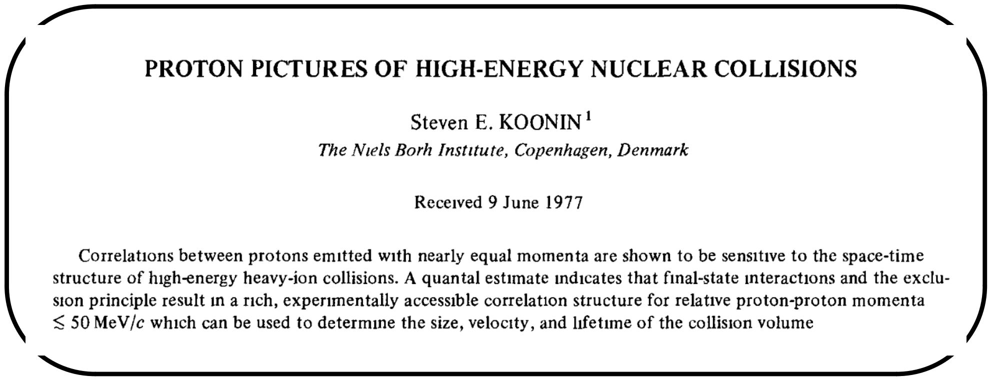
- Original idea

$\longrightarrow$ measure spatial/temporal correlations within (ultra)relativistic heavy-ion collisions (HICs)



$$d = 1.22\lambda/\theta$$



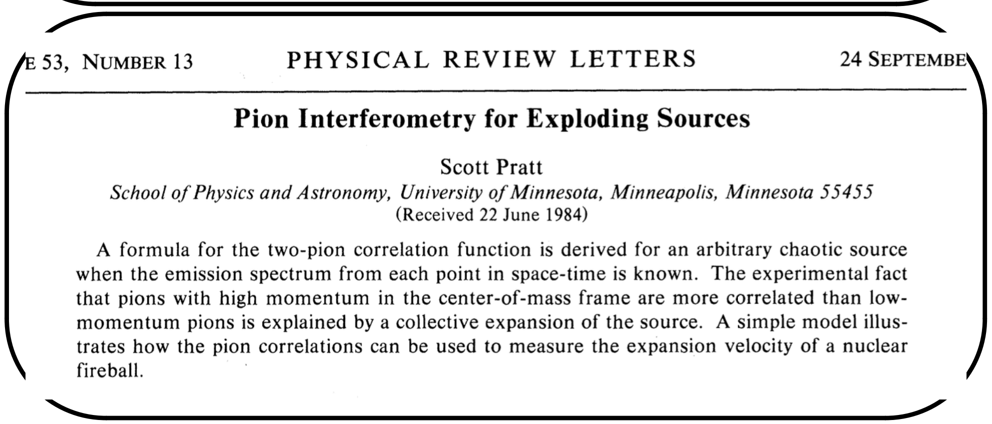


PROTON PICTURES OF HIGH-ENERGY NUCLEAR COLLISIONS

Steven E. KOONIN[1]
The Niels Borh Institute, Copenhagen, Denmark

Received 9 June 1977

Correlations between protons emitted with nearly equal momenta are shown to be sensitive to the space-time structure of high-energy heavy-ion collisions. A quantal estimate indicates that final-state interactions and the exclusion principle result in a rich, experimentally accessible correlation structure for relative proton-proton momenta $\lesssim$ 50 MeV/c which can be used to determine the size, velocity, and lifetime of the collision volume



E 53, NUMBER 13 PHYSICAL REVIEW LETTERS 24 SEPTEMBE

Pion Interferometry for Exploding Sources

Scott Pratt
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 22 June 1984)

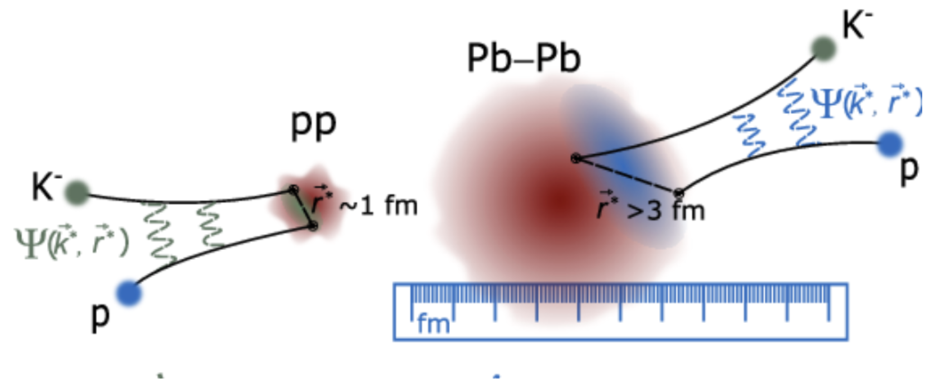
A formula for the two-pion correlation function is derived for an arbitrary chaotic source when the emission spectrum from each point in space-time is known. The experimental fact that pions with high momentum in the center-of-mass frame are more correlated than low-momentum pions is explained by a collective expansion of the source. A simple model illustrates how the pion correlations can be used to measure the expansion velocity of a nuclear fireball.

Koonin, Phys. Lett. **70B** (1977) 1219

Pratt, Phys. Rev. Lett. **53** (1984) 1219

Review: Lisa et al., Ann. Rev. Nucl. Part. Sci. **55** (2005) 357

Basics of Femtoscopy II



- Koonin-Pratt (KP) formula for the correlation function $C(\mathbf{k})$ in the CMS:

$$C(\mathbf{k}) = \int d\mathbf{r}\, S_{12}(\mathbf{r})\, \left|\Psi(\mathbf{r},\, \mathbf{k})\right|^2$$

$\mathbf{k}$ = the relative momentum, $S_{12}(\mathbf{r})$ = the source function

$\Psi(\mathbf{r},\, \mathbf{k})$ = relative wf of the outgoing two-body state (solution of the scattering problem)

Lednicky, Phys. Part. Nucl. **40** (2009) 307

- From experiment to interpretation:

Step 1: measure the correlation functions $C(\mathbf{k})$

Step 2: modeling of the source function $S_{12}(\mathbf{r})$ which is deemed to be **universal**

assume Gaussian form, extract r_0 from one reaction (pp) assuming some interaction model

Step 3: Once $S_{12}(\mathbf{r})$ is fixed, use the KP formula to analyze hadronic interactions

- Note many refinements for coupled channels etc, but let us keep it simple

Using Femtoscopy

- Many claims by the ALICE collaboration:

PHYSICAL REVIEW LETTERS **124**, 092301 (2020)

Scattering Studies with Low-Energy Kaon-Proton Femtoscopy in Proton-Proton Collisions at the LHC

S. Acharya *et al.**
(A Large Ion Collider Experiment Collaboration)

(Received 18 July 2019; revised manuscript received 3 December 2019; accepted 11 February 2020; published 6 March 2020)

The study of the strength and behavior of the antikaon-nucleon ($\bar{K}N$) interaction constitutes one of the key focuses of the strangeness sector in low-energy quantum chromodynamics (QCD). In this Letter a unique high-precision measurement of the strong interaction between kaons and protons, close and above the kinematic threshold, is presented. The femtoscopic measurements of the correlation function at low pair-frame relative momentum of ($K^+p \oplus K^-\bar{p}$) and ($K^-p \oplus K^+\bar{p}$) pairs measured in pp collisions at $\sqrt{s} = 5, 7$, and 13 TeV are reported. A structure observed around a relative momentum of 58 MeV/c in the measured correlation function of ($K^-p \oplus K^+\bar{p}$) with a significance of 4.4σ constitutes the first experimental evidence for the opening of the ($\bar{K}^0 n \oplus K^0 \bar{n}$) isospin breaking channel due to the mass difference between charged and neutral kaons. The measured correlation functions have been compared to Jülich and Kyoto models in addition to the Coulomb potential. The high-precision data at low relative momenta presented in this work prove femtoscopy to be a powerful complementary tool to scattering experiments and provide new constraints above the $\bar{K}N$ threshold for low-energy QCD chiral models.

