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Application of chiral YN interactions to light hypernuclei

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Workshop on „Physics opportunities with proton beams at SIS100"
Wuppertal, Germany, February 6th-9th, 2024

- Motivation
- Chiral YN & YY interactions
- Uncertainty of Λ separation energies and size of chiral 3BF contributions
- Determination of CSB contact interactions and Λn scattering length
- Application to $A=7$ and 8 hypernuclei
- Light $\Lambda\Lambda$ hypernuclei and Ξ hypernuclei
- Conclusions & Outlook

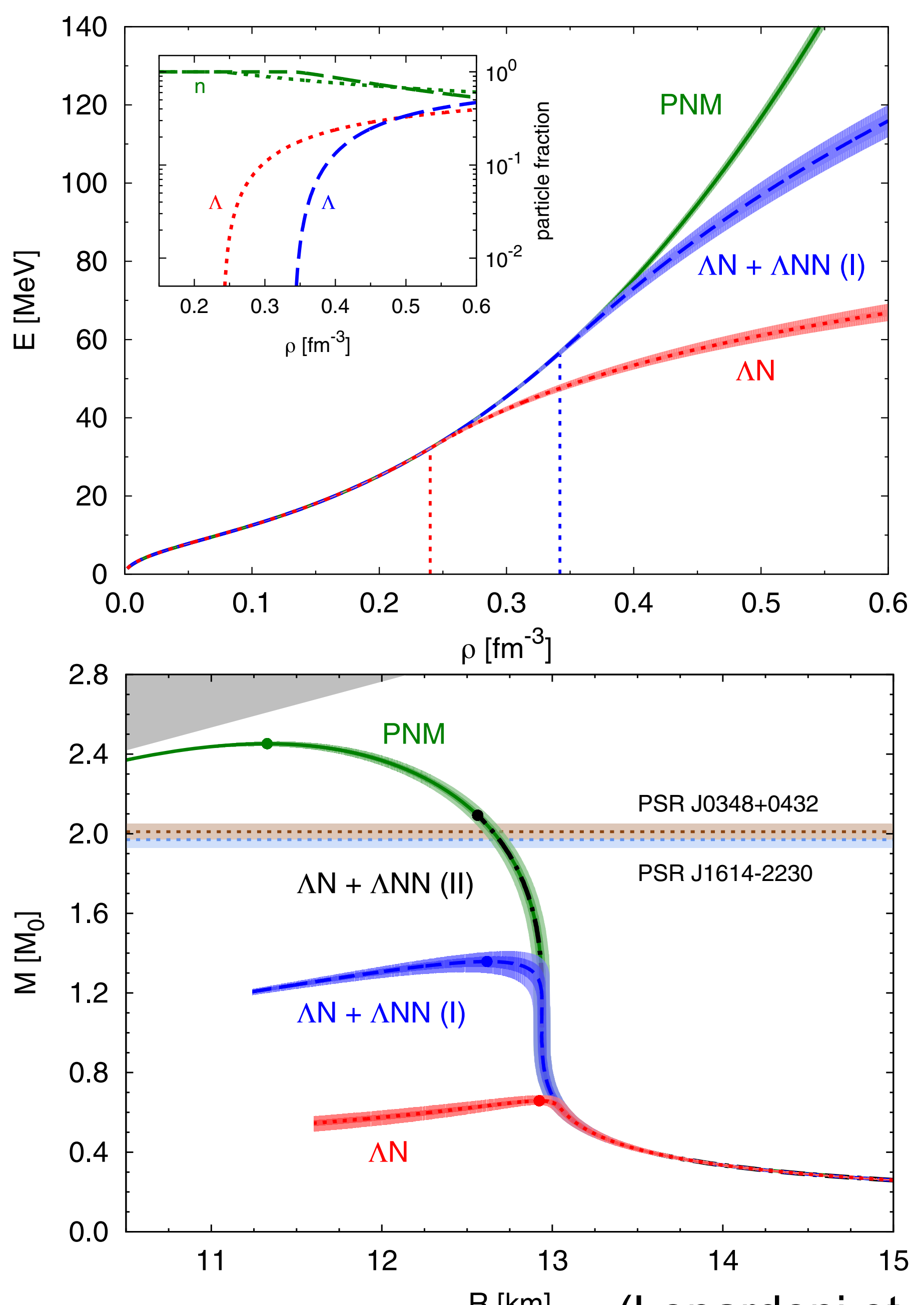
in collaboration with Johann Haidenbauer, **Hoai Le**, Ulf Meißner

February 8th, 2024

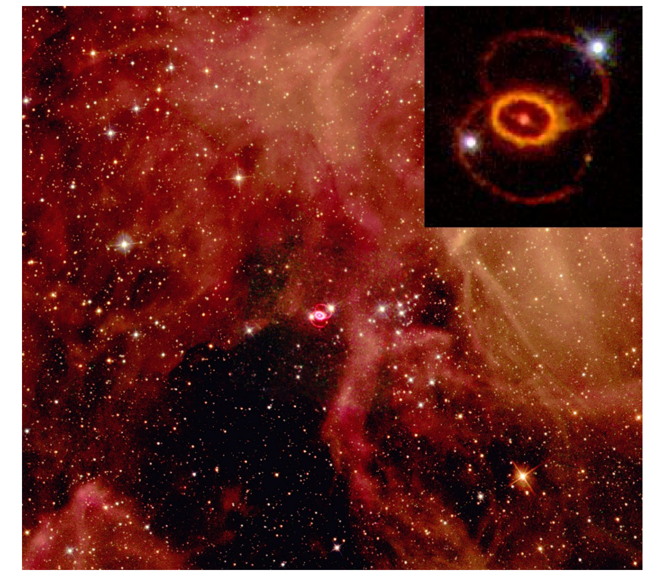
Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

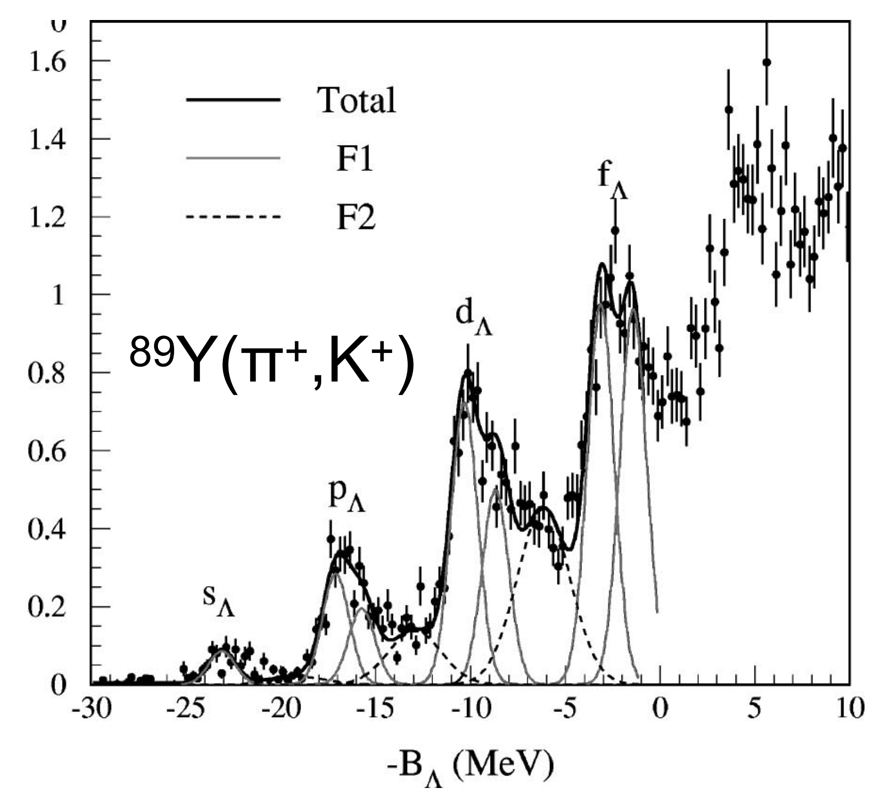
- *hyperon contribution to the EOS, neutron stars, supernovae*
- *"hyperon puzzle"*
- *Λ as probe to nuclear structure*
- *flavor dependence of baryon-baryon interactions*



(Lonardoni et al. (2015))



(SN1987a, Wikipedia)



(Hotchi et al. (2001))

Hypernuclei

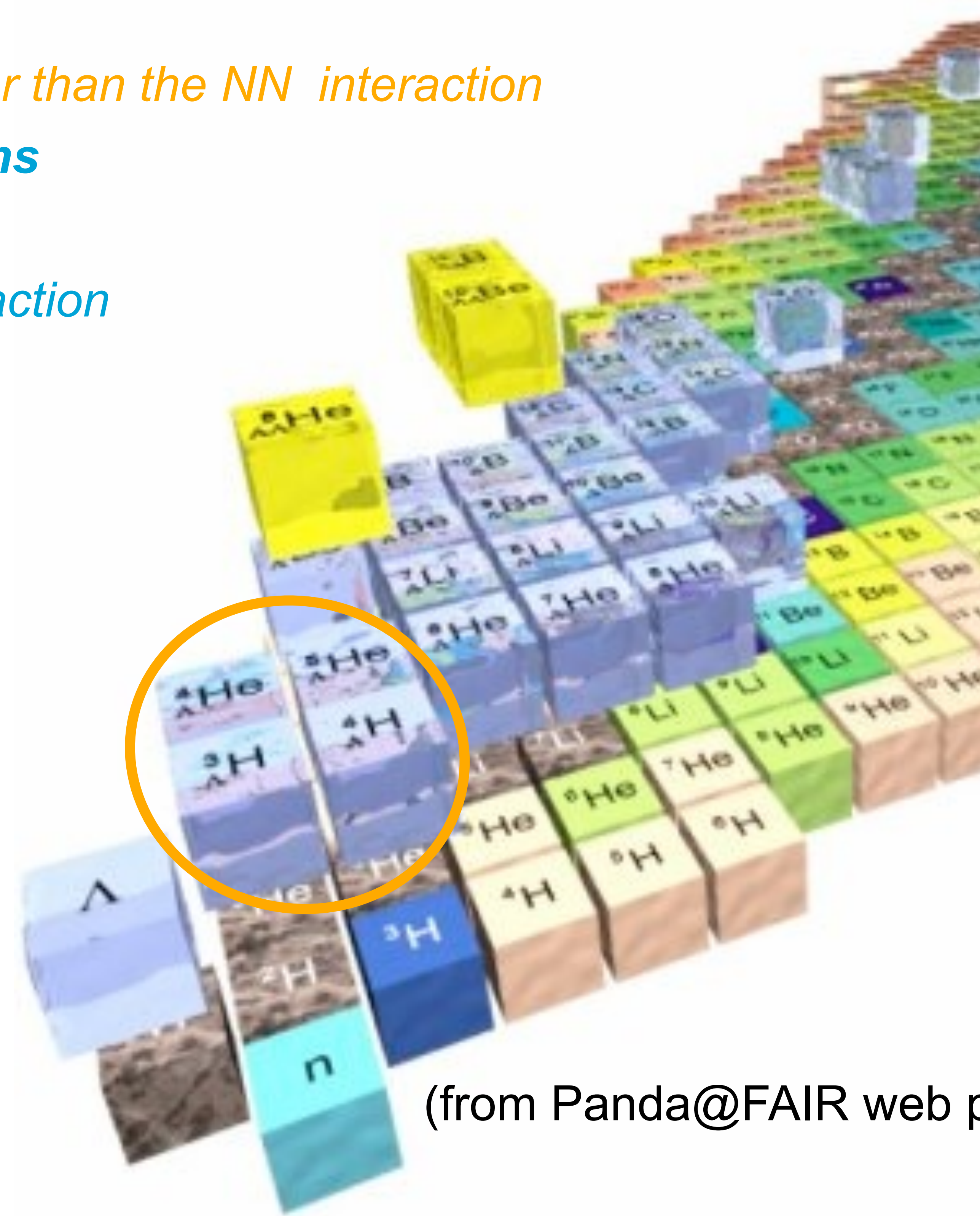
Only few YN data. Hypernuclear data provides additional constraints.

