Hyperon-nucleon interaction and light hypernuclei in chiral effective field theory

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(Hoai Le, Ulf-G. Meißner, Andreas Nogga)

Outline

Hyperon physics - recent developments

- Role of hyperons in neutron stars ("hyperon puzzle")
  Neutron stars with masses $\geq 2M_{\odot} \Rightarrow$ stiff equation of state (EoS)
  With increasing density $n \rightarrow \Lambda \Rightarrow$ softening of the EoS
  $\Rightarrow$ Conventional explanations of observed mass-radius relation fail
- New measurements of Λp cross sections by the CLAS Collaboration at JLab
  New extended measurements of ΣN observables in the E40 experiment at J-PARC
  differential cross sections for $\Sigma^+ p$, $\Sigma^- p$
- Measurements of two-particle momentum correlation functions by the STAR, HADES, and ALICE Collaborations
  (Λp, $\Lambda\Lambda$, $\Xi^- p$, ...)
- HAL QCD: Lattice QCD simulations for YN interactions for quark masses close to the physical point ($M_\pi \approx 145$ MeV)
- Progress in *ab initio* methods like no-core shell model (NCSM)
  microscopic calculations of hypernuclei up to $A \geq 10$

BB interaction in chiral effective field theory

Baryon-baryon interaction in SU(3) χEFT à la Weinberg (1990)

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three-baryon forces and external current operators in a consistent way

- degrees of freedom: octet baryons (N, Λ, Σ, Ξ), pseudoscalar mesons (π, K, η)

- pseudoscalar-meson exchanges
- contact terms – represent unresolved short-distance dynamics
  involve low-energy constants (LECs) that need to be fixed
  by a fit to data

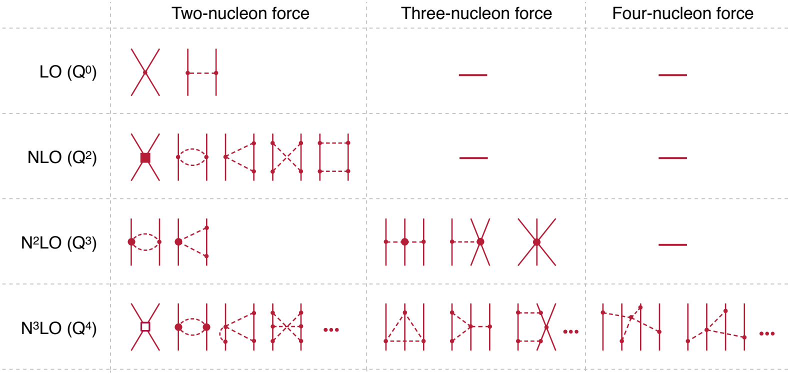
ΛN-ΣN interaction

LO: H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244
NLO13: J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24
NLO19: J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91
SMS NLO, N^{2}LO: J.H., U.-G. Meißner, A. Nogga, H. Le, EPJA 59 (2023) 63

(*BB* systems with strangeness $S = -1$ to -6)

Extension of chiral EFT interaction up to N^{2}LO

(Nucleon-nucleon forces in chiral EFT (E. Epelbaum))

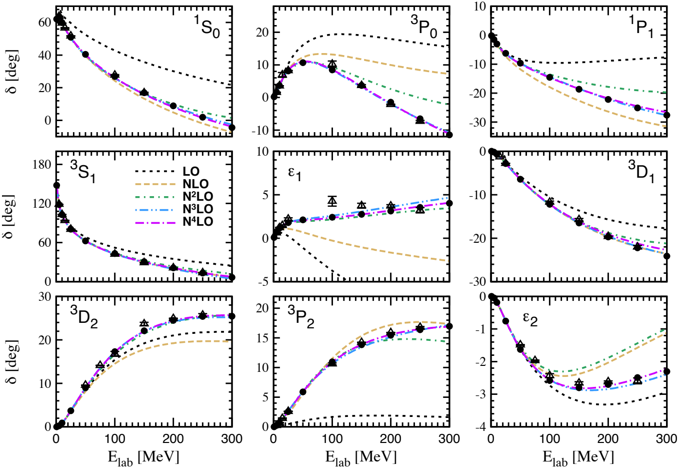


N^{2}LO: no additional *BB* contact terms (no new LECs) in the two-body sector

leading-order three-body forces (3BFs)

NN interaction in chiral EFT

Semilocal momentum-space (SMS) regularized chiral *NN* potential



(Reinert, Krebs, Epelbaum, EPJA 54 (2018) 86) [up to N^{4}LO (N^{4}LO$^+$) !!]

LO to NLO: drastic change in all partial waves

NLO to N^{2}LO: changes mostly in *P*-waves and higher partial waves

chiral YN potential up to N^{2}LO

adopt the framework of Reinert, Krebs, Epelbaum, EPJA 54 (2018) 86:
"Semilocal momentum-space regularized (SMS) chiral NN potentials"

- employ a regulator that minimizes artifacts from cutoff Λ

nonlocal cutoff $\quad (\vec{q} = \vec{p}\,' - \vec{p}) \quad$ (used in NLO13 and NLO19 YN potentials)

$$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{p'^4+p^4}{\Lambda^4}}}{\vec{q}^2 + M_\pi^2} \to \frac{1}{\vec{q}^2 + M_\pi^2}\left[1 - \frac{p'^4 + p^4}{\Lambda^4} + \mathcal{O}(\Lambda^{-8})\right]$$

local cutoff:

$$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{\vec{q}^2+M_\pi^2}{\Lambda^2}}}{\vec{q}^2 + M_\pi^2} \to \frac{1}{\vec{q}^2 + M_\pi^2} - \frac{1}{\Lambda^2} + \frac{\vec{q}^2 + M_\pi^2}{\Lambda^4} + \ldots$$