Article

Unveiling the strong interaction among hadrons at the LHC

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Accepted: 20 October 2020
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Open access
Check for updates

ALICE Collaboration*

One of the key challenges for nuclear physics today is to understand from first principles the effective interaction between hadrons with different quark content. First successes have been achieved using techniques that solve the dynamics of quarks and gluons on discrete space-time lattices[1,2]. Experimentally, the dynamics of the strong interaction have been studied by scattering hadrons off each other. Such scattering experiments are difficult or impossible for unstable hadrons[3–6] and so high-quality measurements exist only for hadrons containing up and down quarks[7]. Here we demonstrate that measuring correlations in the momentum space between hadron pairs[8–12] produced in ultrarelativistic proton–proton collisions at the CERN Large Hadron Collider (LHC) provides a precise method with which to obtain the missing information on the interaction dynamics between any pair of unstable hadrons. Specifically, we discuss the case of the interaction of baryons containing strange quarks (hyperons). We demonstrate how, using precision measurements of proton–omega baryon correlations, the effect of the strong interaction for this hadron–hadron pair can be studied with precision similar to, and compared with, predictions from lattice calculations[13,14]. The large number of hyperons identified in proton–proton collisions at the LHC, together with accurate modelling[15] of the small (approximately one femtometre) inter-particle distance and exact predictions for the correlation functions, enables a detailed determination of the short-range part of the nucleon-hyperon interaction.

Physics Letters B 797 (2019) 134822

Contents lists available at ScienceDirect
Physics Letters B
www.elsevier.com/locate/physletb

Study of the Λ–Λ interaction with femtoscopy correlations in pp and p–Pb collisions at the LHC

ALICE Collaboration

ARTICLE INFO

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ABSTRACT

This work presents new constraints on the existence and the binding energy of a possible Λ–Λ bound state, the H-dibaryon, derived from Λ–Λ femtoscopic measurements by the ALICE collaboration. The results are obtained from a new measurement using the femtoscopy technique in pp collisions at $\sqrt{s} =$ 13 TeV and p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, combined with previously published results from pp collisions at $\sqrt{s} = 7$ TeV. The Λ–Λ scattering parameter space, spanned by the inverse scattering length f_0^{-1} and the effective range d_0, is constrained by comparing the measured Λ–Λ correlation function with calculations obtained within the Lednický model. The data are compatible with hypernuclei results and lattice computations, both predicting a shallow attractive interaction, and permit to test different theoretical approaches describing the Λ–Λ interaction. The region in the (f_0^{-1}, d_0) plane which would accommodate a Λ–Λ bound state is substantially restricted compared to previous studies. The binding energy of the possible Λ–Λ bound state is estimated within an effective-range expansion approach and is found to be $B_{\Lambda\Lambda} = 3.2^{+1.6}_{-2.4}(\text{stat})^{+1.8}_{-1.0}(\text{syst})$ MeV.

Phys. Lett. B 856 (2024) 138915

Contents lists available at ScienceDirect
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Letter

Investigating the composition of the $K_0^*(700)$ state with $\pi^\pm K_S^0$ correlations at the LHC

ALICE Collaboration *

ARTICLE INFO

Editor: M. Doser
Dataset link: https://www.hepdata.net/record/ins2739149

ABSTRACT

The first measurements of femtoscopic correlations with the particle pair combinations $\pi^\pm K_S^0$ in pp collisions at $\sqrt{s} = 13$ TeV at the Large Hadron Collider (LHC) are reported by the ALICE experiment. Using the femtoscopic approach, it is shown that it is possible to study the elusive $K_0^*(700)$ particle that has been considered a tetraquark candidate for over forty years. Source and final-state interaction parameters are extracted by fitting a model assuming a Gaussian source to the experimentally measured two-particle correlation functions. The final-state interaction in the $\pi^\pm K_S^0$ system is modeled through a resonant scattering amplitude, defined in terms of a mass and a coupling parameter, The extracted mass and Breit–Wigner width, derived from the coupling parameter, of the final-state interaction are found to be consistent with previous measurements of the $K_0^*(700)$. The small value and increase of the correlation strength with increasing source size support the hypothesis that the $K_0^*(700)$ is a four-quark state, i.e. a tetraquark state of the form $(q_1, \overline{q_2}, q_3, \overline{q_3})$ in which q_1, q_2 and q_3 indicate the flavor of the valence quarks of the π and K_S^0. This latter trend is also confirmed via a simple geometric model that assumes a tetraquark structure of the $K_0^*(700)$ resonance.

Analysis of the KP formula – a Gedankenexperiment –

Femtoscopy revisited

- Fundamental flaw of the KP formula:

  Combined with the universality assumption for the source function $S_{12}(\mathbf{r})$,
  it implies the measurability of hadronic wave functions
  and thus also of the corresponding interaction potentials

- But: hadronic potentials are **not** observable (scheme-dependent) [as is well known]

- Consider non-relativistic systems:

$$C(\mathbf{k}) = \langle \Psi_{-\mathbf{k}}^{(+)} | \hat{S}_{12} | \Psi_{-\mathbf{k}}^{(+)} \rangle \quad \text{for} \quad \langle \mathbf{r}' | \hat{S}_{12} | \mathbf{r} \rangle = \delta(\mathbf{r}' - \mathbf{r}) S_{12}(\mathbf{r}) \quad \text{(local)}$$

- Consider unitary transformations ($\hat{U}^\dagger \hat{U} = \hat{U} \hat{U}^\dagger = 1$)

$$C(\mathbf{k}) = \left(\langle \Psi_{-\mathbf{k}}^{(+)} | \hat{U}^\dagger \right) \left(\hat{U} \hat{S}_{12} \hat{U}^\dagger \right) \left(\hat{U} | \Psi_{-\mathbf{k}}^{(+)} \rangle \right) = \langle \Psi'^{(+)}_{-\mathbf{k}} | \hat{S}'_{12} | \Psi'^{(+)}_{-\mathbf{k}} \rangle$$

- Universality of the source term means $\hat{S}'_{12} = \hat{S}_{12}$

  $\hookrightarrow$ model dependence of the calculated correlation functions

Gedankenexperiment I

- ALICE and Bob analyze a femtoscopy measurement (two dist. spinless particles, no Coulomb)
- Interaction taken as chiral EFT N^{4}LO$^+$ potential projected on $S = 0$ states

$$\langle p'l|\hat{V}_{\rm Alice}|pl\rangle = \langle p'l|\hat{V}^{S=0}_{\rm np,\, N^4LO+}|p\,l\rangle \;\; (l = 0,1,2,3), \qquad \hat{V}^{S=0}_{\rm np} = \hat{V}_{\rm np}[1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2]/4$$

