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Application of SMS chiral interactions to light hypernuclei

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ROCKSTAR Workshop, ECT*, Trento, Italy

- Motivation
- New chiral YN interactions
- Theoretical uncertainties of Λ-separation energies
- Estimates of 3BF contributions for ${}^{3}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{H}$ / ${}^{4}_{\Lambda}\mathrm{He}$ and ${}^{5}_{\Lambda}\mathrm{He}$
- CSB of the YN interaction
- Conclusions & Outlook

in collaboration with Hoai Le, Johann Haidenbauer and Ulf Meißner

Hoai Le et al. arXiv:2308.01756.

J. Haidenbauer et al. EPJ A 59, 63 (2023).

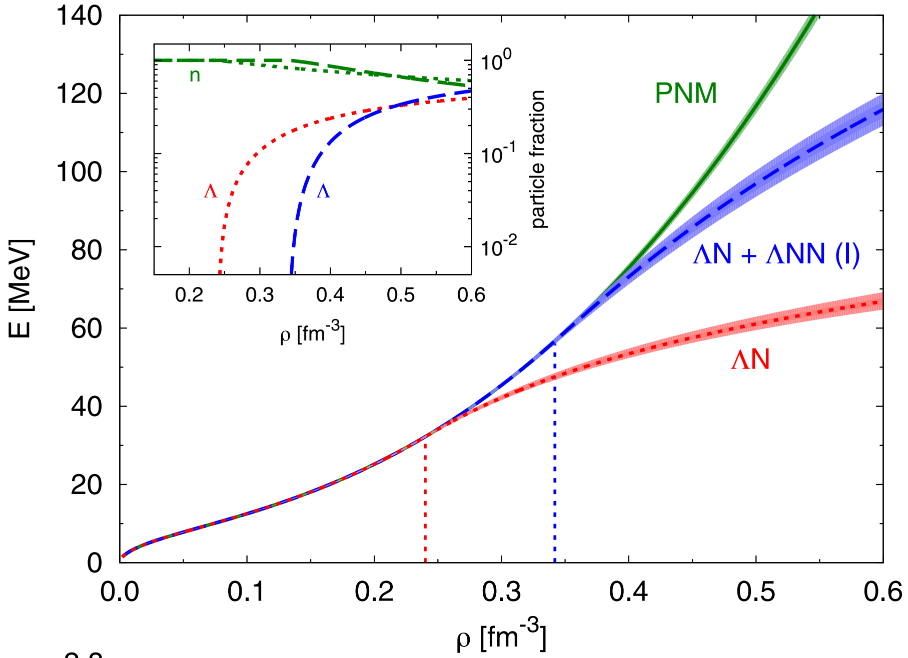
J. Haidenbauer et al. FBS 62, 105 (2021).

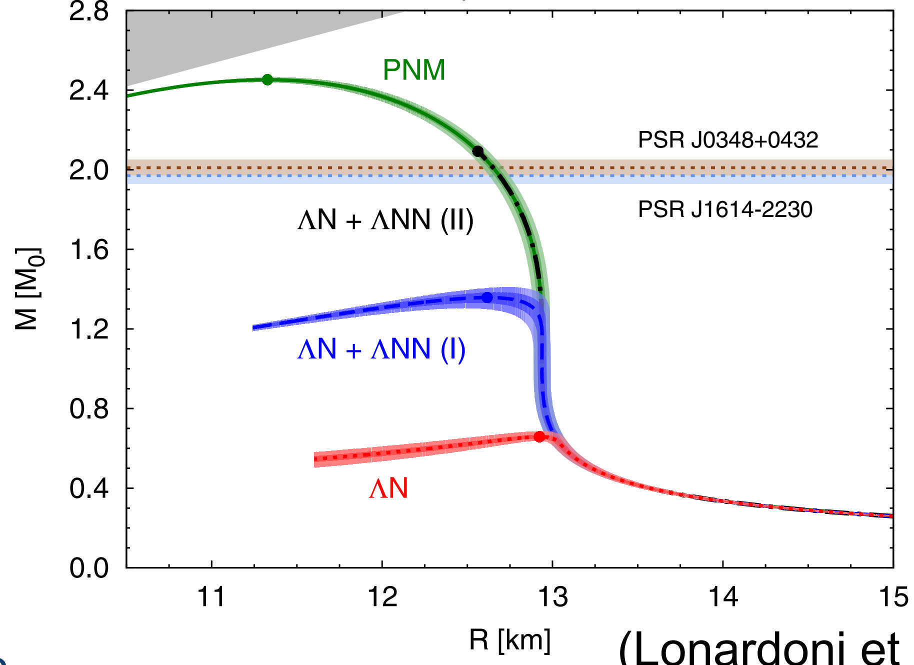
October 9th, 2023

Hypernuclear interactions

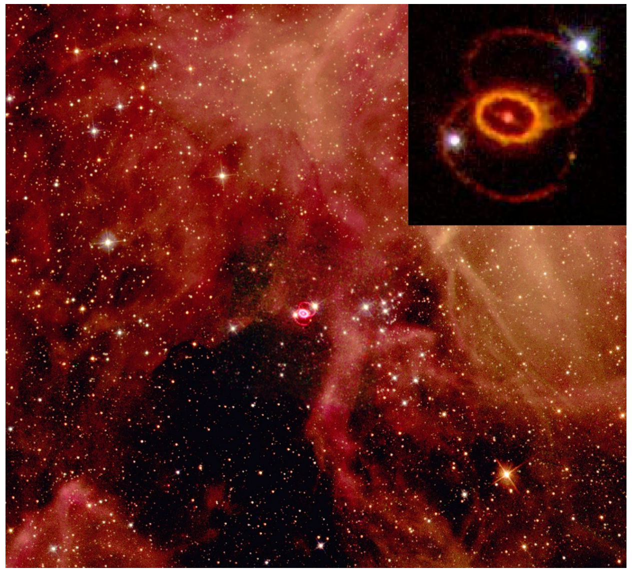
Why is understanding hypernuclear interactions interesting?

- *hyperon contribution to the EOS, neutron stars, supernovae*
- *"hyperon puzzle"*
- *Λ as probe to nuclear structure*
- *flavor dependence of baryon-baryon interactions*

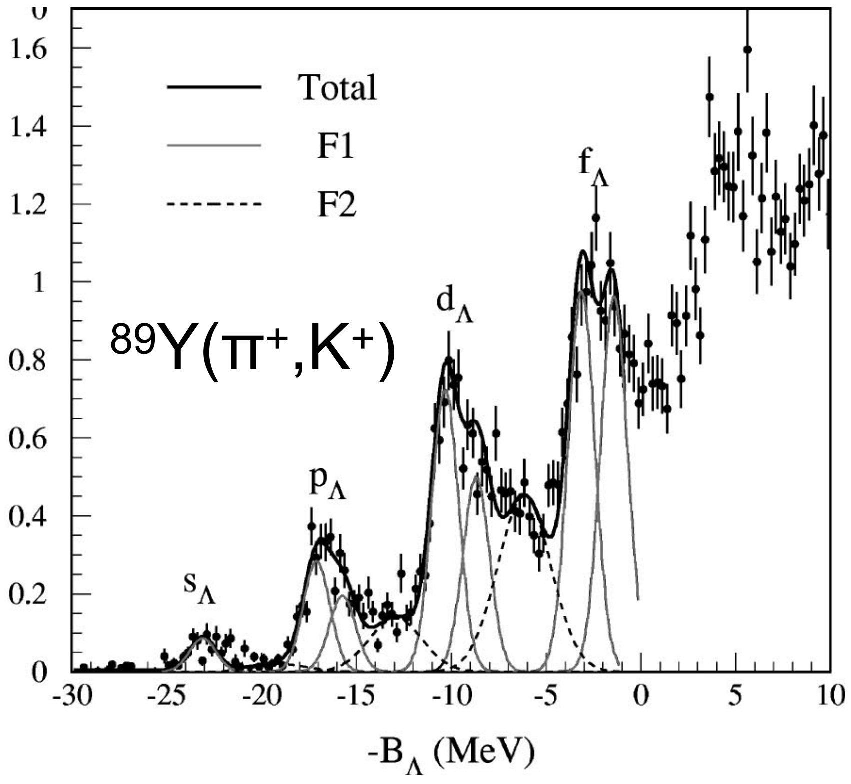




(Lonardoni et al. (2015))



(SN1987a, Wikipedia)

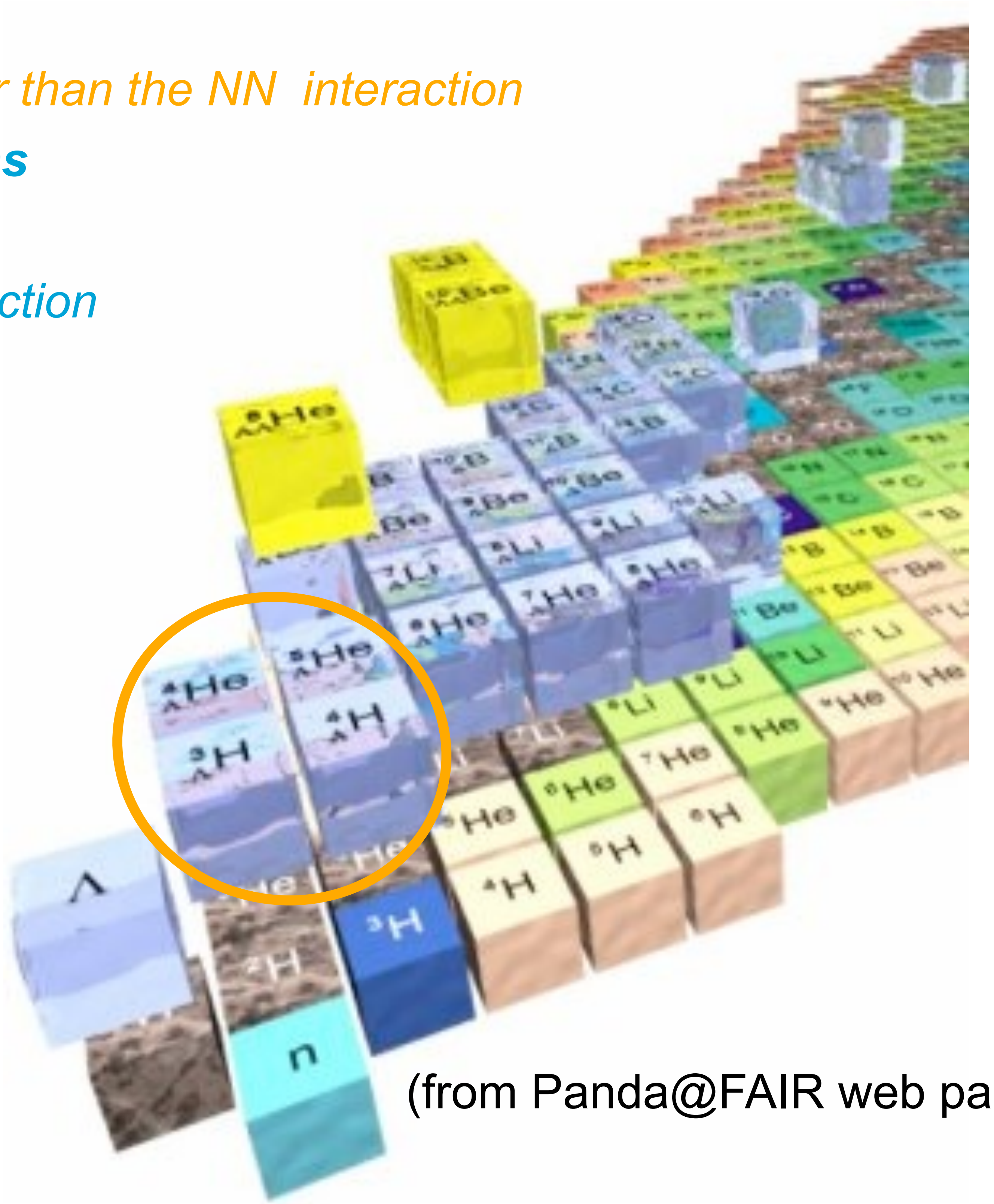


(Hotchi et al. (2001))

