Ab initio calculation of neutron and hyperneutron matter/stars

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by ERC, EXOTIC

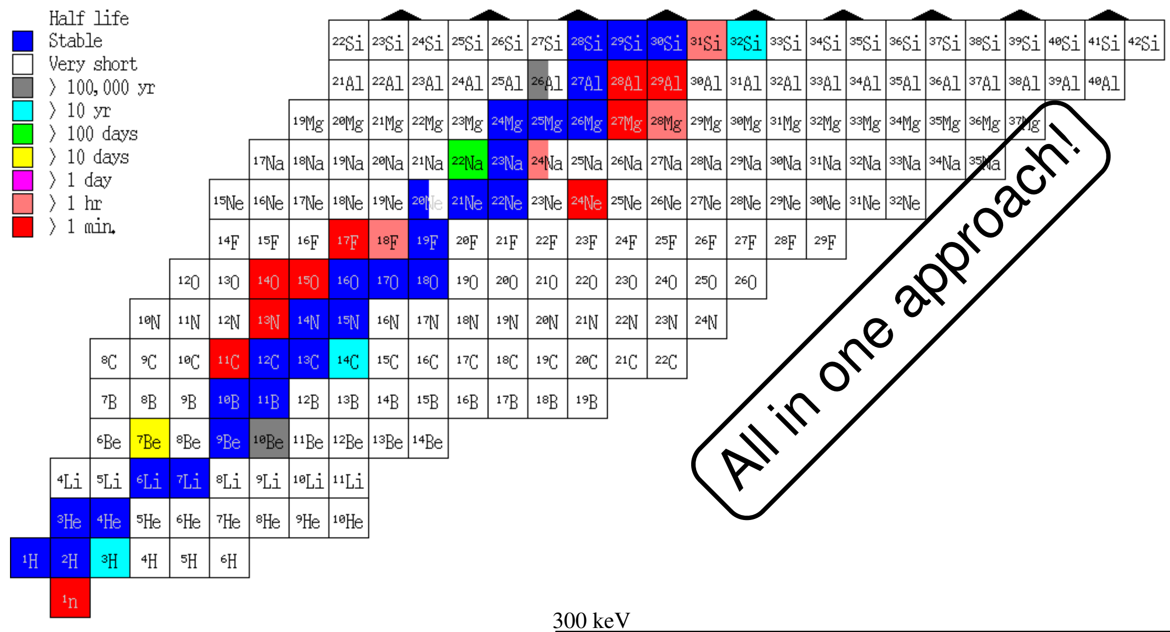
by NRW-FAIR

Contents

Very brief Introduction

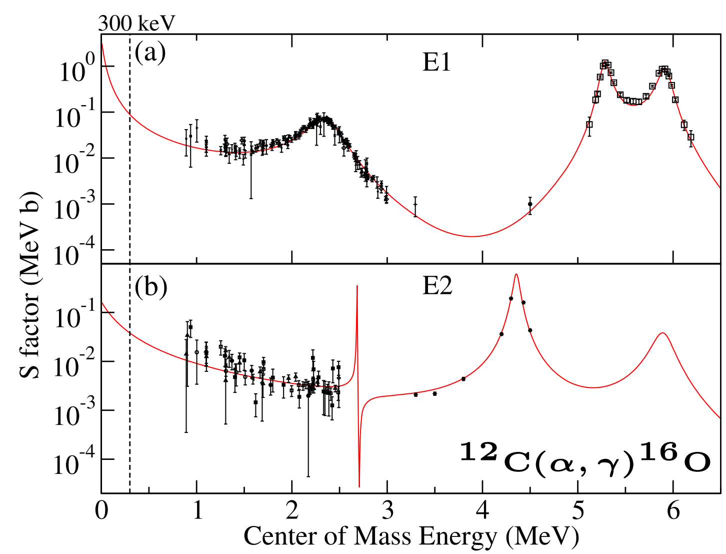
Our goal: Ab initio nuclear structure & reactions

- Nuclear structure:
  - ⋆ limits of stability
  - ⋆ 3-nucleon forces
  - ⋆ alpha-clustering
  - ⋆ EoS & neutron stars
  - ⋮