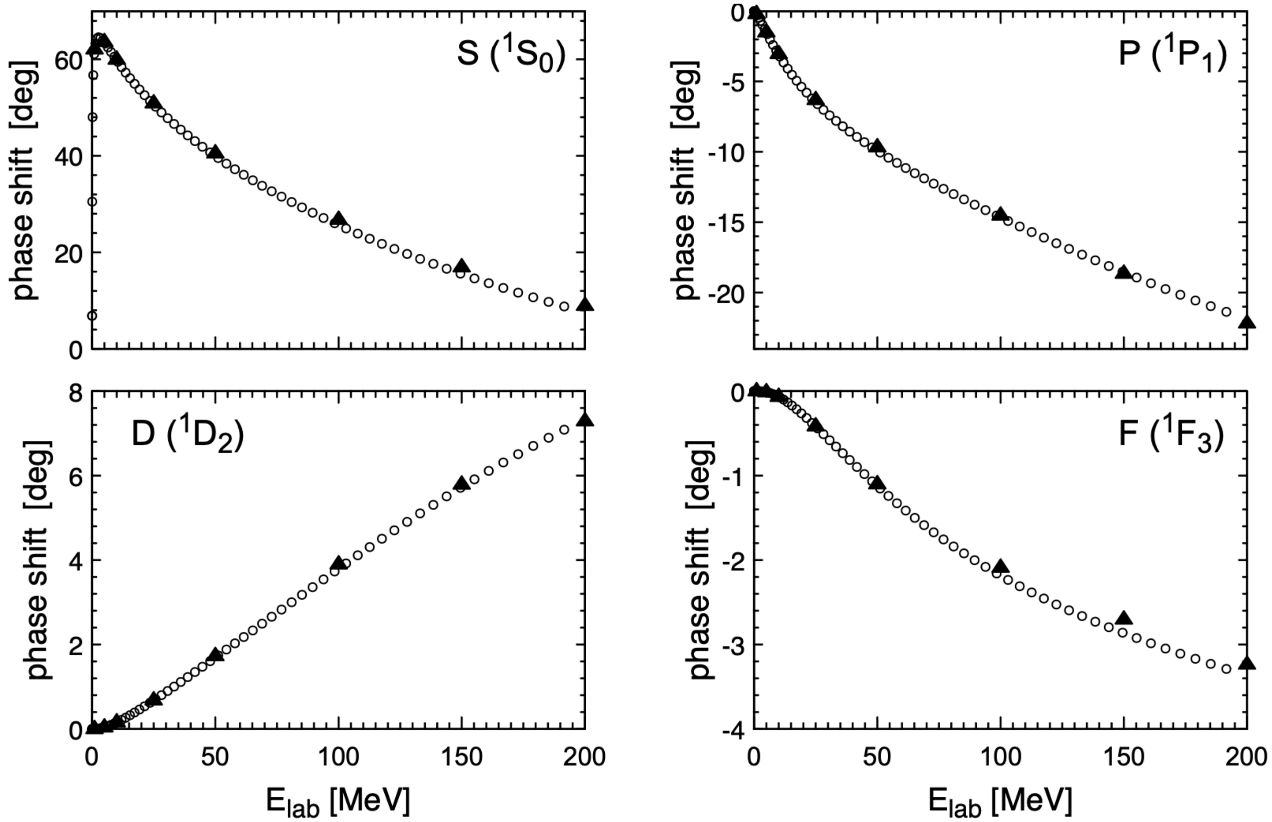


Fig. courtesy E. Epelbaum

Gedankenexperiment II

- $C(p) = \langle \Psi^{(+)} | \hat{S} | \Psi^{(+)} \rangle$ is calculated in the most general way (PW mom. space basis)
  - no restriction to a local source term
  - no need to assume that the interaction is S-wave only
- Choose a static local source

$$S_{12}^{\rm Alice}(\vec{r}) = \frac{e^{-r^2/(4r_0^2)}}{(4\pi r_0^2)^{3/2}}$$

$$\to \langle \vec{p}\,' | \hat{S}_{12}^{\rm Alice} | \vec{p} \rangle = e^{-q^2 r_0^2}$$

$$\vec{q} = \vec{p}\,' - \vec{p}$$

choose $r_0 = 1.5$ fm

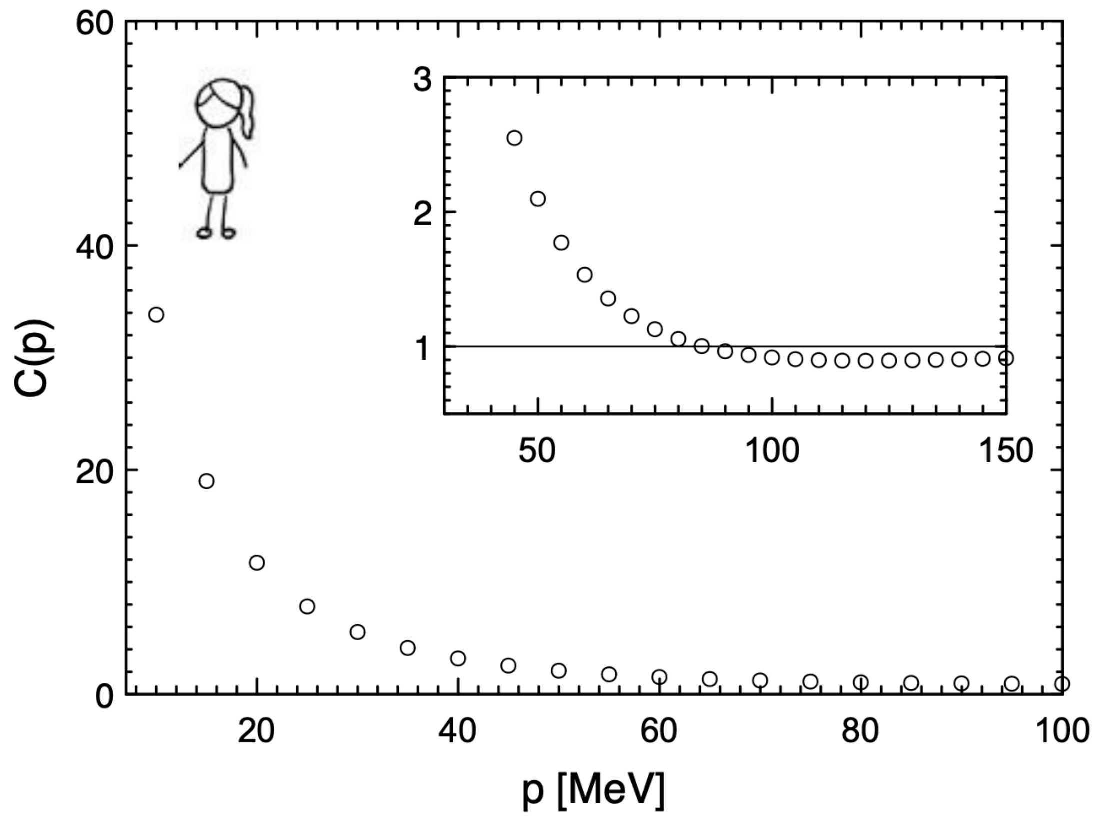


Fig. courtesy E. Epelbaum

Gedankenexperiment III

- Bob chooses another basis: $|\Psi_{\rm Bob}\rangle = \hat{U}|\Psi_{\rm Alice}\rangle$, $\hat{U} = 1 - 2|g\rangle\langle g|$, $\langle g|g\rangle = 1$

$$g(\vec{r}\,) = \langle \vec{r}\,|g\rangle = Cr(1-\beta r)e^{-\alpha r} \longrightarrow g(\vec{p}\,) \sim \frac{p^4 - 3\alpha^3(\alpha - 4\beta) - 2p^2\alpha(\alpha + 6\beta)}{(p^2+\alpha^2)^4}$$

Sauer, Phys. Rev. Lett. **32** (1974) 626

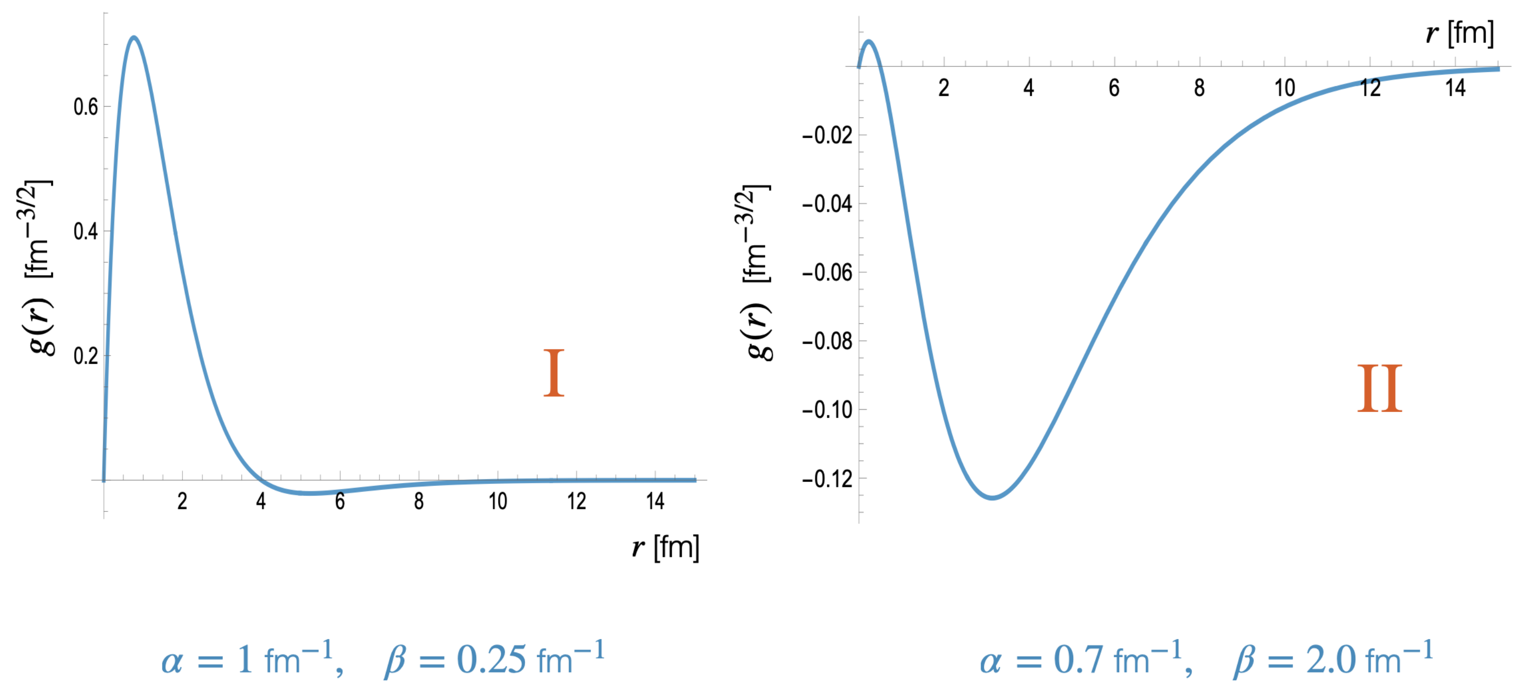


Fig. courtesy E. Epelbaum

Gedankenexperiment IV

- In Bob's notation, the potential takes the form (UT only acts on the S-wave)

