



# Status and perspectives of NLEFT

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

supported by CAS, PIFI



by DFG, EXC 3107



by DFG, SFB 1639



by ERC, EXOTIC



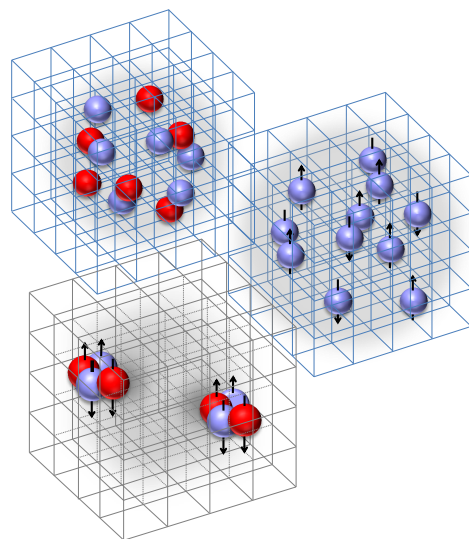
by NRW-FAIR



# Contents

- Nuclear physics on a lattice
  - Foundations
  - The chiral Hamiltonian at N<sup>3</sup>LO
  - Determination of 3NFs
  - Applications to structure, reactions and matter
  - Towards improved 3NFs
- Summary & outlook

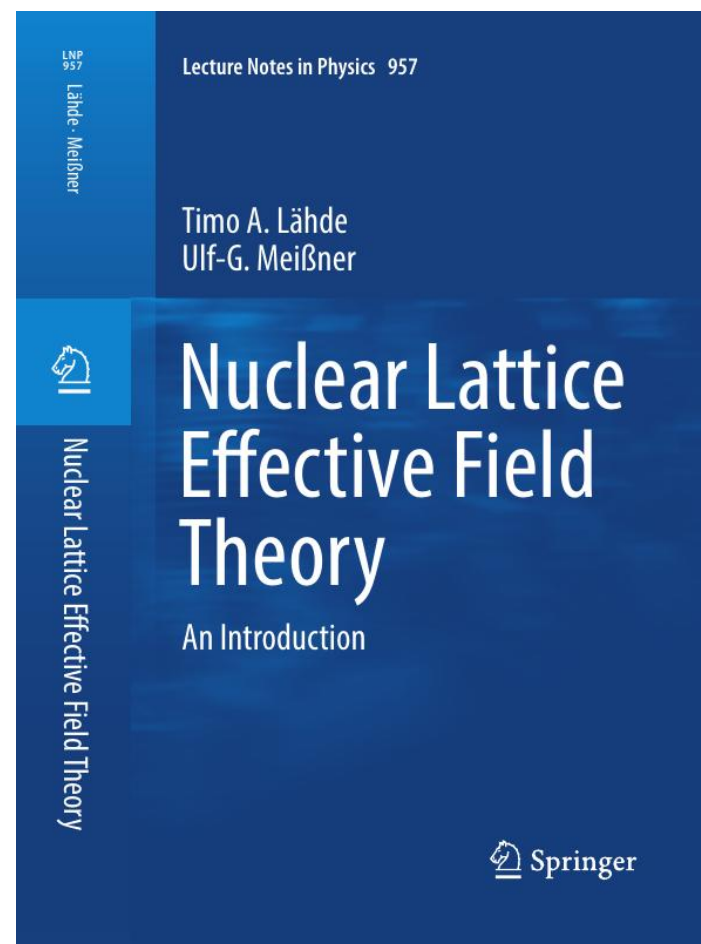
# Nuclear physics on a lattice



T. Lähde & UGM

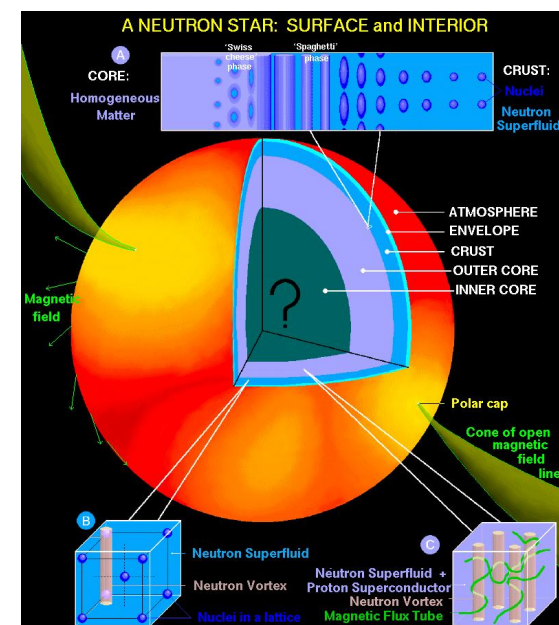
*Nuclear Lattice Effective Field Theory - An Introduction*

Springer Lecture Notes in Physics **957** (2019) 1 - 396 [2nd edition in the works]



# The nucleus as a quantum laboratory

- The nucleus is a challenging and fascinating many-body system
  - ↪ non-perturbative strong interactions balanced by the Coulomb force
  - ↪ many interesting phenomena: drip lines, clustering, reactions, ...
  - ↪ a plethora of few-body/many-body methods already exists
- Macroscopic nuclear matter = neutron stars
  - ↪ gained prominence again in the multi-messenger era
  - ↪ must be able to describe these with the same methods
- I will advocate here a new quantum many-body approach
  - ↪ synthesizes chiral EFT w/ stochastic methods ✓
  - ↪ allows to tackle nuclear structure *and* reactions ✓
  - ↪ allows to access the multiverse



# Foundations

# Nuclear lattice effective field theory (NLEFT)

- *new method* to tackle the nuclear many-body problem
- discretize space-time  $V = L_s \times L_s \times L_s \times L_t$ :  
nucleons are point-like particles on the sites
- discretized chiral potential w/ pion exchanges  
and contact interactions + Coulomb

→ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- EFT on the lattice, maximal momentum:

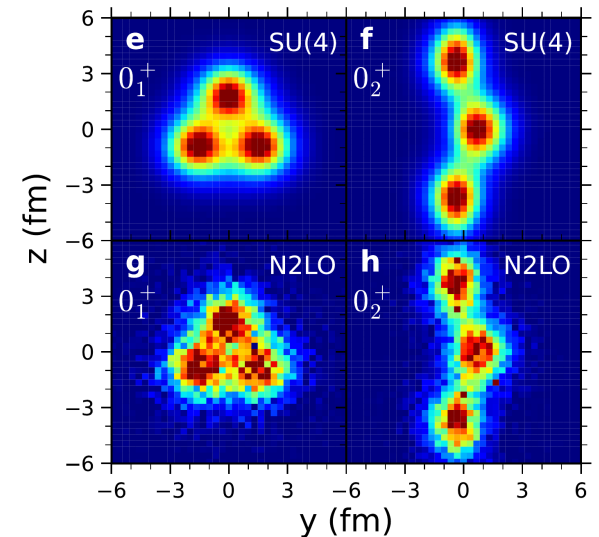
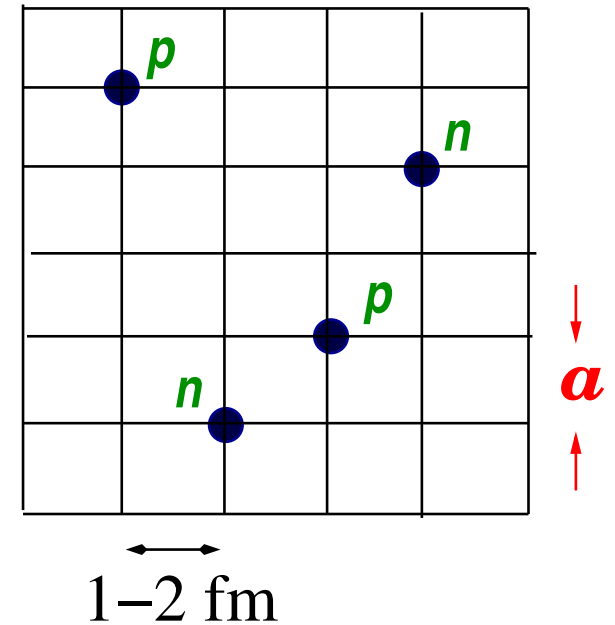
$$p_{\max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$

- strong suppression of sign oscillations SU(4)  
due to approximate Wigner (spin-isospin) symmetry

Wigner, Phys. Rev. **51** (1937) 106; Chen et al., Phys. Rev. Lett. **93** (2004) 242302

↪ works well for even-even nuclei

↪ we still need another method

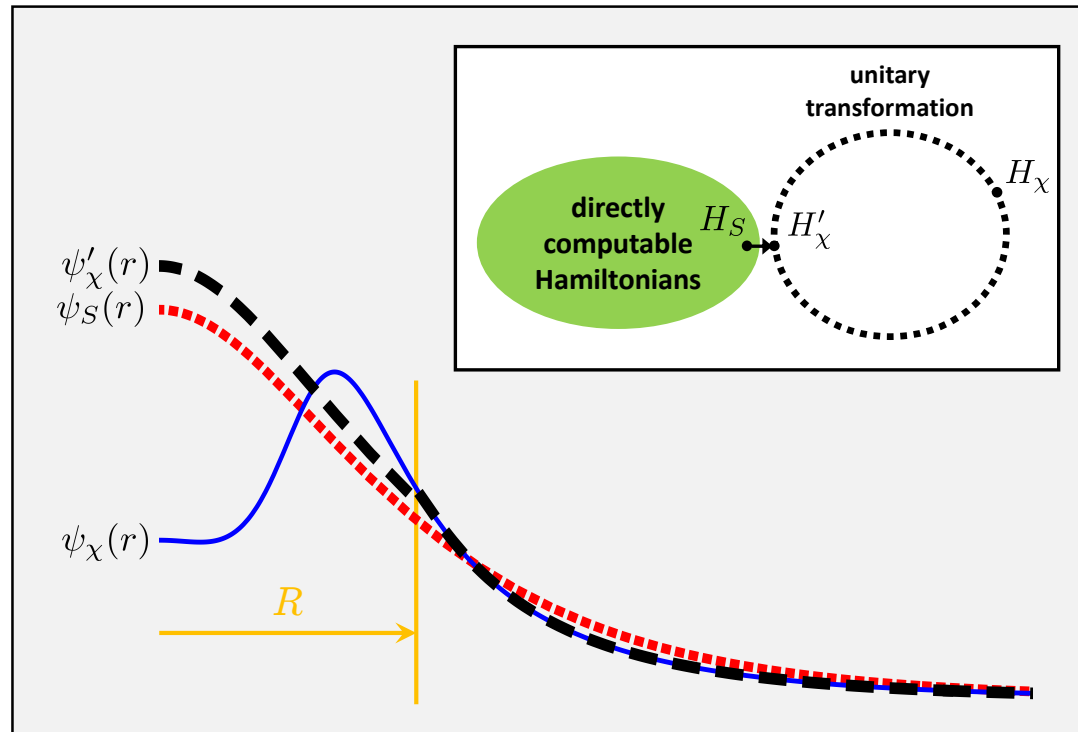


Shen et al., Nature Commun. **14** (2023) 2777

# Wave function matching

Elhatisari et al., Nature **630** (2024) 59

- A new quantum many-body method: Bring a complex Hamiltonian  $H_\chi$  close to a simple one  $H_S$  (with  $H_S$  essentially free of sign oscillations)
  - ↪ treat  $H_S$  non-perturbatively &  $H'_\chi - H_S$  in perturbation theory
- Graphical representation of w.f. matching



⇒ Efficient suppression of sign oscillations, applicable in many fields!

# Transfer matrix method

- Correlation–function for  $A$  nucleons:  $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

with  $\Psi_A$  a Slater determinant for  $A$  free nucleons  
[or a more sophisticated (correlated) initial/final state]

Euclidean time

- Transient energy

$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

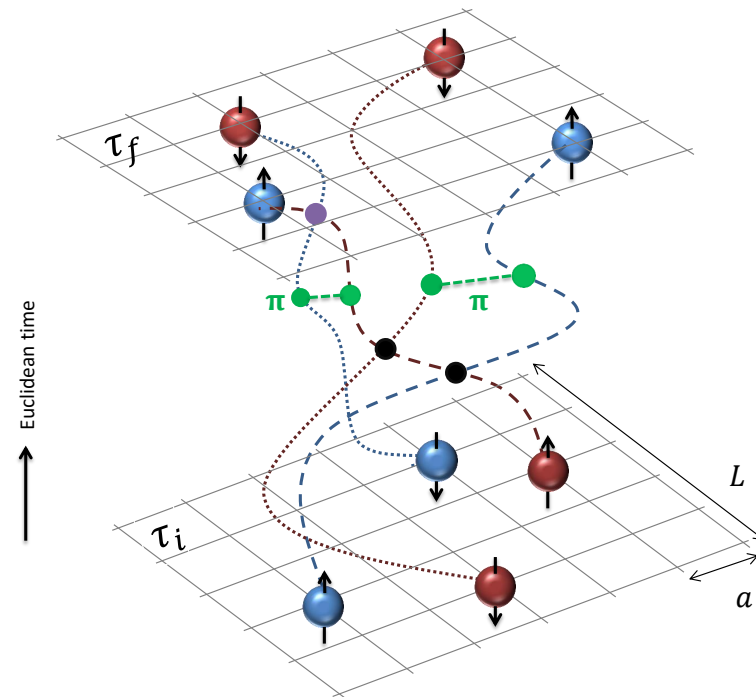
→ ground state:  $E_A^0 = \lim_{\tau \rightarrow \infty} E_A(\tau)$