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively:* ***core nucleus + hyperons***
  - *„separation energies" are quite independent from NN(+3N) interaction*

- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in* ***several spin states***

- ***non-trivial constraints*** *on the YN interaction even from lightest ones*

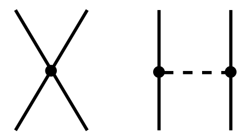
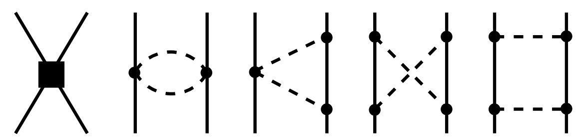
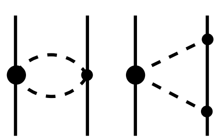
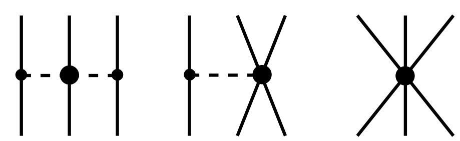
- *size of* ***YNN*** *interactions? need to include Λ-Σ conversion!*



(from Panda@FAIR web page)

Chiral NN & YN & YY interactions

EFT based approaches

	BB force	3B force	4B force
LO		—	—
NLO		—	—
N^2LO			—

5 (+1) NN/YN (YY) short range parameters

23(+5) NN/YN (YY) short range parameters

no additional contact terms in NN/YN (YY)

Chiral EFT implements **chiral symmetry of QCD** (adapted from Epelbaum, 2008)

- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale $\approx 600 - 700\,\mathrm{MeV}$
- Semi-local momentum regularization (SMS) up to N^2LO (for YN, YY within NRW Fair)

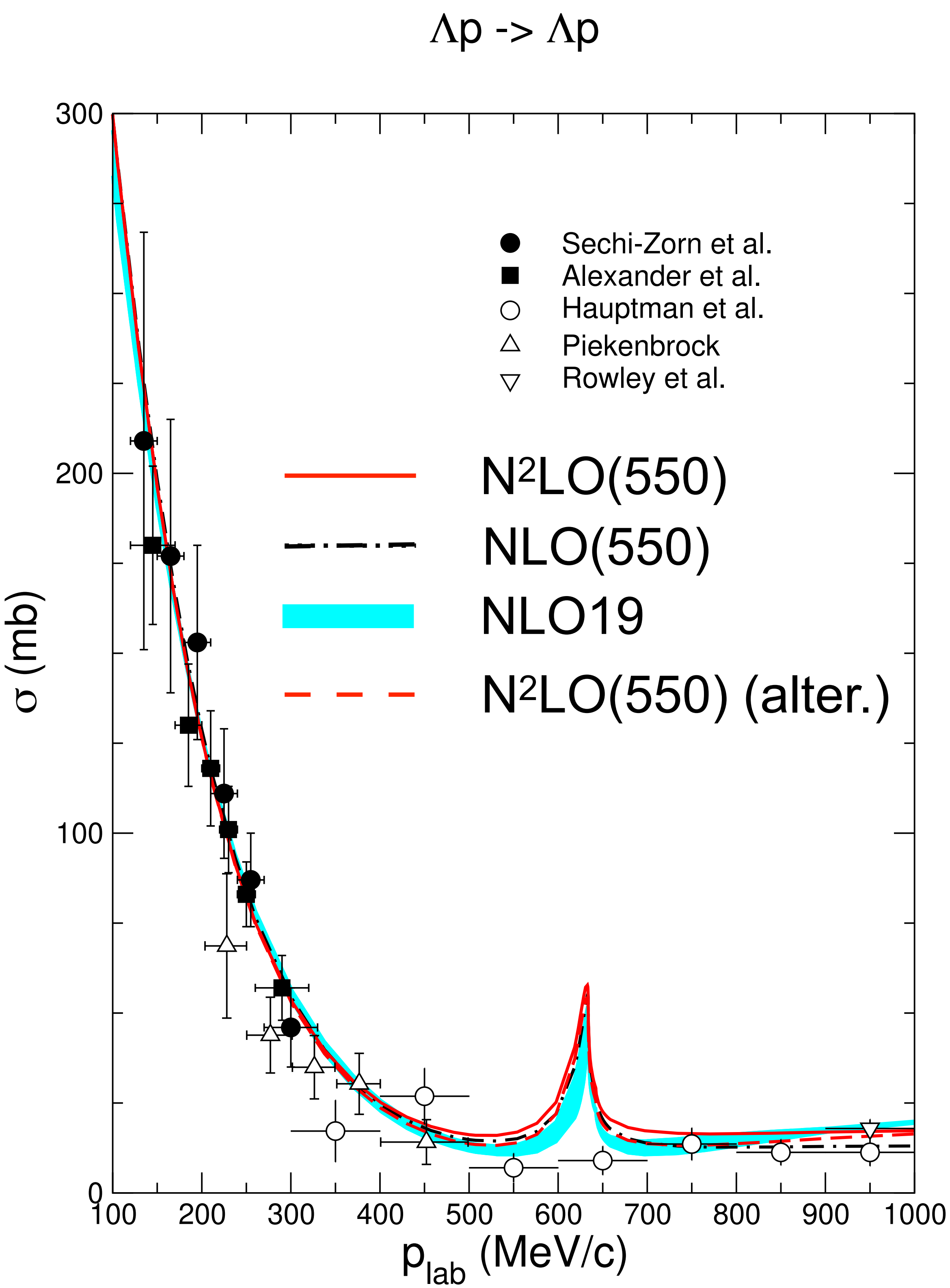
Retain flexibility to adjust to data due to counter terms

Regulator required — cutoff/different orders often used to estimate uncertainty

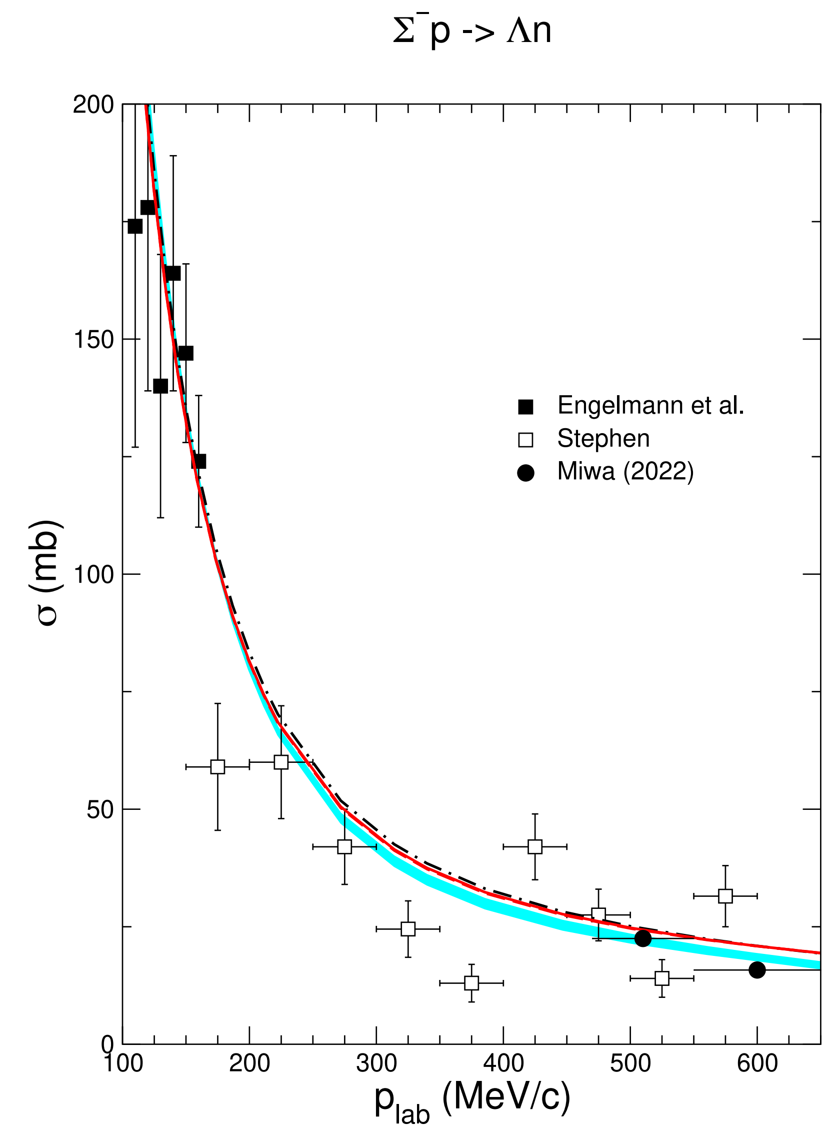
$\Lambda - \Sigma$ and $\Lambda\Lambda - \Sigma\Sigma - \Xi N$ **conversion** is explicitly included (3BFs only in N^2LO)

SMS NLO/N²LO interaction

Selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)