$$\hat{V}_{\rm Bob} = \hat{U}\left(\frac{\hat{p}^2}{2\mu} + \hat{V}_{\rm Alice}\right)\hat{U}^\dagger - \frac{\hat{p}^2}{2\mu}$$

- But the physics is the same!

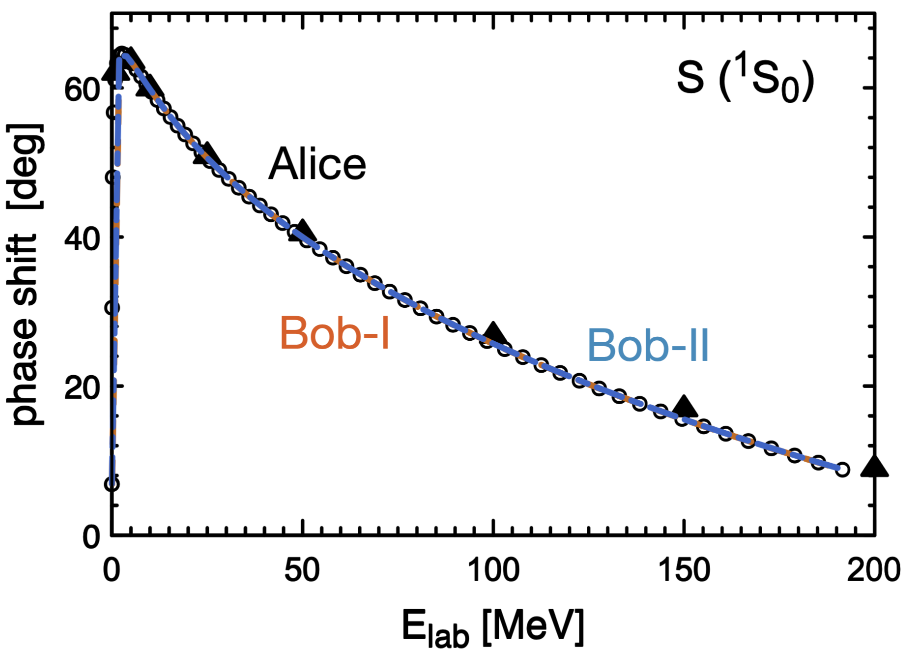
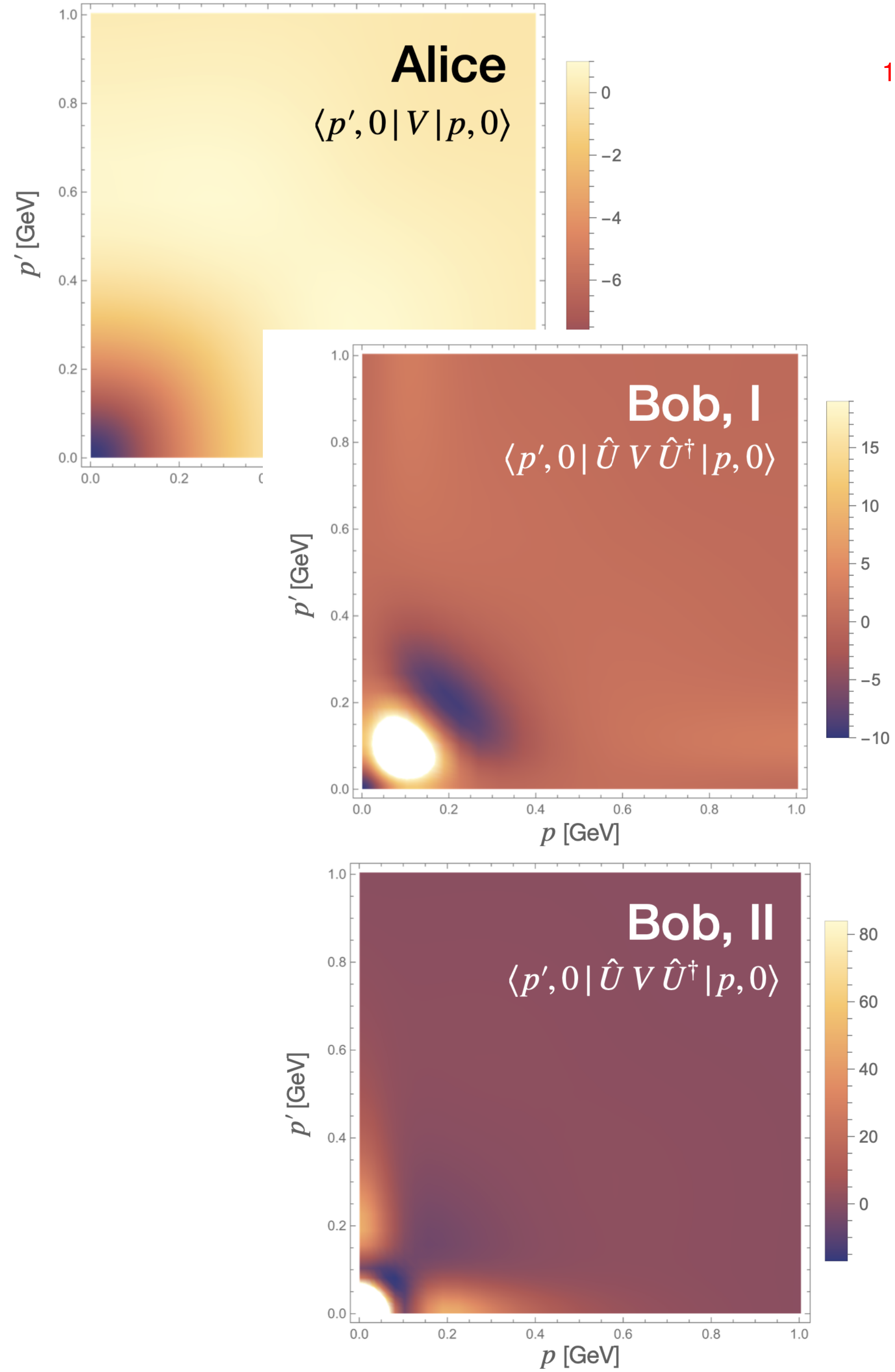
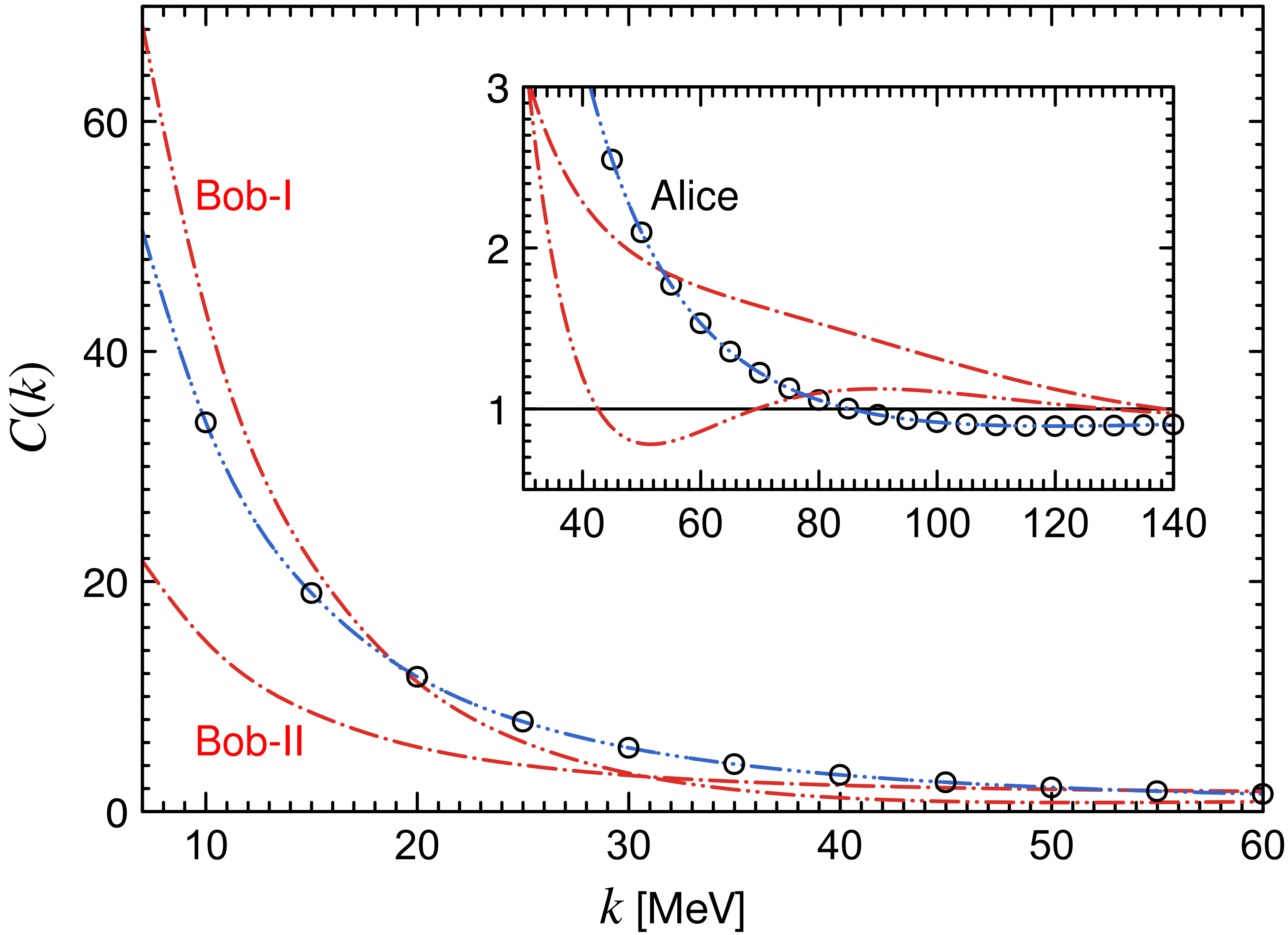