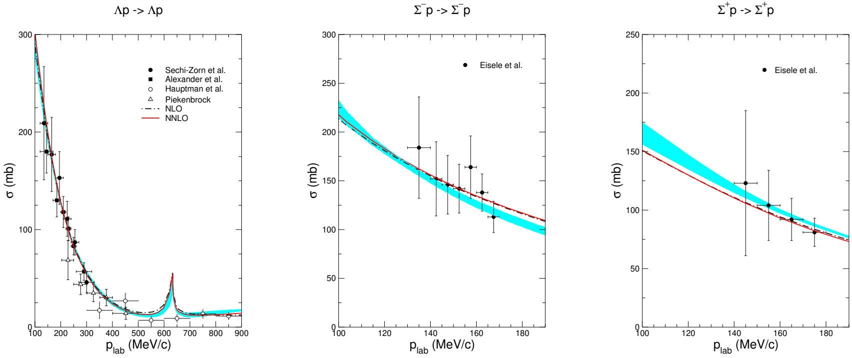
does not affect long-range physics at any order in the $1/\Lambda^2$ expansion

applicable to 2π exchange too:

$$V_{2\pi} = \frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{\vec{q}^2 + \mu^2} \to V_{2\pi}^{\rm reg} = e^{-\frac{\vec{q}^2}{2\Lambda^2}} \frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{\vec{q}^2 + \mu^2} e^{-\frac{\mu^2}{2\Lambda^2}} \quad + \ldots$$

NN: $\Lambda = 350$-550 MeV (π) $\qquad YN$: $\Lambda = 500$-600 MeV (π, K, η)

Results for SMS YN interactions



SMS YN potentials up to NLO, N^{2}LO (with $\Lambda = 550$ MeV)

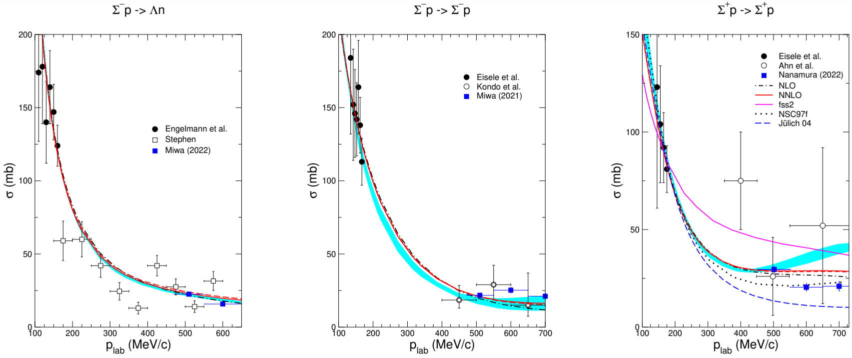
(J.H., U.-G. Meißner, A. Nogga, H. Le, EPJ A 59 (2023) 63)

NLO19: J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91

quality of the fit – total χ^2 (36 data points):
NLO19(600): 16.0   SMS NLO: 15.2   SMS N^{2}LO: 15.6

cross sections dominated by S-waves (are already well described at NLO)
$\rightarrow$ (as expected) practically no change when going to N^{2}LO

Results for SMS YN interactions



integrated cross sections at higher energies not included in the fitting process!

$\Sigma^+ p \to \Sigma^+ p$ and $\Sigma^- p \to \Sigma^- p$ cross sections:

$$\sigma = \frac{2}{\cos\theta_{\max} - \cos\theta_{\min}} \int_{\cos\theta_{\min}}^{\cos\theta_{\max}} \frac{d\sigma(\theta)}{d\cos\theta} d\cos\theta$$

$\cos\theta_{\min} = -0.5$; $\cos\theta_{\max} = 0.5$

fss2 ... Fujiwara et al. (constitutent quark model) Jülich 04, Nijmegen NSC97f ... meson-exchange potentials

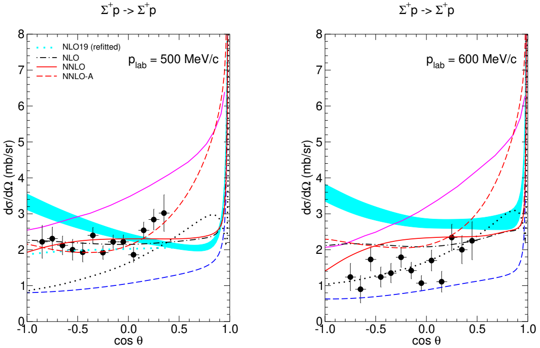
Results for SMS YN interactions

$\Sigma^+ p$

T. Nanamura et al.,
PTEP 2022 (2022) 093D01

$440 \leq p_{lab} \leq 550$ MeV/c
($T_{lab} \approx 100$ MeV)

$550 \leq p_{lab} \leq 650$ MeV/c
($T_{lab} \approx 150$ MeV)



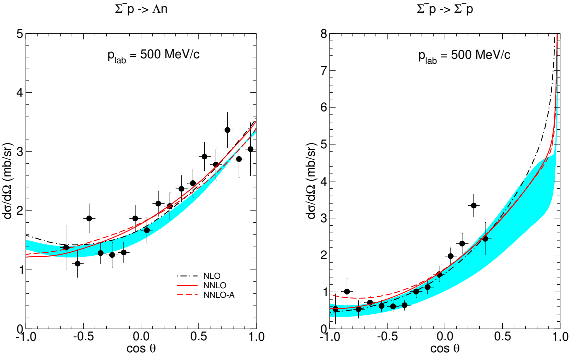
LECs in the $^1S_0$, $^3S_1$-$^3D_1$ fixed from low-energy YN cross sections

SMS NLO: LECs in ^3P-waves taken over from NN fit (RKE)
(strict SU(3) symmetry: $V_{NN} \equiv V_{\Sigma^+ p}$ in the $^1S_0$, $^3P_{0,1,2}$ partial waves!)