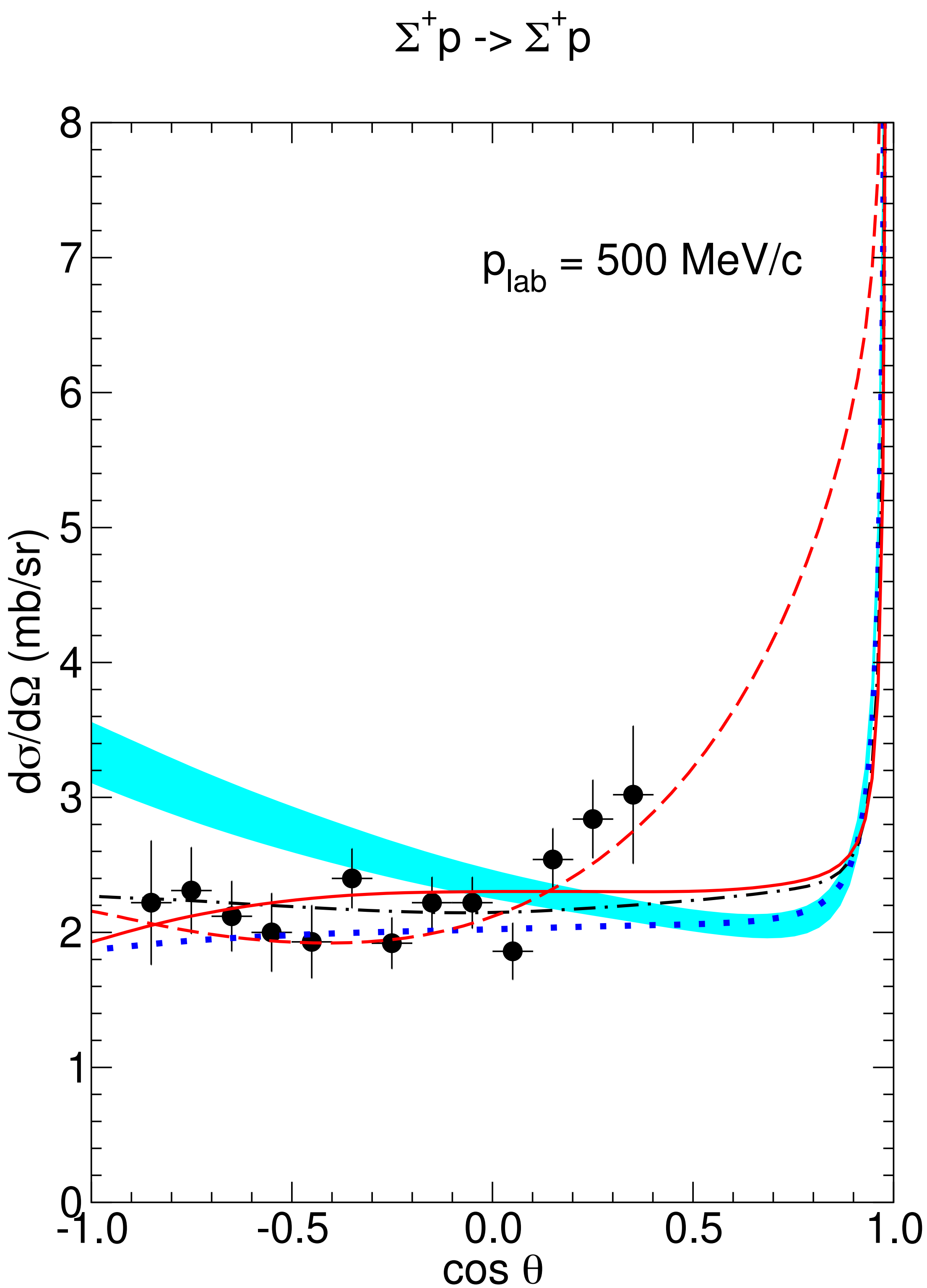
- most relevant cross sections very similar in NLO and N²LO
- similar to NLO19
- alternative fit (see later)

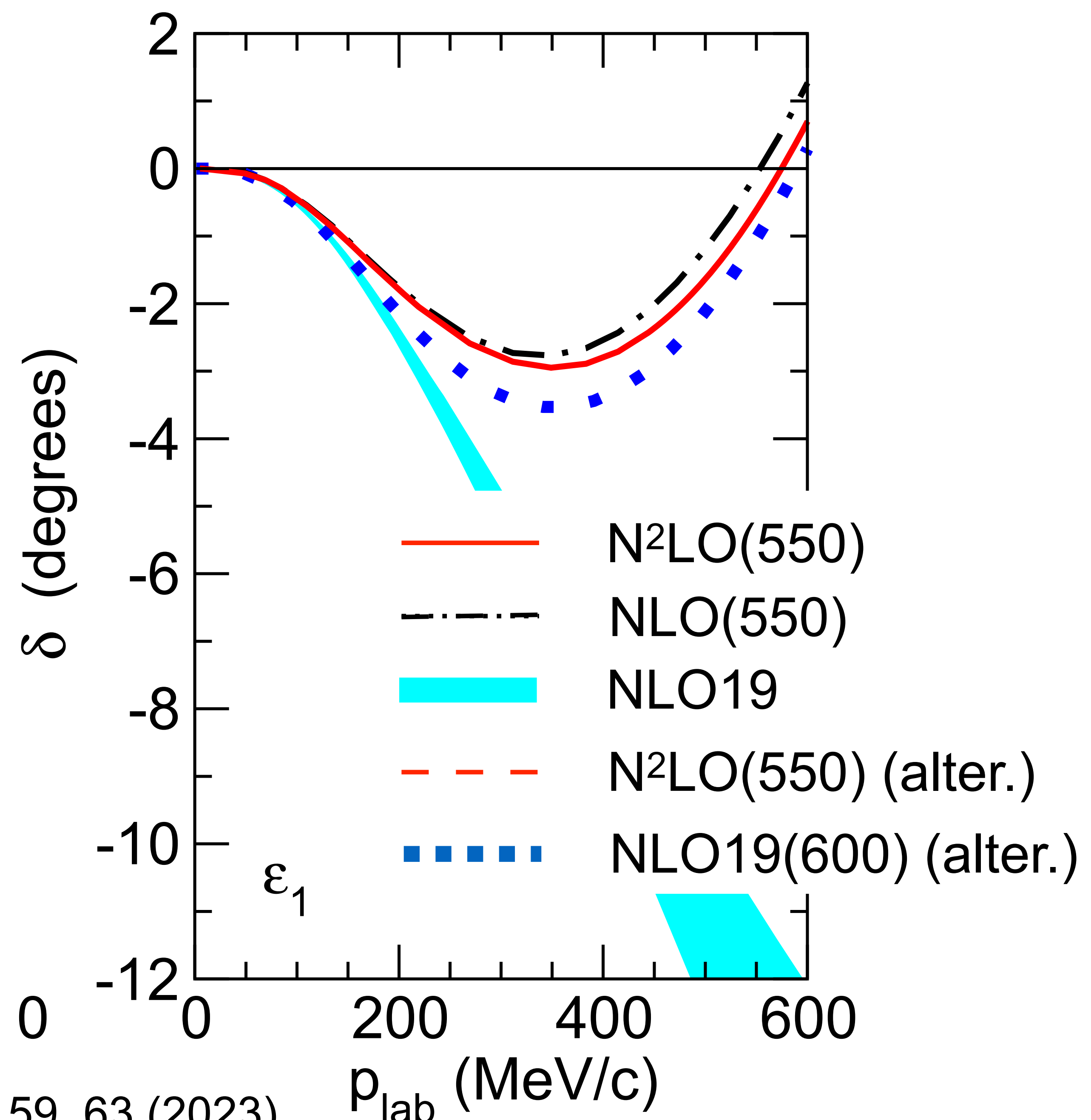


J. Haidenbauer et al. EPJ A 59, 63 (2023).

SMS NLO/N²LO interaction

new data (Miwa(2022)) at higher energies provides **new constraints**!