Hypernuclei

Only few YN data. Hypernuclear data provides additional constraints.

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively:* ***core nucleus + hyperons***
  - *„separation energies“ are quite independent from NN(+3N) interaction
- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in* ***several spin states***
- ***non-trivial constraints*** *on the YN interaction even from lightest ones*
- *size of* ***YNN*** *interactions? need to include Λ-Σ conversion!*



(from Panda@FAIR web page)

Hypernuclei

- new data (J-PARC, Star, Alice, …)
- world averages compiled by Mainz group (Eckert et al.)

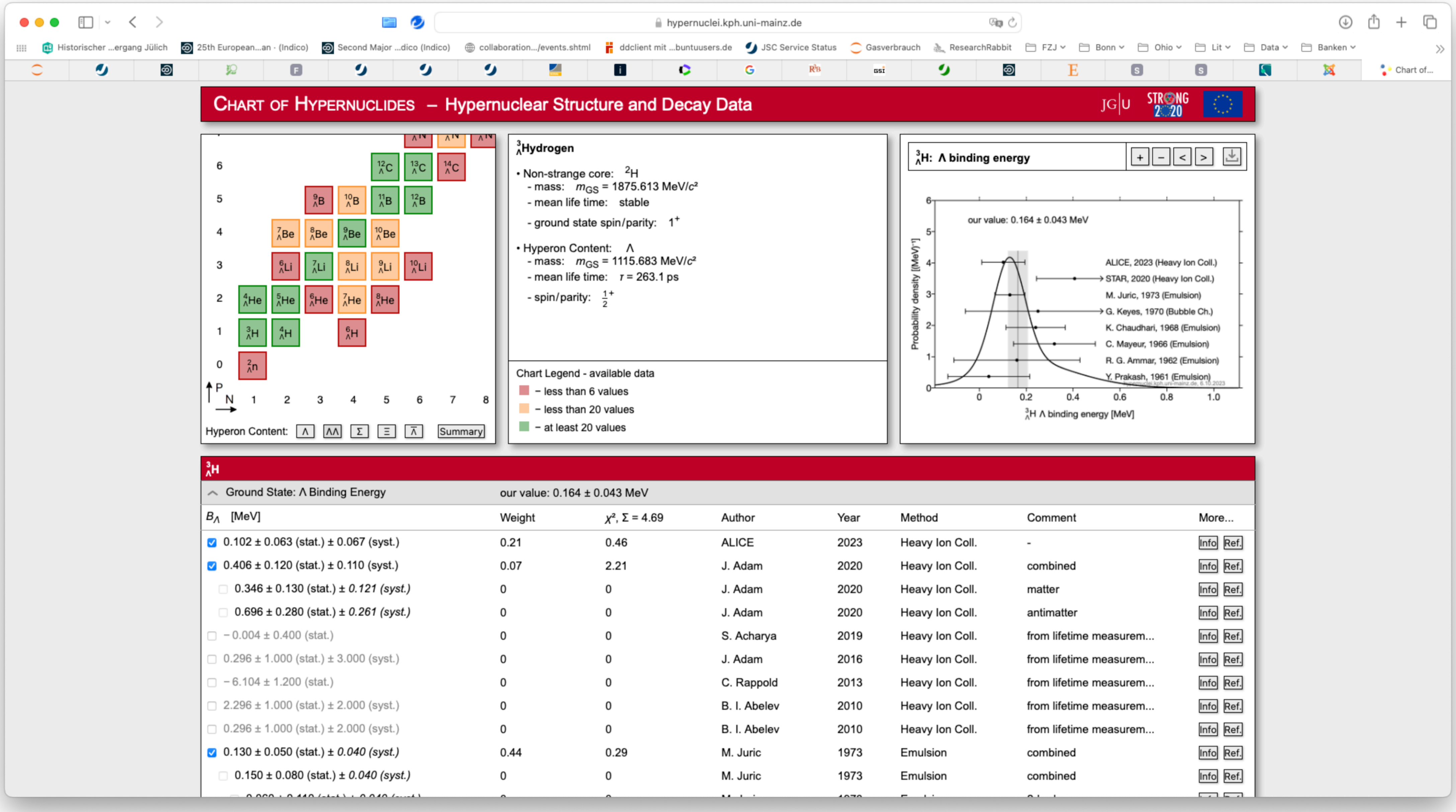


CHART OF HYPERNUCLIDES – Hypernuclear Structure and Decay Data

${}^{3}_{\Lambda}$Hydrogen

- Non-strange core: ${}^{2}H$
  - mass: $m_{GS} = 1875.613$ MeV/c^2
  - mean life time: stable
  - ground state spin/parity: 1^+
- Hyperon Content: Λ
  - mass: $m_{GS} = 1115.683$ MeV/c^2
  - mean life time: $\tau = 263.1$ ps
  - spin/parity: $\frac{1}{2}^+$

Chart Legend - available data

- 🟥 – less than 6 values
- 🟧 – less than 20 values
- 🟩 – at least 20 values

${}^{3}_{\Lambda}H$

Ground State: Λ Binding Energy — our value: 0.164 ± 0.043 MeV

	B_Λ [MeV]	Weight	χ^2, Σ = 4.69	Author	Year	Method	Comment	More...
☑	0.102 ± 0.063 (stat.) ± 0.067 (syst.)	0.21	0.46	ALICE	2023	Heavy Ion Coll.	-	Info Ref.
☑	0.406 ± 0.120 (stat.) ± 0.110 (syst.)	0.07	2.21	J. Adam	2020	Heavy Ion Coll.	combined	Info Ref.
☐	0.346 ± 0.130 (stat.) ± *0.121 (syst.)*	0	0	J. Adam	2020	Heavy Ion Coll.	matter	Info Ref.
☐	0.696 ± 0.280 (stat.) ± *0.261 (syst.)*	0	0	J. Adam	2020	Heavy Ion Coll.	antimatter	Info Ref.
☐	−0.004 ± 0.400 (stat.)	0	0	S. Acharya	2019	Heavy Ion Coll.	from lifetime measurem...	Info Ref.
☐	0.296 ± 1.000 (stat.) ± 3.000 (syst.)	0	0	J. Adam	2016	Heavy Ion Coll.	from lifetime measurem...	Info Ref.
☐	−6.104 ± 1.200 (stat.)	0	0	C. Rappold	2013	Heavy Ion Coll.	from lifetime measurem...	Info Ref.
☐	2.296 ± 1.000 (stat.) ± 2.000 (syst.)	0	0	B. I. Abelev	2010	Heavy Ion Coll.	from lifetime measurem...	Info Ref.
☐	0.296 ± 1.000 (stat.) ± 2.000 (syst.)	0	0	B. I. Abelev	2010	Heavy Ion Coll.	from lifetime measurem...	Info Ref.
☑	0.130 ± 0.050 (stat.) ± *0.040 (syst.)*	0.44	0.29	M. Juric	1973	Emulsion	combined	Info Ref.
☐	0.150 ± 0.080 (stat.) ± *0.040 (syst.)*	0	0	M. Juric	1973	Emulsion	combined	Info Ref.

Hypernuclear interactions

We will use here chiral interactions, start with a brief summary of other approaches to the ΛN (YN) interaction

Long history of ΛN interaction models

- early models (Downs, Iddings, Brown, Dalitz, before 1970)
- Nijmegen group (Nijm D, Nijm F, SC89, SC97 and ESC(ESC16), 1973-2016)
- Jülich model (Jülich 1994, Jülich 2004)
- RGM model of Fujiwara, fss2 (1995, 2002)

models have successfully used to understand binding mechanism

important role of $\Lambda - \Sigma$ conversion

EFT based approaches

pionless (=Goldstone boson less) EFT

- application to ${}^{3}_{\Lambda}\mathrm{H}$ and Λnn (Hammer 2002, Hildenbrand et al. 2019,2020)
- application to $A = 3 - 5$ hypernuclei (Contessi et al. PRL 2018)