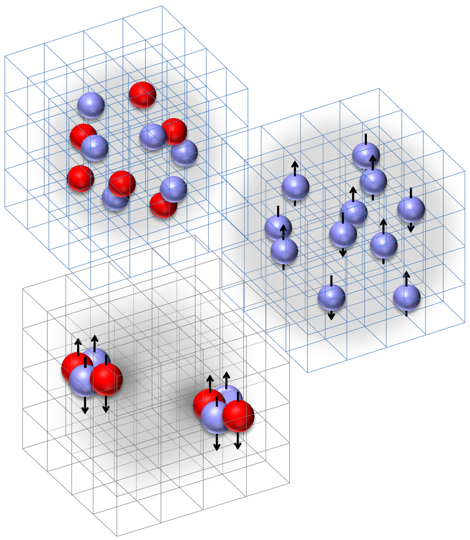
Ⓒ National Nuclear Data Center

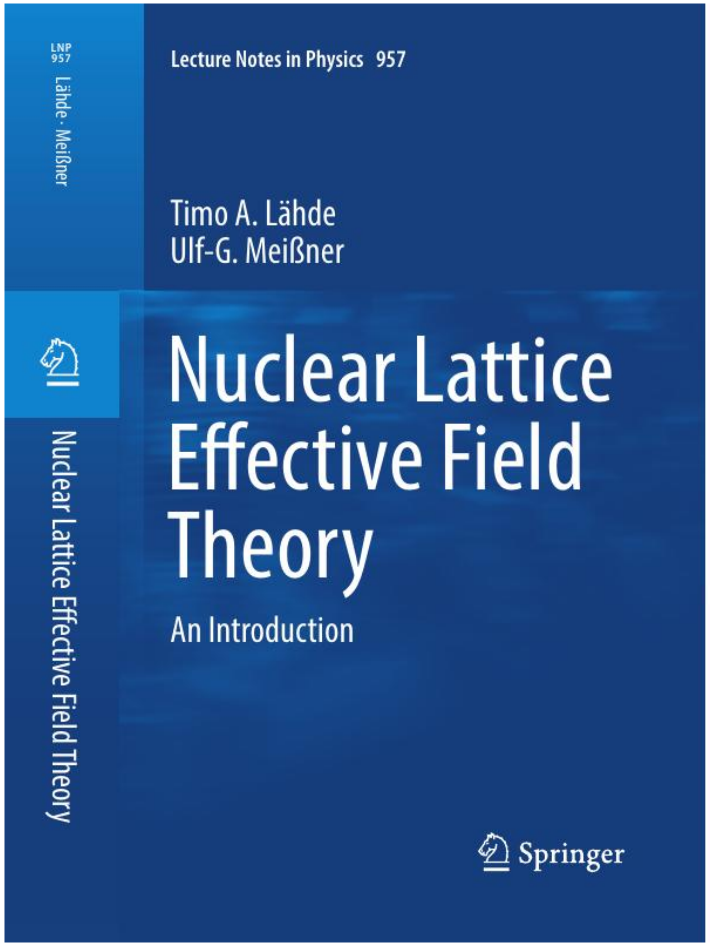
- Nuclear reactions, nuclear astrophysics:
  - ⋆ alpha-particle scattering
  - ⋆ triple-alpha reaction
  - ⋆ alpha-capture on carbon
  - ⋮



de Boer et al, Rev. Mod. Phys. **89** (2017) 035007

Chiral EFT on a lattice





T. Lähde & UGM

Nuclear Lattice Effective Field Theory - An Introduction

Springer Lecture Notes in Physics **957** (2019) 1 - 396

Nuclear lattice effective field theory (NLEFT)

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem

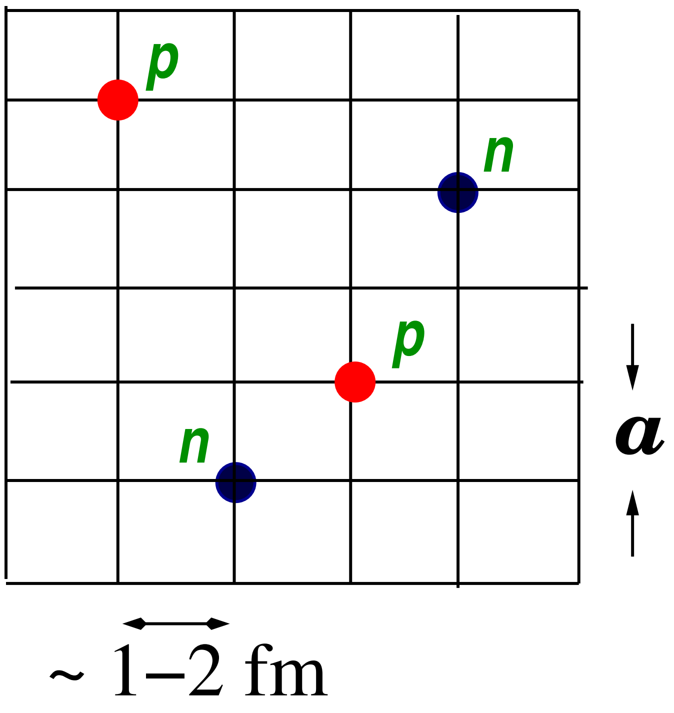
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

  → see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters

$$p_{\max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

  E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

  Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

Transfer matrix method

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons

  [or a more sophisticated (correlated) initial/final state]

Euclidean time

- Transient energy

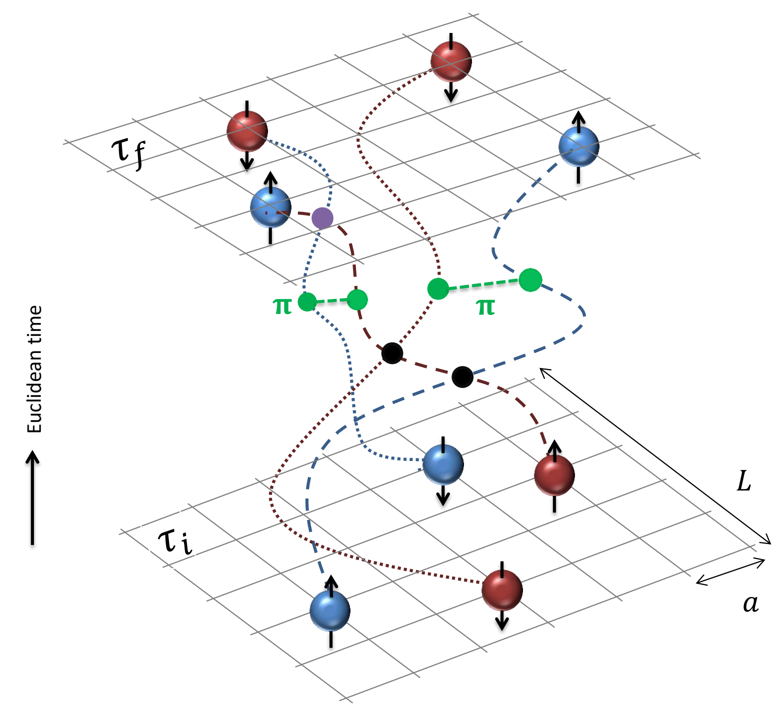
$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