Fig. courtesy E. Epelbaum



Gedankenexperiment V

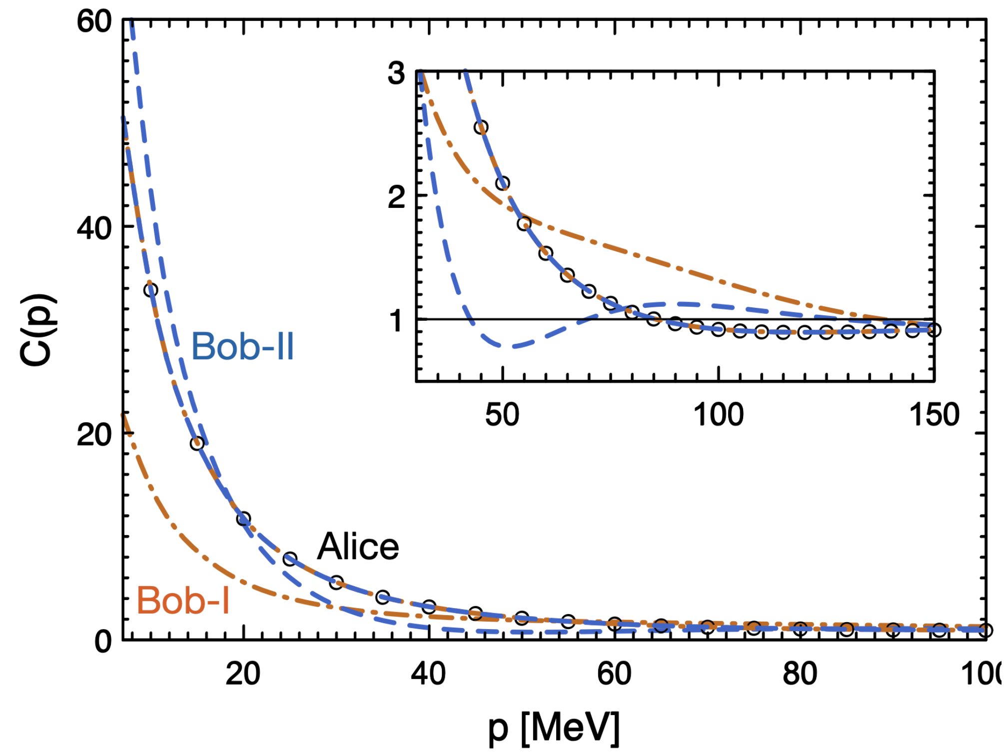
- Naturally, Bob's correlation functions look very different from Alice's

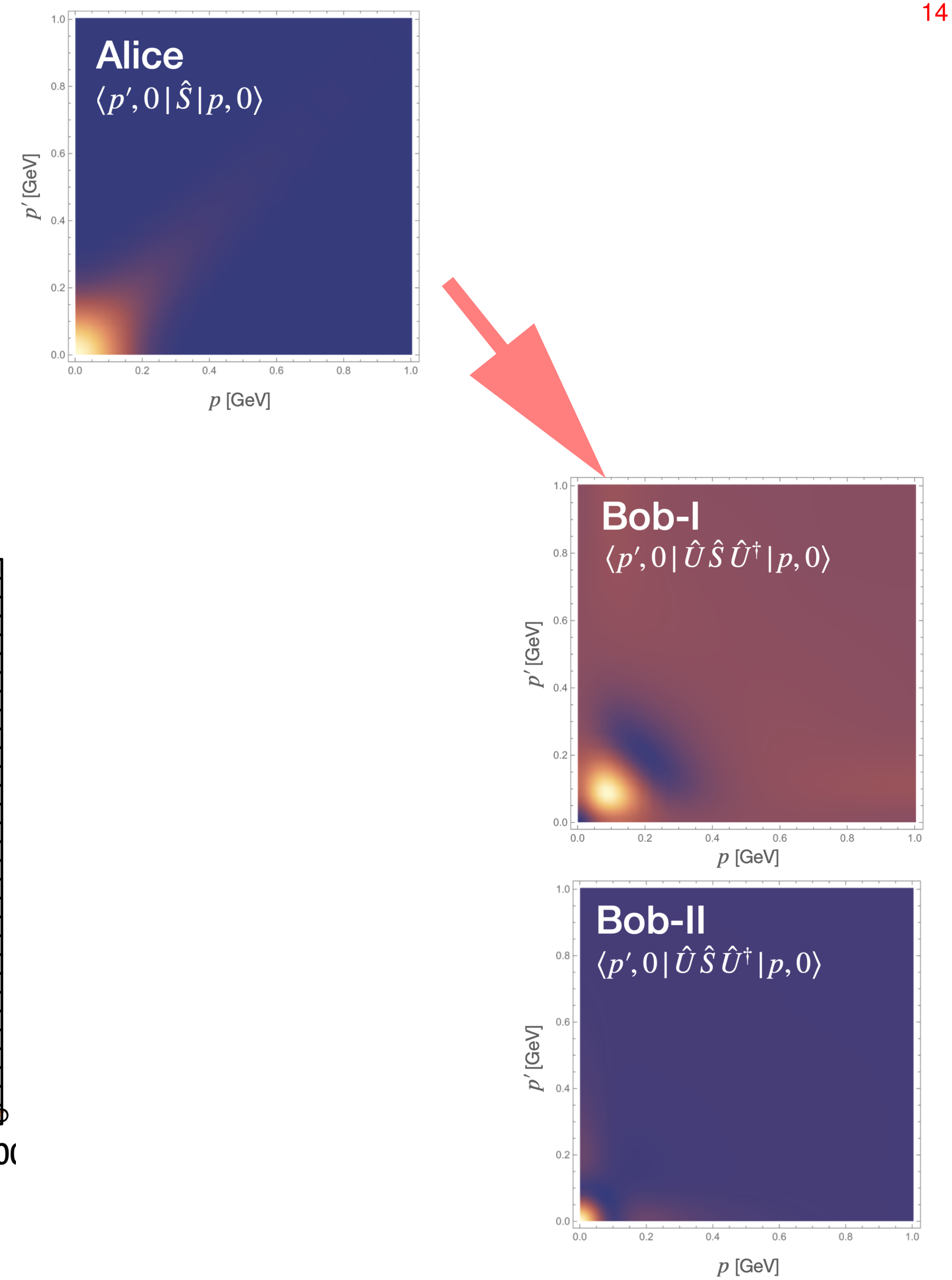


Gedankenexperiment V

- The solution is (of course) trivial:
  The source term needs to be transformed
  into Bob's conventions