SMS N^{2}LO: LECs in P-waves fitted to the E40 data (two trials)!

data suggest a drop from $440 \leq p \leq 550$ MeV/c to $550 \leq p \leq 650$ MeV/c!
effect of $\Lambda p \pi^+$ threshold (≈ 600 MeV/c)?

Results for SMS YN interactions



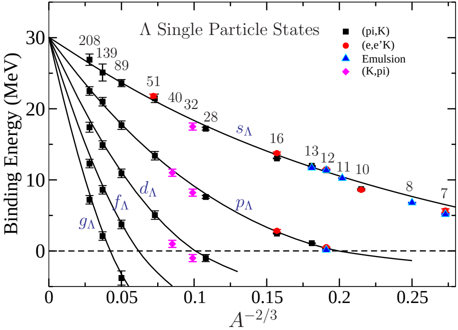
$\Sigma^- p \to \Lambda n$: quite well reproduced by NLO19 (NLO13) and SMS YN potentials
$\Sigma^- p \to \Sigma^- p$: behavior at forward angles remains unclear

$\Sigma^- p$ and $\Sigma^- p \to \Lambda n$ data for $(550 \le p \le 650)$ MeV/c are reproduced with comparable quality

- no unique determination of all P-wave LECs possible
- one needs data from additional channels (Λp, $\Sigma^- p \to \Sigma^0 n$, ...)
- one needs additional differential observables (polarizations, ...)

The Λ in infinite nuclear matter

Gal, Hungerford, Millener, Rev. Mod. Phys. 88 (2016) 035004



Fit to the separation energies of heavy Λ hypernuclei with a standard Woods-Saxon potential V_{WS} representing the Λ-nucleus interaction with depth $V_0 = -30.05$ MeV, radius $R = r_0\, A^{1/3}$, where $r_0 = 1.165$ fm, and diffusivity $a = 0.6$ fm

Λ binding in infinite nuclear matter $\Rightarrow U_\Lambda \approx -30$ MeV !

Λ and Σ in infinite nuclear matter

non-relativistic lowest order Brueckner theory (Bethe-Goldstone equation):

$$
\begin{aligned}
\langle YN|G_{YN}(\zeta)|YN\rangle &= \langle YN|V|YN\rangle \\
&+ \sum_{Y'N} \langle YN|V|Y'N\rangle \, \langle Y'N|\frac{Q}{\zeta - H_0}|Y'N\rangle \, \langle Y'N|G_{YN}(\zeta)|YN\rangle
\end{aligned}
$$

Q ... Pauli projection operator

$$\zeta = E_Y(p_Y) + E_N(p_N)$$

$$E_\alpha(p_\alpha) = M_\alpha + \frac{p_\alpha^2}{2M_\alpha} + U_\alpha(p_\alpha), \qquad \alpha = \Lambda, \Sigma, \; N$$

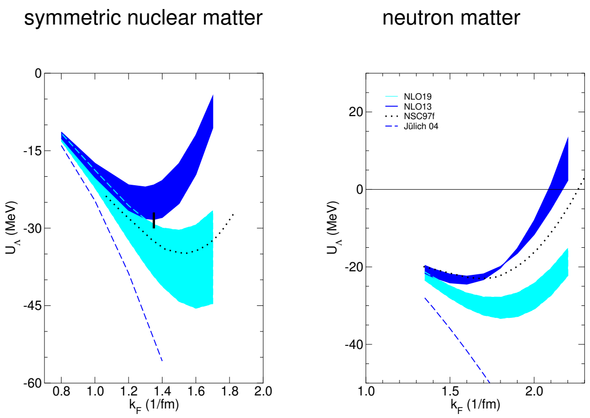
U_α ... single-particle potential

$$U_Y(p_Y) = \int_{p_N \le k_F} d^3 p_N \, \langle YN|G_{YN}(\zeta(U_Y))|YN\rangle$$

$B_Y(\infty) = -U_Y(p_Y = 0)$ - evaluated at saturation point of nuclear matter

⇒ J.H., U.-G. Meißner, NPA 936 (2015) 29; S. Petschauer, et al., EPJA 52 (2016) 15
J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91

k_F dependence of $U_\Lambda(p_\Lambda = 0)$



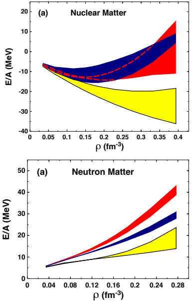
- NLO13 and NLO19 are almost phase equivalent
- NLO13 characterized by a stronger ΛN-ΣN coupling potential ($^3S_1$-$^3D_1$)

- Bethe-Goldstone equation for coupled channels: strong dispersive effects
- contributions from three-body forces are missing

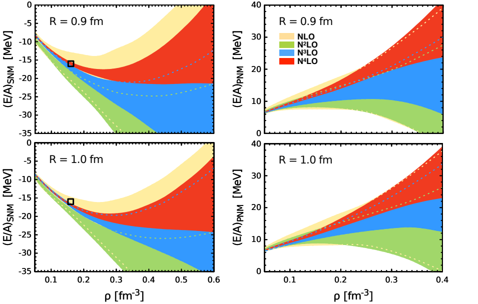
(S. Petschauer et al., NPA 957 (2017) 347): ΛNN force $\rightarrow$ density-dependent effective ΛN interaction