- Exp. value of any normal–ordered operator  $\mathcal{O}$

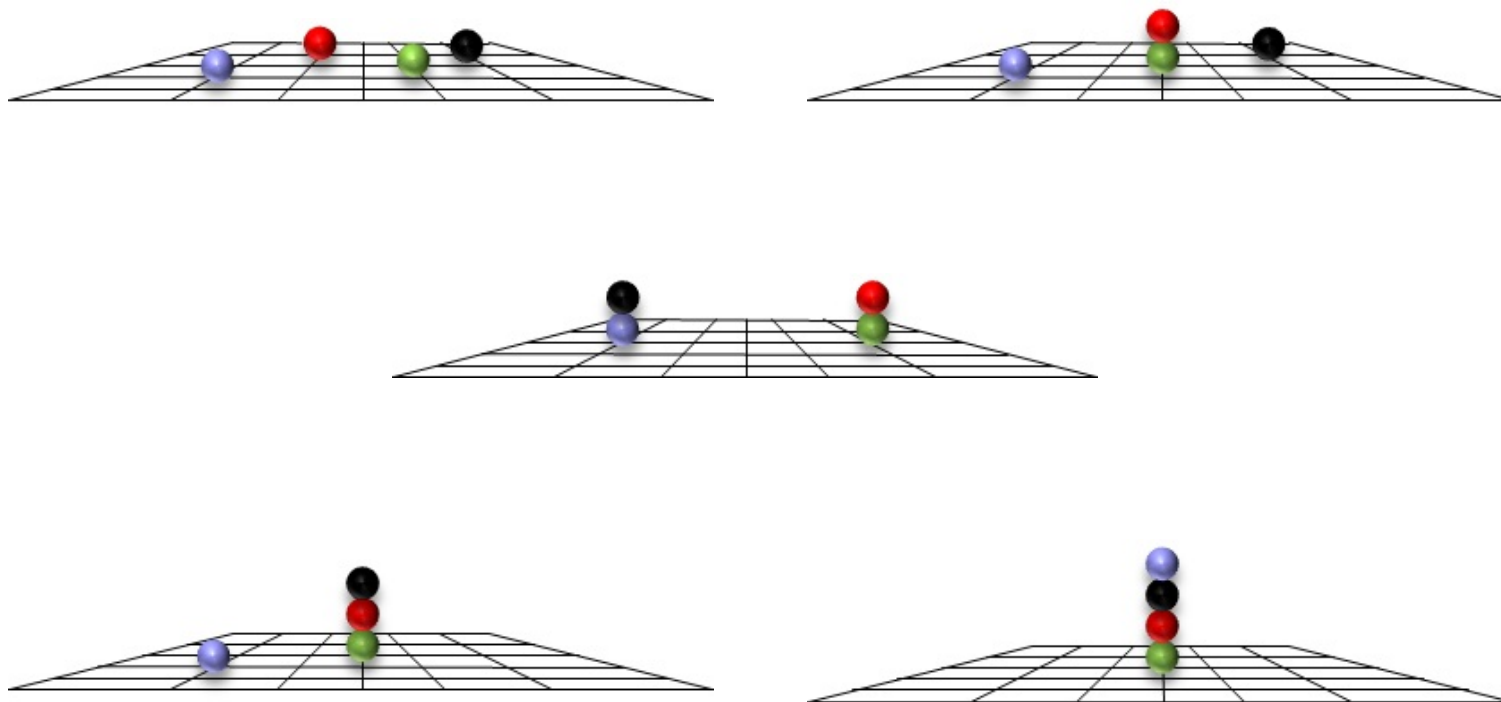
$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2) \mathcal{O} \exp(-\tau H/2) | \Psi_A \rangle$$

$$\lim_{\tau \rightarrow \infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} | \Psi_A \rangle$$

- Excited states:  $Z_A(\tau) \rightarrow Z_A^{ij}(\tau)$ , diagonalize, e.g.  $0_1^+$ ,  $0_2^+$ ,  $0_3^+$ , ... in  $^{12}\text{C}$



# Configurations

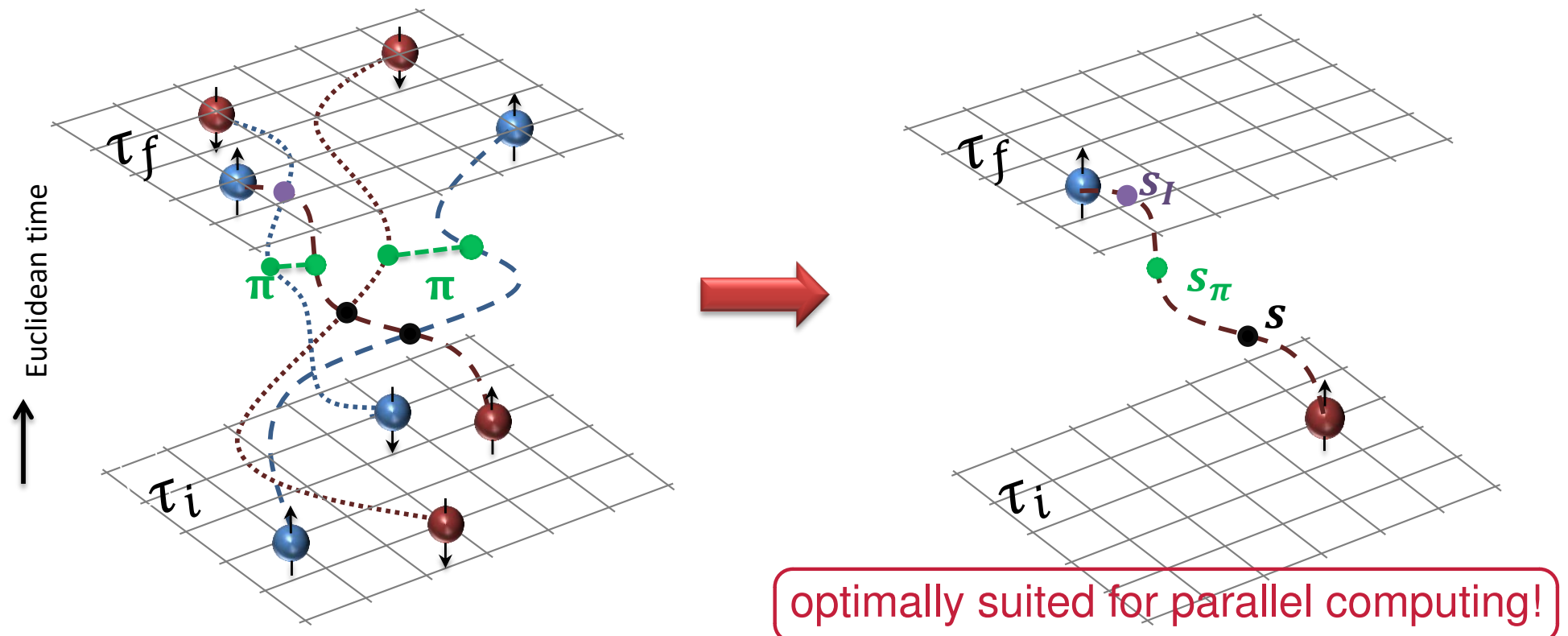


- ⇒ all *possible* configurations are sampled
- ⇒ preparation of *all possible* initial/final states
- ⇒ *clustering* emerges *naturally*

# Auxiliary field method

- Represent interactions by auxiliary fields (Gaussian completion):

$$\exp \left[ -\frac{C}{2} (N^\dagger N)^2 \right] = \sqrt{\frac{1}{2\pi}} \int ds \exp \left[ -\frac{s^2}{2} + \sqrt{C} s (N^\dagger N) \right]$$



# Pinhole algorithm

Elhatisari et al., Phys. Rev. Lett. **119** (2017) 222505

- Solution to the CM-problem:  
track the individual nucleons using the *pinhole algorithm*

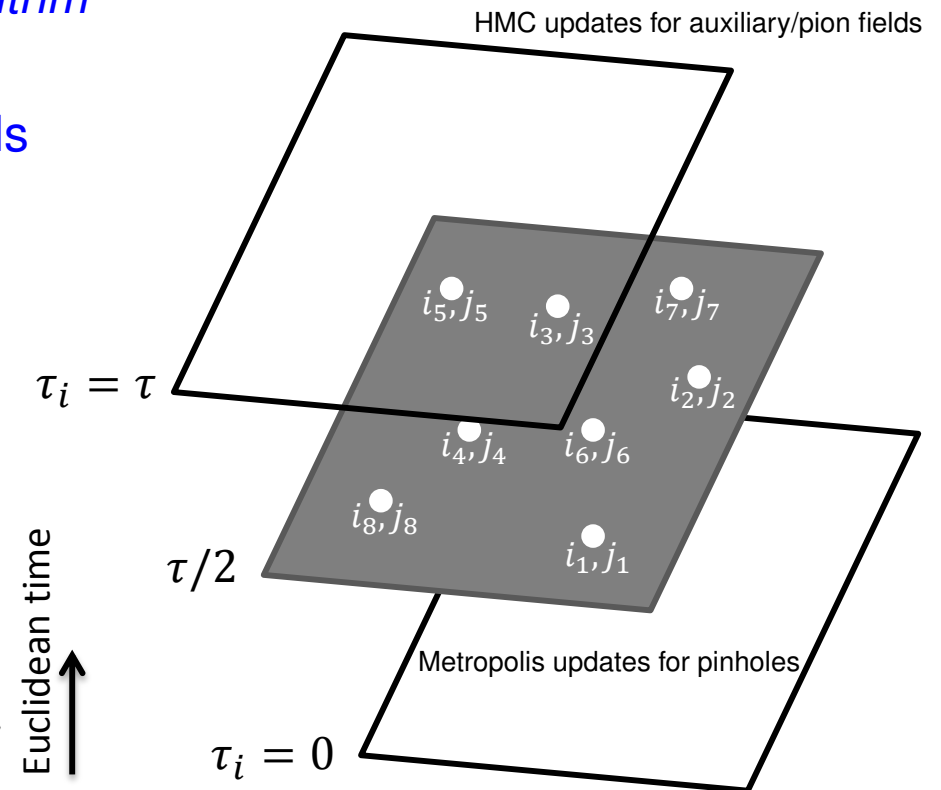
- Insert a screen with pinholes with spin & isospin labels that allows nucleons with corresponding spin & isospin to pass = insertion of the A-body density op.:

$$\begin{aligned} & \rho_{i_1, j_1, \dots, i_A, j_A}(\mathbf{n}_1, \dots, \mathbf{n}_A) \\ & = : \rho_{i_1, j_1}(\mathbf{n}_1) \cdots \rho_{i_A, j_A}(\mathbf{n}_A) : \end{aligned}$$

- MC sampling of the amplitude:

$$\begin{aligned} & A_{i_1, j_1, \dots, i_A, j_A}(\mathbf{n}_1, \dots, \mathbf{n}_A, L_t) \\ & = \langle \Psi_A(\tau/2) | \rho_{i_1, j_1, \dots, i_A, j_A}(\mathbf{n}_1, \dots, \mathbf{n}_A) | \Psi_A(\tau/2) \rangle \end{aligned}$$

- Allows to measure proton and neutron distributions & A-body correlations
- Resolution scale  $\sim a/A$  as cm position  $\mathbf{r}_{\text{cm}}$  is an integer  $\mathbf{n}_{\text{cm}}$  times  $a/A$
- Induces some sign oscillations, particularly in p/n-rich nuclei



# Partial pinhole algorithm

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Nuclear radii can be expressed in terms of two-particle correlations  $G_{pp}$ ,  $G_{pn}$ ,  $G_{nn}$

$$\langle r^2 \rangle = \sum_{c_1, c_2} \langle : \rho_{i_1 j_1}(\mathbf{n}_1) \rho_{i_2 j_2}(\mathbf{n}_2) : \rangle (\mathbf{n}_1 - \mathbf{n}_2)^2, \quad c_k = (i_k, j_k, \mathbf{n}_k)$$

↪ the number of pinholes needed in such a calculation can be reduced:

$$\rho_M(c_1, \dots, c_M) = \frac{(A - M)!}{A!} : \rho_{i_1 j_1}(\mathbf{n}_1) \cdots \rho_{i_M j_M}(\mathbf{n}_M) :,$$

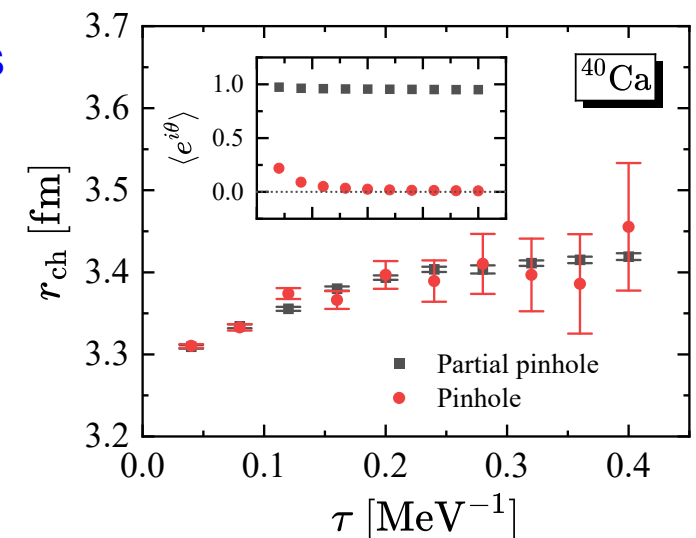
$$\text{with } \sum_{c_1, \dots, c_M} \rho_M = 1$$

- However,  $\rho_M |\Psi_A\rangle$  is not a Slater determinant, even if  $|\Psi_A\rangle$  is

↪ use the rank-one operator method for  $\rho_M$  Ma et al. (2024)

$$\begin{aligned} & : \rho_{i_1 j_1}(\mathbf{n}_1) \cdots \rho_{i_M j_M}(\mathbf{n}_M) : \\ &= \lim_{\epsilon \rightarrow \infty} \frac{1}{\epsilon^M} : e^{\epsilon[\rho_{i_1 j_1}(\mathbf{n}_1) + \cdots]} : \end{aligned}$$

↪ much improved signal allows for a more precise extraction



# Computational equipment

- Present = JUPITER + FRONTIER + ...