→ ground state: $E_A^0 = \lim_{\tau \to \infty} E_A(\tau)$

- Exp. value of any normal–ordered operator $\mathcal{O}$

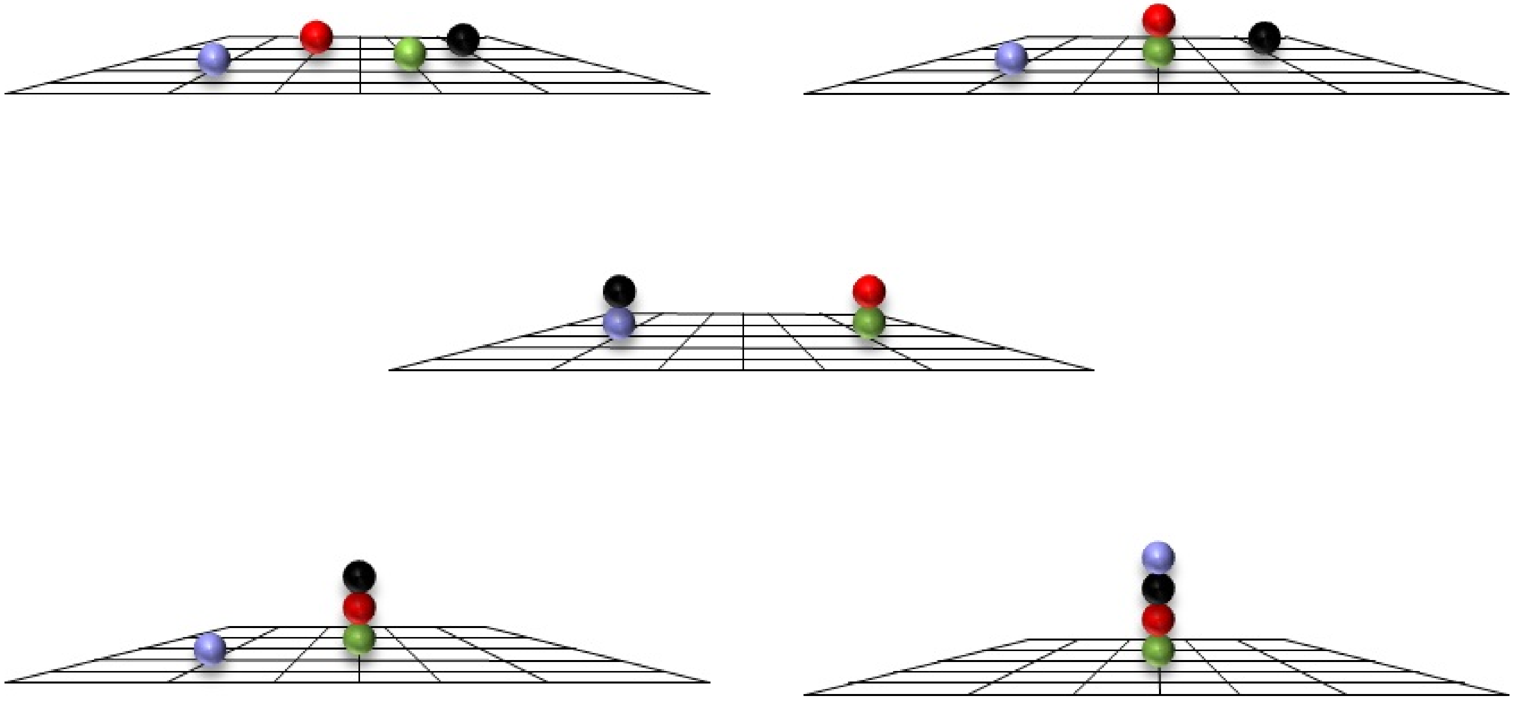
$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2)\, \mathcal{O}\, \exp(-\tau H/2)\, | \Psi_A \rangle$$

$$\lim_{\tau \to \infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} \, | \Psi_A \rangle$$



- Excited states: $Z_A(\tau) \to Z_A^{ij}(\tau)$, diagonalize, e.g. $0_1^+, 0_2^+, 0_3^+, \ldots$ in ^{12}C