Uncertainties in the purely nucleonic case

F. Sammarruca et al., PRC 91 (2015) 054311
(NLO, N^{2}LO, N^{3}LO)



J. Hu et al., PRC 96 (2017) 034307
(*NN* potentials from Epelbaum et al., EPJA 51 (2015) 53)



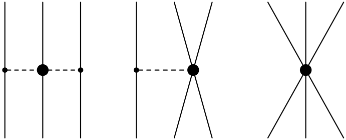
Alternative schemes:

- Holt, Kaiser, Weise (PRC 81 (2010) 024002): consider density-dependent corrections to the in-medium *NN* interaction that result from three-nucleon forces
- Hebeler et al. (PRC 83 (2011) 031301): Use low-momentum interaction (soft Hamiltonians) so that high-order corrections to the Hartree-Fock scheme can be evaluated easily
- J. Oller (JPG: Nucl. Part. Phys. 46 (2019) 073001): power counting scheme for in-medium chiral perturbation theory
- phenomenological approaches: mean field models, ...

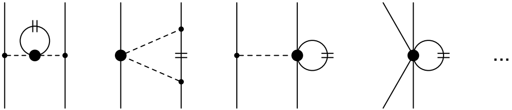
Density dependent effective YN interaction

(for application to heavy hypernuclei and hyperons in infinite nuclear matter)

three-body force:



$\Rightarrow$ density dependent effective YN interaction:



close two baryon lines by sum over occupied states within the Fermi sea
arising 3BF LECs can be constrained by resonance saturation (via decuplet baryons)
($\rightarrow$ 1 for ΛNN, 2 for ΣNN, ΞNN, ...)

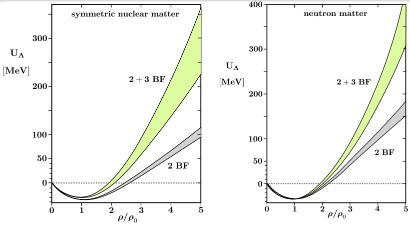
J.W. Holt, N. Kaiser, W. Weise, PRC 81 (2010) 064009 (for NNN)
S. Petschauer et al., NPA 957 (2017) 347 (for ΛNN)
D. Gerstung et al., EPJA 56 (2020) 175 (ΛNN, ΣNN)

Implications for neutron stars (incl. chiral 3BF)

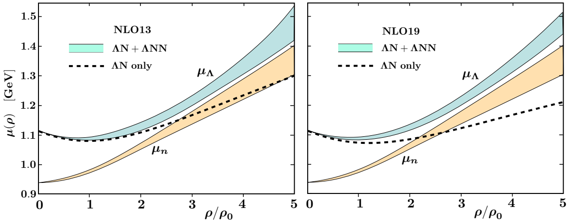
D. Gerstung et al.,
EPJA 56 (2020) 175
(NLO13 & NLO19)

include ΛNN, ΣNN 3BFs

(in form of density-dependent effective
ΛN, ΣN potentials)



Chemical potentials of the Λ hyperon (μ_Λ) and the neutron (μ_n)



$\mu_\Lambda(\rho) \leq \mu_n(\rho) \Rightarrow$ energetically favorable to replace n by Λ $\quad (\mu_\Lambda(\rho) = M_\Lambda + U_\Lambda(\rho))$

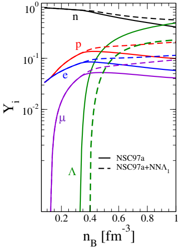
ΛN only: equation-of-state becomes too soft to support 2 $M_\odot$ neutron stars (“hyperon puzzle”)

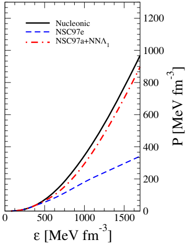
ΛN + ΛNN: appearance of Λs can be avoided

Logoteta, Vidaña, Bombaci,
EPJA 55 (2019) 207
(Nijmegen NSC97 potentials)

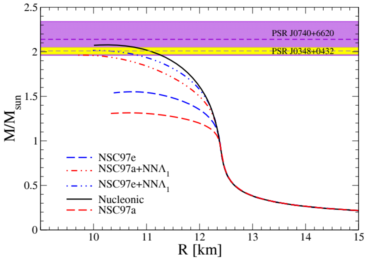
Composition and EoS
of neutron star matter

($n_B \equiv \rho$)





Mass-radius relation without and with chiral ΛNN force



	$M_{max}(M_\odot)$	R (km)	n_c (fm^{-3})
Nucleonic	2.08	10.26	1.15
NSC97a	1.31	10.60	1.40
NSC97a+NNΛ_1	1.96	9.80	1.30
NSC97a+NNΛ_2	1.97	9.87	1.28
NSC97e	1.54	10.81	1.18
NSC97e+NNΛ_1	2.01	10.10	1.20
NSC97e+NNΛ_2	2.02	10.15	1.19

Hypernuclei within the no-core shell model (NCSM)

Basic idea: use harmonic oscillator states and soft interactions

- m-scheme uses single particle states (center-of-mass motion not separated)
- antisymmetrization for nucleons easily performed (Slater determinant)
- larger dimensions
  (applications to p-shell hypernuclei by Wirth & Roth; $A \leq 13$)