>1 Exaflop

# The chiral Hamiltonian at N<sup>3</sup>LO

# The chiral Hamiltonian

- Chiral Hamiltonian with **smear**ed N3LO (2N) and N2LO (3N) interactions

$$H = K + V_{\text{OPE}} + V_{\text{C}} + V_{3\text{N}}^{\text{Q}^3} + V_{2\text{N}}^{\text{Q}^4} + W_{2\text{N}}^{\text{Q}^4}$$

- $K$  – kinetic energy term with correct nucleon dispersion relation (FFT)
  - $V_{\text{OPE}}$  – regulated OPE (similar to continuum case) Reinert et al. (2018)
  - $V_{\text{C}}$  – Coulomb interaction
  - $V_{3\text{N}}^{\text{Q}^3}$  – 3N interaction at N2LO → to be discussed later
  - $V_{2\text{N}}^{\text{Q}^4}$  – 2N short-range interaction at N3LO ( $2\pi$  exchange absorbed) → next<sup>2</sup> slide
  - $W_{2\text{N}}^{\text{Q}^4}$  – 2N Galilean invariance restoration (GIR) interaction at N3LO Li et al. (2019)
- Smearing explained in what follows
  - Throughout, we use  $a = 1.32$  fm, corresponding to the magic momentum cutoff  $\Lambda \simeq 470$  MeV Lee et al. (2021)

# Smearred operators & densities

- Annihilation/creation operators  $a_{i,j}, a_{i,j}^\dagger$ , spin  $i = 0, 1(\uparrow, \downarrow)$  and isospin  $j = 0, 1(p, n)$
- Non-locally smeared annihilation and creation operators, velocity-independent:

$$\tilde{a}_{i,j}^{(\dagger)}(\vec{n}) = a_{i,j}^{(\dagger)}(\vec{n}) + s_{\text{NL}} \sum_{|\vec{n}' - \vec{n}|=1} a_{i,j}^{(\dagger)}(\vec{n}'), \quad s_{\text{NL}} = 0.1 - 0.5 \quad \text{Lu et al. (2019,2020)}$$

- Most general form of the point-like density operators, velocity-dependent:

$$\tilde{\rho}^{(d)}(\vec{n}) = \sum_{i,j} \tilde{a}_{i,j}^\dagger(\vec{n}) \tilde{a}_{i,j}(\vec{n}) + s_{\text{L}} \sum_{|\vec{n} - \vec{n}'|^2=1}^d \sum_{i,j} \tilde{a}_{i,j}^\dagger(\vec{n}') \tilde{a}_{i,j}(\vec{n}'), \quad s_{\text{L}} = 0.05 - 0.15$$

Elhatisari et al., (2016), Lu et al. (2019,2020)

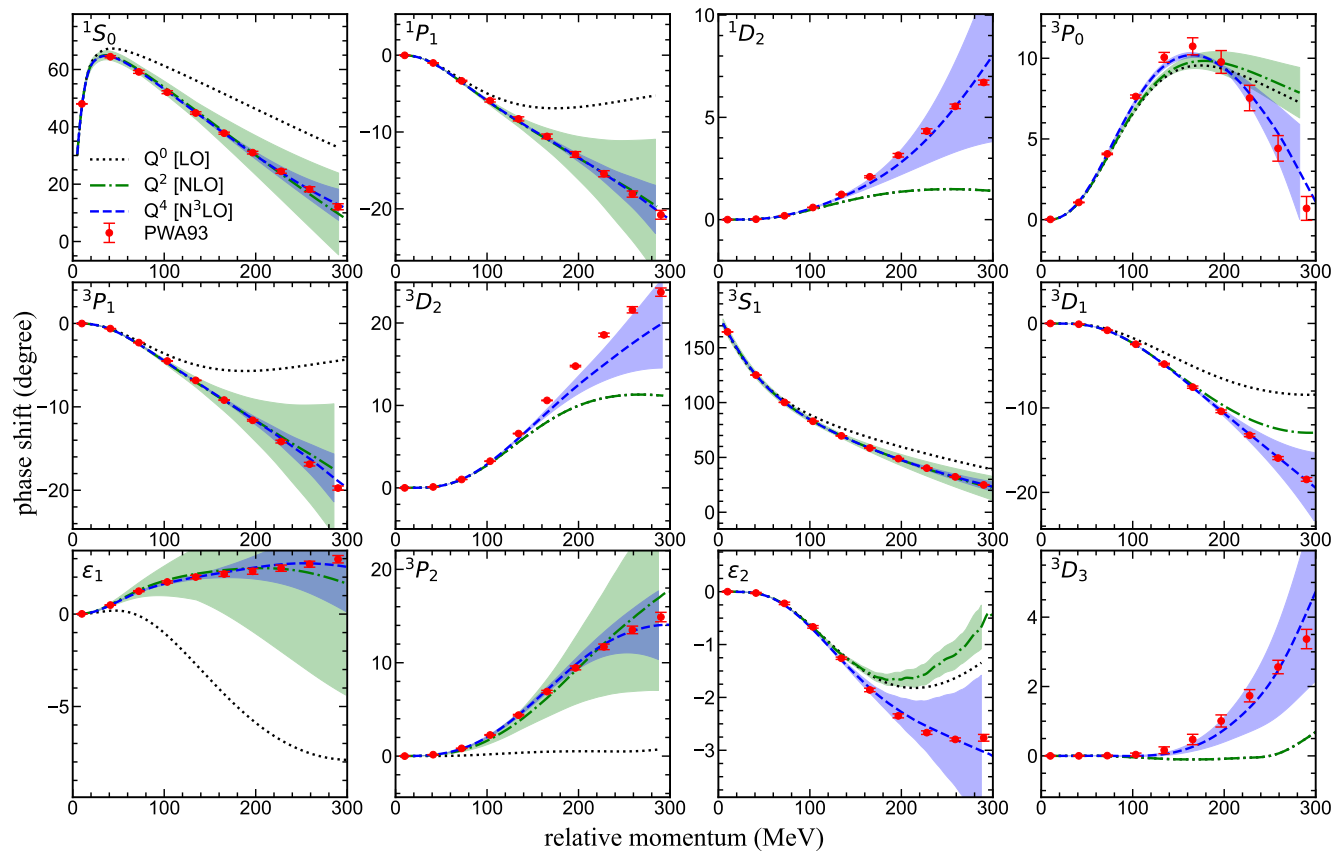
- In the NN system, local and non-local interactions are equally valid
- However,  $\alpha$ - $\alpha$  scattering is very sensitive to the degree of locality of the NN interaction  
Elhatisari et al., (2016)

↪ purely nonlocal forces suppress the  $\alpha$ - $\alpha$  interaction

↪ thus require stronger 3NFs in heavier nuclei to compensate for this

# Two-pion exchange potential

- Here, we absorb the TPEP in the short distance LECs → works, see also Li et al. (2018)



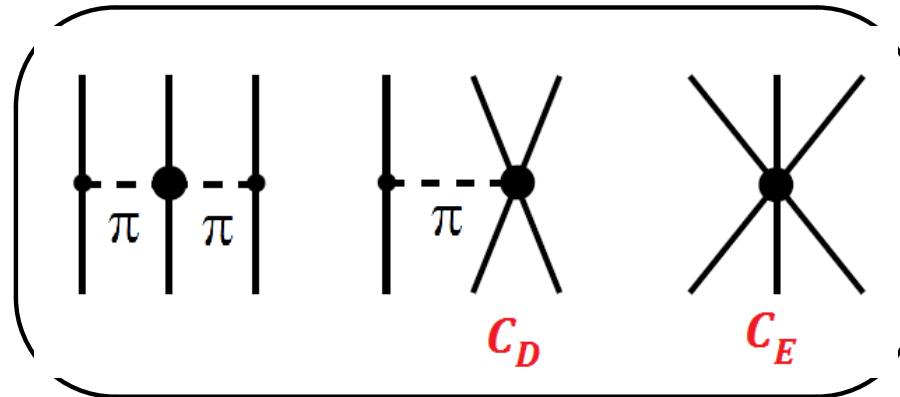
↪ needs to be improved (work underway)

↪ especially important for getting consistent currents beyond LO

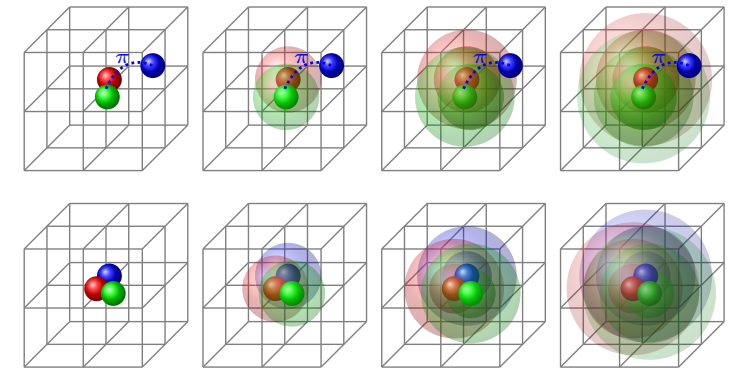
# Determination of 3NFs

# Three-nucleon forces

- Basic topologies at N2LO:



- 2-pion topology entirely fixed from  $\pi N$  scattering
- Smearing of the one-pion and contact term topologies
  - $\hookrightarrow$  more LECs (see below)
  - $\hookrightarrow$  effectively higher-order ops



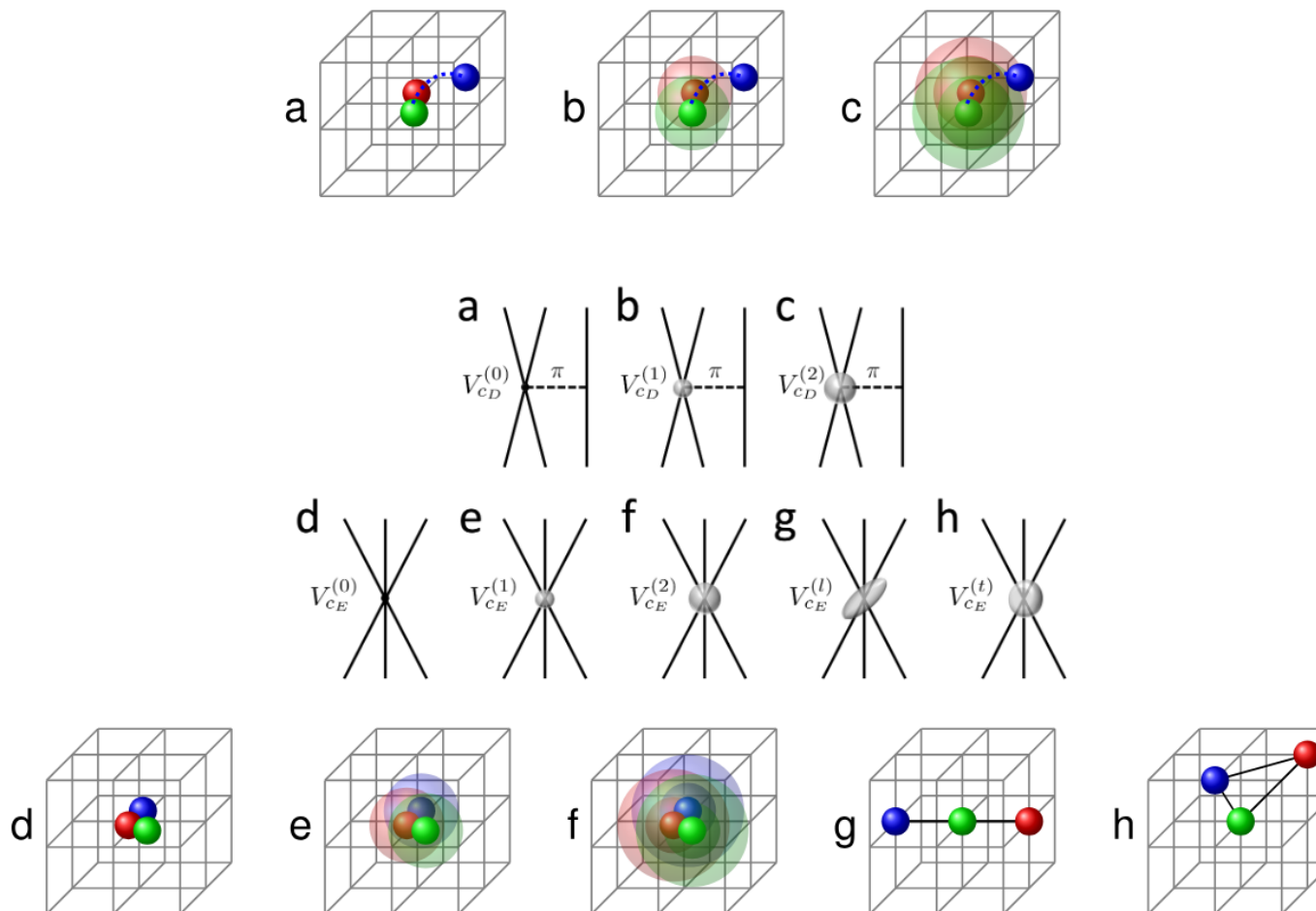
- On the market so far (more in the works!)
  - $\hookrightarrow \mathcal{F}_p = 8$ , only local smearings (next slide)
  - $\hookrightarrow \mathcal{F}_p = 6$ , a few non-locally smeared ops

Elhatisari et al. (2024)

Ren et al. (2025)

# Locally smeared three-nucleon forces

- Most used set with  $\mathcal{F}_p = 8$  locally smeared op's:



- Note the two additional SU(4) symmetric terms  $V_{cE}^{(l,t)}$  motivated by cluster EFT

# Determination of the 3NFs

- Main ingredients to fit the parameters of the 3NFs:

- **Objective function:** Gaussian likelihood for selected binding energies

$$\mathcal{L}(\vec{\beta}, \sigma^2) \propto \prod_{n \in \mathcal{S}} \exp \left\{ -\frac{1}{2\sigma_n^2} \left[ z_n^{\text{exp}} - z_{n,\text{NP}}^{\text{th}} - \sum_{k \in \mathcal{F}_p} \beta_k \frac{\partial z_n^{\text{th}}}{\partial \beta_k} \right]^2 \right\}$$

$\beta_k$  = 3NF LECs,  $\mathcal{S}$  = subset of nuclei,  $\mathcal{F}_p$  = chosen set of  $p \in \{6, 7, 8\}$  3NF ops

- **Number & sampling of observables:** to avoid bias, perform MCMC over  
(i) subsets  $\mathcal{S}$  and (ii) operator sets  $\mathcal{F}_p$  with update of  $\vec{\beta}$
- **Choice-of-observables uncert.:** sensitivity to chosen set of energies & subset of 3NFs
- **Expected N3LO accuracy:** EFT truncation uncertainties either using history matching or the EKM procedure with  $Q \simeq 0.3 - 0.4$