Configurations



⇒ all *possible* configurations are sampled

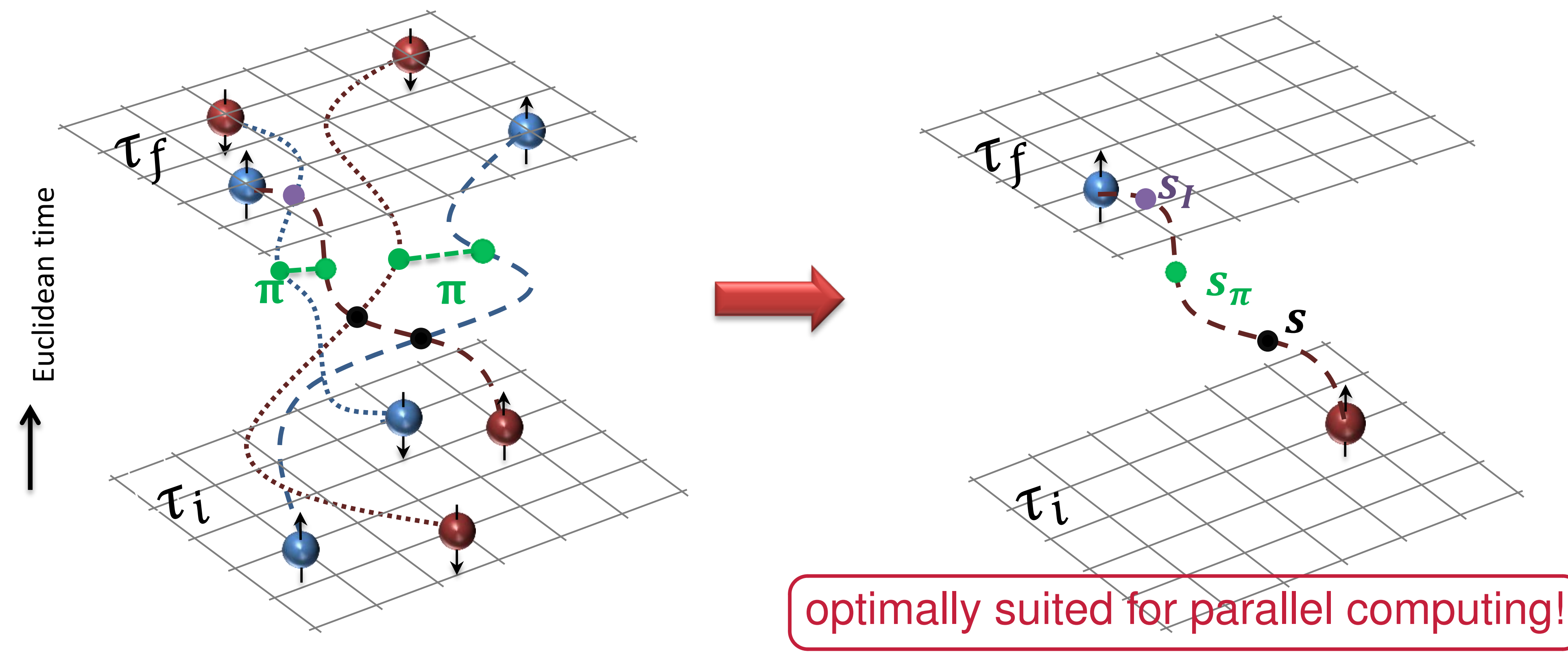
⇒ preparation of *all possible* initial/final states

⇒ *clustering* emerges *naturally*

Auxiliary field method

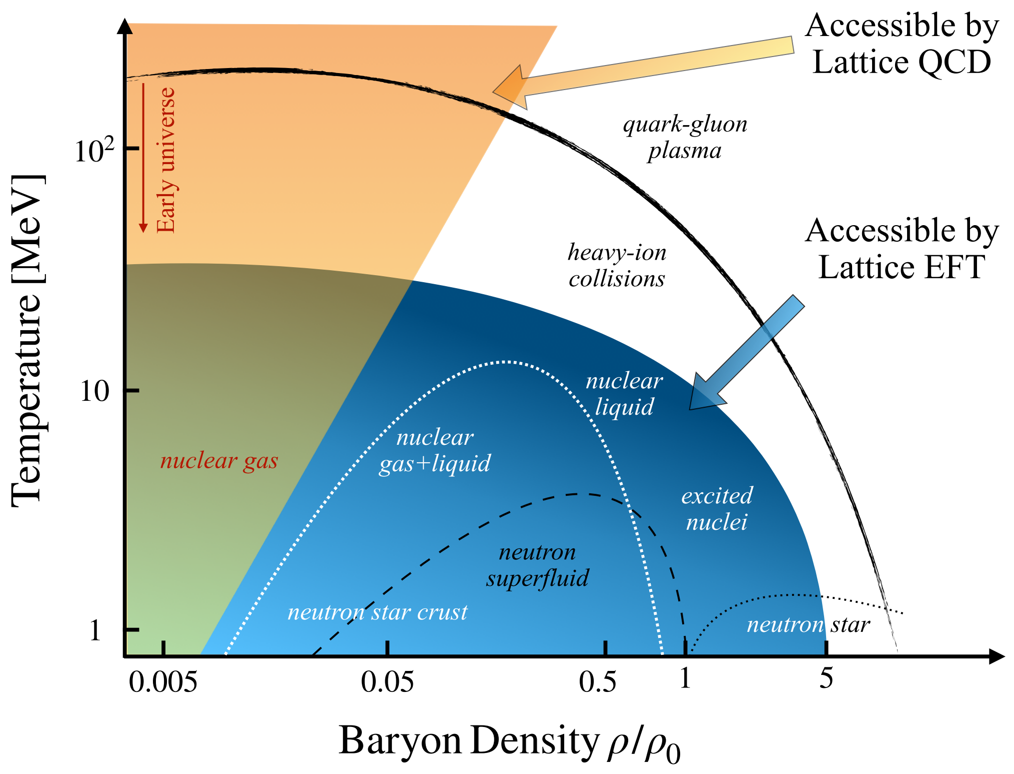
- Represent interactions by auxiliary fields (Gaussian completion):