Jacobi-NCSM

- uses relative (Jacobi) coordinates (Hoai Le et al., EPJA 56 (2020) 301)
- explicit separation of center-of-mass motion possible
- antisymmetrization for nucleons difficult but feasible for $A \leq 9$
- small dimensions

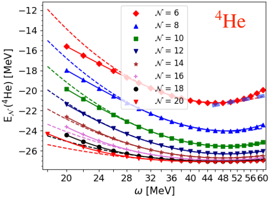
Soft interactions: Similarity renormalization group (SRG) (unitary transformation)

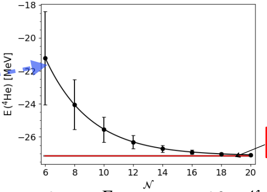
$$\frac{dH(s)}{ds} = [[T, H(s)], H(s)] \qquad H(s) = T + V(s) \qquad V(s) : V^{NN}(s), V^{YN}(s)$$

- Flow equations are solved in momentum space
- parameter (cutoff) $\lambda = \left(4\mu_{BN}^2/s\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- $V(s)$ is phase equivalent to original interaction
- transformation leads to induced 3BFs, 4BFs, ...
  (induced 3BFs included in the work of Wirth & Roth and in our recent studies)
  (induced 4BFs are most likely very small)

slide from Hoai Le:

- **extrapolation of energies:**
  - $E_b(\omega, \mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$
  - $E_{\mathcal{N}} = E_\infty + Ae^{-b\mathcal{N}}$





NN: SMS N^{4}LO$^+$(450)
$\lambda = 7\ \mathrm{fm}^{-1}$

$E_{FY} = -27.15 \pm 0.02$ MeV

$E_\infty = -27.146 \pm 0.062$ MeV

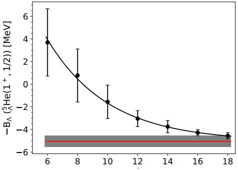
$\delta E = E_\infty - E_{\mathcal{N}_{max}}$

- lowest $E_{\mathcal{N},\omega_{opt}}$ are used for $\mathcal{N}$-space extrapolation ✓
- estimated uncertainties are rather conservative

- **extrapolation of Λ separation energies:** $B_\Lambda = E_{nucl} - E_{hyp}$
  - strong correlations between $E_{nucl}(\mathcal{N})$, $E_{hypnucl}(\mathcal{N})$

→ $B_{\Lambda,\mathcal{N}} = E_{nucl}(\mathcal{N}) - E_{hypnucl}(\mathcal{N})$

$$B_{\Lambda,\mathcal{N}} = B_{\Lambda,\infty} + A_1 e^{-b_1\mathcal{N}}$$



YN: SMS N^{2}LO(550)

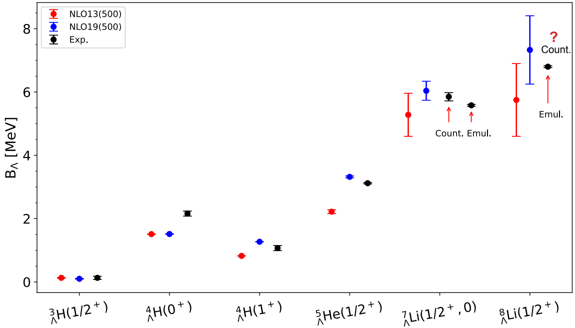
$\lambda_{YN} = 7\ \mathrm{fm}^{-1}$

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56 (2020)

Results for $B_\Lambda (A \leq 8)$

Hoai Le et al., PRC 107 (2023) 024002

- NLO13 and NLO19 are almost phase equivalent
- NLO13 characterized by a stronger ΛN-ΣN coupling potential ($^3S_1$-$^3D_1$)

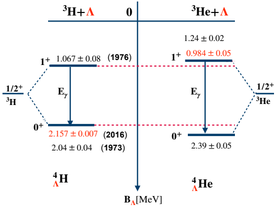


Experiment: M. Jurič et al. NPB 52 (1973); E.Botta et al., NPA 960 (2017) 165

NN: SMS N^{4}LO$^+$(450) + 3NF: N^{2}LO(450)
YN: NLO13(19) + SRG-induced YNN force – but no chiral YNN forces!

- NLO13 underestimates separation energies
- NLO19 describes $^4_\Lambda$He(1$^+$), $^5_\Lambda$He, $^7_\Lambda$Li fairly well

CSB in the ${}^4_\Lambda$He - ${}^4_\Lambda$H hypernuclei



Schulz et al (2016); Yamamoto et al (2015); Juric et al (1973); Bedjidian et al (1976,1979)

$$\Delta E(1^+) = B_\Lambda({}^4_\Lambda\text{He}, 1^+) - B_\Lambda({}^4_\Lambda\text{H}, 1^+) = -83 \pm 94 \text{ keV}$$

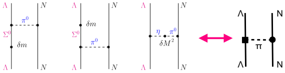
$$\Delta E(0^+) = B_\Lambda({}^4_\Lambda\text{He}, 0^+) - B_\Lambda({}^4_\Lambda\text{H}, 0^+) = 233 \pm 92 \text{ keV}$$

$$\Delta E({}^3\text{H}, {}^3\text{He}) \sim 683 + 81 \text{ keV}$$ **(R. Brandenburg et al NPA 294(1978))**

683: **Coulomb**; 81: $\boldsymbol{\Delta M(p,n)}$

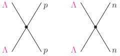
CSB YN interactions at NLO **(J. Haidenbauer, U.-G. Meißner, A. Nogga FBS 62(2021))**

- sub-leading contributions are dominant:



$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle\Sigma^0|\delta m|\Lambda\rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle\pi^0|\delta M^2|\eta\rangle}{M_\eta^2 - M_{\pi^0}^2}\right] f_{\Lambda\Sigma\pi}$$

(Dalitz, von Hippel, 1964)



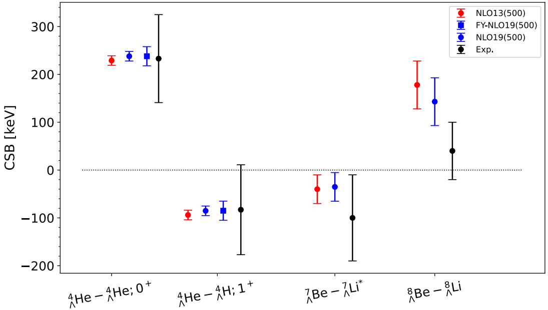
C_s^{CSB}, C_t^{CSB} adjusted to $\Delta E(0^+, 1^+)$

(fm//keV)	$a_s^{\Lambda p}$	$a_s^{\Lambda n}$	δa_s	$a_t^{\Lambda p}$	$a_t^{\Lambda n}$	δa_t
NLO19(**500**) no CSB	**-2.91**	**-2.91**	**0**	**-1.42**	**-1.41**	**-0.01**
CSB(**500**)	**-2.65**	**-3.20**	**0.55**	**-1.58**	**-1.47**	**-0.11**
CSB(550)	-2.64	-3.21	0.57	-1.52	-1.41	-0.11
CSB(600)	-2.63	-3.23	0.6	-1.47	-1.36	-0.09
CSB(650)	-2.62	-3.23	0.61	-1.46	-1.37	-0.09

→ **cutoff (and YN) independent prediction for $a(\Lambda n)$**

CSB results for A=4,7,8 hypernuclei

Hoai Le, J.H., U.-G. Meißner, A. Nogga, PRC 107 (2023) 024002



NN:SMS N^{4}LO+(450)

+3N: N^{2}LO(450)

+YN: NLO13,19(CSB)

+SRG-induced YNN

- NLO13 & NLO19 CSB results for A=7 **are comparable to experiment.**
- two potentials predict a somewhat **larger CSB in A=8 doublet as compared to experiment**

→ ▸ experimental CS splitting for A=8 could be **larger than 40 ± 60 keV?**

- CSB estimate for $A = 4$ too large? different spin-dependence?

STAR Collaboration (M. Abdallah et al., PLB 834 (2022) 137449)

$\Delta B_\Lambda(^4_\Lambda\text{He - }^4_\Lambda\text{H}; 0^+) = 160 \pm 140$ keV; $\Delta B_\Lambda(^4_\Lambda\text{He - }^4_\Lambda\text{H}; 1^+) = -160 \pm 140$ keV

Separation energies for A=3-8 Λ hypernuclei (MeV)

- NLO13(19), SMS NLO,N^{2}LO are phase equivalent ($\chi^2 \approx 16$ for 36 YN data points)
- NLO13 characterized by a stronger ΛN-ΣN coupling potential ($^3S_1$-$^3D_1$)

	$^3_\Lambda\mathrm{H}$ [Faddeev]	$^4_\Lambda\mathrm{He}(0^+)$	$^4_\Lambda\mathrm{He}(1^+)$	$^5_\Lambda\mathrm{He}$	$^7_\Lambda\mathrm{Li}$	$^8_\Lambda\mathrm{Li}$
NLO13	0.090	1.48 ± 0.02	0.58 ± 0.02	2.22 ± 0.06	5.28 ± 0.68	5.75 ± 1.08
NLO19	0.091	1.46 ± 0.02	1.06 ± 0.02	3.32 ± 0.03	6.04 ± 0.30	7.33 ± 1.15
SMS NLO	0.124	2.10 ± 0.02	1.10 ± 0.02	3.34 ± 0.01		
SMS N^{2}LO	0.139	2.02 ± 0.02	1.25 ± 0.02	3.71 ± 0.01		
Exp.*	0.164 ± 0.04	2.347 ± 0.036	0.942 ± 0.036	3.102 ± 0.03	5.85 ± 0.13	6.80 ± 0.03
					5.58 ± 0.03	

NLO19 (600): $^4_\Lambda\mathrm{He}(1^+)$, $^5_\Lambda\mathrm{He}$, $^7_\Lambda\mathrm{Li}$ fairly well described
NLO13 (600) underestimates most separation energies
SMS NLO,N^{2}LO (550): $^4_\Lambda\mathrm{He}(0^+, 1^+)$, $^5_\Lambda\mathrm{He}$ fairly well described
($^3_\Lambda\mathrm{H}$ is used to constrain the strength of the ΛN singlet/triplet interaction!)

are the variations due to (missing) chiral YNN forces?
chiral YNN forces appear at N^{2}LO
$\Rightarrow$ estimate size of YNN forces from truncation error in the chiral expansion

* Chart of Hypernuclides *https://hypernuclei.kph.uni-mainz.de/*

Uncertainty quantification for EFTs

- Uncertainty for a given observable $X(p)$:
  (EKM: Epelbaum, Krebs, Meißner, EPJA 51 (2015) 53, S. Binder et al., PRC 93 (2016) 044002)

  estimate uncertainty via
  - the expected size of higher-order corrections
  - the actual size of higher-order corrections

$$
\begin{aligned}
\Delta X^{LO} &= Q^2 |X^{LO}| \qquad (X^{NLO} \approx Q^2 X^{LO}) \qquad \text{expansion parameter}: \; Q \sim M_\pi / \Lambda_b \approx 140/600 \\
\Delta X^{NLO} &= \max\left(Q^3 |X^{LO}|, Q^1 |\delta X^{NLO}| \right); \quad \delta X^{NLO} = X^{NLO} - X^{LO} \\
\Delta X^{N^2LO} &= \max\left(Q^4 |X^{LO}|, Q^2 |\delta X^{NLO}|, Q^1 |\delta X^{N^2LO}| \right); \quad \delta X^{N^2LO} = X^{N^2LO} - X^{NLO} \\
&\cdots
\end{aligned}
$$