interactions given by contact interactions, usually in leading order

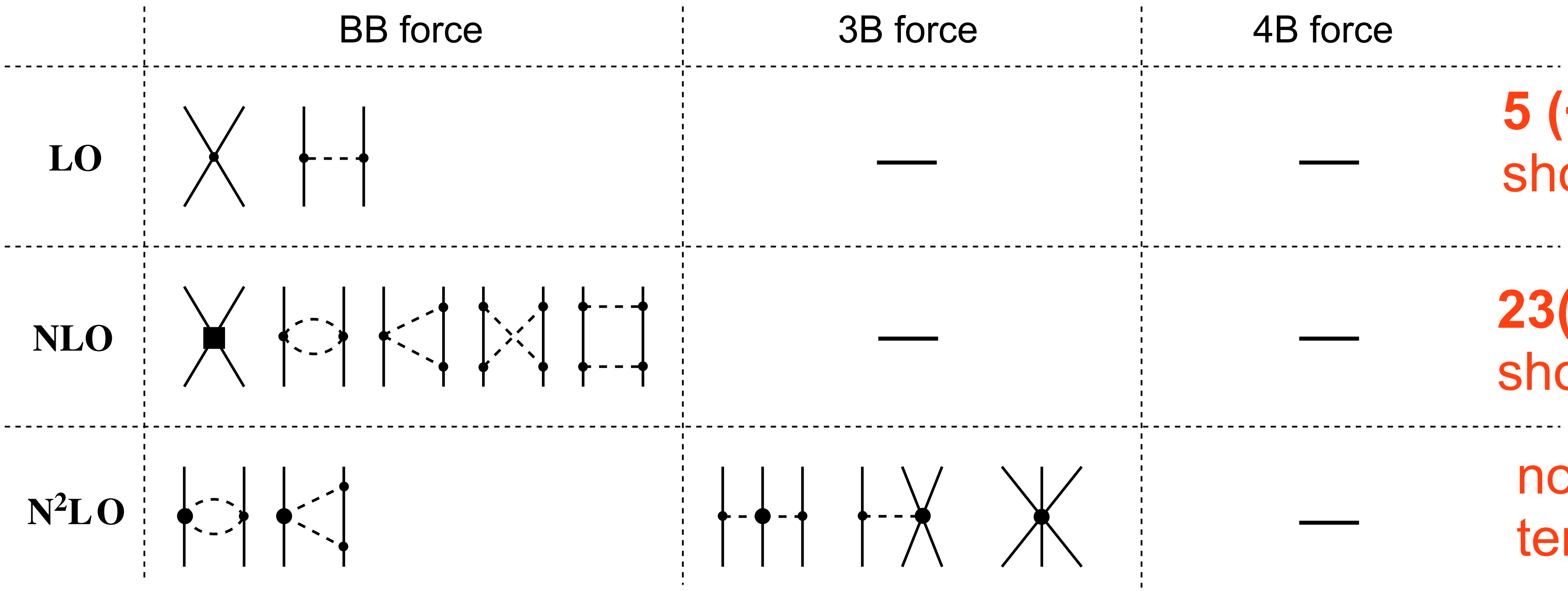
only Λ explicitly considered

EFT requires **3BFs** in leading order (three additional parameters)

expansion parameter

Chiral NN & YN interactions

EFT based approaches (cont')



5 (+1) NN/YN (YY)
short range parameters

23(+5) NN/YN (YY)
short range parameters

no additional contact
terms in NN/YN (YY)

(adapted from Epelbaum, 2008)

Chiral EFT implements **chiral symmetry of QCD**

- perturbative expansion for the interaction
- non-perturbative solution of Schrödinger eq.
- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale $\approx 600 - 700\,\mathrm{MeV}$

Retain flexibility to adjust to data due to counter terms

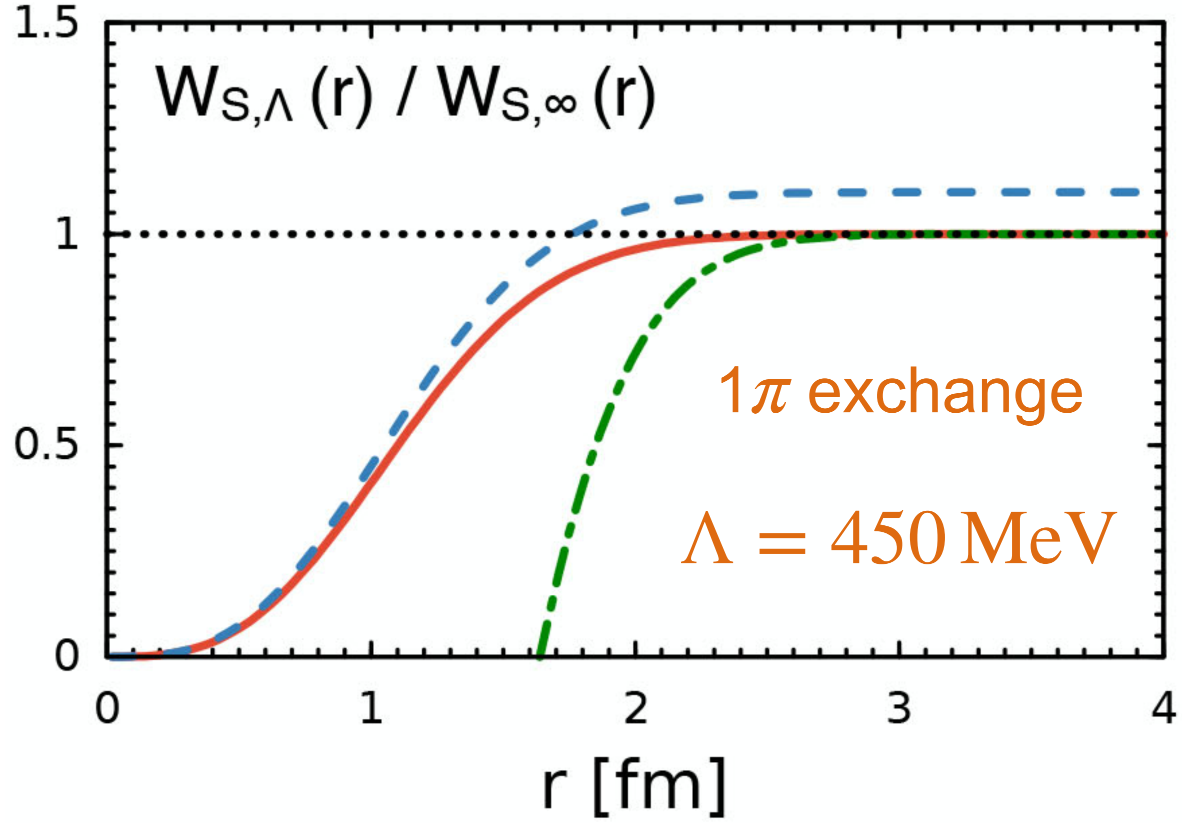
Regulator required — cutoff/different orders often used to estimate uncertainty

Λ-Σ conversion is explicitly included (size of 3BFs expected to be N²LO)

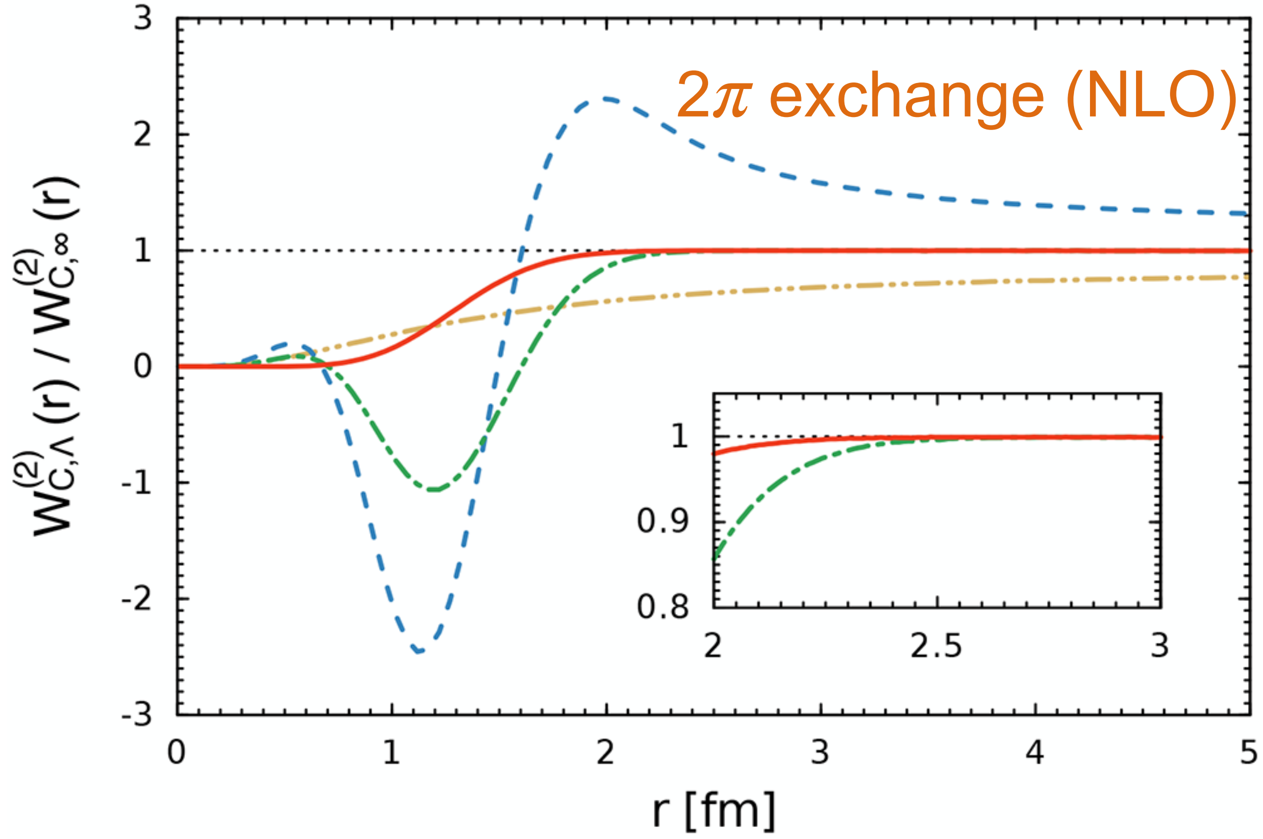
Choice of regulator

- trad. regularized (Entem et al. 2005, Epelbaum 2001, Ordonez et al. 1994)
- spectral function (SFR) regularization (Epelbaum 2005)
- semilocal coordinate-space (SCS) regularization (Epelbaum et al. 2015)
- semilocal momentum-space (SMS) regularization (Reinert et al. 2018)

$$W_{C,\Lambda}^{(2)}(r) = \frac{1}{2\pi^2 r}\int_{2M_\pi}^{\infty} d\mu\, \mu\; e^{-\mu r}\eta_C^{(2)}(\mu)\, e^{-\frac{q^2+\mu^2}{2\Lambda^2}} + \text{subtraction}$$



(Reinert et al. 2018)



(Reinert et al. 2018)

NLO13/NLO19 YN based on trad. reg.