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds\,\exp\left[-\frac{s^2}{2} + \sqrt{C}\,s\left(N^\dagger N\right)\right]$$



Comparison to lattice QCD

LQCD (quarks & gluons)	NLEFT (nucleons & pions)
relativistic fermions	non-relativistic fermions
renormalizable th'y	EFT
continuum limit	no continuum limit
(un)physical masses	physical masses
Coulomb - difficult	Coulomb - easy
high T/small ρ	small T/nuclear densities
sign problem severe	sign problem moderate



- For nuclear physics, NLEFT is the far better methodology!

Computational equipment

- Present = JUWELS (modular system) + JUPITER + ...

The minimal nuclear interaction: Foundations

A minimal nuclear interaction

- Basic problem: Straightforward application of chiral EFT forces at N2LO leads to problems when one goes beyond light and alpha-cluster nuclei (sign problem)

- Main idea: Construct a minimal nuclear interactions that reproduces the ground state properties of light nuclei, medium-mass nuclei, and neutron matter simultaneously with no more than a few percent error in the energies and charge radii

- This can be achieved by making use of Wigner's SU(4) spin-isospin symmetry

Wigner, Phys. Rev. **C 51** (1937) 106

- If the nuclear Hamiltonian does not depend on spin and isospin, then it is obviously invariant under SU(4) transformations [really $U(4) = U(1) \times SU(4)$]:

$$N \to UN\,, \quad U \in SU(4)\,, \quad N = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$N \to N + \delta N\,, \quad \delta N = i\epsilon_{\mu\nu}\sigma^\mu\tau^\nu\, N\,, \quad \sigma^\mu = (1, \sigma_i)\,, \quad \tau^\mu = (1, \tau_i)$$

Remarks on Wigner's SU(4) symmetry

- Wigner SU(4) spin-isospin symmetry in the context of pionless nuclear EFT

  $\hookrightarrow$ large scattering lengths — Mehen, Stewart, Wise, Phys. Rev. Lett. **83** (1999) 931

- Wigner SU(4) spin-isospin symmetry is particularly beneficial for NLEFT

  $\hookrightarrow$ suppression of sign oscillations — Chen, Lee, Schäfer, Phys. Rev. Lett. **93** (2004) 242302

  $\hookrightarrow$ provides a very much improved LO action when smearing is included $\to$ this talk
  Lu, Li, Elhatisari, Lee, Epelbaum, UGM, Phys. Lett. B **797** (2019) 134863

- Initimately related to α-clustering in nuclei

  $\hookrightarrow$ cluster states in ^{12}C like the famous Hoyle state
  Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501

  $\hookrightarrow$ nuclear physics is close to a quantum phase transition
  Elhatisari et al., Phys. Rev. Lett. **117** (2016) 132501

Essential elements for nuclear binding

Lu, Li, Elhatisari, Epelbaum, Lee, UGM, Phys. Lett. B **797** (2019) 134863 [arXiv:1812.10928]

- Highly SU(4) symmetric LO action without pions, only **four** parameters

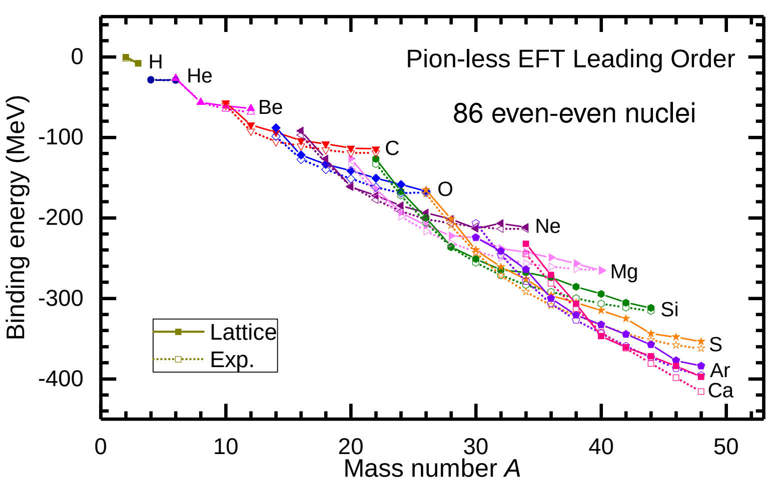
$$H_{\rm SU(4)} = H_{\rm free} + \frac{1}{2!} C_2 \sum_n \tilde\rho(n)^2 + \frac{1}{3!} C_3 \sum_n \tilde\rho(n)^3$$