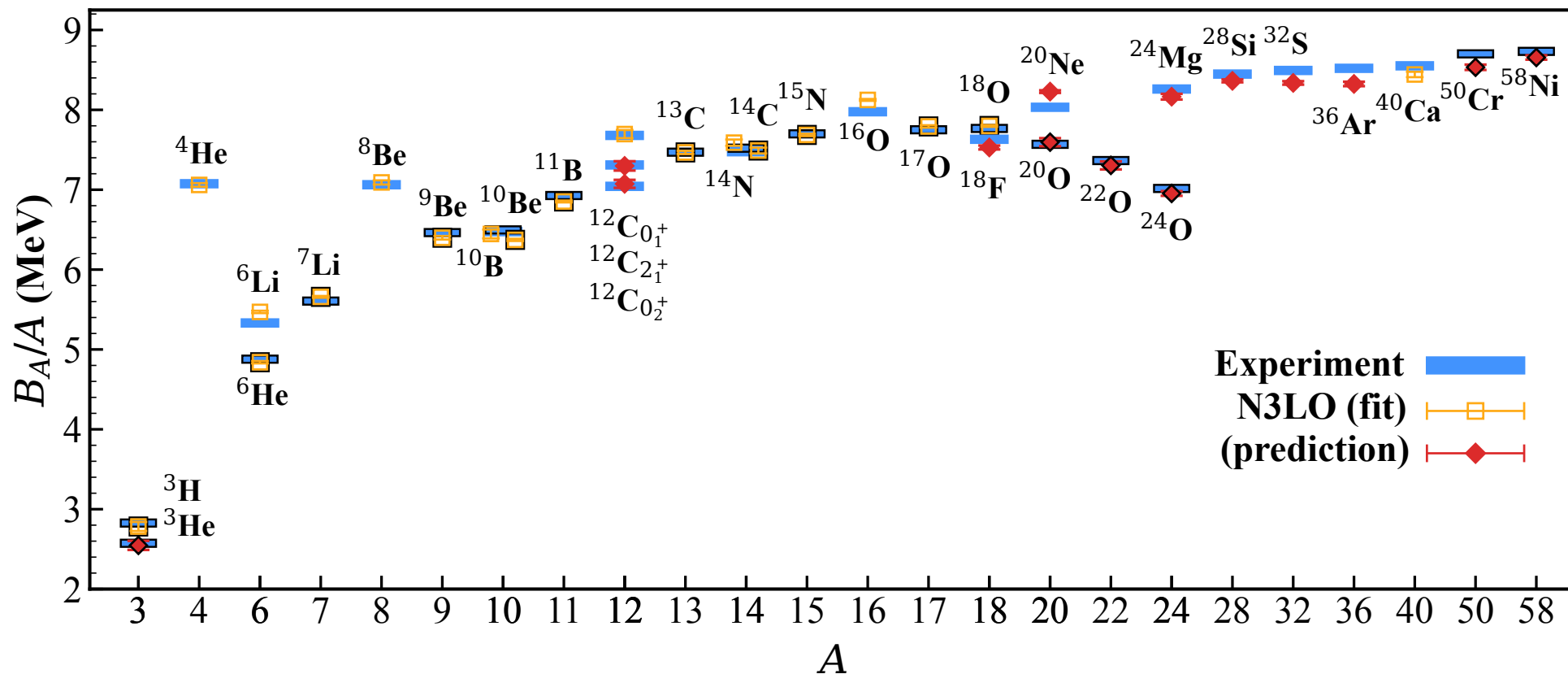
HM: Vernon et al. (2018), Hu et al. (2022), Elhatisari et al. (2024)

EKM: Epelbaum et al. (2015)

# Fixing the 3NF LECs: Binding Energies at N3LO

- Quality of the fit:  $\text{RMSD}(\mathcal{S}) = \sqrt{\frac{1}{M_{\mathcal{S}}} \sum_{i \in \mathcal{S}} \left( \frac{E_i - E_i^{\text{exp}}}{A_i} \right)^2}$  with size  $M_{\mathcal{S}}$

↪ here,  $\text{RMSD} = 0.079 \text{ MeV} = 1.11\%$  of the average BE per nucleon

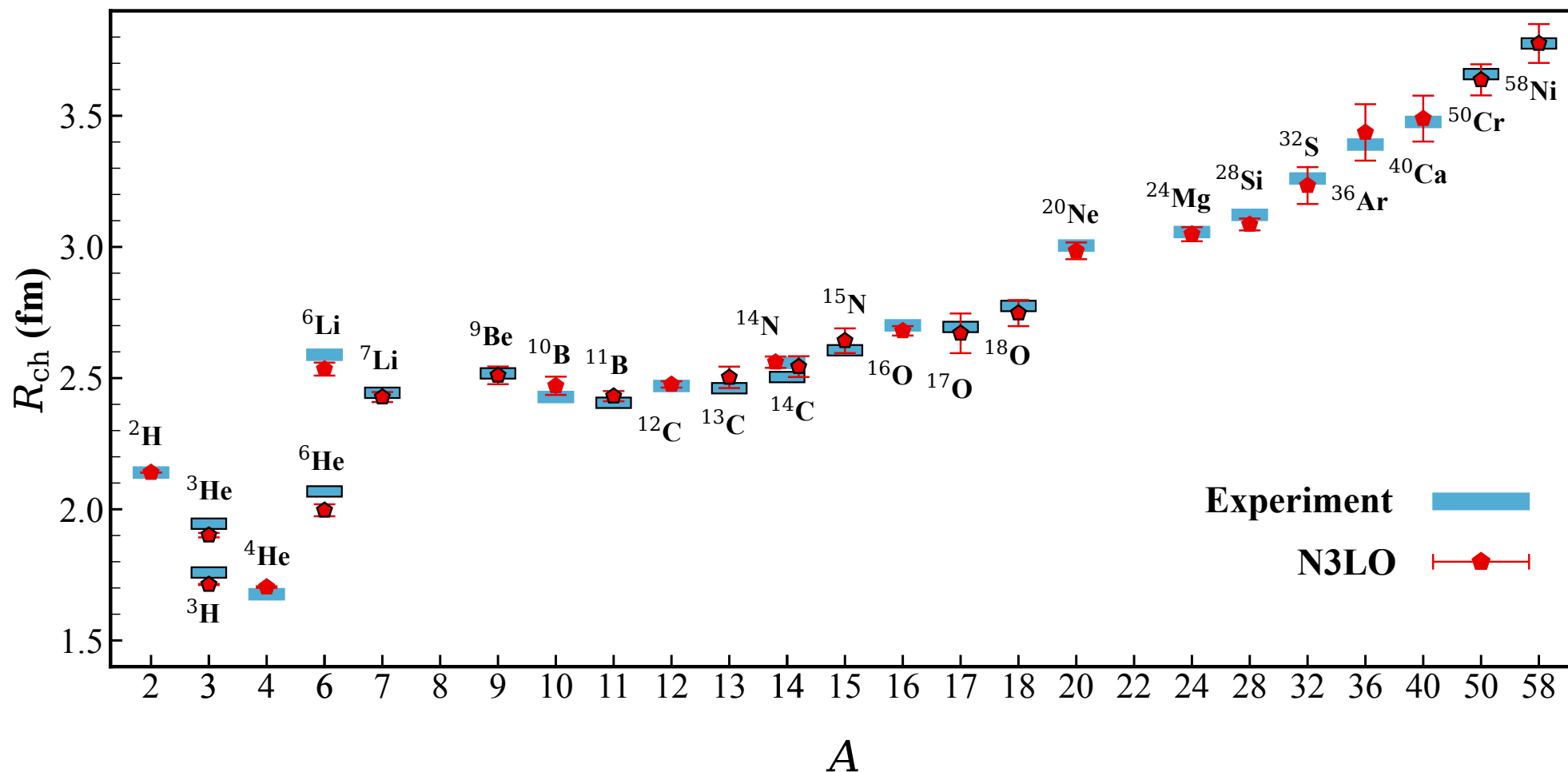


# Applications to structure, reactions and matter

# Prediction: Charge radii at N3LO

Elhatisari et al., Nature **630** (2024) 59

- Charge radii ( $a = 1.32$  fm, statistical errors can be reduced)

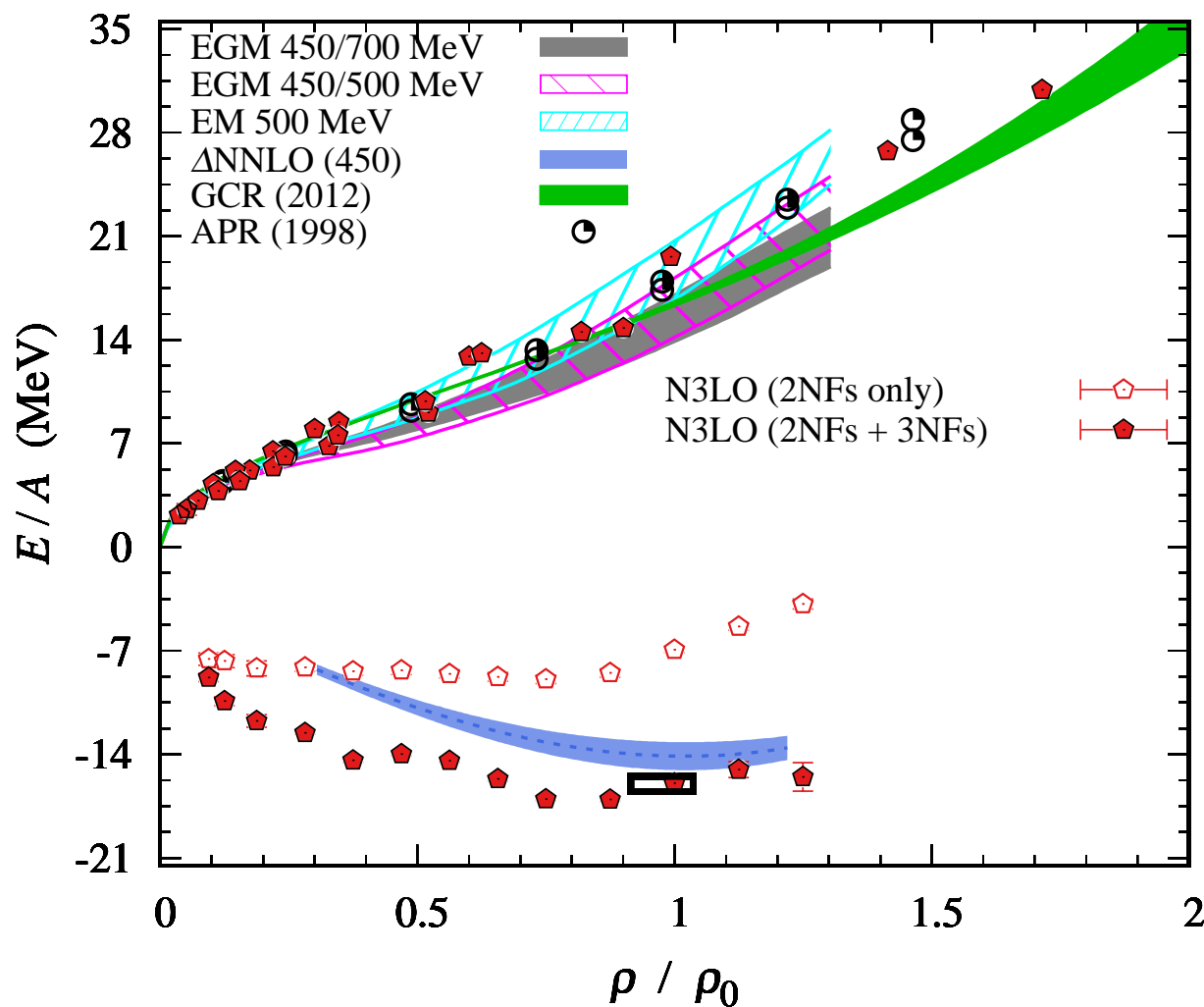


↔ no radius problem! but radii to be improved, see later

# Prediction: Neutron & nuclear matter at N3LO

Elhatisari et al., Nature **630** (2024) 59

- Equation of State (EoS) of pure neutron matter & nuclear matter ( $a = 1.32$  fm)



↪ can be improved using twisted b.c.'s

PHYSICAL REVIEW LETTERS 132, 232502 (2024)

## Structure Factors for Hot Neutron Matter from *Ab Initio* Lattice Simulations with High-Fidelity Chiral Interactions

Yuan-Zhuo Ma<sup>1,2</sup>, Zidu Lin<sup>3</sup>, Bing-Nan Lu<sup>4</sup>, Serdar Elhatisari<sup>5,6</sup>, Dean Lee<sup>7,\*</sup>, Ning Li<sup>7</sup>, Ulf-G. Meißner<sup>6,8,9</sup>, Andrew W. Steiner<sup>3,10</sup> and Qian Wang<sup>1</sup><sup>1</sup>Key Laboratory of Atomic and Subatomic Structure and Quantum Control (MOE),

Guangdong Basic Research Center of Excellence for Structure and Fundamental Interactions of Matter, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China

<sup>2</sup>Facility for Rare Isotope Beams and Department of Physics and Astronomy, Michigan State University, Michigan 48824, USA<sup>3</sup>Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee, USA<sup>4</sup>Graduate School of China Academy of Engineering Physics, Beijing 100193, China<sup>5</sup>Faculty of Natural Sciences and Engineering, Gaziantep Islam Science and Technology University, Gaziantep 27010, Turkey<sup>6</sup>Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany<sup>7</sup>School of Physics, Sun Yat-Sen University, Guangzhou 510275, China<sup>8</sup>Institute for Advanced Simulation, Institut für Kernphysik, and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany<sup>9</sup>Thilisi State University, 0186 Thilisi, Georgia<https://www.elsevier.com/locate/physletb>

Phys. Lett. B 869 (2025) 139839



ELSEVIER

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: [www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

Letter

## Lattice simulation of nucleon distribution and shell closure in the proton-rich nucleus <sup>22</sup>Si

Shuang Zhang<sup>10,a</sup>, Serdar Elhatisari<sup>10,b,c</sup>, Ulf-G. Meißner<sup>10,d,e,\*</sup>, Shihang Shen<sup>a,c</sup><sup>a</sup>Institute for Advanced Simulation (IAS-4) Forschungszentrum Jülich, D-52425, Jülich, Germany<sup>b</sup>King Fahd, University of Petroleum and Minerals (KFUPM), 31261, Dhahran, Saudi Arabia<sup>c</sup>Faculty of Natural Sciences and Engineering, Gaziantep Islam Science and Technology University, 27010, Gaziantep, Turkey<sup>d</sup>Bethe Center for Theoretical Physics and Helmholtz-Institut für Strahlen- und Kernphysik, Universität Bonn, D-53115, Bonn, Germany<sup>e</sup>Peng Huanwu Collaborative Center for Research and Education, International Institute for Interdisciplinary and Frontiers, Beihang University, 100191, Beijing, China

ARTICLE INFO

Editor: Dr A Schwenk

Keywords:  
Effective field theory  
Lattice  
Silicon  
Shell closure

ABSTRACT

The proton-rich nucleus <sup>22</sup>Si is studied using Nuclear Lattice Effective Field Theory with high-fidelity chiral forces. Our results indicate that <sup>22</sup>Si is more tightly bound than <sup>20</sup>Mg, thereby excluding the possibility of two-proton emission. The Z = 14 shell closure in <sup>22</sup>Si is investigated through the evolution of the 2<sup>+</sup> state in the neighboring nuclei. We then focus on the charge radius and spatial distribution information of <sup>22</sup>Si, considering the novel phenomena that may emerge due to the small two-proton separation energy and the shell closure. We present the distribution of the 14 protons and 8 neutrons obtained from our lattice simulation, revealing insights into the spatial arrangement of the nucleons. Moreover, the spatial localization of the outermost proton and neutron suggests that <sup>22</sup>Si is a doubly magic nucleus. Furthermore, we develop the pinhole method based on the harmonic oscillator basis, which gives insight into the nuclear structure in terms of the shell model picture from lattice simulations. Our calculated occupation numbers support that Z = 14 and N = 8 are the shell closures and show that the π1s<sub>1/2</sub> orbital component is minor in <sup>22</sup>Si.