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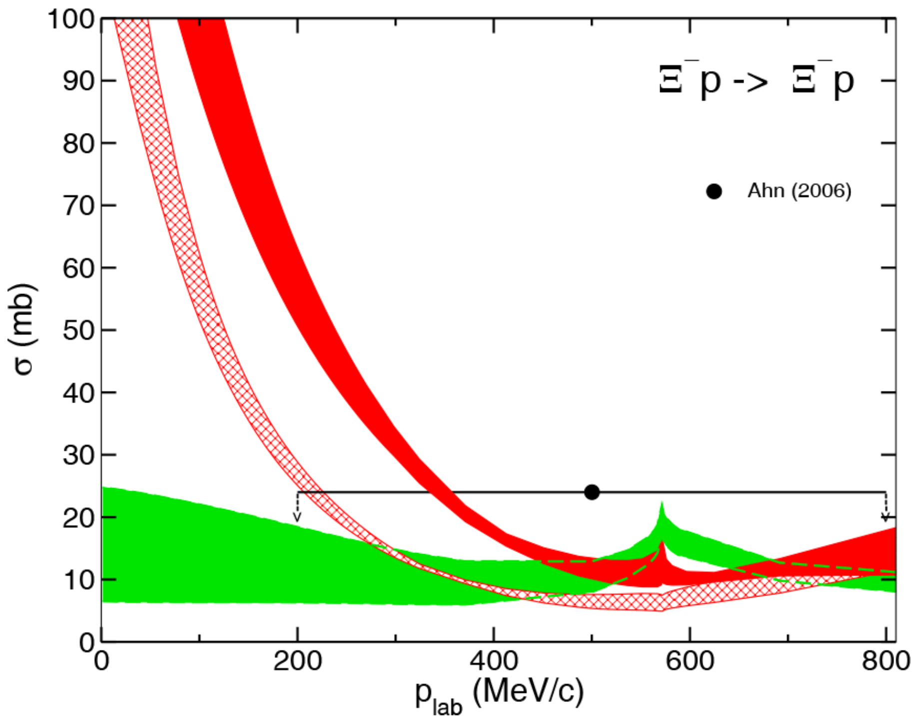
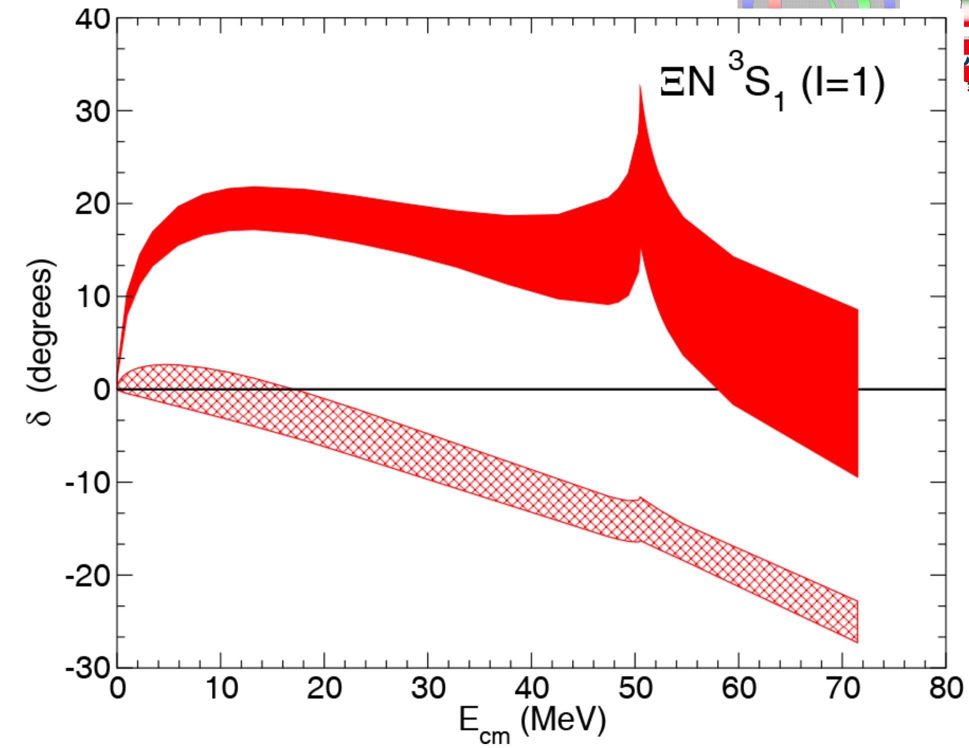
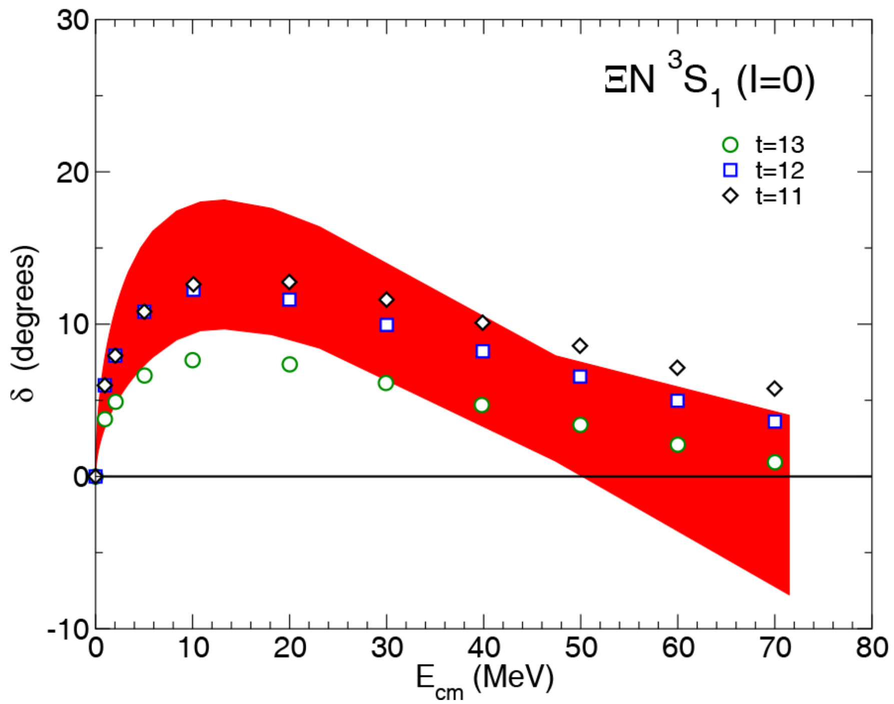
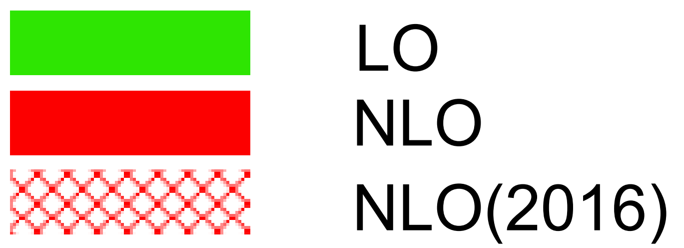
YY interaction









(Haidenbauer at al., 2019)

→ adjusted to data & LQCD (HAL QCD)

updated version consistent with Ξ-nuclei (only change in $\Xi N\ {}^3S_1$)

Tools

Need reliable predictions for hypernuclei to further constrain interactions

Faddeev-Yakubovsky (FY) equations for $A = 3$ and 4 (momentum space)

- long distance tails of wave functions can be well represented
- uses Jacobi coordinates separating off CM motion
- chiral interactions can be directly used
- hugh linear eigenvalue problem (dimension 10^9x10^9) even for A=4 systems
- is feasible only for A ≤ 4

(see AN, Glöckle, Kamada, 2002))

Jacobi-no core shell model (J-NCSM) for $A \geq 4$ (HO space)

- smaller dimensions allow to tackle p-shell nuclei
- exact antisymmetrization of wave functions can be prepared
- uses Jacobi coordinates separating off CM motion
- chiral interactions require similarity renormalization group (SRG) evolution
- long distance wave functions require large HO model spaces

(see Liebig et al., 2016; Le et al., 2020 & 2021)

Uncertainty analysis to $A = 3$ to 5

Order N²LO requires combination of chiral NN, YN, 3N and YNN interaction

Need calculation of separation energies (use Faddeev, Yakubovsky eq. or J-NCSM) and use **different orders** for uncertainty estimate.

Assuming a negligible numerical uncertainty and the following ansatz for the order by order convergence

$$X_K = X_{ref} \sum_{k=0}^{K} c_k \, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, ...})$$

a **Bayesian analysis** of the uncertainty is possible (see Melendez et al. 2017,2019)

Extracting c_k for $k \leq K$ **from calculations** and assuming identical probability distributions for c_k for $k > K$ the uncertainty is given by the distribution of

$$\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k \, Q^k$$

Numerical uncertainties negligible (carefully checked!).

Uncertainty due to missing higher orders is most relevant!

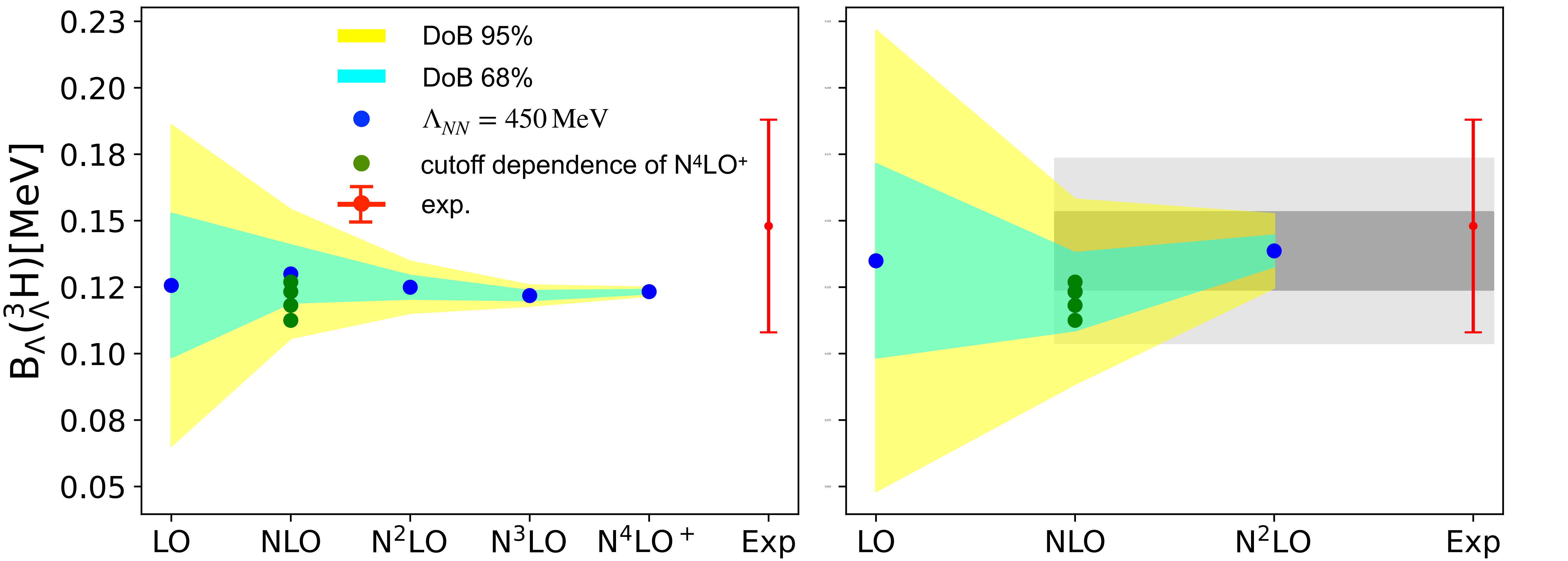
Application to ${}^{3}_{\Lambda}\mathrm{H}$

- Q, ν_0 and τ_0 are chosen using all available data (NN and YN convergence)
- uncertainties are extracted using c_k for NN or YN convergence
- use c_k of individual hypernuclei

→ individual uncertainties for NN and YN convergence for each separation energy

consistent with experimental data

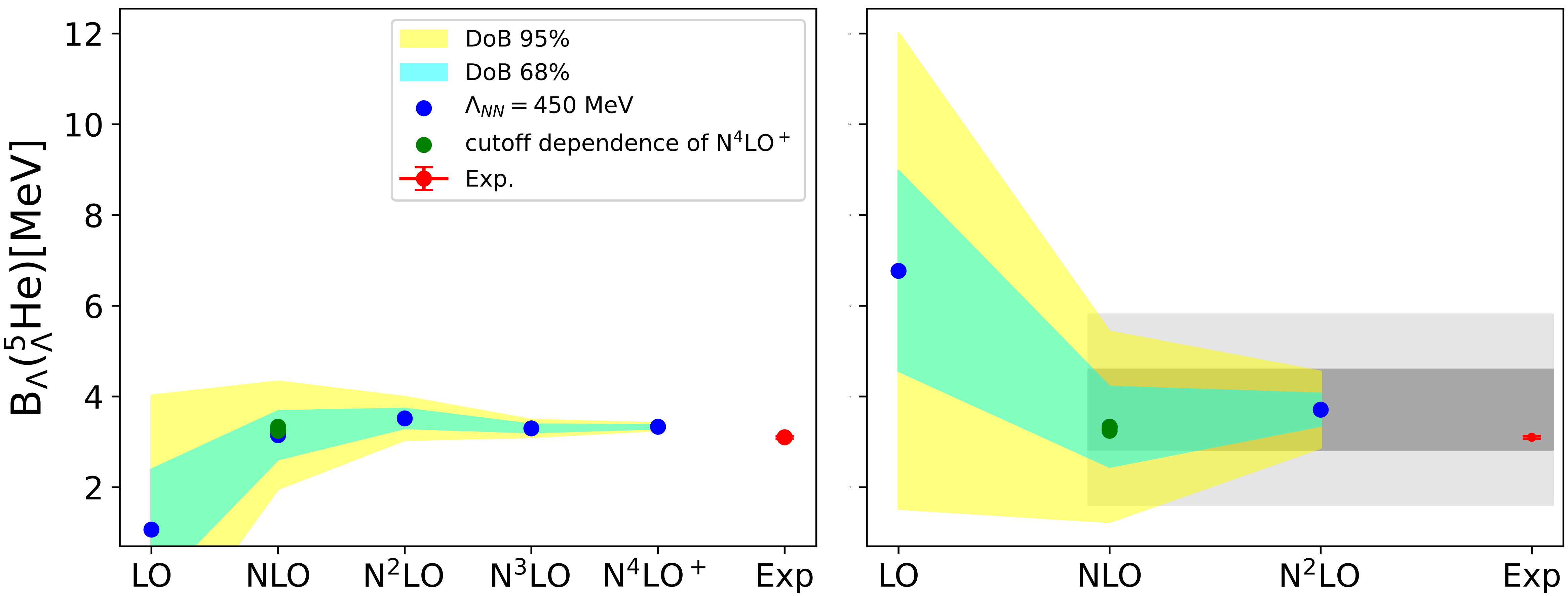
cutoff dependence always at least NLO (YNN missing!)