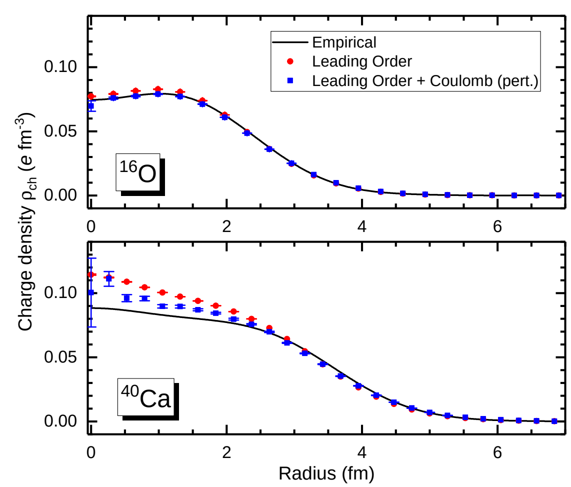
$$\tilde\rho(n) = \sum_i \tilde a_i^\dagger(n)\tilde a_i(n) + s_L \sum_{|n'-n|=1} \sum_i \tilde a_i^\dagger(n')\tilde a_i(n')$$

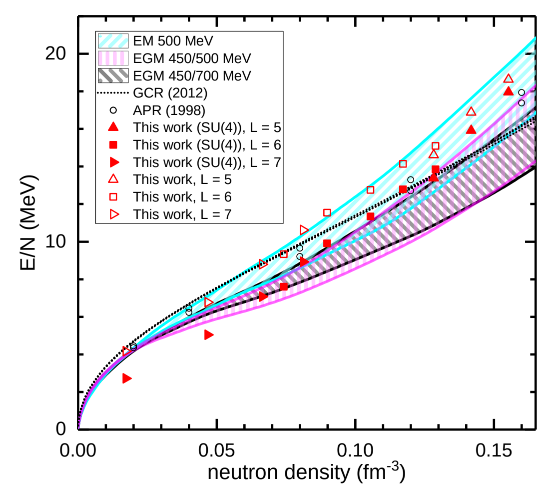
$$\tilde a_i(n) = a_i(n) + s_{NL} \sum_{|n'-n|=1} a_i(n')$$

s_L controls the locality of the interactions, s_{NL} the non-locaity of the smearing

$\rightarrow$ describes binding energies, radii, charge densities and the EoS of neutron matter







The minimal nuclear interaction: Applications

Wigner's SU(4) symmetry and the carbon spectrum

- Study of the spectrum (and other properties) of ^{12}C

  ↪ spin-orbit splittings are known to be weak

  Hayes, Navratil, Vary, Phys. Rev. Lett. **91** (2003) 012502 Johnson, Phys. Rev. C **91** (2015) 034313

  ↪ start with cluster and shell-model configurations → next slide

- Fit the four parameters:

  C_2, C_3 – ground state energies of ^{4}He and ^{12}C

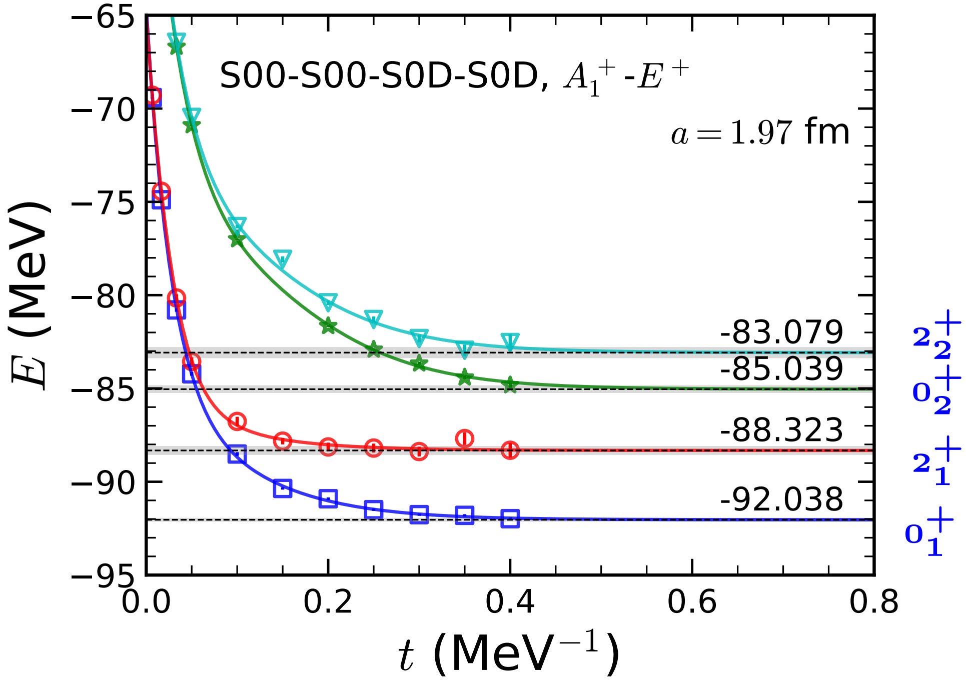
  $s_{\rm L}$ – radius of ^{12}C around 2.4 fm