N²LO based on SMS (incl. subtractions)

SMS N^2LO interaction

In order to check uncertainties realization in

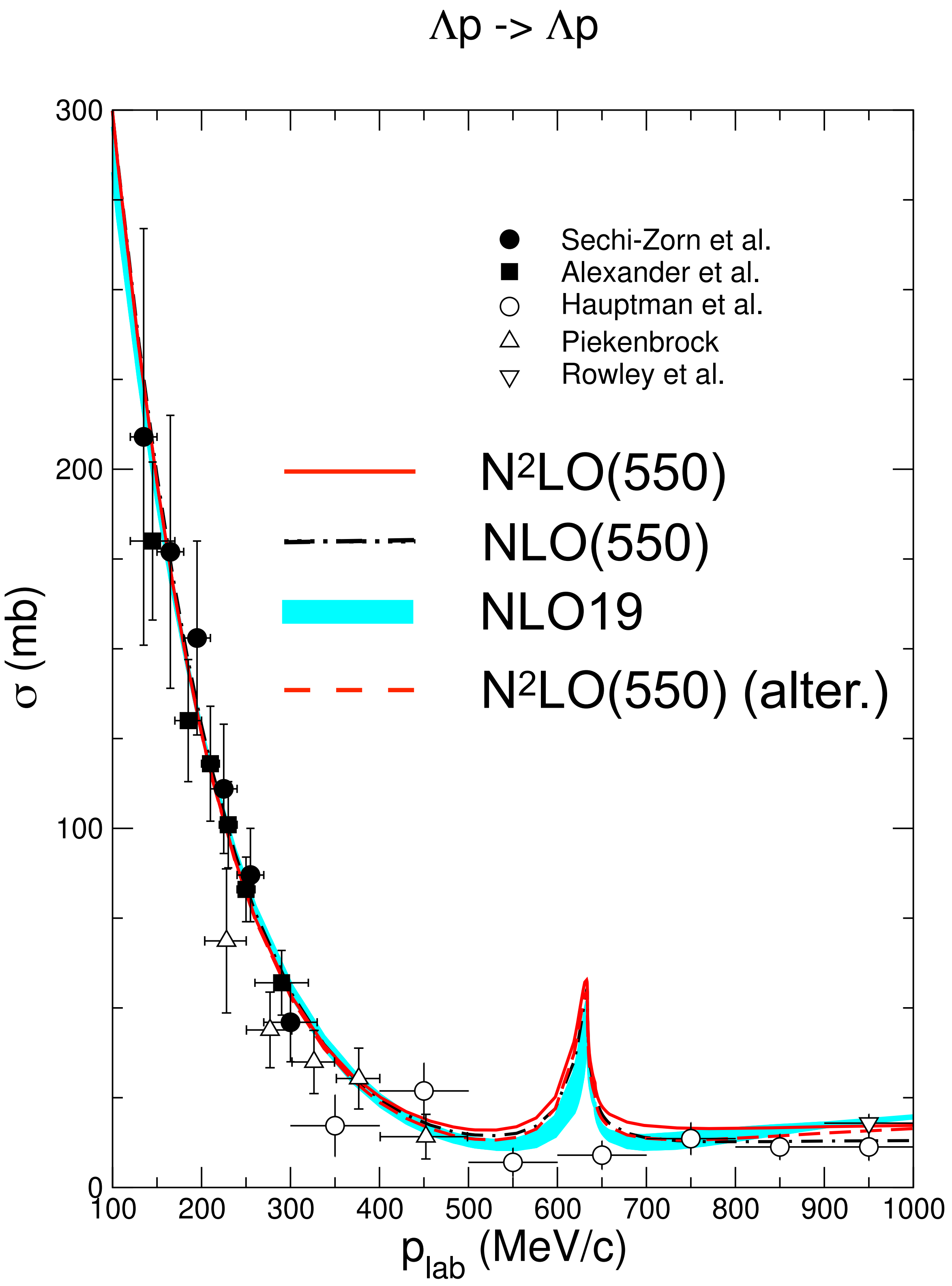
LO(700) NLO(500), **NLO(550)**, NLO(600) and N^2LO(500), **N^2LO(550)**, N^2LO(600)

Details on fitting procedure using partly flavor-SU(3):

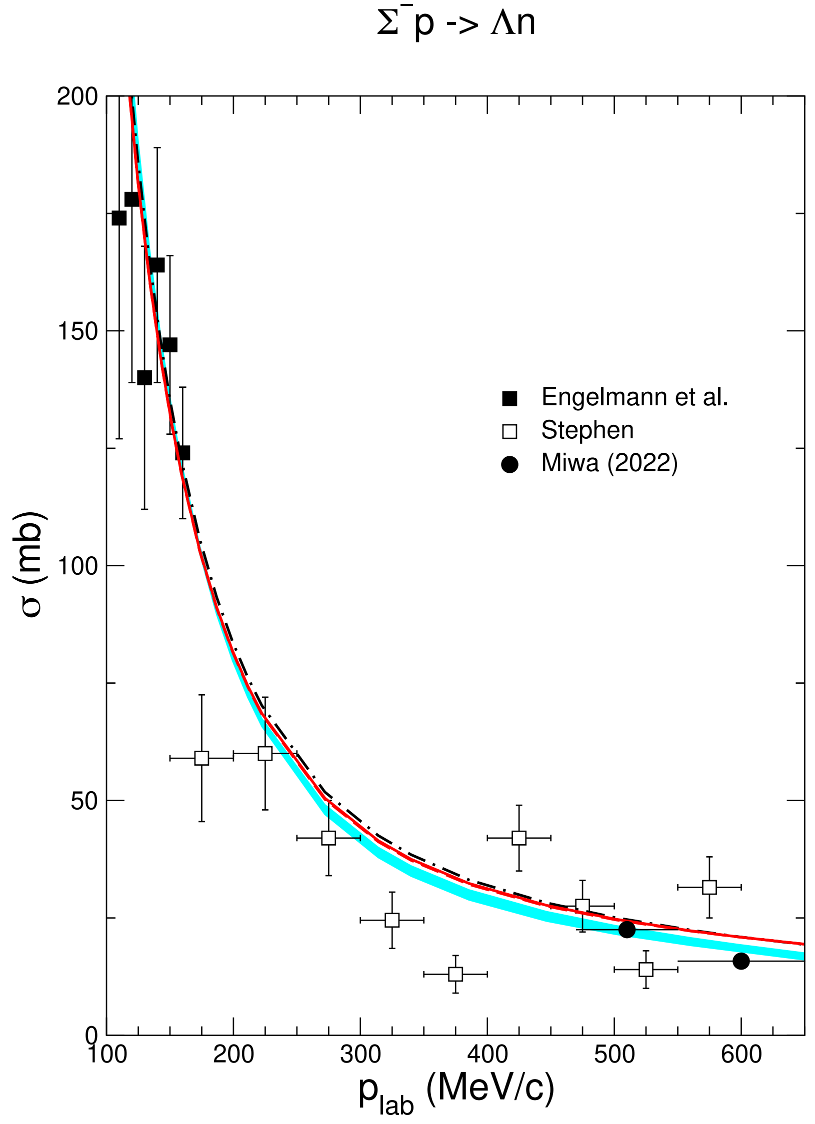
- flavor SU(3) is broken by using physical meson and baryon masses!
- retain only the 2π exchange from the 2 Goldstone boson exchanges
  → should be absorbed in SU(3)-breaking counter terms
- use **36 data** at low energy to determine **s-wave counter** terms
- hypertriton is required to be bound (binding energy **roughly** correct)
  → $a_s(\Lambda p) = -2.8\,\mathrm{fm}$ in NLO and N^2LO
- include **SU(3) breaking in LO** counter terms (necessary to avoid bound states in YN)
- assume **SU(3) symmetry for p-waves** counter terms in NLO
  values for p-wave counter terms of NN
- fit to differential cross section in N^2LO
  → two versions for N^2LO(550) differ for differential cross sections
  none is clearly preferred

SMS NLO/N^{2}LO interaction

Selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)



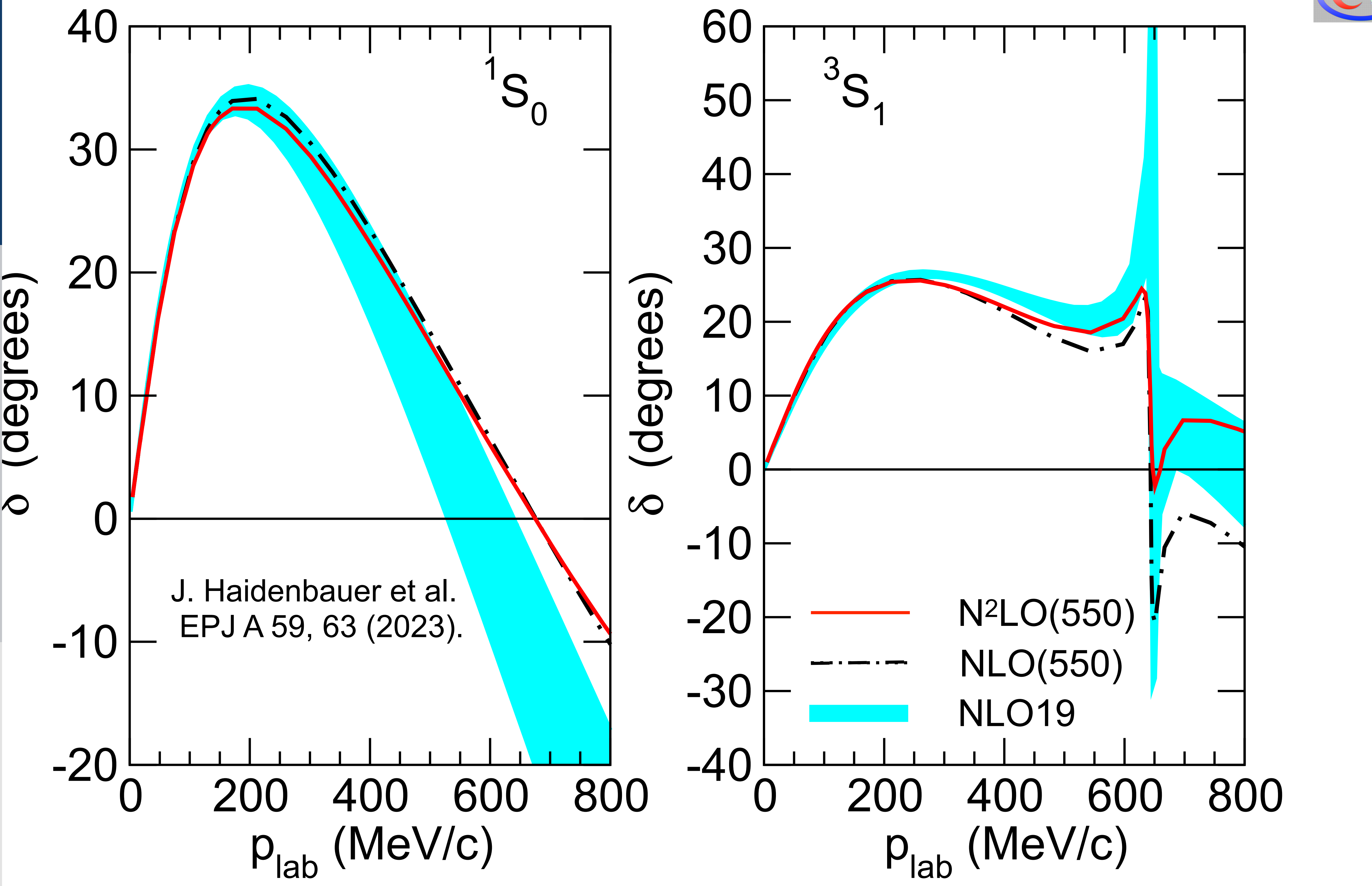
- most relevant cross sections very similar in NLO and N^{2}LO
- similar to NLO19
- alternative fit (see later)