PHYSICAL REVIEW LETTERS 134, 162503 (2025)

## *Ab Initio* Study of the Beryllium Isotopes <sup>7</sup>Be to <sup>12</sup>Be

Shihang Shen<sup>1,2,3</sup>, Serdar Elhatisari<sup>4,5,6</sup>, Dean Lee<sup>7</sup>, Ulf-G. Meißner<sup>6,8,9,1,\*</sup> and Zhengxue Ren<sup>3</sup><sup>1</sup>Peng Huanwu Collaborative Center for Research and Education, International Institute for Interdisciplinary and Frontiers, Beihang University, Beijing 100191, China<sup>2</sup>School of Physics, Beihang University, Beijing 102206, China<sup>3</sup>Institute for Advanced Simulation (IAS-4), Forschungszentrum Jülich, D-52425 Jülich, Germany<sup>4</sup>Interdisciplinary Research Center for Industrial Nuclear Energy (IRC-INE),

King Fahd University of Petroleum and Minerals (KFUPM), 31261 Dhahran, Saudi Arabia

<sup>5</sup>Faculty of Natural Sciences and Engineering, Gaziantep Islam Science and Technology University, Gaziantep 27010, Turkey<sup>6</sup>Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany<sup>7</sup>Facility for Rare Isotope Beams and Department of Physics and Astronomy, Michigan State University,

East Lansing, Michigan 48824, USA

<sup>8</sup>Tbilisi State University, 0186 Tbilisi, Georgia

(Received 26 November 2024; revised 20 February 2025; accepted 3 April 2025; published 25 April 2025)

Phys. Lett. B 872 (2026) 140086



ELSEVIER

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: [www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

Letter

## *Ab initio* calculations of the carbon and oxygen isotopes: Energies, correlations, and superfluid pairing

Young-Ho Song<sup>10,a</sup>, Myungkuk Kim<sup>10,b,\*</sup>, Youngman Kim<sup>10,b</sup>, Kihyeon Cho<sup>10,c</sup>, Serdar Elhatisari<sup>10,d,e</sup>, Dean Lee<sup>10,f</sup>, Yuan-Zhuo Ma<sup>10,g</sup>, Ulf-G Meißner<sup>10,g,h,i</sup><sup>a</sup>Institute for Rare Isotope Sciences, Institute for Basic Science, Daejeon, 34000, Korea<sup>b</sup>Center for Exotic Nuclear Studies, Institute for Basic Science, Daejeon, 34126, Korea<sup>c</sup>Korea Institute of Science and Technology Information, Daejeon, 34141, Korea<sup>d</sup>Faculty of Natural Sciences and Engineering, Gaziantep Islam Science and Technology University, Gaziantep, 27010, Turkey<sup>e</sup>King Fahd University of Petroleum and Minerals (KFUPM), 31261, Dhahran, Saudi Arabia<sup>f</sup>Facility for Rare Isotope Beams and Department of Physics and Astronomy, Michigan State University, MI 48824, USA<sup>g</sup>Helmholtz-Institut für Strahlen- und Kernphysik (Theorie) and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany<sup>h</sup>Institute for Advanced Simulation (IAS-4), D-52425 Jülich, Germany<sup>i</sup>Peng Huanwu Collaborative Center for Research and Education, International Institute for Interdisciplinary and Frontiers, Beihang University, Beijing, 100191, China

ARTICLE INFO

Editor: Dr. A. Schwenk

Keywords:  
Nuclear lattice effective field theory  
Neutron-rich isotopes  
Superfluid pairings  
*Ab initio* calculation

ABSTRACT

We perform *ab initio* nuclear lattice calculations of the neutron-rich carbon and oxygen isotopes using high-fidelity chiral interactions. We find good agreement with the observed binding energies and compute correlations associated with each two-nucleon interaction channel. For the isospin T = 1 channels, we show that the dependence on T<sub>2</sub> provides a measure of the correlations among the extra neutrons in the neutron-rich nuclei. For the spin-singlet S-wave channel, we observe that any paired neutron interacts with the nuclear core as well as its neutron pair partner, while any unpaired neutron interacts primarily with only the nuclear core. For the other partial waves, the correlations among the extra neutrons grow more slowly and smoothly with the number of neutrons. These general patterns are observed in both the carbon and oxygen isotopes and may be universal features that appear in many neutron-rich nuclei.

# Prediction: Tin isotopes close to the proton dripline

27

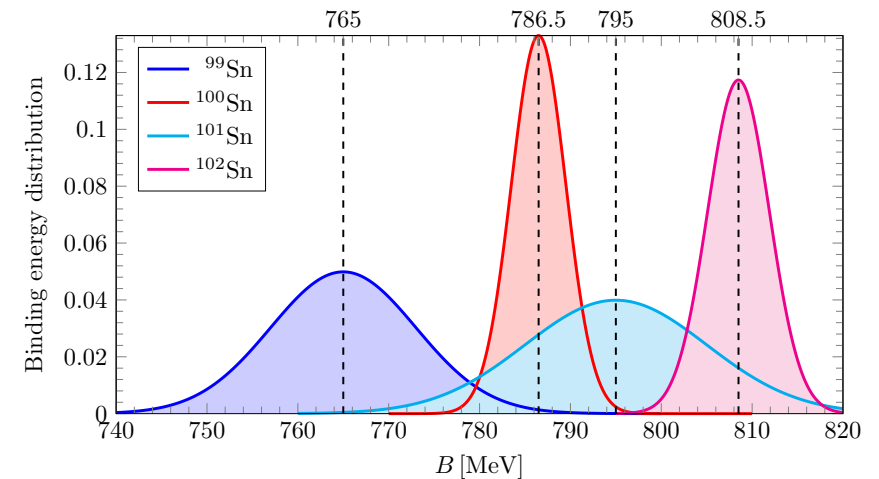
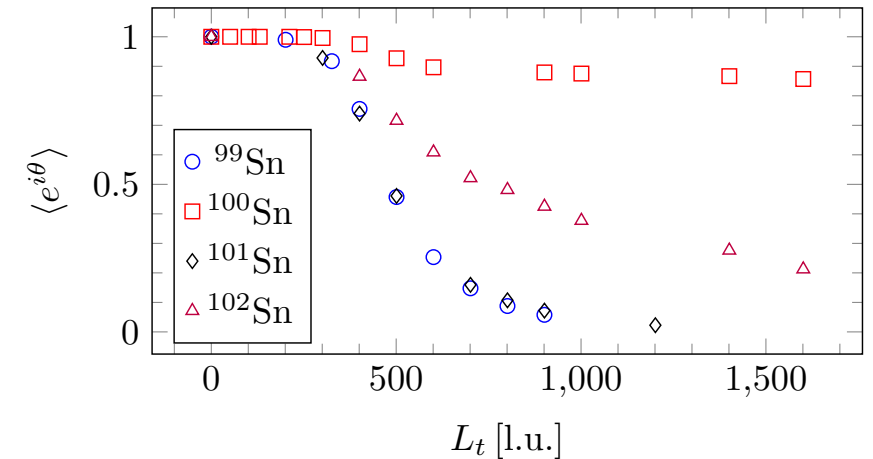
Hildenbrand et al., PRL **136** (2026) 062501

- First application using the Exascale system  
JUPITER @ FZJ
- Signal still strong enough for even-odd isotopes

↪ can extract the BEs of  $^{99}\text{Sn}$  to  $^{102}\text{Sn}$

Nucleus	N3LO	N3LO*	Exp.
$^{99}\text{Sn}$	$765 \pm 8$	$804 \pm 8$	$807.9 \pm 0.6$
$^{100}\text{Sn}$	$786.5 \pm 3.0$	$825.2 \pm 3.0$	$825.16 \pm 0.24$
$^{101}\text{Sn}$	$795 \pm 10$	$834 \pm 10$	$836.39 \pm 0.30$
$^{102}\text{Sn}$	$808.5 \pm 3.4$	$848.1 \pm 3.4$	$849.09 \pm 0.10$

Isotopes	N3LO	N3LO*	Exp.
102-101	$13.5 \pm 10.6$	$14.1 \pm 10.6$	$12.7 \pm 0.3$
102-100*	$22.5 \pm 4.6$	$22.9 \pm 4.6$	$23.9 \pm 0.3$
102-99	$43.5 \pm 8.7$	$44.1 \pm 8.7$	$41.2 \pm 0.6$
101-100	$8.5 \pm 10.4$	$8.9 \pm 10.4$	$11.2 \pm 0.4$
101-99*	$30.0 \pm 12.8$	$30.1 \pm 12.8$	$28.5 \pm 0.7$
100-99	$21.5 \pm 8.6$	$21.2 \pm 8.6$	$17.3 \pm 0.7$



# Prediction: Multi-neutron correlations in light nuclei I

28

Zhang, Elhatisari, UGM, 2512.18849 [nucl-th]

- $3n$  and  $4n$  clusters much thought about
- ↳ breakup, knockout & double charge exchange reactions
- ↳ investigate multi- $n$  correlations in light nuclei

- Use 282 chiral interactions to predict:

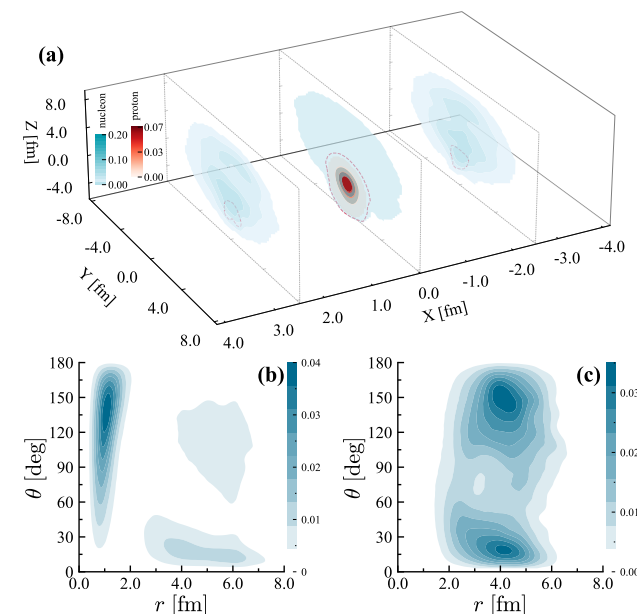
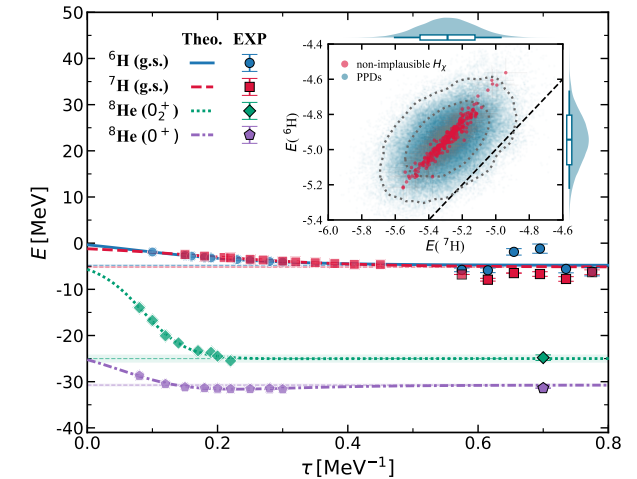
$$E_{\text{gs}}(^6\text{H}) = -4.94 \pm 0.14 \text{ MeV}$$

$$E_{\text{gs}}(^7\text{H}) = -5.29 \pm 0.16 \text{ MeV}$$

- Marginal posteriors suggest:

$$S_n(^7\text{H}) = 0.35 \pm 0.32 \text{ MeV}$$

- ↳ sequential  $^6\text{H} + n$  decay disfavored
- ↳ enhances the relevance of multi- $n$  correlations
- ↳ two-neutron correlations clearly visible

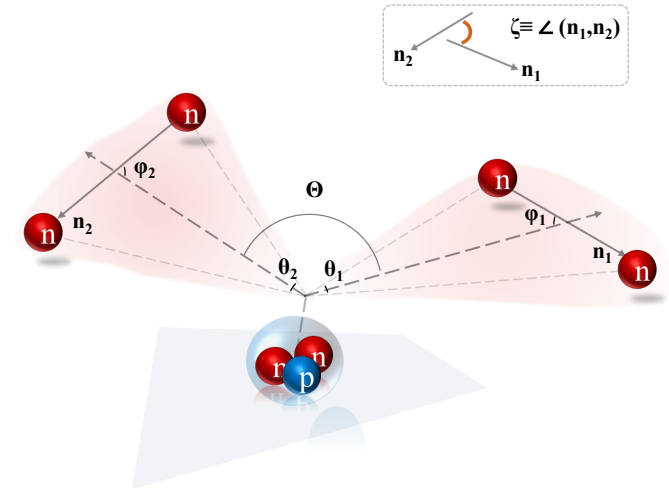


# Prediction: Multi-neutron correlations in light nuclei II <sup>29</sup>

Zhang, Elhatisari, UGM, 2512.18849 [nucl-th]

- Define four-neutron correlations via:

$$\begin{aligned} \rho_4(\theta_1, \varphi_1, \theta_2, \varphi_2; \Theta, \zeta) &= \sum_{i < j, k < l}^{(ij), (kl)} \left\langle \delta(\theta_{ij} - \theta_1) \delta(\varphi_{ij} - \varphi_1) \right. \\ &\quad \times \delta(\theta_{kl} - \theta_2) \delta(\varphi_{kl} - \varphi_2) \\ &\quad \left. \times \delta(\Theta_{ij,kl} - \Theta) \delta(\zeta_{ij,kl} - \zeta) \right\rangle, \end{aligned}$$

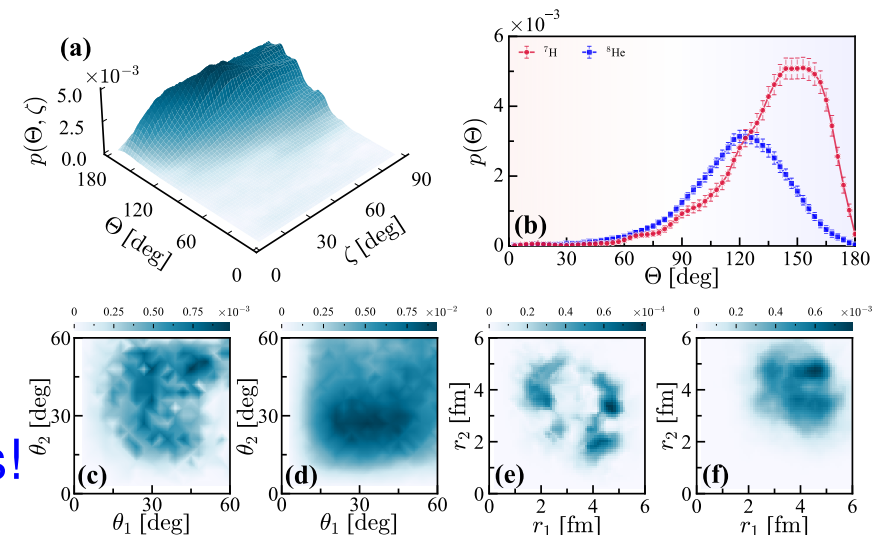


- The four-neutron correlations show:

↪ symmetric  $2n-2n$  configuration is dominant  
( $\sim 95\%$ ,  $\Theta \simeq 140^\circ - 160^\circ$ )

↪ compact tetraneutron configuration small  
( $\sim 5\%$ ,  $\Theta \simeq 60^\circ - 90^\circ$ )

↪ must consider the  $2n-2n$  config. in exp analysis!