  $s_{\rm NL}$ – best overall description of the transition rates

- Calculation of em transitions

  requires coupled-channel approach

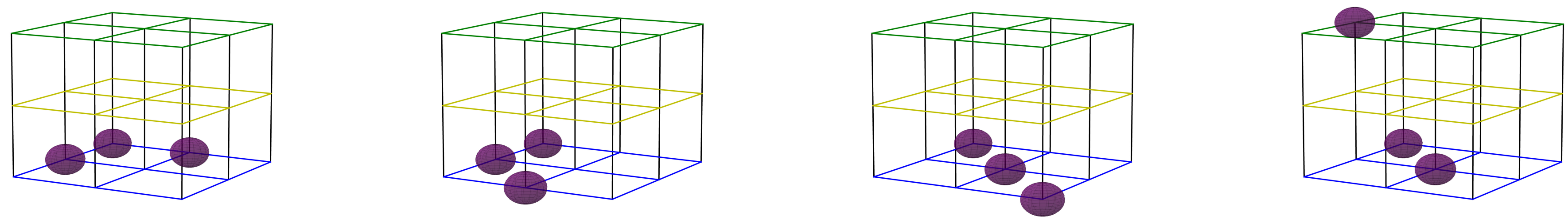
  e.g. 0^+ and 2^+ states



Configurations

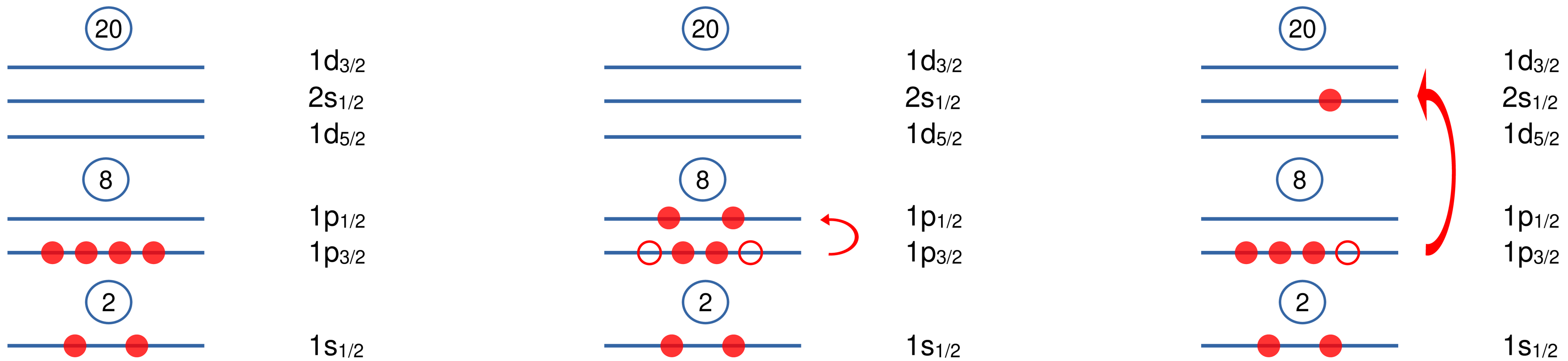
- Cluster and shell model configurations

S1 S2 S3 S4



— isoscele right triangle — "bent-arm" shape — linear diagonal chain — acute isoscele triangle

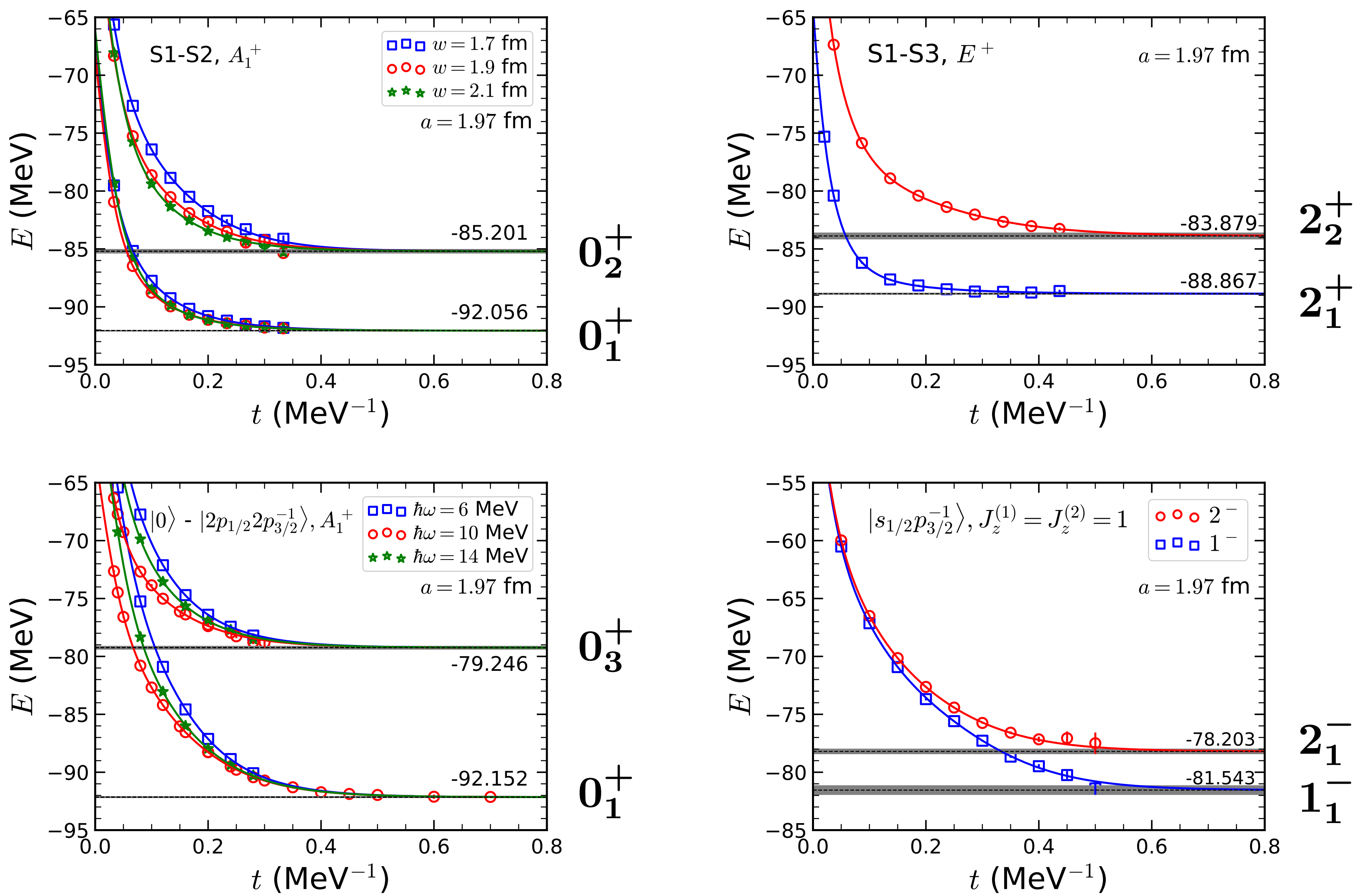
Gaussian wave packets $w = 1.7 - 2.1$ fm