$\Rightarrow$ Correlation functions coincide

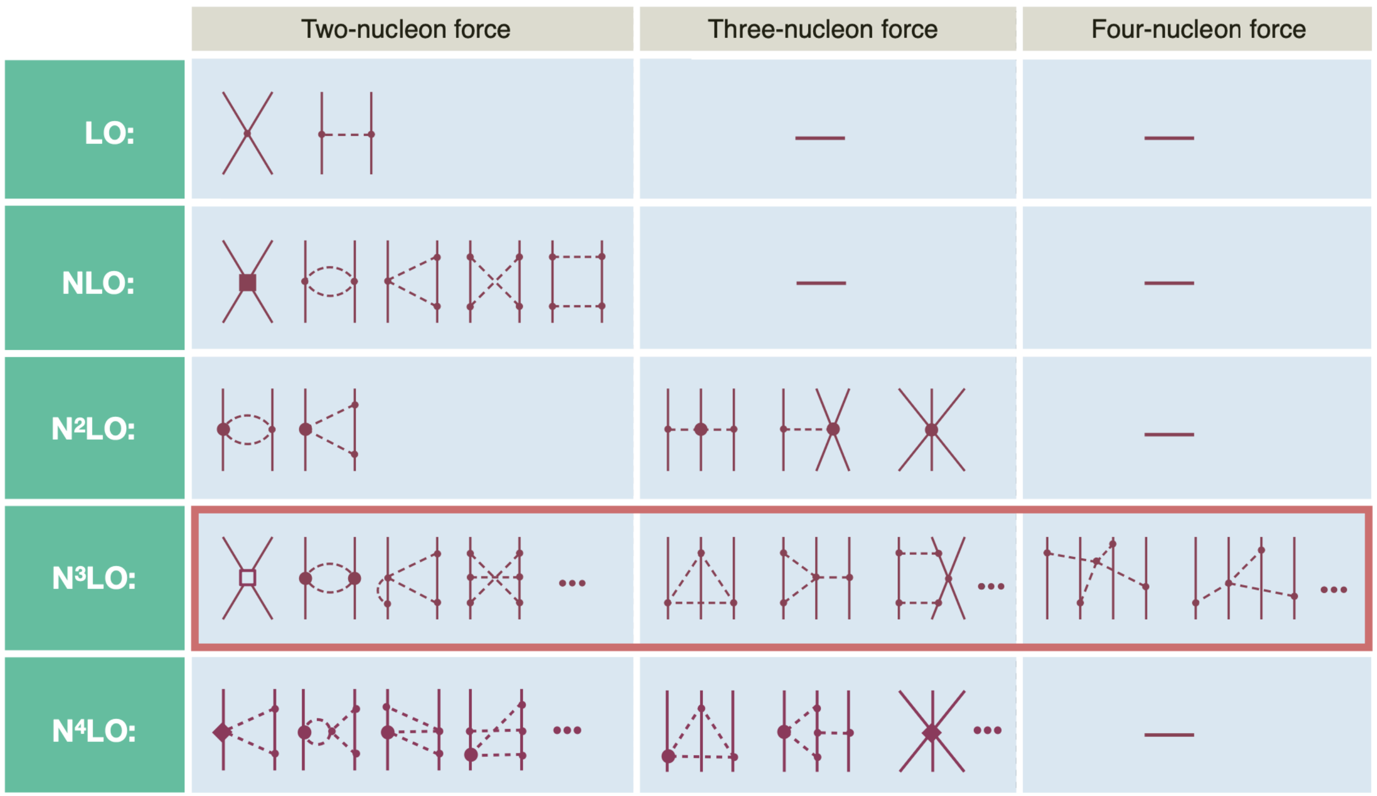




Scheme-dependence in chiral EFT

Scheme dependence in chiral EFT

- Where does the scheme dependence (off-shell effects) appear in chiral EFT?



- Most UTs fixed from renormalizability, but ambiguities remain:
  - two phases in $1/m$ corrections, three phases in the contact terms

Friar (1999), Bernard et al. (2011), Reinert et al., EPJA **54** (2018) 86

  - potentially large scheme-dependence in the 3N force!

Scheme dependence of the two-nucleon interaction

- Consider the ambiguities in the N^{3}LO contacts ($^1S_0$, $^3S_1$, $^3S_1$-$^3D_1$)

$$\langle p', {}^1S_0 | V_{\rm cont} | p, {}^1S_0 \rangle = \underbrace{\tilde{C}_{1S0}}_{\text{from } a} + \underbrace{C_{1S0}(p'^2 + p^2)}_{\text{from } r} + \underbrace{D_{1S0} p^2 p'^2 + D^{\rm off}_{1S0}(p'^2 - p^2)^2}_{\text{from } v_2}$$

$\hookrightarrow$ $D^{\rm off}_{1S0}$ can not be fixed from NN data!

Hammer, Furnstahl (2000), Beane, Savage (2001), Reinert, Krebs, Epelbaum (2018)

$\hookrightarrow$ these off-shell contacts can be eliminated via suitable UTs

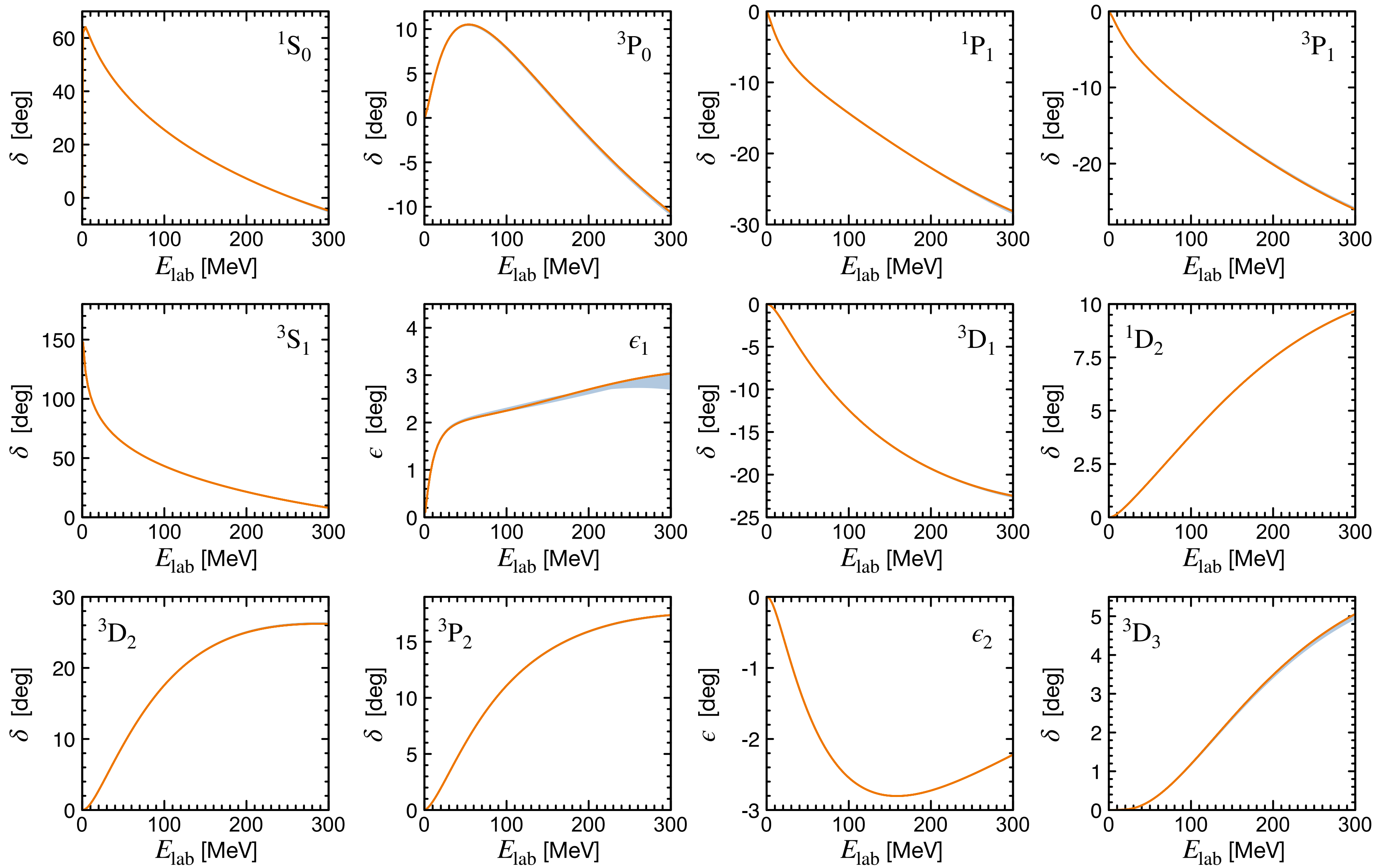
$\hookrightarrow$ thus, in chiral EFT one usually makes the choice $D^{\rm off}_{1S0} = D^{\rm off}_{3S1} = D^{\rm off}_{\epsilon_1} = 0$