Application to ${}^{5}_{\Lambda}\mathrm{He}$ and summary

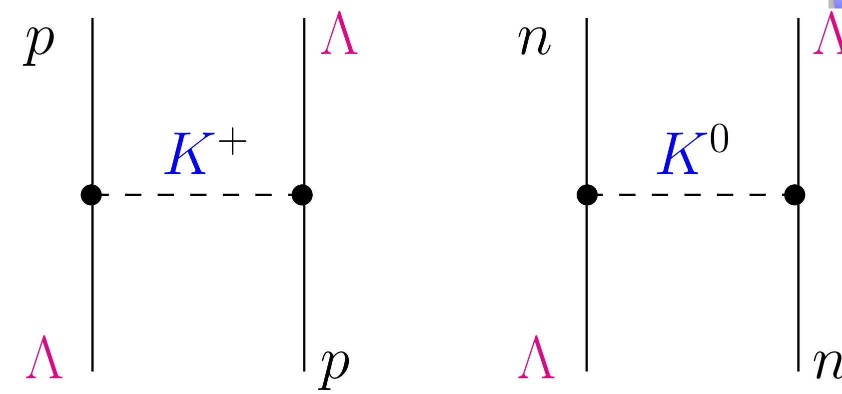
- without YNN: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$

nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
${}^{3}_{\Lambda}\mathrm{H}$	0.011	0.015
${}^{4}_{\Lambda}\mathrm{He}\,(0^+)$	0.157	0.239
${}^{4}_{\Lambda}\mathrm{He}\,(1^+)$	0.114	0.214
${}^{5}_{\Lambda}\mathrm{He}$	0.529	0.881



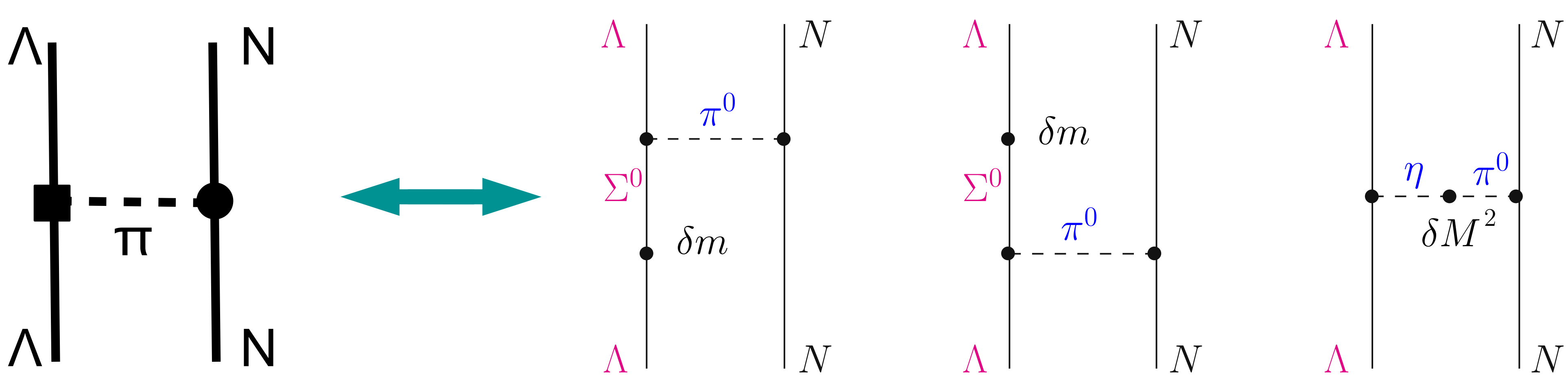
CSB contributions to YN interactions

- **formally leading** contributions:
  Goldstone boson mass difference
  - — very small due to the small relative difference of kaon masses

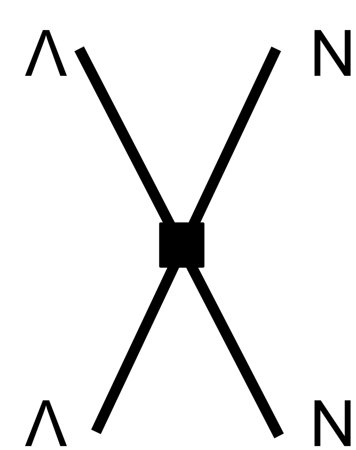


- **subleading but most important**
  — effective CSB ΛΛπ coupling constant (Dalitz, van Hippel, 1964)

$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle \pi^0 | \delta M^2 | \eta \rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi}$$



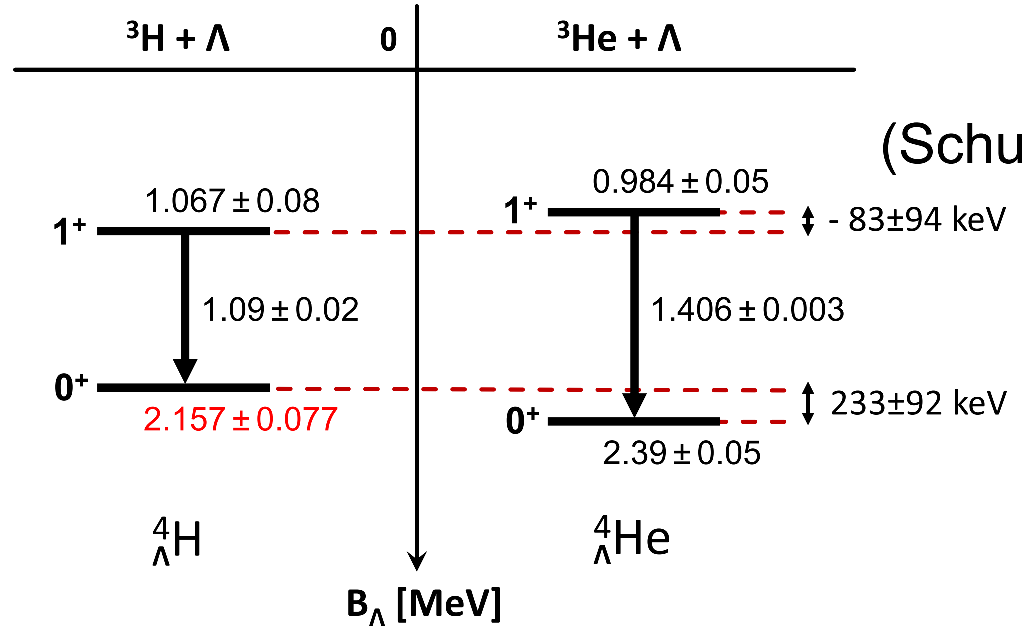
- **so far less considered, but equally important**
  — CSB contact interactions (for singlet and triplet)



Aim: use A=4 hypernuclei to determine the two unknown CSB LECs and predict Λn scattering

(so far: NLO13 and NLO19)

Fit of contact interactions



(Schulz et al.,2016; Yamamoto, 2015)

- Adjust the two CSB contact interactions to one main scenario (**CSB1**)

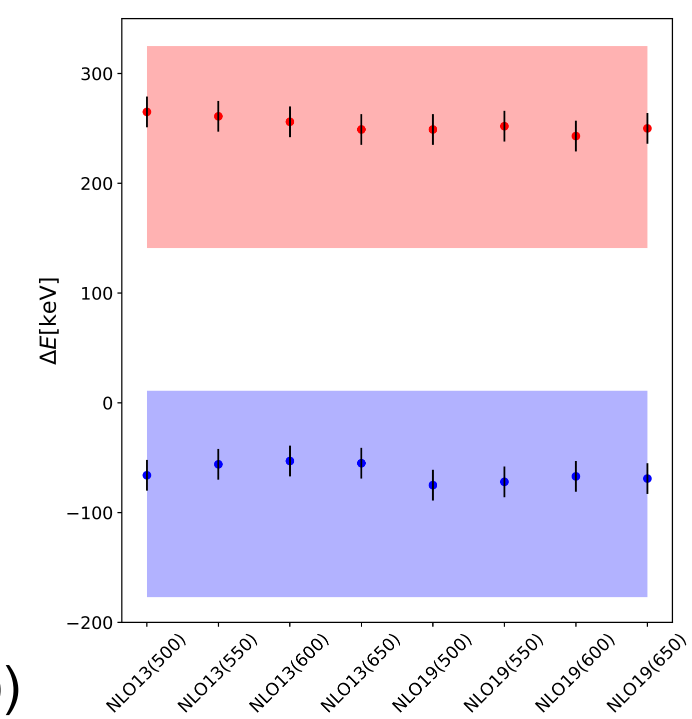
Λ	NLO13		NLO19	
	C_s^{CSB}	C_t^{CSB}	C_s^{CSB}	C_t^{CSB}
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

The values of the LECs are in 10^4 GeV^{-2}

- Size of LECs as expected by power counting

$$\frac{m_d - m_u}{m_u + m_d}\left(\frac{M_\pi}{\Lambda}\right)^2 \quad C_{S,T} \approx 0.3 \cdot 0.04 \cdot 0.5 \cdot 10^4\,\mathrm{GeV} \propto 6 \cdot 10^{-3} \cdot 10^4\,\mathrm{GeV}$$