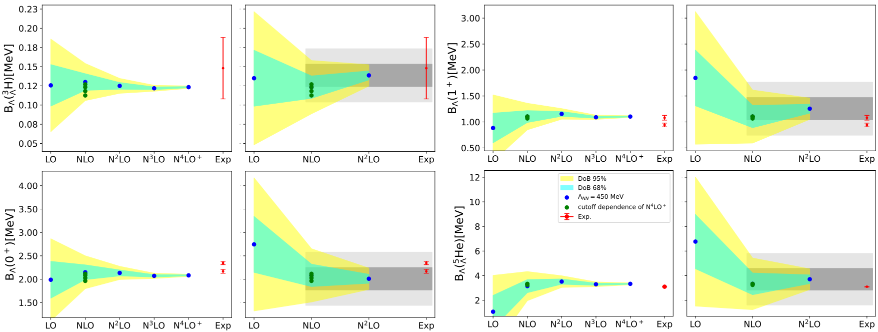
- Bayesian approach (Furnstahl, Klco, Phillips, Melendez):
  (Furnstahl et al., PRC 92 (2015) 024005; Melendez et al., PRC 100 (2019) 044001)

$$
\begin{aligned}
X^{(k)} &= X^{(0)} + \sum_{i=2}^{k} \delta X^{(i)} =: X_{ref}(c_0 + c_2 Q^2 + c_3 Q^3 + \dots) \\
\Delta X^{(k)} &= X_{ref} \left(\sum_{n=k+1}^{\infty} c_n Q^n \right); \quad c_n \sim \mathcal{O}(1); \; c_n | \bar{c}^2 \sim \mathcal{N}(0, \bar{c}^2); \; \bar{c}^2 \sim \chi^{-2}(\nu_0, \tau_0^2)
\end{aligned}
$$

$\bar{c}^2$ ... marginal variance; ν_0 ... prior degrees-of-freedom; τ_0^2 ... prior scale (pointwise model)
Q, $\bar{c}^2$, etc. ... deduced from order-by-order calculations, prior expectations, consistency plots

Truncation error within the Bayesian approach

Hoai Le et al., arXiv:2308.01756



- *NN*: SMS LO - N^{4}LO$^+$ (+ N^{2}LO *NNN* force)
- *YN*: SMS LO, NLO, N^{2}LO

- excellent convergence for *NN* interaction
- uncertainty is dominated by the truncation in *YN* interaction

- effect of *YNN* 3BF $\simeq$ half of 68% DoB interval for NLO result

Truncation error for separation energies B_Λ (MeV)

Truncation error at NLO provides an estimate (upper limit) for the contribution of the leading order ΛNN (and ΣNN) 3BF to the separation energies B_Λ

$$\Delta X^{NLO} \sim |X^{N^2LO}_{YN} - X^{NLO}_{YN}|, \; |X^{N^2LO}_{YNN}|$$

	Bayesian approach		EKM			
	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$	$\Delta(NN)$	$\Delta(YN)$	$\Delta(NN)$	$\Delta(YN)$
			Q = 0.31		Q = 0.40	
${}^{3}_{\Lambda}$H	0.01	0.02	0.01	0.02	0.01	0.02
${}^{4}_{\Lambda}$He (0^+)	0.16	0.24	0.06	0.30	0.13	0.39
${}^{4}_{\Lambda}$He (1^+)	0.11	0.21	0.07	0.36	0.09	0.47
${}^{5}_{\Lambda}$He	0.53	0.88	0.64	1.1	0.83	1.4

$\Rightarrow$ expect YNN 3BF contributions of 20 keV (${}^{3}_{\Lambda}$H), 250 keV (${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He), 900 keV (${}^{5}_{\Lambda}$He)

- Kamada et al. (PRC 108 (2023) 024004): explicit inclusion of 2π exchange ΛNN 3BF

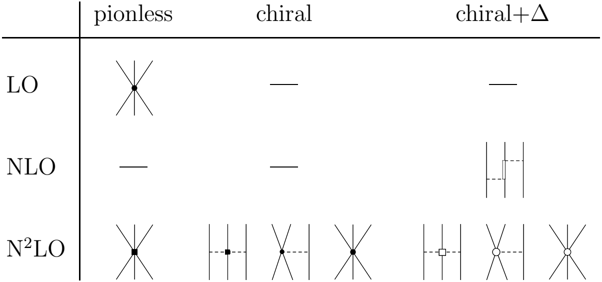
$\Rightarrow$ $\Delta B_\Lambda \approx 20$ keV (and repulsive!) (based on NLO13, NLO19)

Three-body forces are not observables!

two-body off-shell ambiguities ⇔ three-body forces (Polyzou & Glöckle, 1990)

depend on degrees of freedom considered in the calculations
(N, Λ only ... or Σ, Δ, Σ^*, ...)