# Prediction: Searching for the tetraneutron

Wu, Elhatisari, UGM, Shen, Geng, Kim, PRL **136** (2026) 142502

- Latest tetraneutron sighting in  ${}^4\text{He}({}^8\text{He}, {}^8\text{Be})4n$

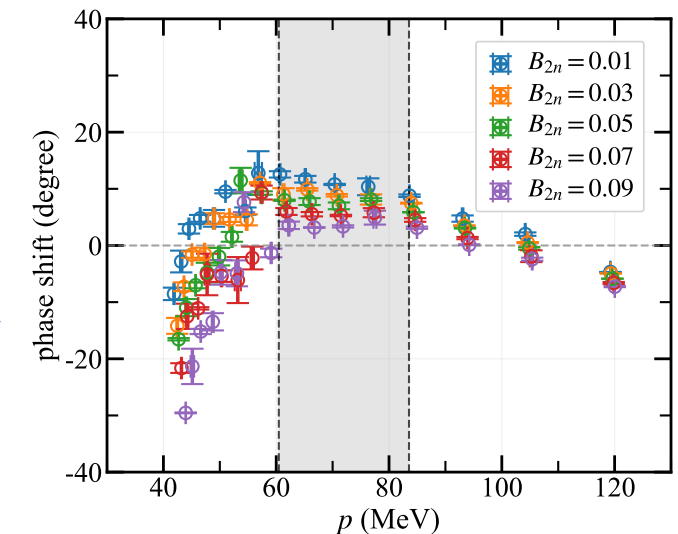
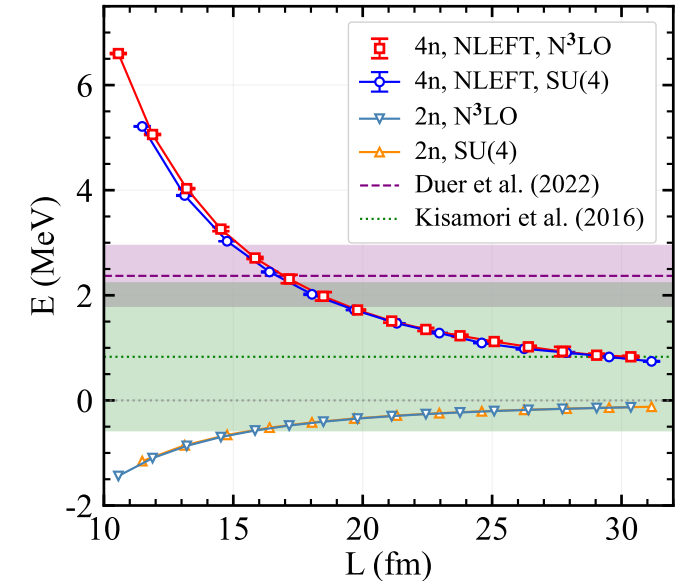
Duer et al., Nature **606** (2022) 678

- ↳ analysis presumably too simplified, various re-analyses show no resonance
- ↳ most theory analyses do not give a  $4n$  resonance

- Search on the lattice (SU(4) and N<sup>3</sup>LO interactions)

- ↳ volume-dependence shows no sign of a resonance

- ↳  $2n$ - $2n$  scattering shows a shallow attraction in the momentum range  $p \simeq 60 - 85$  MeV corresponds to confined  $4n$  energies of  $1.7 - 3.3$  MeV similar to the  $4n$  peak energies in experiment



# Prediction: Neutron-alpha scattering

Elhatisari, Hildenbrand, UGM, J. Phys. G **52** (2025) 125102

- $n$ - $\alpha$  scattering know to be a good testbed of 3NFs

Pieper, Pandharipande (1993), Quaglioni, Navratil (2008), Kravvaris et al. (2020), ...

- Use the time-honoured Lüscher approach

Lüscher, Nucl.Phys.B **354** (1991) 531

- Compare to  $R$ -matrix results

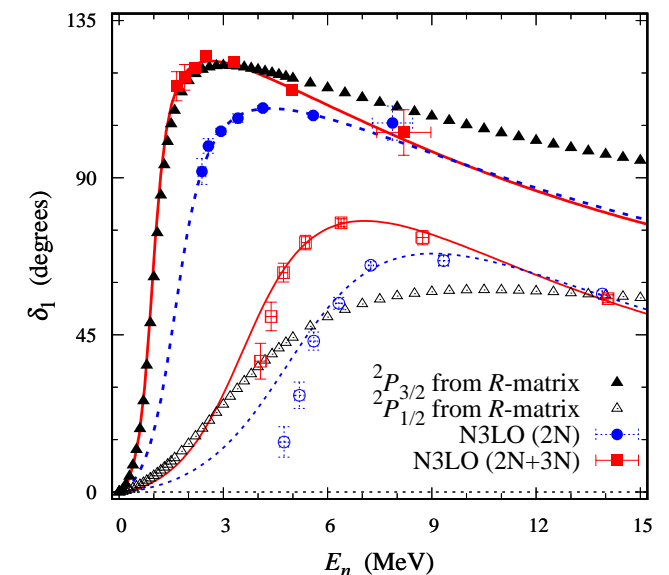
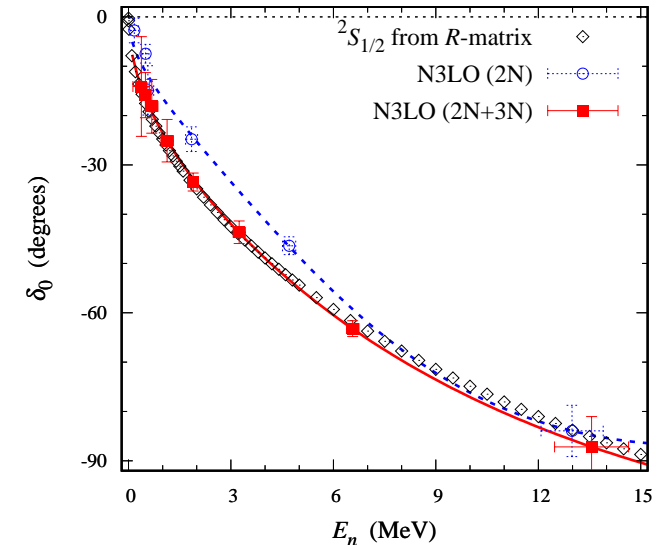
G. M. Hale, private communication (2023)

↪ S-wave ( ${}^2S_{1/2}$ ) well described when 3NFs are included

↪ Large  ${}^2P_{3/2}$  well described when 3NFs are included

↪ Visible deviation in the  ${}^2P_{1/2}$  wave for  $E_n \geq 3$  MeV

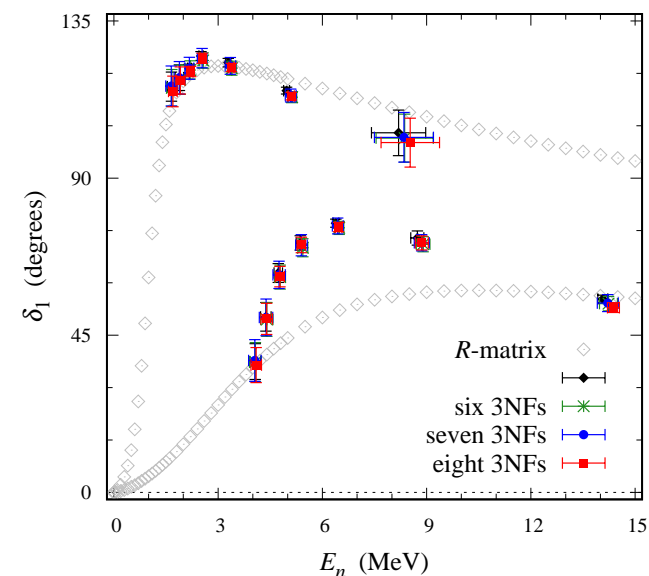
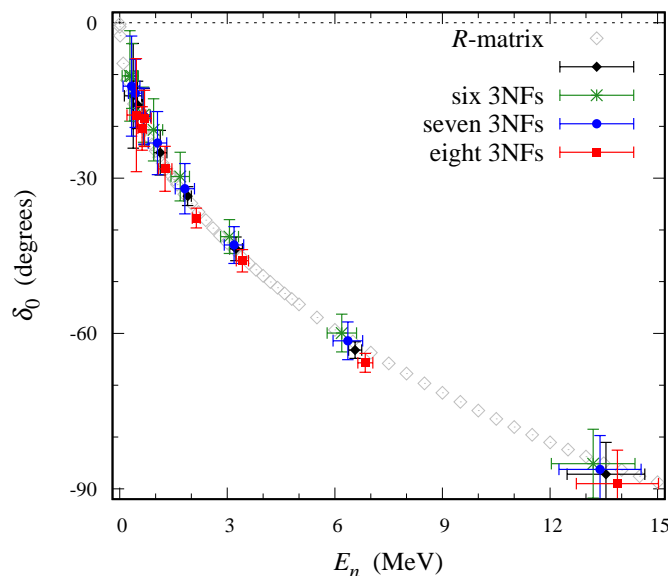
- What is the reason for the deviations in the  ${}^2P_{3/2}$ - ${}^2P_{1/2}$  splitting?



# Neutron-alpha scattering revisited

Elhatisari, Hildenbrand, UGM, J. Phys. **G 52** (2025) 125102

- Investigate one possible reason: Choice of nuclei to fix the LECs?
- Perform MCMC searches to explore systematically the set of 8 3NF ops and also reduce to sets with 6 and 7 ops and quantify the uncertainty using RSMD



↪ clearly, this is not the source of the problem

↪ must improve the 3NFs!

# Towards improved 3NFs

# A minimal set of 3NFs

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Consider sets of 6 3NF op's (called minimal set)

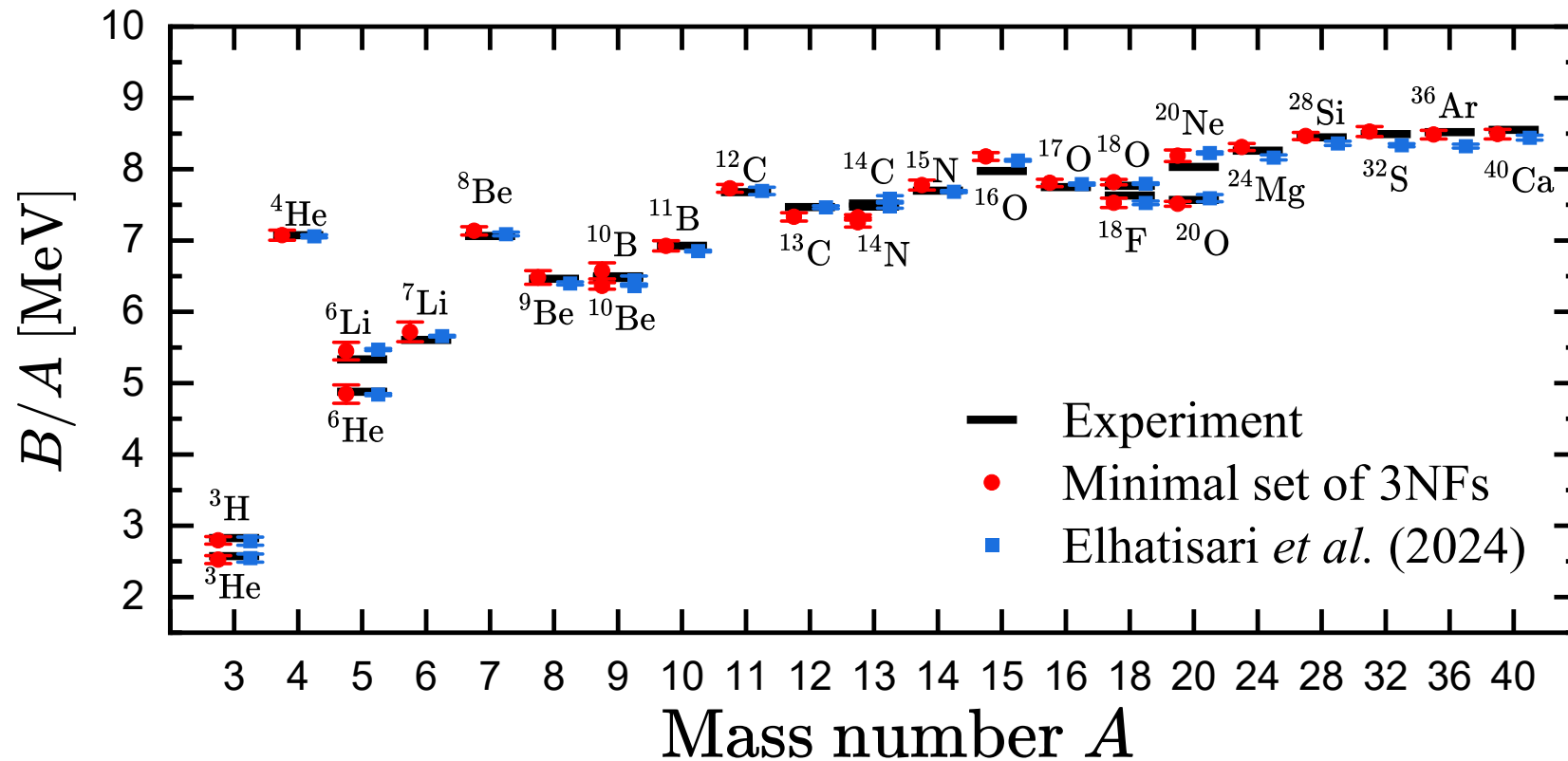
No.	$V_{3N}$ 's
1	$V_{cE,sNL1}^{(0)}$ , $V_{cD}^{(3)}$ , $[V_{cE}^{(0)}]_{044}$ , $[V_{cE}^{(0)}]_{224}$ , $[V_{cE}^{(0)}]_{125}$ , $[V_{cE}^{(0)}]_{145}$
2	$[V_{cE}^{(1)}]_{000}$ , $V_{cD}^{(3)}$ , $[V_{cE}^{(0)}]_{044}$ , $[V_{cE}^{(0)}]_{222}$ , $[V_{cE}^{(0)}]_{114}$ , $[V_{cE,sL}]_{033}$
3	$V_{cE,sNL1}^{(0)}$ , $V_{cD}^{(3)}$ , $[V_{cE}^{(0)}]_{044}$ , $[V_{cE}^{(0)}]_{224}$ , $[V_{cE}^{(0)}]_{125}$ , $[V_{cE}^{(0)}]_{235}$
4	$V_{cD}^{(0)}$ , $V_{cD}^{(2)}$ , $[V_{cE}^{(0)}]_{044}$ , $[V_{cE}^{(0)}]_{222}$ , $[V_{cE}^{(0)}]_{224}$ , $[V_{cE}^{(0)}]_{125}$
5	$V_{cD}^{(1)}$ , $V_{cD}^{(3)}$ , $[V_{cE}^{(0)}]_{033}$ , $[V_{cE}^{(0)}]_{044}$ , $[V_{cE}^{(0)}]_{222}$ , $[V_{cE}^{(0)}]_{125}$
6	$V_{cE,sNL1}^{(0)}$ , $V_{cD}^{(0)}$ , $V_{cD}^{(3)}$ , $[V_{cE}^{(0)}]_{044}$ , $[V_{cE}^{(0)}]_{224}$ , $[V_{cE}^{(0)}]_{125}$
7	$V_{cD}^{(1)}$ , $V_{cD}^{(3)}$ , $V_{cD,sNL2}^{(0)}$ , $[V_{cE}^{(0)}]_{044}$ , $[V_{cE}^{(0)}]_{224}$ , $[V_{cE}^{(0)}]_{125}$
8	$V_{cD}^{(0)}$ , $V_{cD}^{(2)}$ , $[V_{cE}^{(0)}]_{044}$ , $[V_{cE}^{(0)}]_{224}$ , $[V_{cE}^{(0)}]_{125}$ , $[V_{cE}^{(0)}]_{145}$
9	$[V_{cE}^{(0)}]_{000}$ , $V_{cD}^{(3)}$ , $[V_{cE}^{(0)}]_{224}$ , $[V_{cE}^{(0)}]_{125}$ , $[V_{cE,sL}]_{044}$ , $[V_{cE}^{(0)}]_{334}$
10	$[V_{cE}^{(0)}]_{000}$ , $V_{cE,sNL3}^{(0)}$ , $V_{cD}^{(1)}$ , $V_{cD,sNL3}^{(0)}$ , $[V_{cE}^{(0)}]_{224}$ , $[V_{cE}^{(0)}]_{125}$