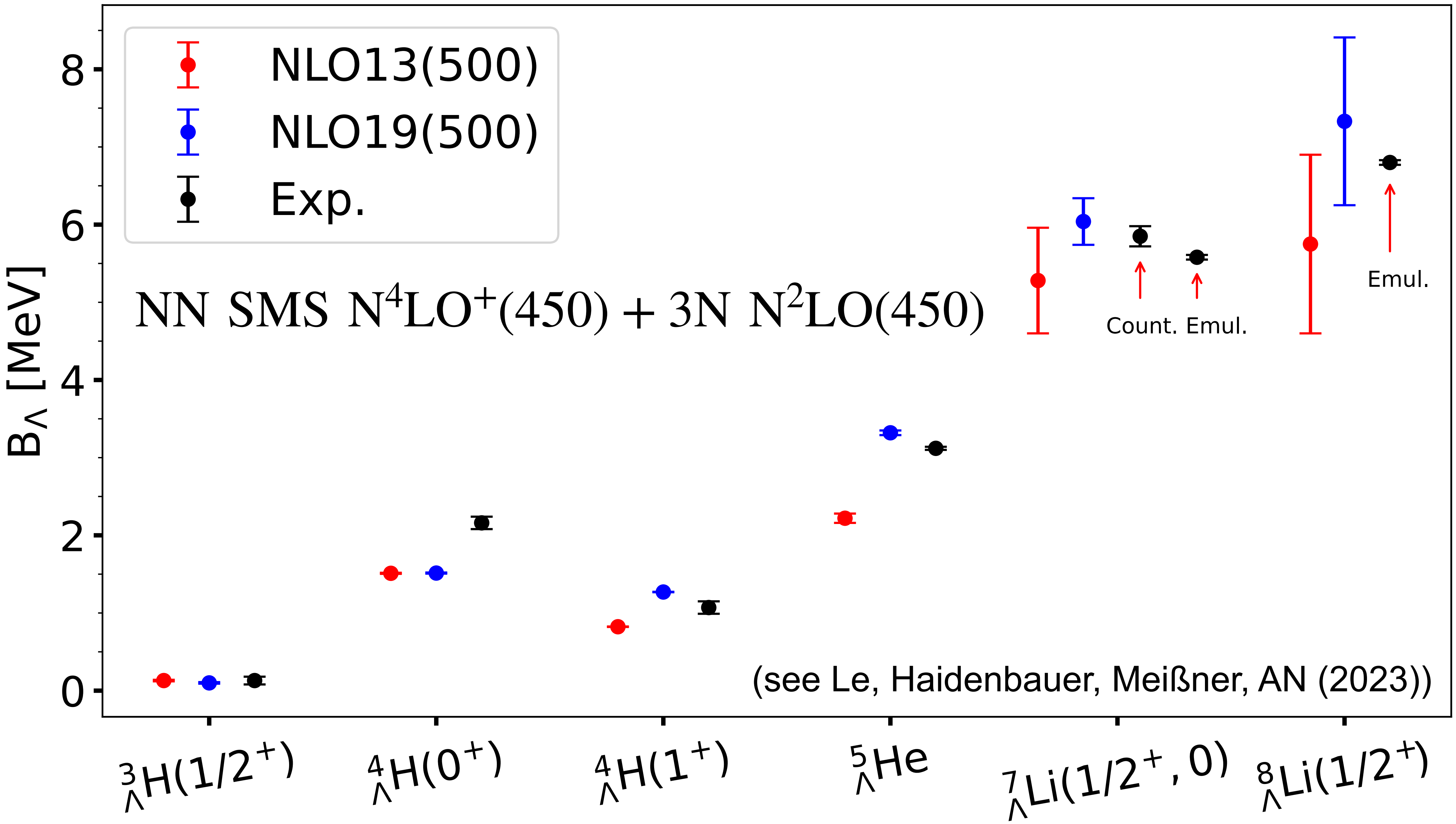
- Problem: large experimental uncertainty of experiment
- here only **fit to central values** to test theoretical uncertainties



(see Haidenbauer, Meißner, AN (2021))

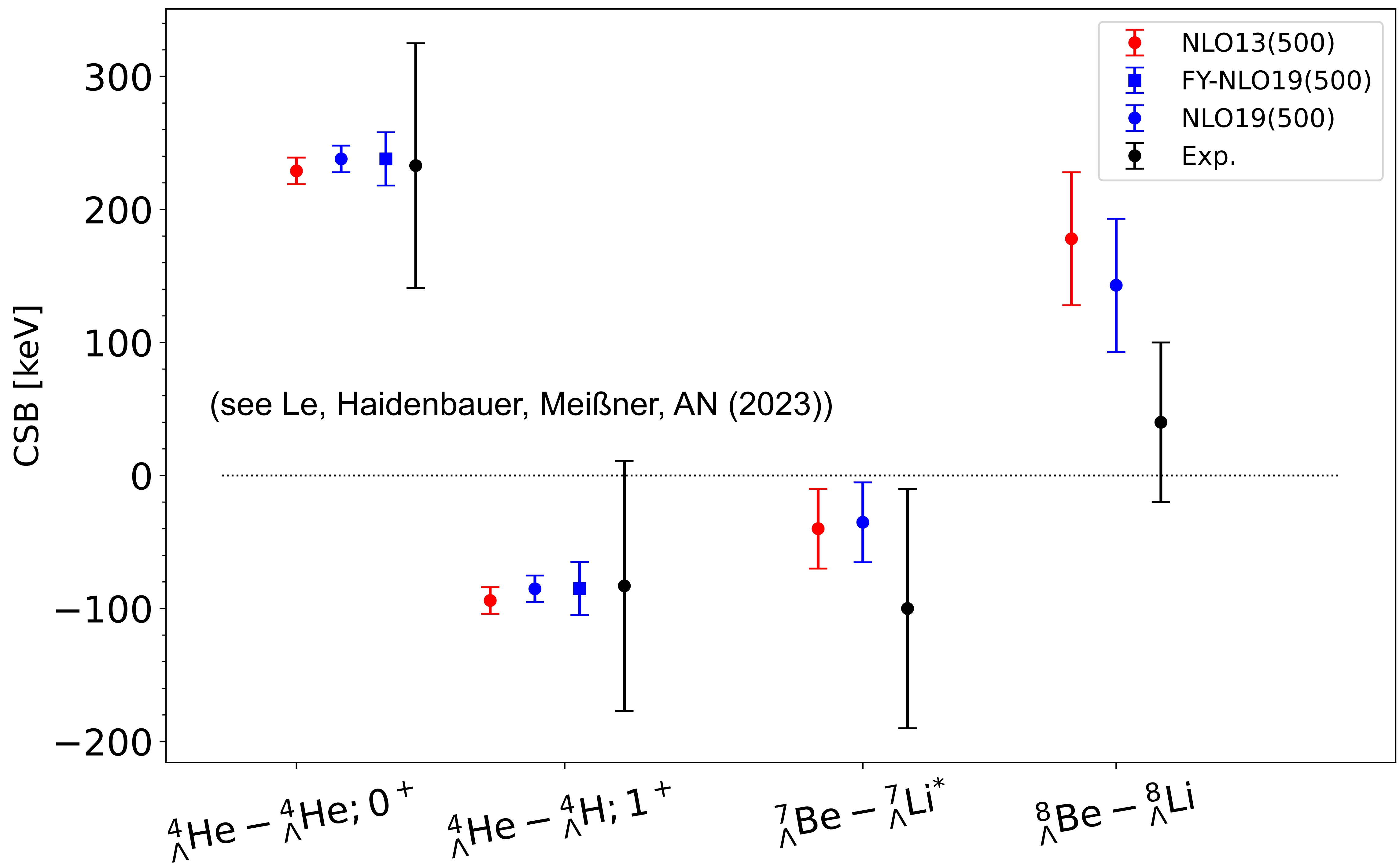
Application to $A = 7$ and 8

- YN interaction adjusted to the hypertriton — YNN is small
- based only on YN interactions: splitting for ${}^4_\Lambda\mathrm{H}$ is not well reproduced — YNN(?)
- NLO19 gives better results for ${}^5_\Lambda\mathrm{He}$ and heavier hypernuclei
  — accidentally small YNN interaction?
- uncertainties are numerical — no estimate of chiral uncertainties yet



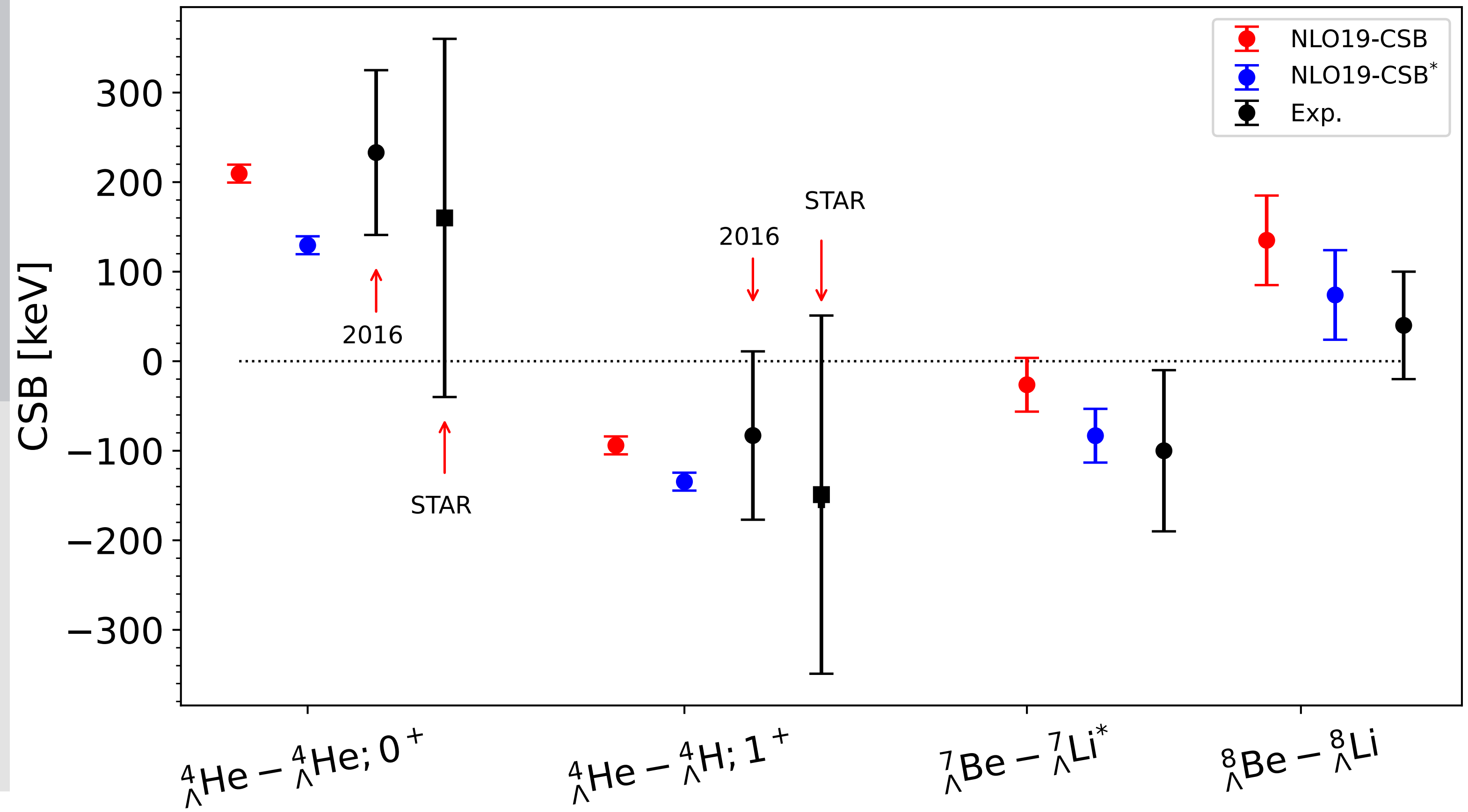
Application to $A = 7$ and 8

- CSB of singlet and triplet states interferes differently
- CSB still not fixed — experimental uncertainty is large
- scenario studied here is only marginally consistent with CSB in $A = 8$