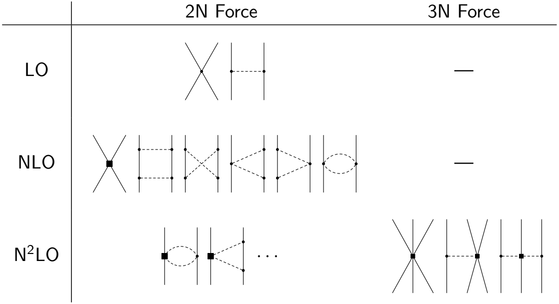
different degrees of freedom in the effective field theory



- different counting schemes
- different hierarchy of 3BFs

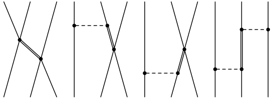
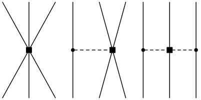
(Hammer, Nogga, Schwenk, Rev. Mod. Phys. 85 (2013) 197)

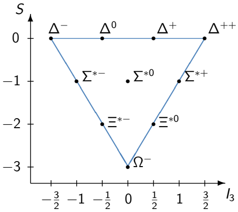
Three-baryon forces in chiral EFT



- 3N force (van Kolck, PRC 49 (1994) 2932; ... E. Epelbaum et al., PRC 66 (2002) 064001)
  2 LECs in 3N force: $D\,(c_D)$, $E\,(c_E)$ $\rightarrow$ have to be fixed in 3N sector
  (e.g., ^{3}H binding energy + ^{4}He binding energy)
  (2π exchange 3N force: c_1, c_3, c_4 ... fixed from πN scattering)
  number of LECs small because of the Pauli principle
- *BBB* force in SU(3) chiral EFT (S. Petschauer et al., PRC 93 (2016) 014001)
  BBB contact terms: 18 LECs (ΛNN: 3 LECs)
  one-meson exchange terms: 14 LECs (ΛNN: 2 LECs)
  two-meson exchange terms: 10 LECs ... (b_0, b_D, b_F, $b_{1,2,3,4}$, $d_{1,2,3}$)

Three-baryon forces with decuplet baryons

	3B Force with decuplet baryons
LO	—
NLO	
N^2LO	 $+ \cdots$

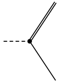


NNN: inclusion of the $\Delta(1232)$ resonance

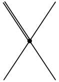
Epelbaum, Krebs, Meißner, NPA 806 (2008) 65; Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81 (2009) 1773

Decuplet (resonance) saturation + SU(3) symmetry

- new vertices:



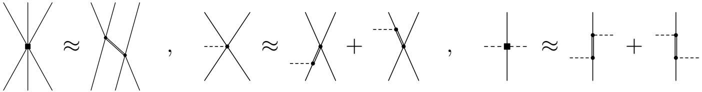
one constant ($C = \frac{3}{4} g_A \approx 1$ from $\Delta \to N\pi$)



two constants (Pauli-forbidden in nucleonic sector)

	tensor products in *flavor* space	and in *spin* space
final state	$\mathbf{10} \otimes \mathbf{8} = \mathbf{35} \oplus \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{8}$	$\mathbf{3/2} \otimes \mathbf{1/2} = \mathbf{1} \oplus \mathbf{2}$
initial state	$\mathbf{8} \otimes \mathbf{8} = \underbrace{\mathbf{27} \oplus \mathbf{8_s} \oplus \mathbf{1}}_{\text{symmetric}} \oplus \underbrace{\mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8_a}}_{\text{antisymmetric}}$	$\mathbf{1/2} \otimes \mathbf{1/2} = \underbrace{\mathbf{0}}_{\text{a.sym.}} \oplus \underbrace{\mathbf{1}}_{\text{sym.}}$

- estimate chiral three-baryon forces via decuplet saturation:



ΛNN: 1 LEC ($\Lambda N \leftrightarrow \Sigma(1385)N$ contact term)

ΛNN-ΣNN, ΣNN: 1 additional LEC ($\Sigma N \leftrightarrow \Sigma(1385)N$ contact term)

$\Rightarrow$ 3BF involves only 2 LECs ... to be fixed from $B_\Lambda(^4_\Lambda\text{H})$, ...

Summary

Hyperon-nucleon interaction within chiral EFT

- ΛN-ΣN interaction within semilocal momentum-space regularized chiral EFT
  confirm our previous YN results (up to NLO) based on a nonlocal regulator
  successful extension to N^{2}LO
  new $\Sigma^{\pm}p$ differential cross sections around $p_{lab} \approx$ 500 MeV/c can be described
  unique determination of the P-waves is not yet possible

Λ in infinite nuclear matter

- $U_\Lambda(p_\Lambda = 0) \approx -30$ MeV ... but large cutoff dependence
- effective (density dependent) YN contributions from three-body forces
  stabilize the results
  provide sufficient repulsion to resolve the so-called hyperon puzzle

Hypernuclei

- three-body forces: are small for ${}^{3}_{\Lambda}\mathrm{H}$, as expected
  moderate for ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$, ${}^{5}_{\Lambda}\mathrm{He}$ ... needs to be quantified/confirmed by explicit inclusion of 3BFs
  $\rightarrow$ LECs of 3BF could be fixed from B(${}^{4}_{\Lambda}$H), ...
- charge-symmetry breaking in ${}^{4}_{\Lambda}\mathrm{H} - {}^{4}_{\Lambda}\mathrm{He}$
  can be reproduced when taking into account the full leading CSB potential within chiral EFT
- charge-symmetry breaking in $A = 7 - 8$ Λ-hypernuclei
  predicted CSB splitting for ${}^{7}_{\Lambda}$Be, ${}^{7}_{\Lambda}$Li*, ${}^{7}_{\Lambda}$He is in line with experiments
  CSB splitting for ${}^{8}_{\Lambda}$Be, ${}^{8}_{\Lambda}$Li is overestimated