- contains also non-local operators (marked in blue)
- only some sets contain the SU(4) symmetric  $V_{cE}^{(l,t)}$  [here:  $[V_{cE}^{(0)}]_{144}$ ,  $[V_{cE}^{(0)}]_{222}$ ]

# Determination of the 3NF LECs

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Fit to the BE of a selected number of nuclei and also include the  $^4\text{He}$  charge radius



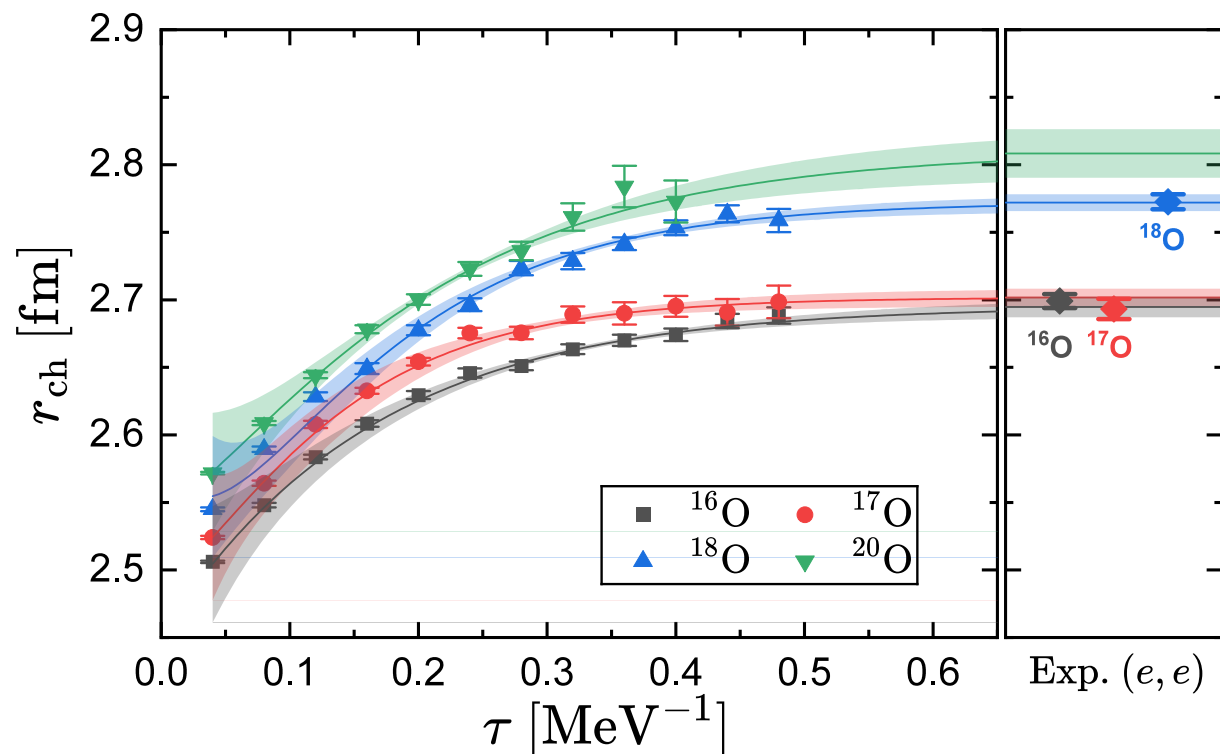
↪ Quality similar to the WFM 3NFs: RMSD = 0.101 MeV (1.4%) [0.093 MeV (1.1%)]

↪ use these now for the calculation of selected oxygen radii

# Radii of selected oxygen isotopes

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Use the partial pinhole algorithm to study the radii of  $^{16,17,18,20}\text{O}$  with  $M = 4$



Isotope	NLEFT	Exp.
$^{16}\text{O}$	2.704(17)	2.699(5)
$^{17}\text{O}$	2.709(15)	2.693(8)
$^{18}\text{O}$	2.768(17)	2.776(2)
$^{20}\text{O}$	2.810(32)	

all radii in [fm]

- Accuracy much improved (cf. WFM paper)
- Trend of the data reproduced:  $r_{\text{ch}}(^{16}\text{O}) \simeq r_{\text{ch}}(^{17}\text{O}) < r_{\text{ch}}(^{18}\text{O})$
- Prediction for  $r_{\text{ch}}(^{20}\text{O}) \rightarrow$  nice test

# Summary & outlook

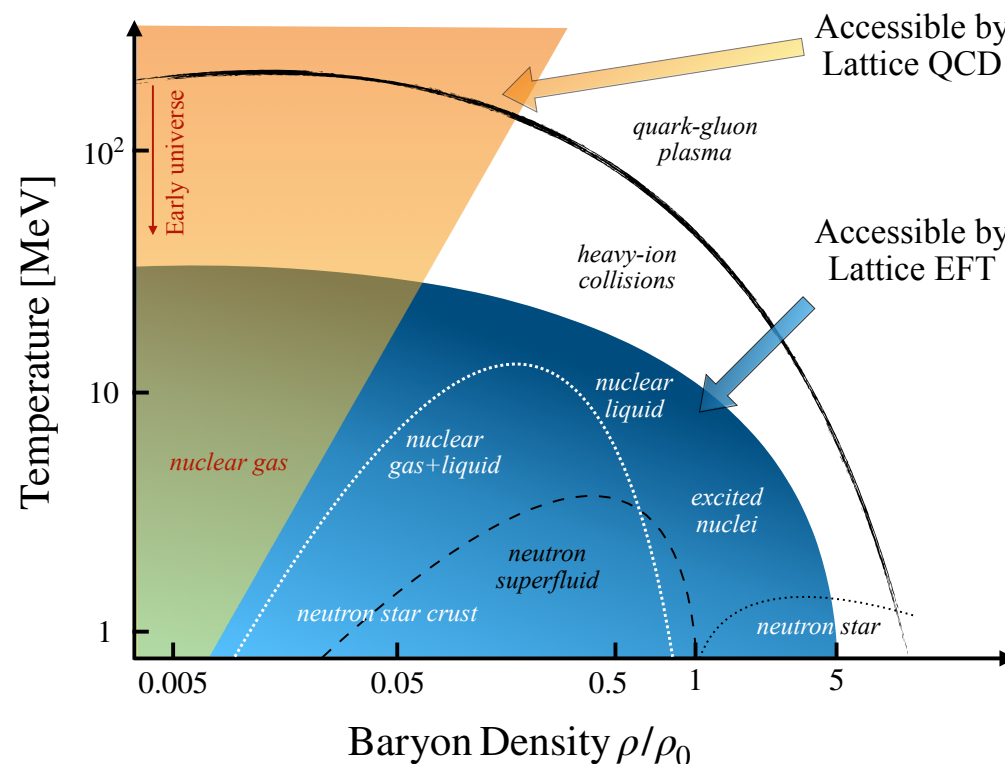
- Nuclear lattice simulations: a new quantum many-body approach
  - based on the successful continuum nuclear chiral EFT
  - a number of highly visible results already obtained
  
- Recent developments
  - NN(N) interaction at N<sup>3</sup>LO w/ wave function matching
    - ↔ first promising results for nuclear structure, matter and scattering
    - ↔ first results for  $\beta$ -decays [ultimately  $0\nu 2\beta$  decays]
      - Elhatisari, Hildenbrand, UGM, Phys. Lett. B **859** (2024) 139086 + PKU group
    - ↔ hypernuclei & neutron stars are under investigation
      - Hildenbrand et al., Eur. Phys. J. A **60** (2024) 215; Tong et al., 2509.26148
  
- Improved 3NFs are being worked out ↔ stay tuned!



SPARES

# Comparison to lattice QCD

LQCD (quarks & gluons)	NLEFT (nucleons & pions)
relativistic fermions	non-relativistic fermions
renormalizable th'y	EFT
continuum limit	no continuum limit
(un)physical masses	physical masses
Coulomb - difficult	Coulomb - easy
high T/small $\rho$	small T/nuclear densities
sign problem severe	sign problem moderate

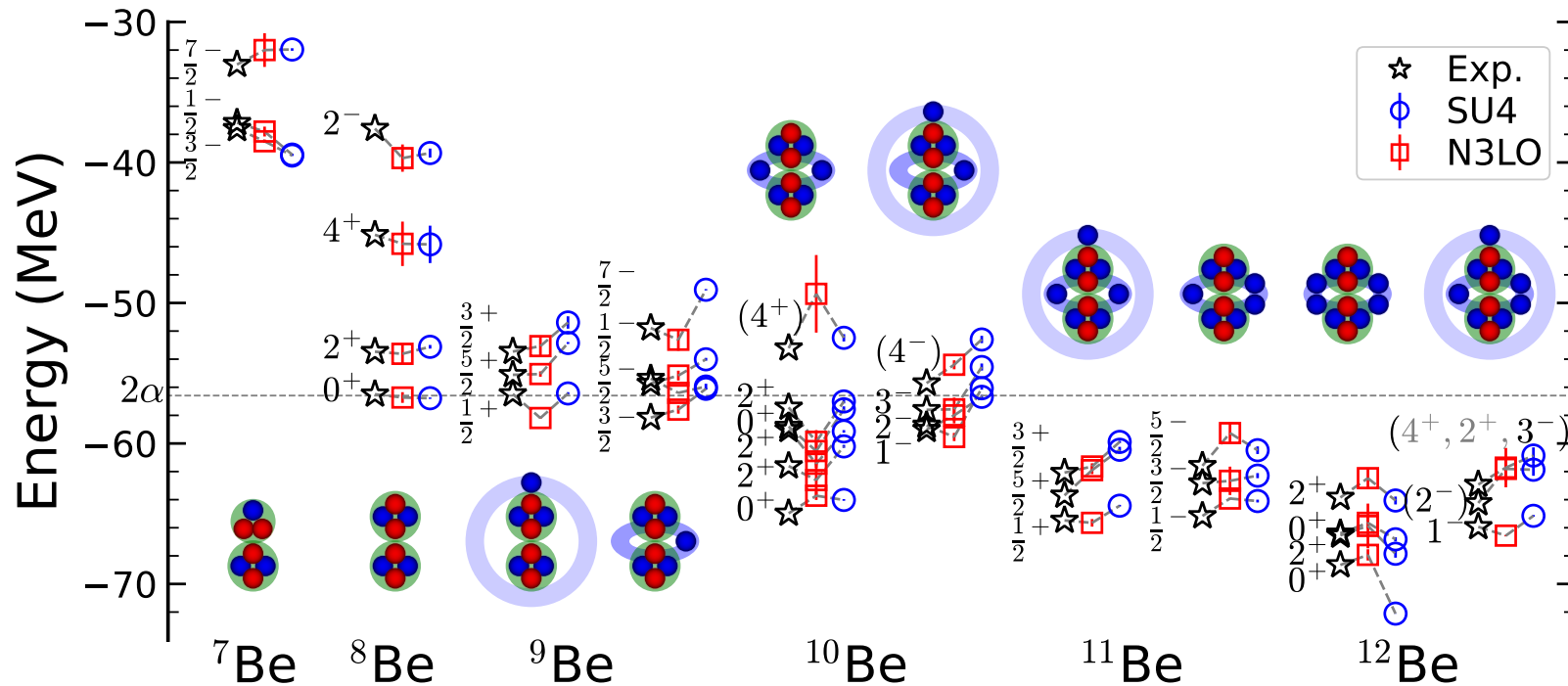


- For nuclear physics, NLEFT is the far better methodology!