J. Haidenbauer et al. EPJ A 59, 63 (2023).

SMS NLO/N²LO interaction

Selected phase shifts

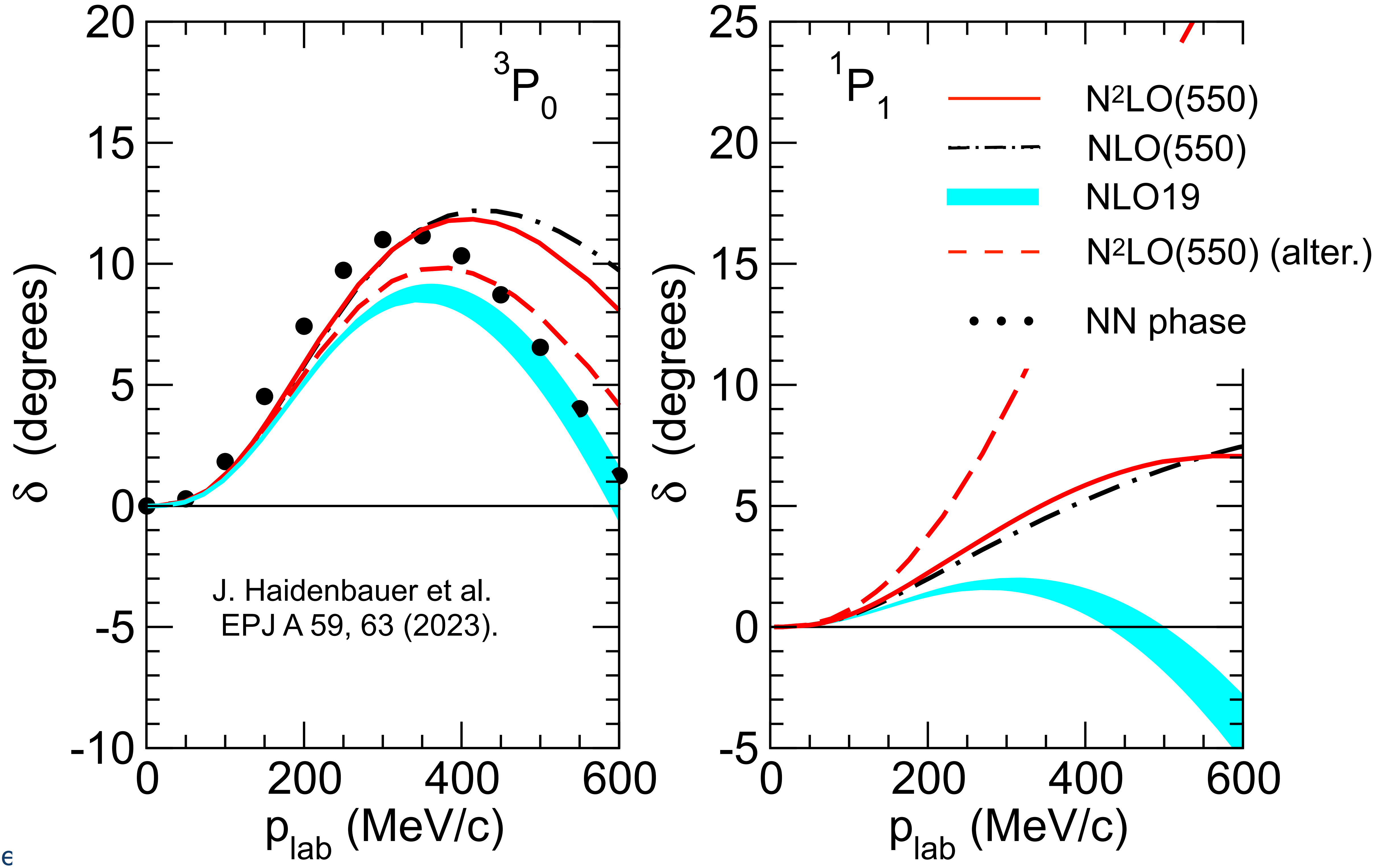


- s-phase shifts very similar in NLO and N²LO
- similar to NLO19 although different tail at high momenta

SMS NLO/N^2LO interaction

$\Sigma^+ - p$ scattering can be strongly related to NN in $^3P_0$,$^3P_1$,$^3P_2$ (at least in NLO)

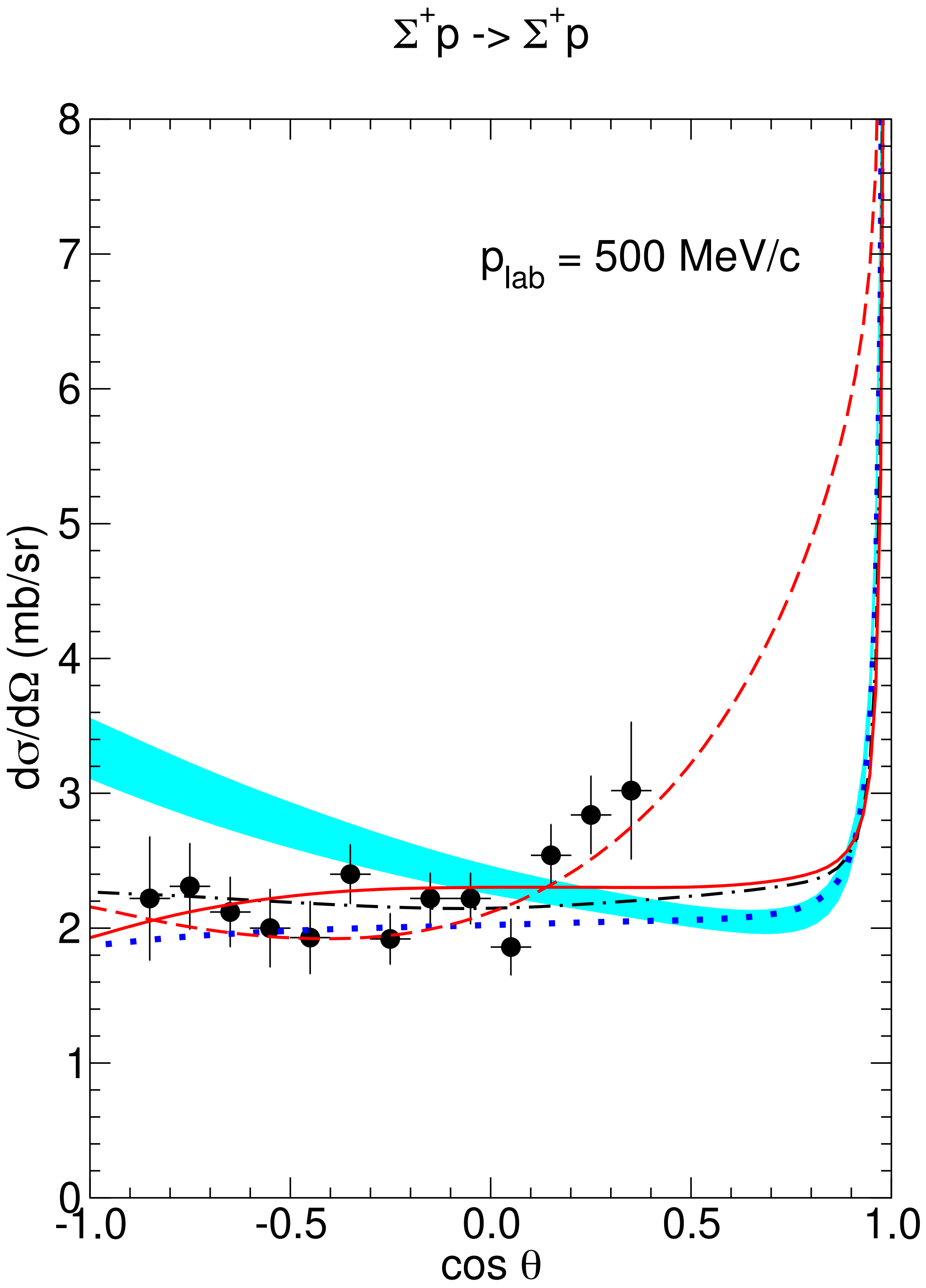
In $^1P_1$, in NLO, setting the counter term to zero is OK.