New STAR data for $A = 4$ CSB

- fit to STAR data only
- only slight adjustment required
- improves description to p-shell CSB
- higher experimental accuracy is desirable
- good example of using hypernuclei to determine YN interactions



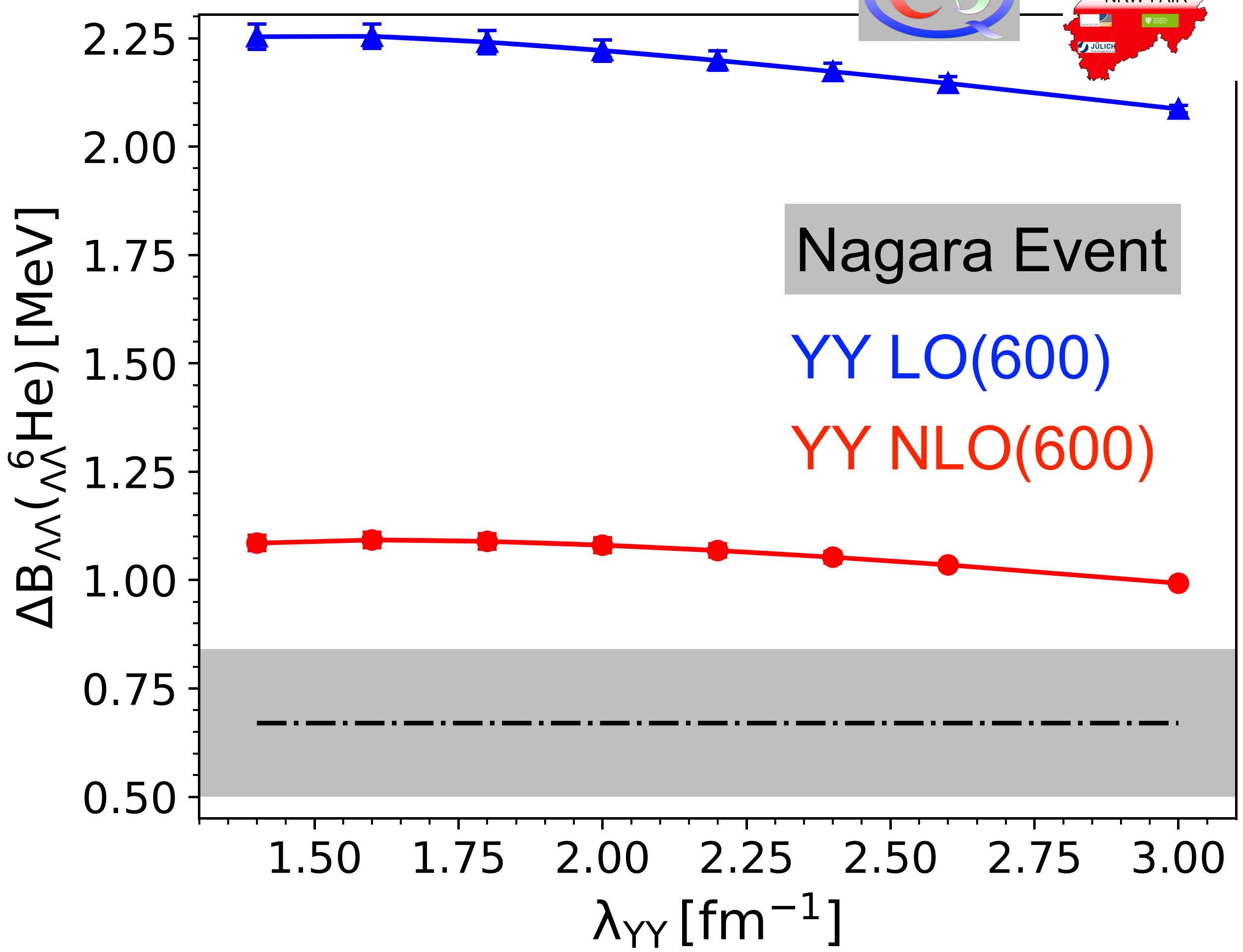
	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

(see Le, Haidenbauer, Meißner, AN (2023))

$S=-2$ hypernuclei — ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

- $\Lambda\Lambda$ excess binding energy

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$$
$$= 2E\left({}^{A-1}_{\Lambda}X\right) - E\left({}^{A}_{\Lambda\Lambda}X\right) - E\left({}^{A-2}X\right)$$



- NN, YN and YY interactions contribute
- use NN and YN that describe nuclei and single Λ hypernuclei

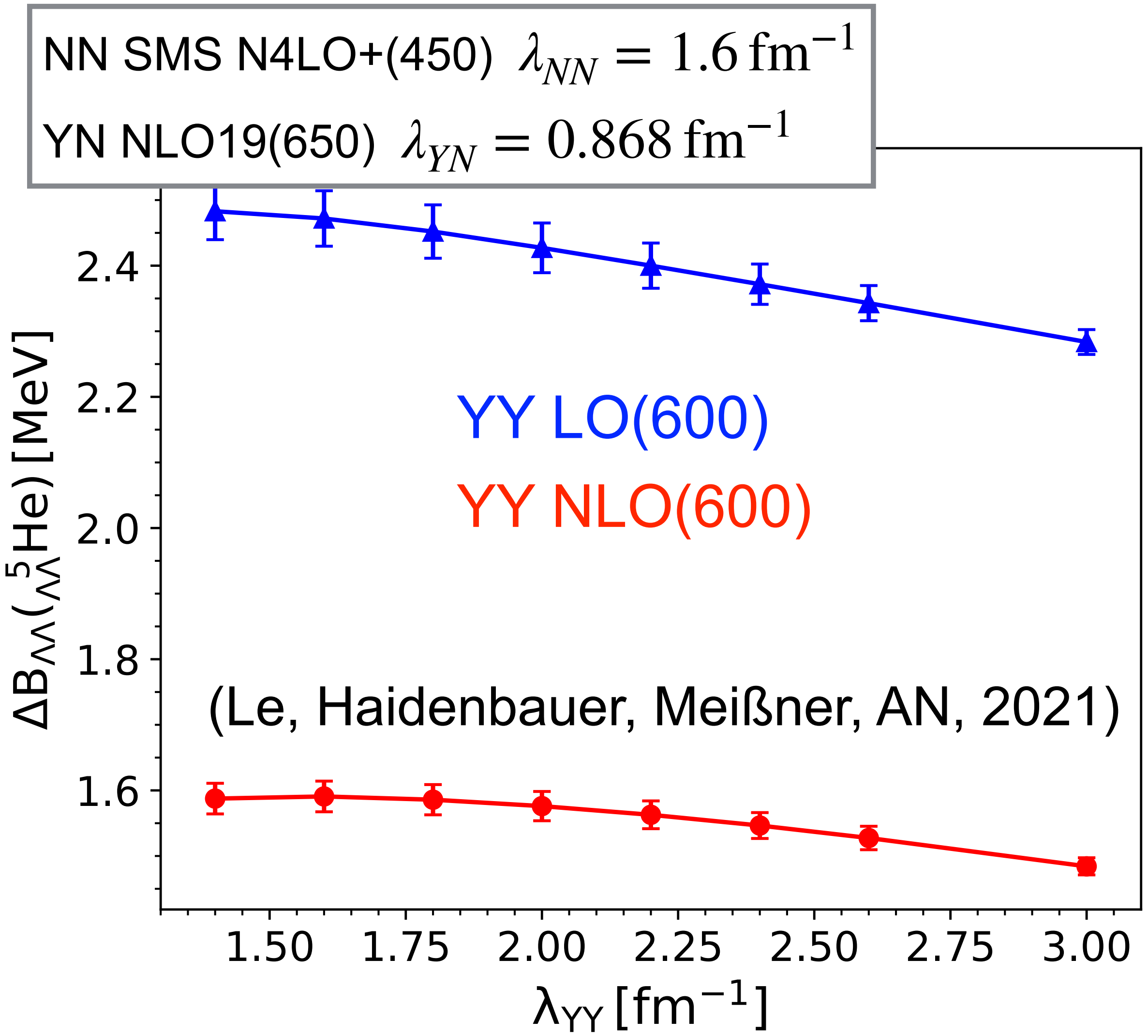
NN SMS N4LO+(450) $\lambda_{NN} = 1.6\ \mathrm{fm}^{-1}$
YN NLO19(650) $\lambda_{YN} = 0.868\ \mathrm{fm}^{-1}$