- but one does not **have** to make this choice! Any values of natural size of these off-shell LECs is as good as setting them to zero!

$\hookrightarrow$ construct 27 N^{4}LO$^+$ potentials with $D^{\rm off}_{1S0,3S1} = \{\pm 3, 0\}$, $D^{\rm off}_{\epsilon_1} = \{\pm 1, 0\}$

Phase shift equivalence

- Phase shifts of these 27 NN interactions



Deuteron properties

- Deuteron observables and not so observables

	N^{4}LO$^+$ (no os LECs)	N^{4}LO$^+$ (w/ os LECs)	Empirical
B_d [MeV]	2.2246*	2.2246*	2.22456614(41)
A_S [fm$^{-1/2}$]	0.8846	0.8845...0.8848	0.8845(8)
η	0.0261	0.0260...0.0263	0.0256(4)
r_m [fm]	1.9662	1.9588...1.9709	—
Q_0 [fm^2]	0.275	0.269...0.280	—
P_D [%]	4.79	3.80...6.33	—

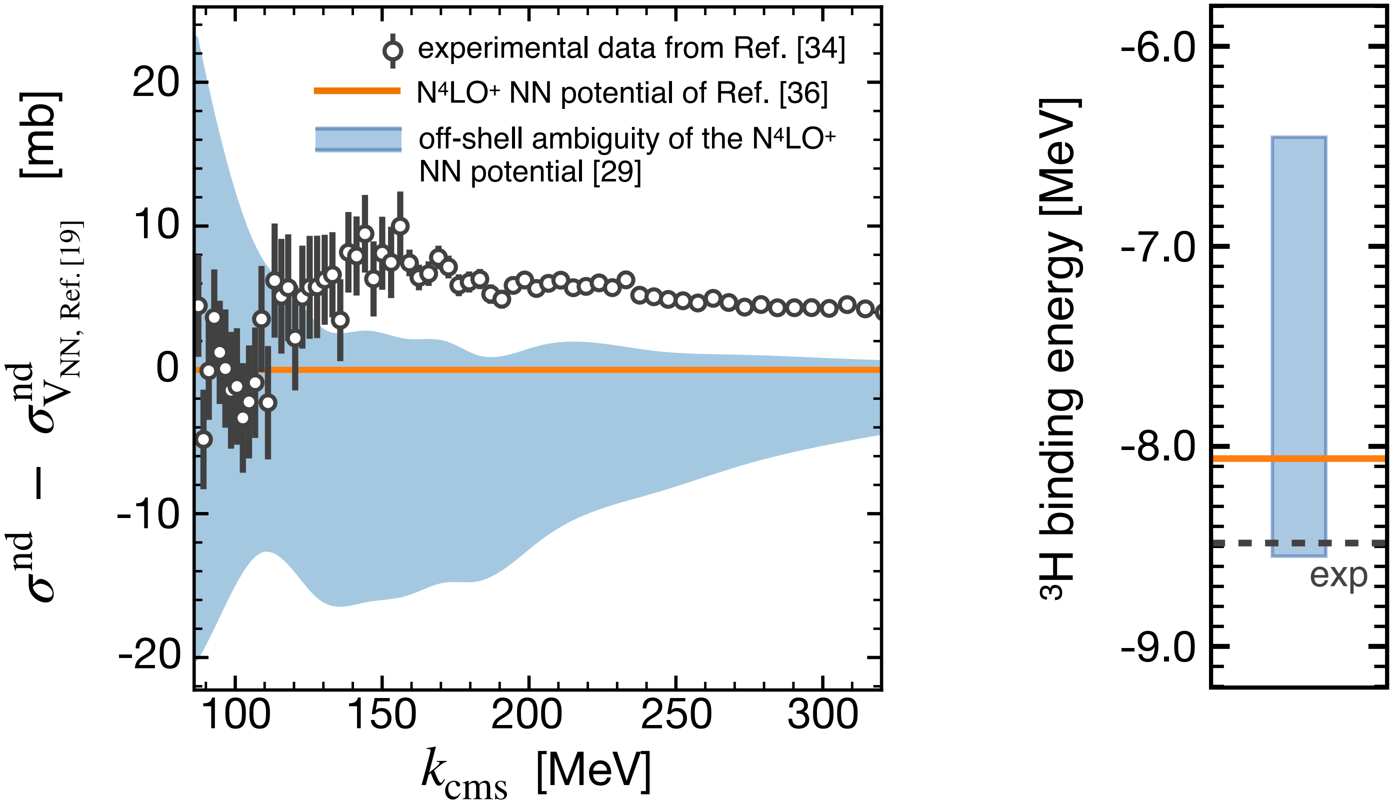
- As expected, observables stay put, non-obervables do not

Three-nucleon interactions

- ALICE claims to be able to determine 3-hadron interactions such as ppp or $p\Lambda\Lambda$

Acharya et al., Eur. Phys. J. A **59** (2023)145, 2023; Phys. Rev. X **14** (2024) 031051; Kievsky et al., Phys. Rev. C **109** (2024) 034006

- calculate 3N observables with the 27 phase-equivalent NN potentials



↪ an illustration of the Polyzou-Glöckle theorem Few Body Syst. **9** (1990) 97

↪ which particular 3BF is then to be measured in femtoscopy?

Proton-proton scattering

Basis of proton-proton scattering

- pp scattering involves the infinitely-ranged Coulomb interaction

$$|\Psi(r,\,k)|^2 = \sum_{j=0}^{j_{\rm max,int}} \frac{2j+1}{2} \sum_{s,l',l} \left|\Psi^j_{s,l',l}(r,\,k)\right|^2 + \sum_{j=j_{\rm max,int}}^{j_{\rm max,free}} \frac{2j+1}{2} \sum_{s,l'} \left|\frac{F_{l'}(\eta, rk)}{rk}\right|^2$$

  - $F_l(\eta, rk)$ = regular Coulomb function
  - $\eta = m_p\alpha_{\rm EM}$ = Sommerfeld factor, $\alpha_{\rm EM} = e^2/(4\pi)$
  - $j_{\rm max,int} = 2$ and $j_{\rm max,free} = 25$ here

- Use the Vincent-Phatak method w/ a screened Colomb potential to calculate $\Psi^j_{s,l',l}(r,\,k)$

Vincent, Phatak, Phys. Rev. C **10** (1974) 391

$$V_C^R(r) = \frac{\alpha_{\rm EM}}{r} \text{ for } r \leq R \text{ and } V_C^R(r) = 0 \text{ for } r > R$$