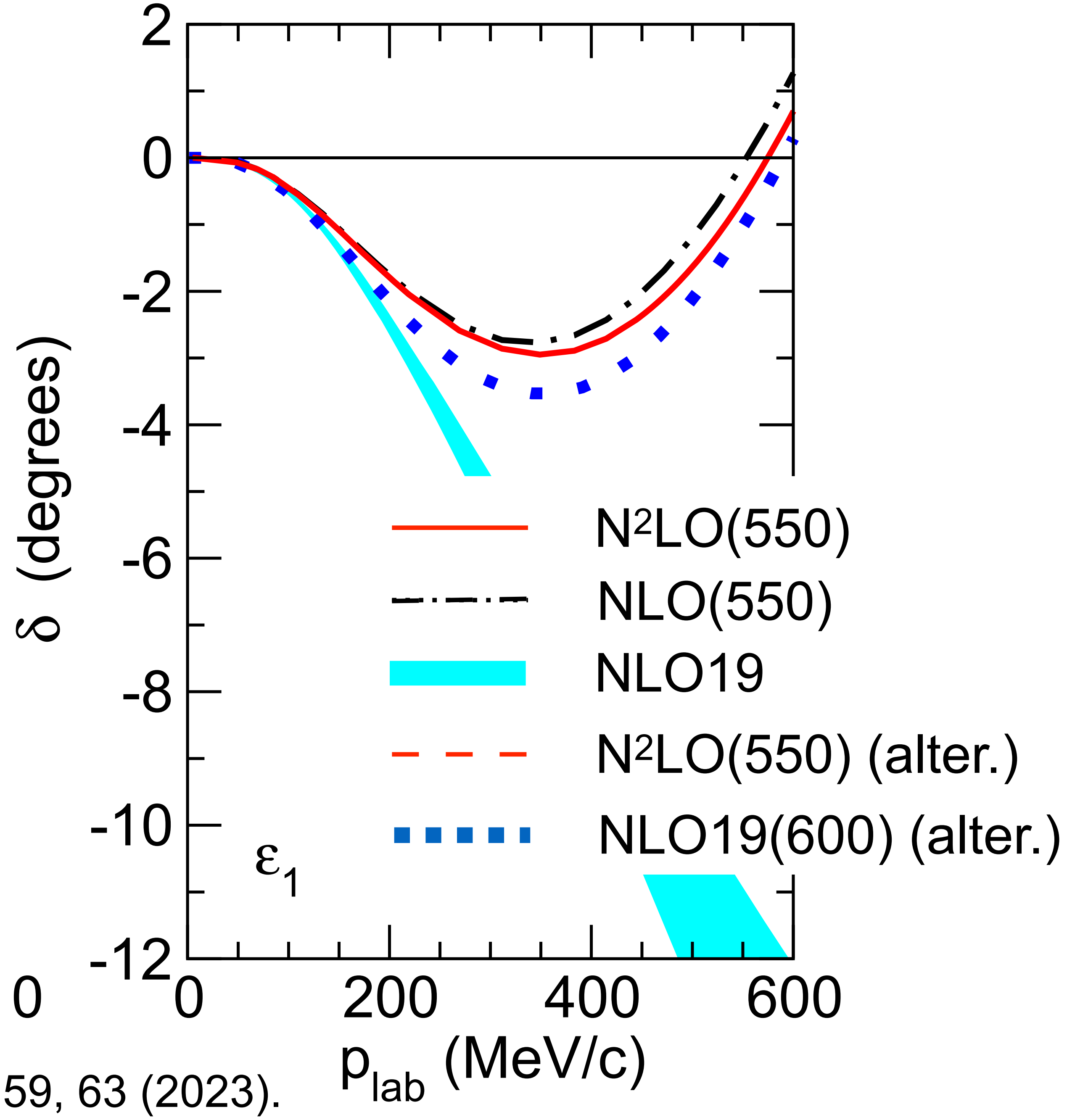


J. Haidenbauer et al.
EPJ A 59, 63 (2023).

SMS NLO/N^{2}LO interaction

new data (Miwa(2022)) at higher energies provides **new constraints**!





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SMS NLO/N^{2}LO interaction

Notes on the current status of the SMS interactions

- so far **no YNN forces** (N^{2}LO) have been used in A>2 calculations (see in a moment)
- p-waves are **not uniquely** determined
  more accurate hypernuclear calculations and/or additional differential observables (polarizations, more cross differential cross sections, …)
- data from additional channels will be helpful $(\Lambda p, \Sigma^- p \to \Sigma^0 n, \ldots)$
- calculation for single particle energies in nuclear matter yields results similar to NLO19
  — dependence on order and cutoff indicates need to include **YNN forces**
- even ratio of spin singlet/spin triplet strength requires ${}^3_\Lambda\mathrm{H}$

Is assumption of negligible YNN force valid for this hypernucleus?

→ **uncertainty analysis** to

1. pin down dependence on NN force
   (motivated by recent work of Gazda et al 2022, Htun et al. 2021)
2. estimate N2LO contribution which quantifies the expected YNN force contribution

Uncertainty analysis to $A = 3$ to 5

Order N^{2}LO requires combination of chiral NN, YN, 3N and YNN interaction

Need calculation of separation energies (use Faddeev, Yakubovsky eq. or J-NCSM) and use **different orders** for uncertainty estimate.

Assuming a negligible numerical uncertainty and the following ansatz for the order by order convergence

$$X_K = X_{ref} \sum_{k=0}^{K} c_k\, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, …})$$

a **Bayesian analysis** of the uncertainty is possible (see Melendez et al. 2017,2019)

Extracting c_k for $k \leq K$ **from calculations** and assuming identical probability distributions for c_k for $k > K$ the uncertainty is given by the distribution of

$$\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k\, Q^k$$

Uncertainty analysis to $A = 3$ to 5

How to obtain the distribution for c_k ?

EFT expectation: c_k **are natural-sized**, i.e. of order 1.

→ defines prior distribution (usually normal distribution with width $\bar{c}$)

$\bar{c}$ is distributed using an inverse-χ^2 distribution (parameters ν_0, τ_0)

For this choice, the posterior then follows the same distribution (conjugate prior) with shifted parameters given the data:

$$\nu = \nu_0 + n_c \qquad \nu\tau^2 = \nu_0\tau_0^2 + \vec{c}_k^{\,2} \qquad (\vec{c}_k^{\,2} = \sum c_k^2 \text{ for } n_c \text{ values extracted})$$

→ uncertainty follows so-called student t distribution (analytically known)

allows to extract degree of believe intervals (DoB)

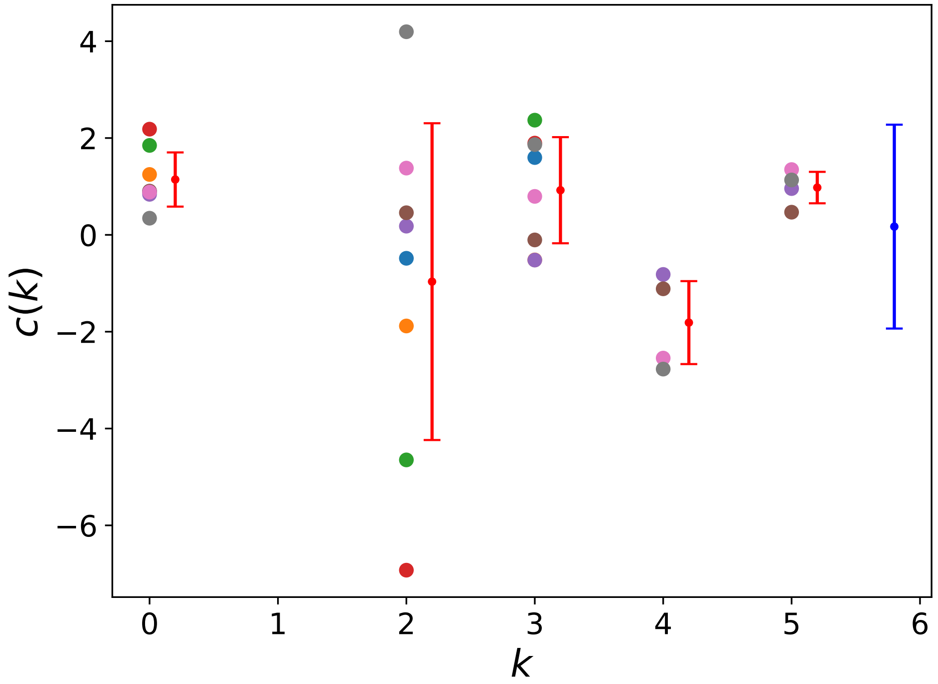
dependence on choice of prior will be less for large n_c !

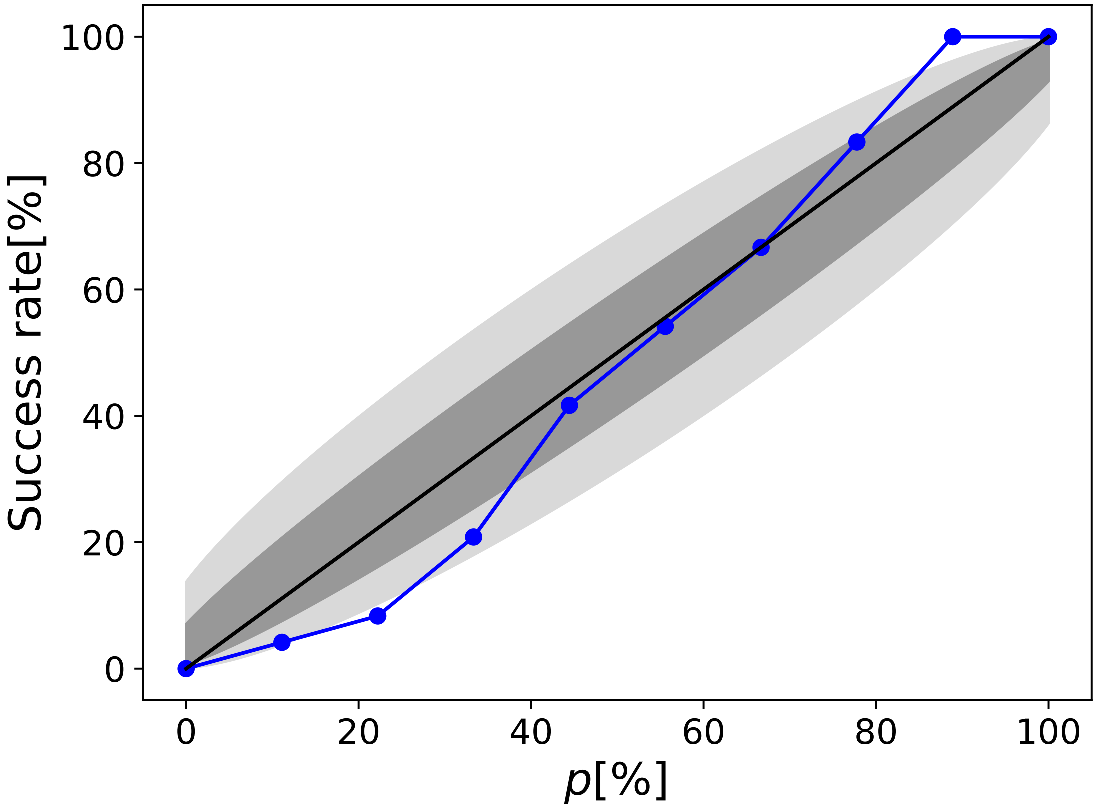
Uncertainty analysis to $A = 3$ to 5

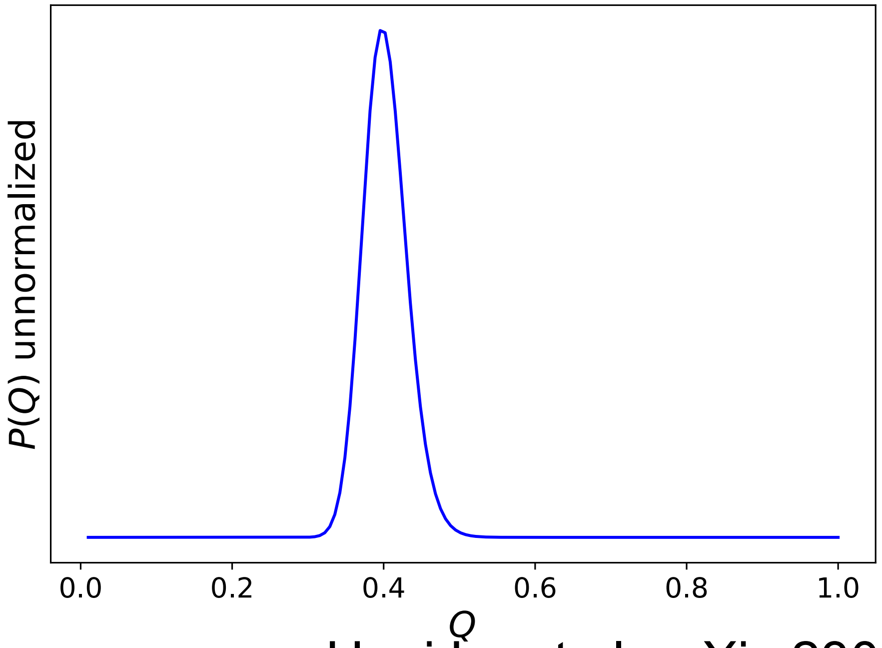
- expansion parameter Q should be consistent with assumption of k independent distribution of c_k
- distribution of of prior should be consistent with observed pattern for c_k
- few orders used cannot entirely remove prior dependence

$$Q = 0.4$$
$$\tau_0^2 = 2.25$$
$$\nu_0 = 1.5$$

(see also Maris et al. 2022)







Hoai Le et al. arXiv:2308.01756.