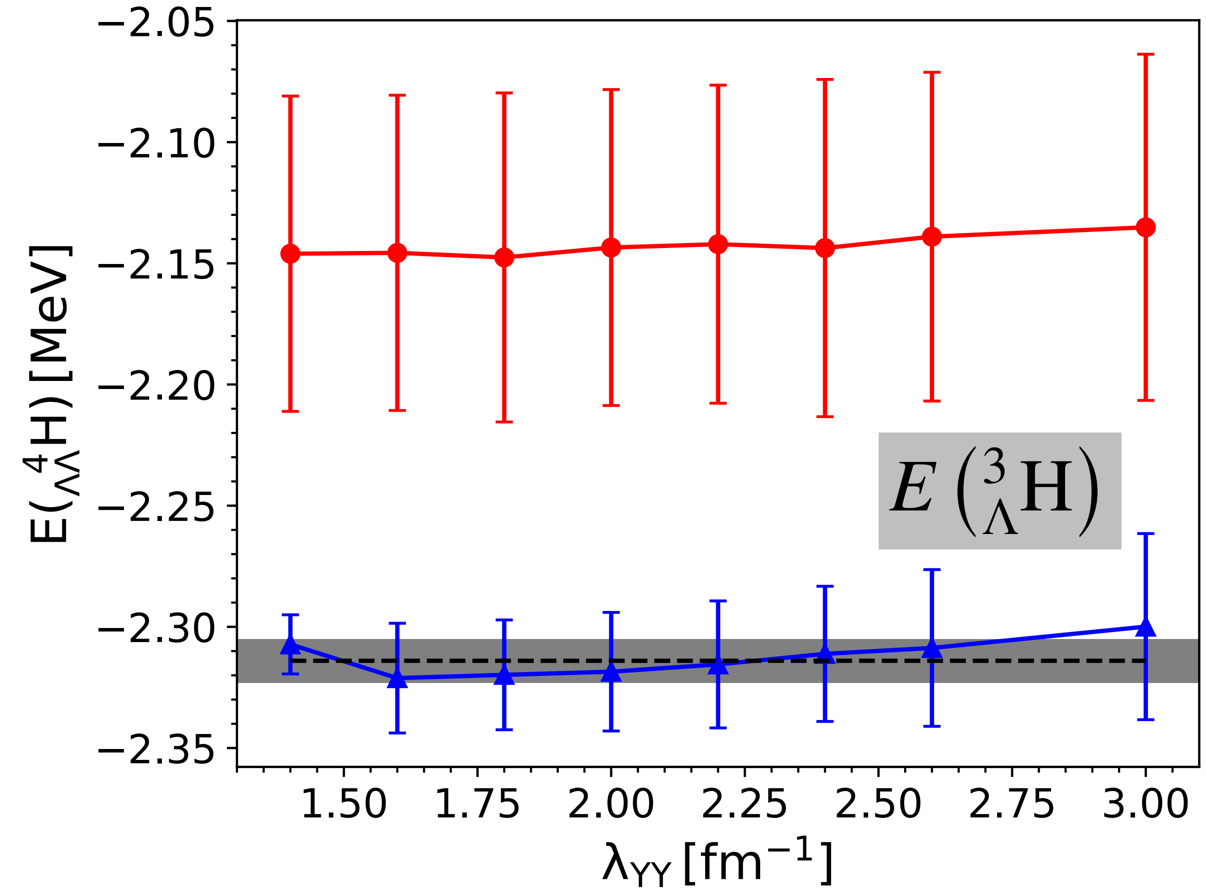
- small λ_{YY} dependence (no induced YYN forces used!)
- LO overbinds YY
- **NLO predicts binding fairly well**

Can an $S=-2$ bound state for $A=4,5$ be expected?

(Le, Haidenbauer, Meißner, AN, 2021)

$S = -2$ hypernuclei — ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ & ${}^{4}_{\Lambda\Lambda}\mathrm{H}$





- $A = 5$: $\Lambda\Lambda$ excess binding energy & $A = 4$: binding energy
- $A = 5$: LO & NLO predicts bound state
- $A = 4$: NLO unbound, LO at threshold to binding (see also Contessi et al., 2019)
- excess energy larger for $A = 5$ than for $A = 6$ (in contrast to Filikhin et al., 2002!)

$S = -2$ bound state for $A = 5$ can be expected,

for $A = 4$ less likely but not ruled out!

Ξ hypernuclei

- experimentally accessible: Ξ^- capture process (experimental data for ${}^{15}_{\Xi}\mathrm{C}$ and ${}^{12}_{\Xi}\mathrm{Be}$)
- $\Xi N - \Lambda\Lambda$ conversion channel open: possibly short life times/difficult calculations
- HAL QCD & chiral YY interactions indicate suppression $\Xi N - \Lambda\Lambda$ transition
- ΞN interaction relevant: Ξ is often the second hyperon to appear in neutron matter

Identify possibly interesting states:

calculations based on chiral interactions neglecting $\Xi N - \Lambda\Lambda$ transitions

(keeping $\Xi N - \Lambda\Sigma, \Sigma\Sigma$) → states are bound states

finetuning of ${}^{11}S_0$ interaction to correct for missing $\Lambda\Lambda$ channel

neglect YN interaction to avoid transitions to $\Lambda\Lambda$

perturbative width estimates indicate small widths ✔

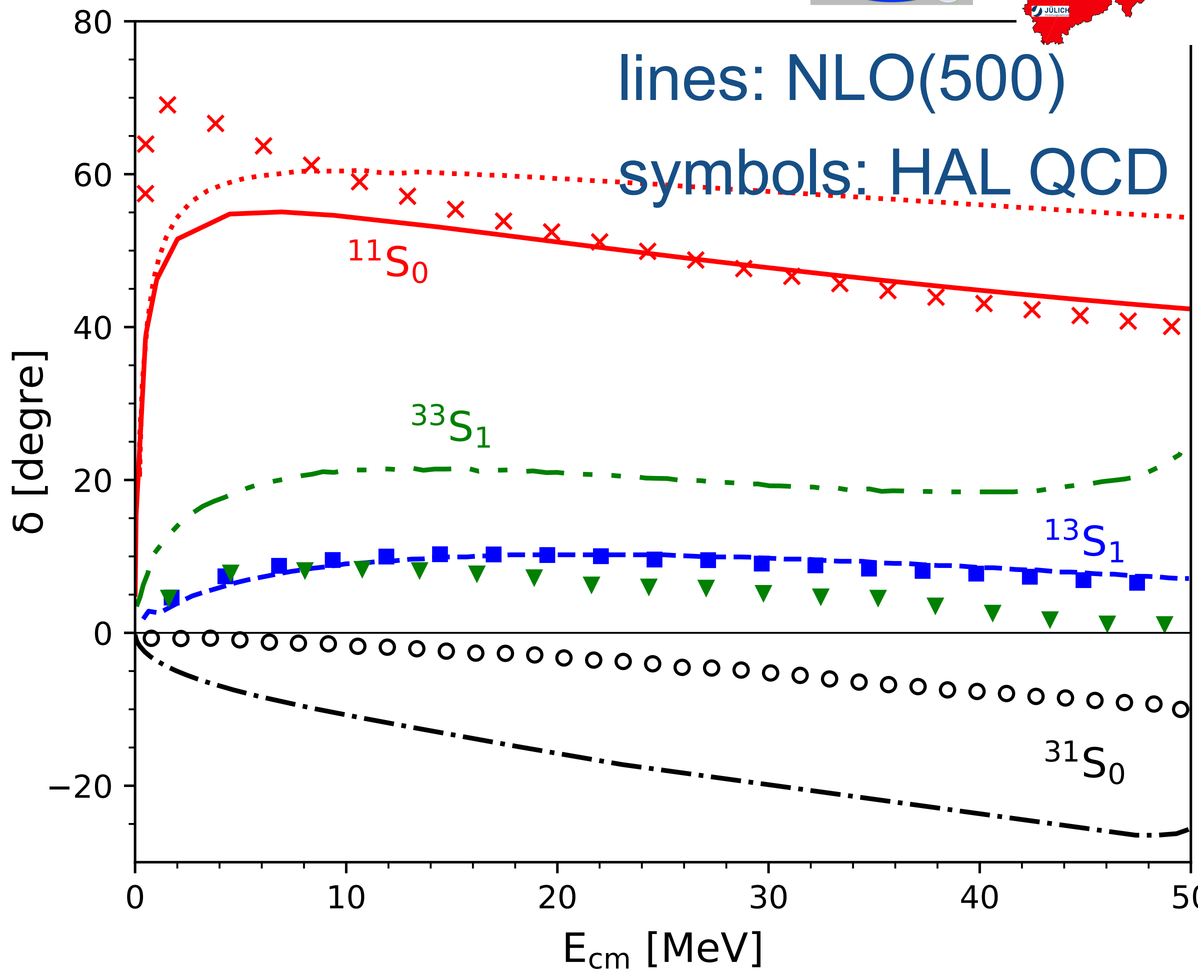
Here: look at ${}^{7}_{\Xi}\mathrm{H}$ (exp. expected), ${}^{5}_{\Xi}\mathrm{H}$, ${}^{4}_{\Xi}\mathrm{H}$ and ${}^{4}_{\Xi}\mathrm{n}$

explore possible bound states and their widths

Ξ hypernuclei

Ξ separation energies (NLO(500) and SMS $N^4LO^+(450)$)

	B_Ξ [MeV]	Γ [MeV]
${}^4_\Xi H(1^+, 0)$	0.48 ± 0.01	0.74
${}^4_\Xi n(0^+, 1)$	0.71 ± 0.08	0.2
${}^4_\Xi n(1^+, 1)$	0.64 ± 0.11	0.01
${}^4_\Xi H(0^+, 0)$	–	–
${}^5_\Xi H(\frac{1}{2}^+, \frac{1}{2})$	2.16 ± 0.10	0.19
${}^7_\Xi H(\frac{1}{2}^+, \frac{3}{2})$	3.50 ± 0.39	0.2



	$V^{S=-2}$ ${}^{11}S_0$	${}^{31}S_0$	${}^{13}S_1$	${}^{33}S_1$
${}^4_\Xi H(1^+, 0)$	– 1.95	0.02	– 0.7	– 2.31
${}^4_\Xi n(0^+, 1)$	– 0.6	0.25	– 0.004	– 0.74
${}^4_\Xi n(1^+, 1)$	– 0.02	0.16	– 0.13	– 1.14
${}^4_\Xi H(0^+, 0)$	– 0.002	0.08	– 0.01	– 0.006
${}^5_\Xi H(1/2^+, 1/2)$	– 0.96	0.94	– 0.58	– 3.63
${}^7_\Xi H(1/2^+, 3/2)$	– 1.23	1.79	– 0.79	– 6.74

(Le, Haidenbauer, Meißner, AN, 2021)

Conclusions & Outlook

- **YN & YY interactions not well understood**
  - *scarce YN data, almost no YY data*
  - *more information necessary to solve "hyperon puzzle"*
- **Hypernuclei provide important constraints**
  - *CSB of ΛN scattering &* $^4_\Lambda\mathrm{He}$ / $^4_\Lambda\mathrm{H}$
  - $^3_\Lambda\mathrm{H}$ *is used to constrain the spin dependence*
  - *Light $S = -2$ hypernuclei provide important information on $\Lambda\Lambda - \Sigma\Sigma - \Xi\mathrm{N}$ forces*
  - *new experiments planned at J-PARC, MAMI, J-Lab, FAIR,…*
- **New SMS YN interactions**
  - *give an accurate description low energy YN data*
  - *order LO, NLO and N²LO allow uncertainty quantification*
  - *have a non-unique determination of contact interactions (data necessary)*
- **Chiral 3BF** need to be included
  - *NLO uncertainty is sizable in $A = 4$ and beyond*
  - *chiral 3BFs are formulated (Petschauer et al., (2016)) and the implementation is currently checked*