# Prediction: Be isotopes

Shen et al., Phys. Rev. Lett. **134** (2025) 162503

- Systematic study of the Be isotopes & their radii & their em transitions:



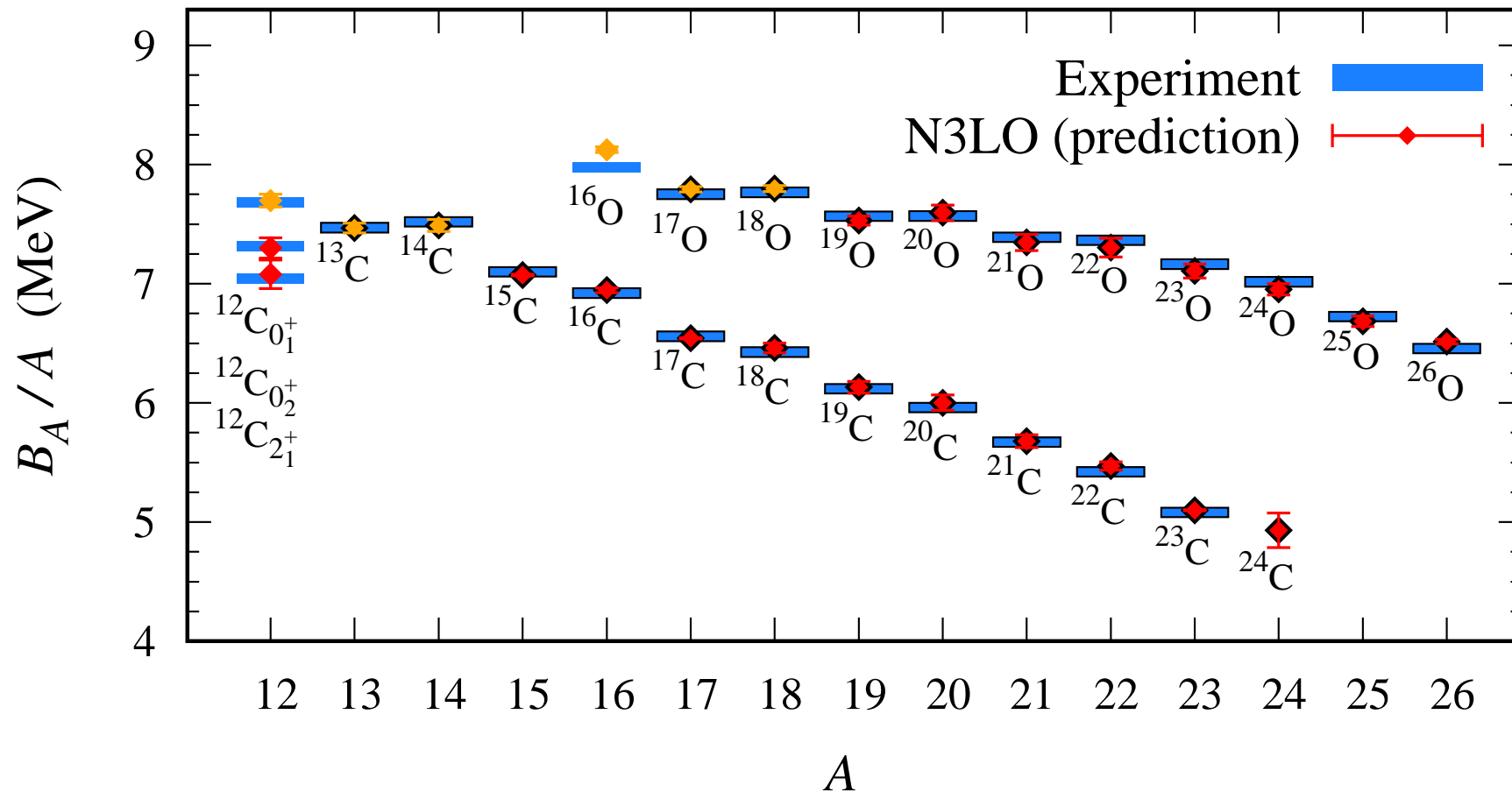
↪ new method to quantify nuclear shapes

↪ clusters, halos, molecular orbitals in **one shot**

# Prediction: Isotope chains of carbon & oxygen

Song et al., Phys. Lett. **B 872** (2026) 140086

- Towards the neutron drip-line in carbon and oxygen:



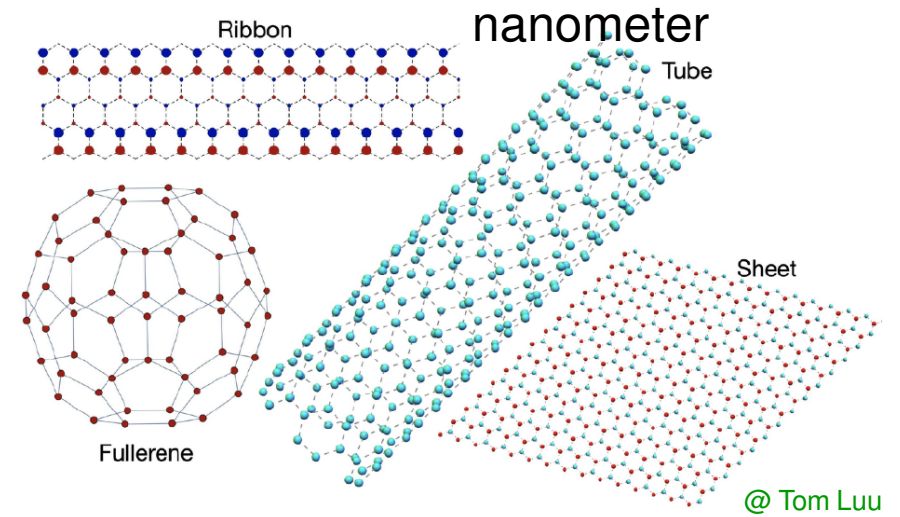
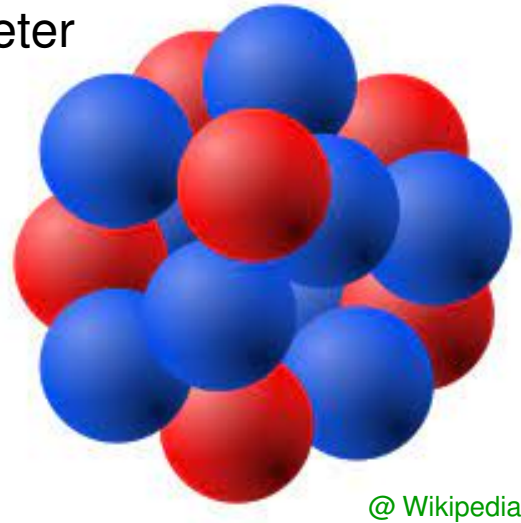
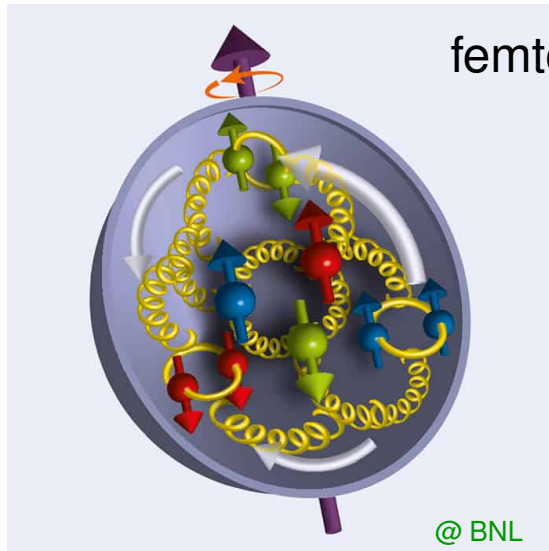
↪ 3NFs of utmost importance for the n-rich isotopes!

↪ universal features of neutron correlations

# Introduction: Why and how

# Strongly correlated fermionic systems

- Strongly correlated fermionic systems come in different forms, shapes and sizes



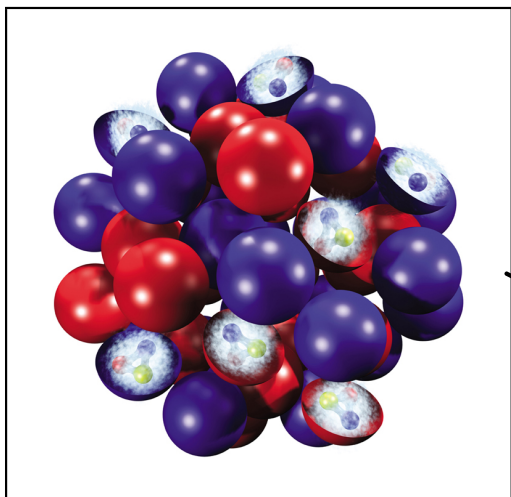
- ... and are a challenge in particle & nuclear & condensed matter physics as well as material science, quantum chemistry, ...

↔ the Pauli principle combined with strong interactions / correlations makes such systems difficult to compute / calculate / investigate theoretically

↔ I discuss here the marriage of Effective Field Theories w/ Monte Carlo simulations

# Intro to EFTs: Resolution matters

- Dynamics at long distances does not depend on what goes on at short distances
- Equivalently, low-energy interactions do not care about the details of high-energy interactions
- Or: you don't need to understand nuclear physics to build a bridge

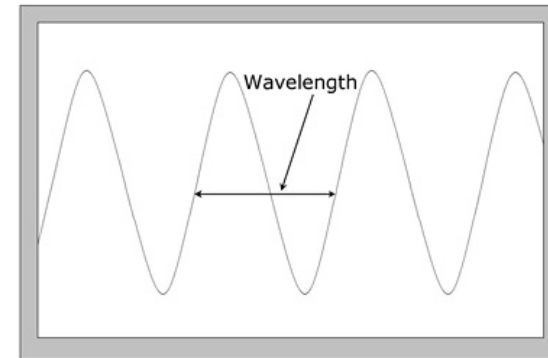


# Intro to EFTs: Organisation

- This is quite true, but how to make the idea precise and quantitative?
- necessary & sufficient ingredients to construct an **Effective Field Theory**:
  - ★ *scale separation* – what is low, what is high?
  - ★ *active degrees of freedom* – what are the building blocks?
  - ★ *symmetries* – how are the interactions constrained by symmetries?
  - ★ *power counting* – how to organize the expansion in low over high?
- a note on units for a quantum particle ( $\hbar = c = 1$ )

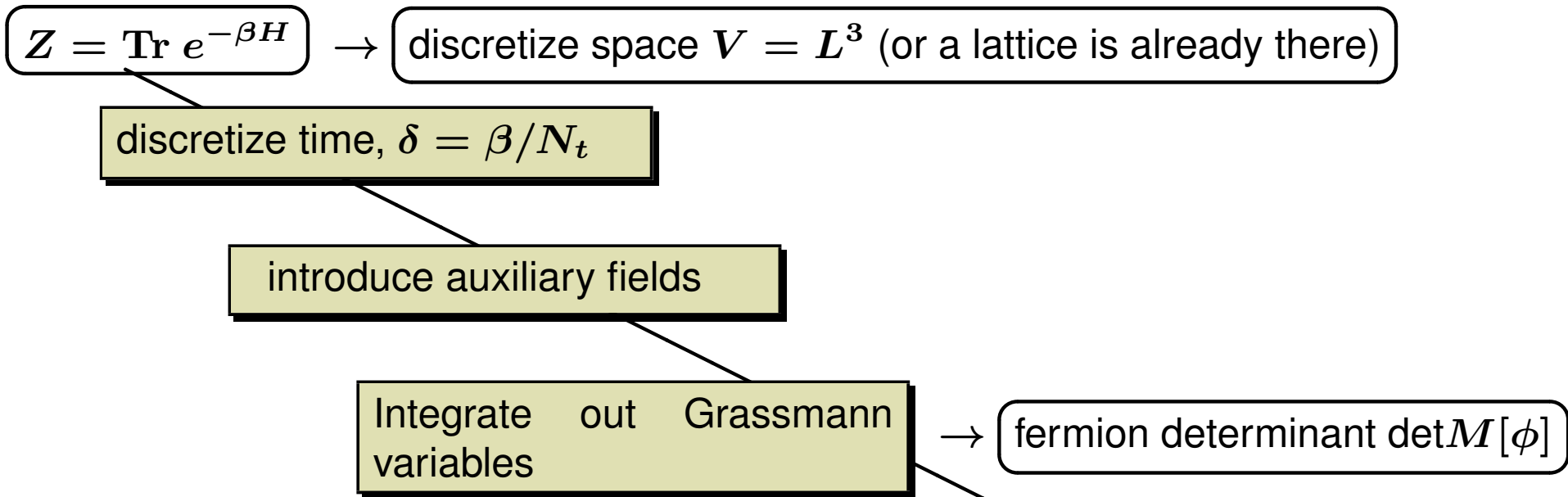
$$p \sim \frac{1}{\lambda}, \quad E = p \quad \text{or} \quad E = \frac{p^2}{2m}$$

→ long wavelength ↔ low momentum

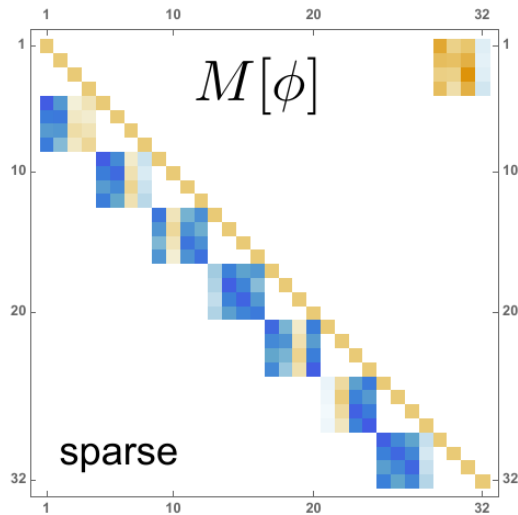


# Intro to Monte Carlo simulations: Basics

- Just outline schematically the basic steps:



... and so on ...



$$\underbrace{\int \mathcal{D}[\phi] \det M[\phi] \exp \left( -\frac{1}{2V_{\text{int}} \delta} \sum_{x,t} \phi^2(x,t) \right)}_{\int \mathcal{D}[\phi] \exp(-S[\phi])}$$

# Intro to Monte Carlo simulations: The sign problem

---

- At finite chemical potential (density) or doping,  $\det M$  is no longer positive definite

↪ the basic assumption of a probability distribution no longer holds, the method fails

↪ this is an **NP hard** problem

Troyer, Wiese, Phys. Rev. Lett. **94** (2005) 170201

↪ must tailor problem-dependent solutions to mitigate this

- discuss three methods here:

- \* Wigner's SU(4) symmetry in nuclear physics

Wigner, Phys. Rev. **51** (1937) 106

- \* Wave function matching (applied to nuclear physics here but more general)

Elhatisari et al., Nature **630** (2024) 8015, 59

- \* Lefschetz thimbles (contour deformations) (applied to low-d materials)

Cristoforetti et al., Phys. Rev. D **88** (2013) 051501(R)