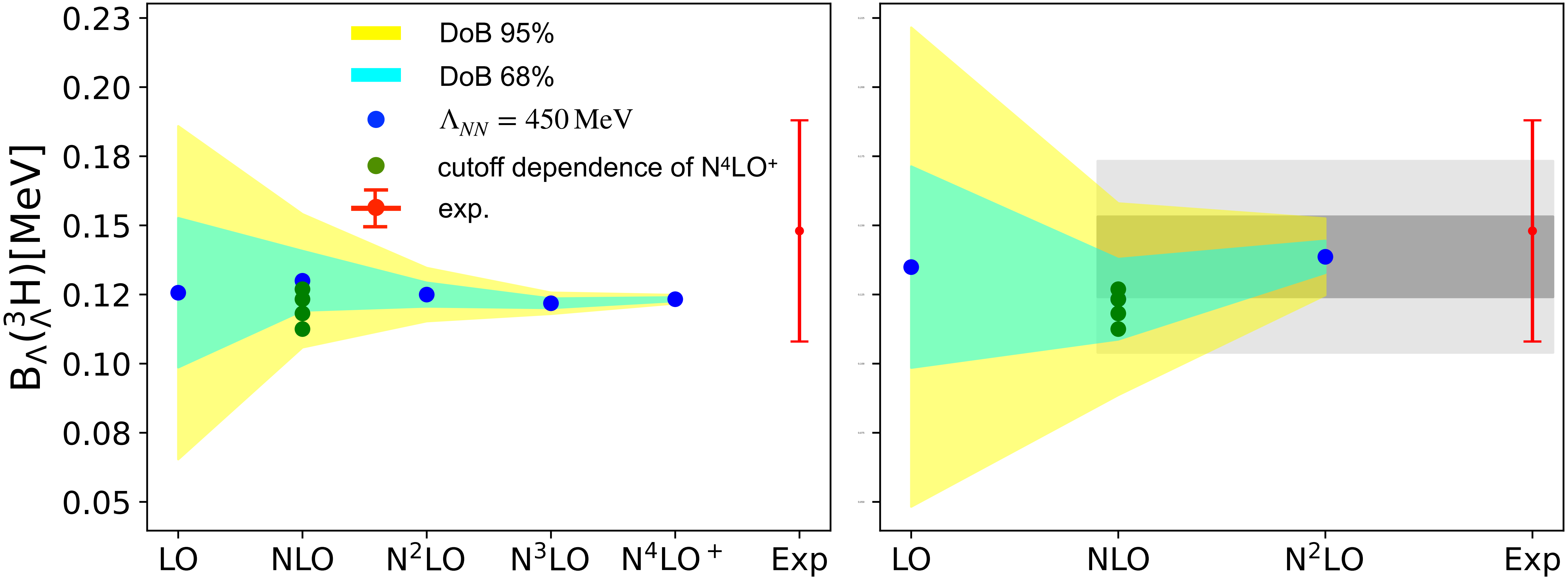
Application to ${}^{3}_{\Lambda}\mathrm{H}$

- Q, ν_0 and τ_0 are chosen using all available data (NN and YN convergence)
- uncertainties are extracted using c_k for NN or YN convergence
- use c_k of individual hypernuclei

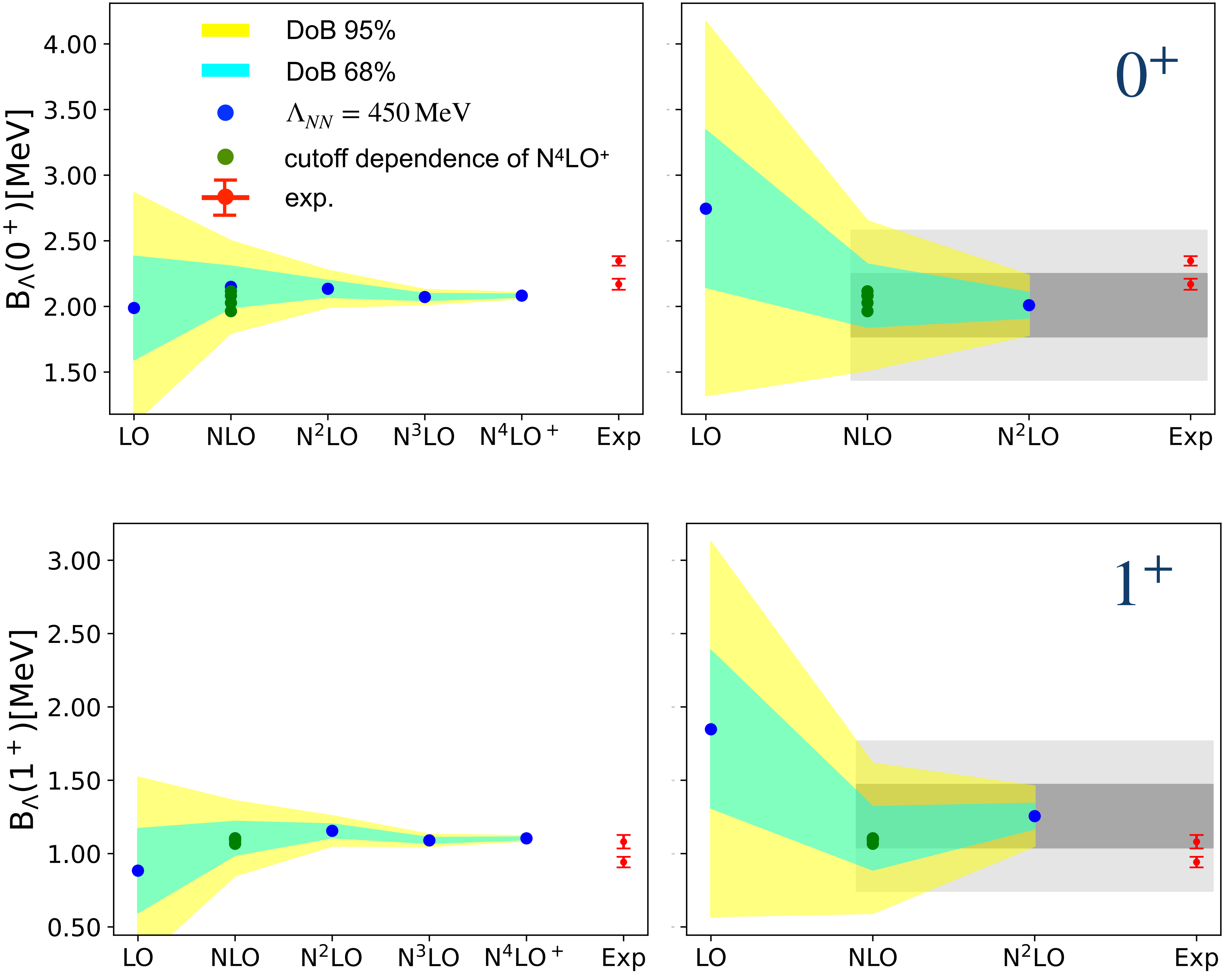
→ individual uncertainties for NN and YN convergence for each separation energy

consistent with experimental data

cutoff dependence always at least NLO (YNN missing!)



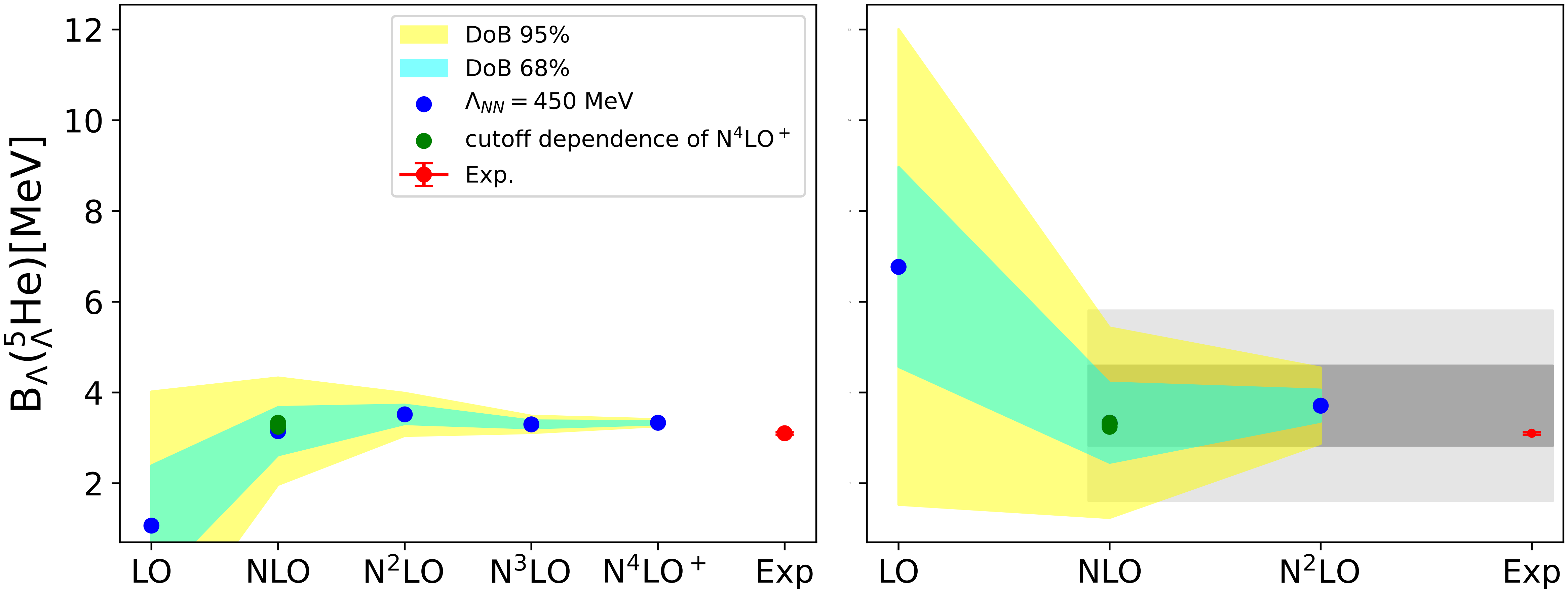
Application to ${}^{4}_{\Lambda}\mathrm{He}$