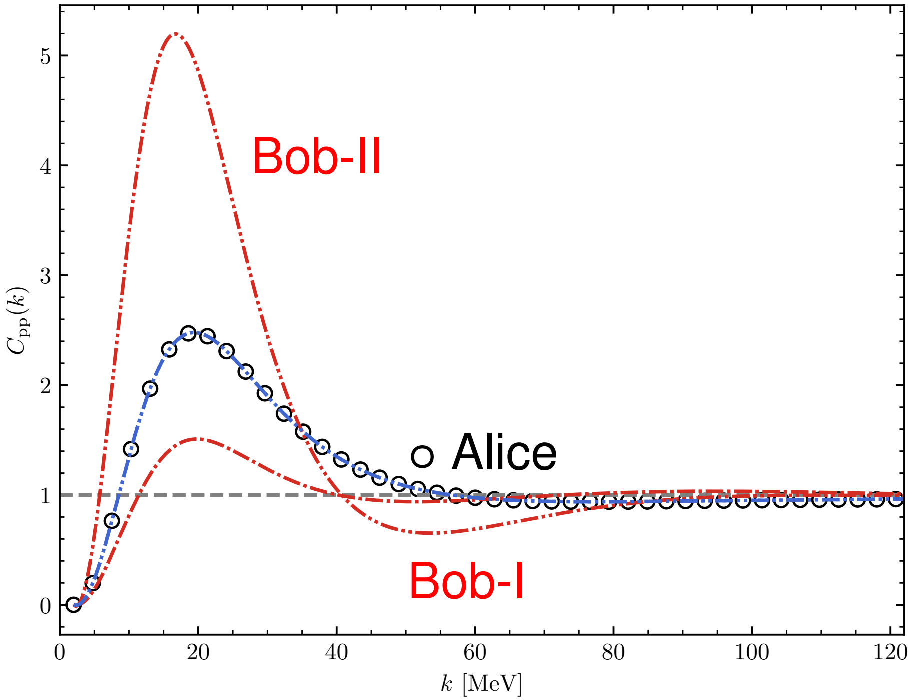
  - screening radius R much bigger than the range of the strong interactions
  - here $R = 11 - 12$ fm

↪ ready to solve the LS equation for pp scattering

Proton-proton correlation function

- Relation between Alice's and Bob's potential

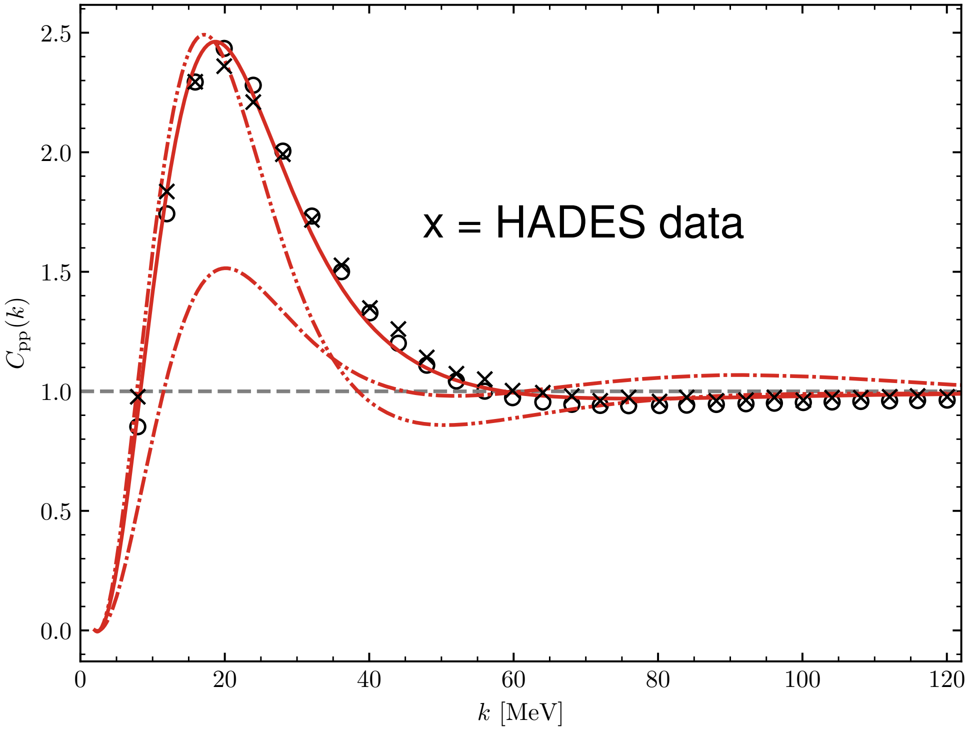
$$\hat{V}_{\rm Bob} = \hat{U}\left(\frac{\hat{p}^2}{2\mu} + \hat{V}_{\rm Alice} + \hat{V}_C^R\right)\hat{U}^\dagger - \frac{\hat{p}^2}{2\mu} - \hat{V}_C^R$$



$\hookrightarrow$ same as before, must transfrom the source, too

Extraction of the source radius

- Fit to the HADES data from p+Nb collisions — Adamczewski-Musch et al. (HADES), PRC **94** (2016) 025201
- Introduce $V_{\mathrm{Bob-III}}$ with $\alpha = 1.15$ fm^{-1} and $\beta = 0.5$ fm^{-1}



$r_0^{\mathrm{Alice}} = 2.02$ fm

$r_0^{\mathrm{Bob-I}} = 1.75$ fm

$r_0^{\mathrm{Bob-II}} = 3.31$ fm

$r_0^{\mathrm{Bob-IIII}} = 3.02$ fm

$\hookrightarrow$ assumption of a universal source leads to significant ambiguities

Summary

Takeaways

- Nuclear interactions are scheme-dependent (esp. at short distances)

- They can be calculated / determined provided one fixes the convention and keeps it consistently in all applications (as done in chiral EFT)

$\hookrightarrow$ Can this be achieved in femtoscopy???

- Any model of the source term must at least comply with the principles of QM

$\Rightarrow$ Claims of high-precision determinations of hadronic interactions based on femtoscopy are thus (at least) questionable

- For the two-hadron case (πK scattering and the nature of the $K_0^*(700)$)

$\hookrightarrow$ much better treatment of the FSI leads to very different conclusions!

Albaladejo, Canoa, Nieves, Pelaez, Ruiz-Arriola, de Elvira, Phys. Lett. B **866** (2025) 139552

SPARES

Interferometry

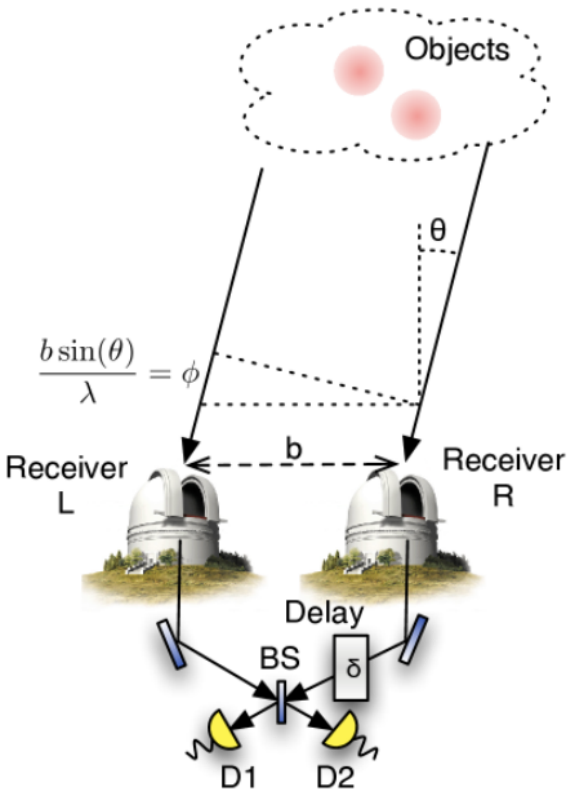
- Hanbury Brown–Twiss intensity interferometry

$\longrightarrow$ measure the (angular) size of an object (star)

or spatial/temporal correlations within such an object

Hanbury Brown, Twiss, Phil. Mag. **45** (1954) 663

Nice intro: Baym, Acta Phys. Pol. **29** (1998) 1839



- Intensity correlations (photon fields)

$$g^{(2)}(t_1,t_2,r_1,r_2) = \frac{\langle : I(t_1,r_1) I_2(t_2,r_2) : \rangle}{\langle I(t_1,r_1)\rangle \langle I(t_2,r_2)\rangle}$$

- can also be explained classically (current with intensity I at relative separation/times)

$\hookrightarrow$ $g^{(2)}(0,0) = 2$ for thermal light (Gaussian)

$g^{(2)}(0,0) = 1$ for coherent light (Dirac delta)