Application to ${}^{5}_{\Lambda}\mathrm{He}$ and summary

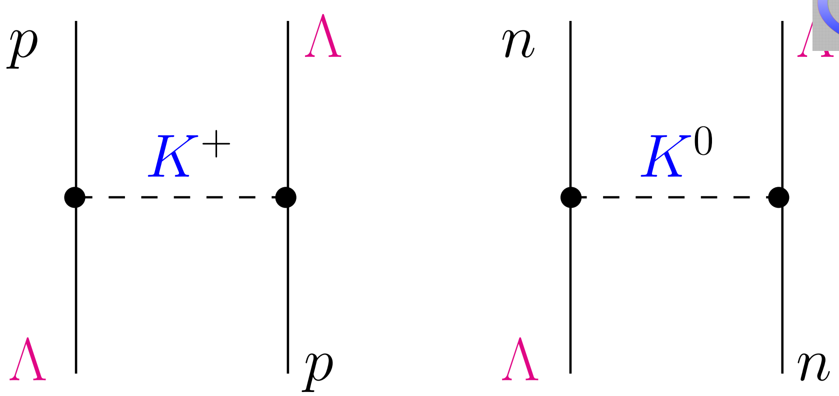
- without YNN: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$

nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
${}^{3}_{\Lambda}\mathrm{H}$	0.011	0.015
${}^{4}_{\Lambda}\mathrm{He}\,(0^{+})$	0.157	0.239
${}^{4}_{\Lambda}\mathrm{He}\,(1^{+})$	0.114	0.214
${}^{5}_{\Lambda}\mathrm{He}$	0.529	0.881



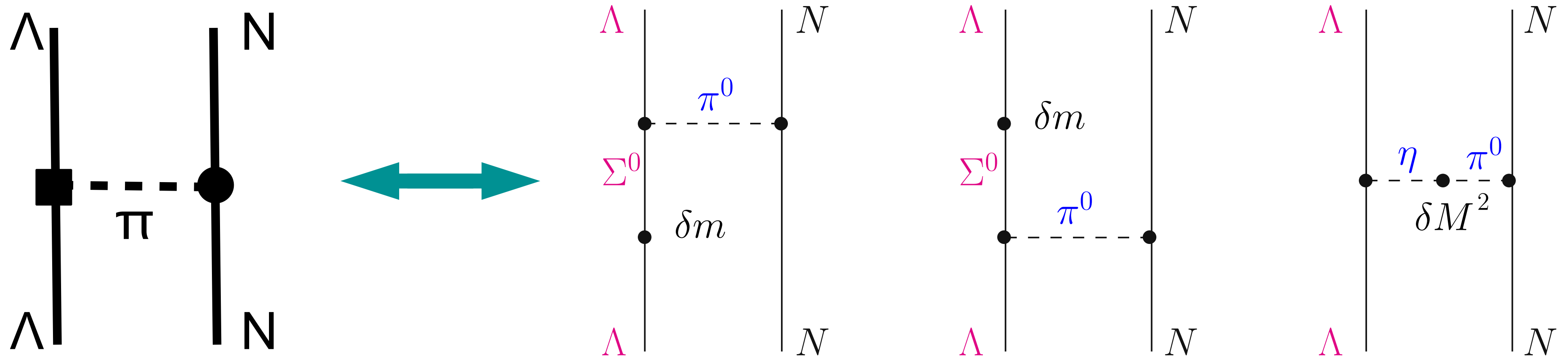
CSB contributions to YN interactions

- **formally leading** contributions:
  Goldstone boson mass difference
  - very small due to the small relative difference of kaon masses

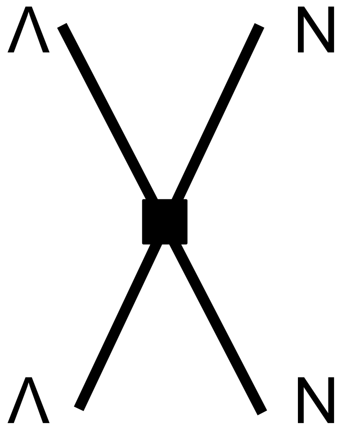


- **subleading but most important**
  - effective CSB ΛΛπ coupling constant (Dalitz, van Hippel, 1964)

$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle \pi^0 | \delta M^2 | \eta \rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi}$$



- **so far less considered, but equally important**
  - CSB contact interactions (for singlet and triplet)



Aim: use A=4 hypernuclei to determine the two unknown CSB LECs and predict Λn scattering

Fit of contact interactions

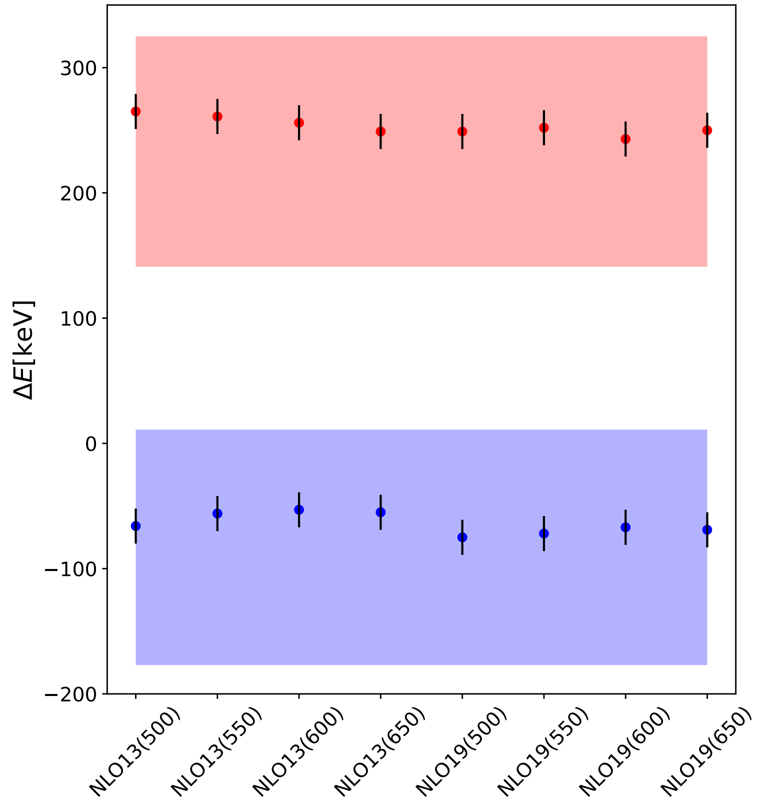
- Adjust the two CSB contact interactions to one main scenario (**CSB1)**
- Size of LECs as expected by power counting

$$\frac{m_d - m_u}{m_u + m_d}\left(\frac{M_\pi}{\Lambda}\right)^2 \quad C_{S,T} \approx 0.3 \cdot 0.04 \cdot 0.5 \cdot 10^4\,\mathrm{GeV} \propto 6 \cdot 10^{-3} \cdot 10^4\,\mathrm{GeV}$$

Λ	NLO13		NLO19	
	C_s^{CSB}	C_t^{CSB}	C_s^{CSB}	C_t^{CSB}
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

The values of the LECs are in $10^4\ \mathrm{GeV}^{-2}$

- Problem: large experimental uncertainty of experiment
- here only **fit to central values** to test theoretical uncertainties



Prediction for Λn scattering

- assuming the current experimental situation for ${}^4_\Lambda\mathrm{H}$ / ${}^4_\Lambda\mathrm{He}$
- **without** CSB: $a_s^{\Lambda n} \approx 2.9\ fm$ **with** CSB1: $a_s^{\Lambda n} \approx 3.3\ fm$
- improved description of Λp data
- almost independent of cutoff & NLO variant
- CSB of triplet is smaller than of singlet

for "**CSB1**": currently accepted experimental values

	$a_s^{\Lambda p}$	$a_t^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda n}$	$\chi^2(\Lambda p)$	$\chi^2(\Sigma N)$	$\chi^2(\text{total})$
NLO13(500)	-2.604	-1.647	-3.267	-1.561	4.47	12.13	16.60
NLO13(550)	-2.586	-1.551	-3.291	-1.469	3.46	12.03	15.49
NLO13(600)	-2.588	-1.573	-3.291	-1.487	3.43	12.38	15.81
NLO13(650)	-2.592	-1.538	-3.271	-1.452	3.70	12.57	16.27
NLO19(500)	-2.649	-1.580	-3.202	-1.467	3.51	14.69	18.20
NLO19(550)	-2.640	-1.524	-3.205	-1.407	3.23	14.19	17.42
NLO19(600)	-2.632	-1.473	-3.227	-1.362	3.45	12.68	16.13
NLO19(650)	-2.620	-1.464	-3.225	-1.365	3.28	12.76	16.04

An accurate prediction for the Λn interaction is possible using hypernuclei!

remeasurement of ${}^4_\Lambda\mathrm{H}$ excitation energy to match accuracy for ${}^4_\Lambda\mathrm{He}$?

measurement of ${}^4_\Lambda\mathrm{He}$ ground state at J-PARC

Conclusions & Outlook

- **YN interactions not well understood**
  - *scarce YN data*
  - *more information necessary to solve "hyperon puzzle"*
- **Hypernuclei provide important constraints**
  - *CSB of* ΛN *scattering &* ${}^{4}_{\Lambda}\mathrm{He}\ /\ {}^{4}_{\Lambda}\mathrm{H}$
  - ${}^{3}_{\Lambda}\mathrm{H}$ *is used to constrain the spin dependence*
  - *new experiments planned at J-PARC, MAMI, J-Lab, FAIR,…*
- **New SMS YN interactions**
  - *give an accurate description low energy YN data*
  - *order LO, NLO and N²LO allow uncertainty quantification*
  - *have a non-unique determination of contact interactions (data necessary)*
- **Chiral 3BF** need to be included
  - *NLO uncertainty is sizable in* $A = 4$ *and* 5
  - *chiral 3BFs are formulated (Petschauer et al., (2016)) and the implementation is currently checked*