— ground state $|0\rangle$ — $2p$-$2h$ state, $J_z = 0$ — $1p$-$1h$ state, $J_z^{(1)} = J_z^{(2)} = 1$

Transient energies

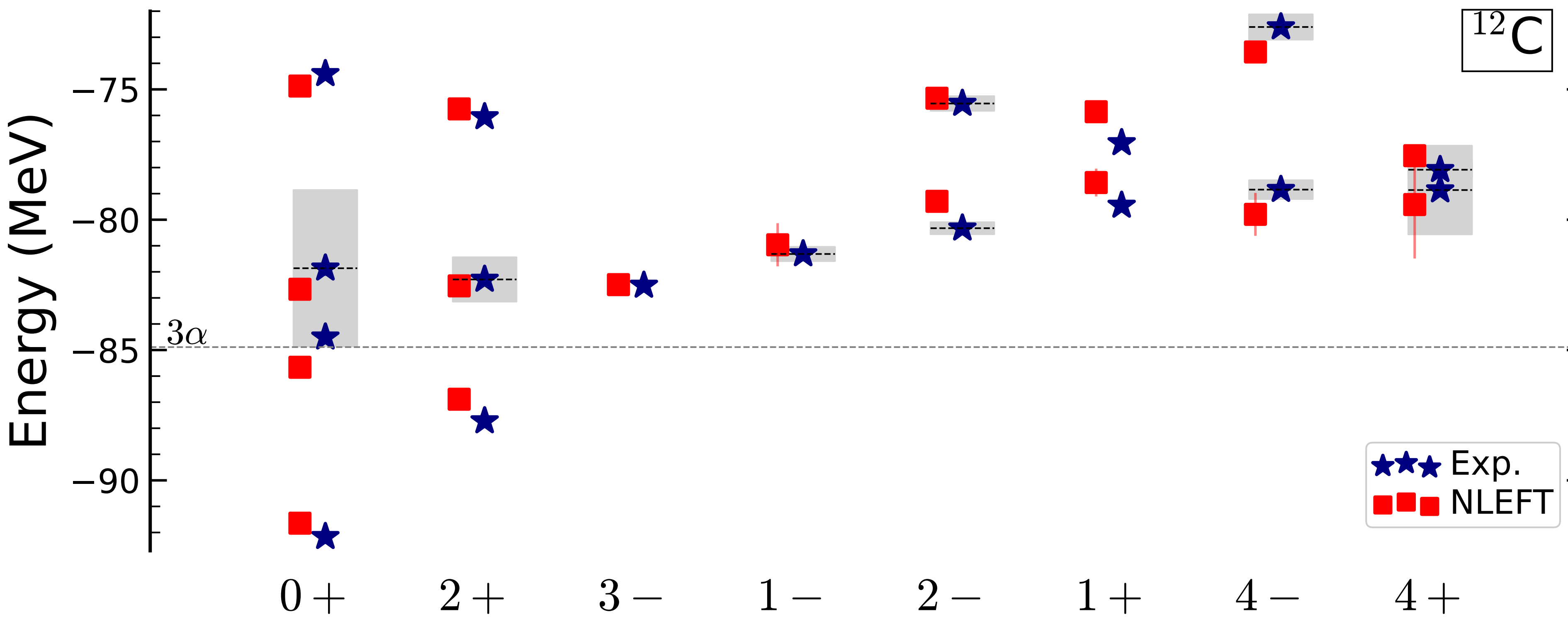
- Transient energies from cluster and shell-model configurations



Spectrum of ^{12}C

Shen, Elhatisari, Lähde, Lee, Lu, UGM, Nature Commun. **14** (2023) 2777

- Improved description when 3NFs are included, amazingly good

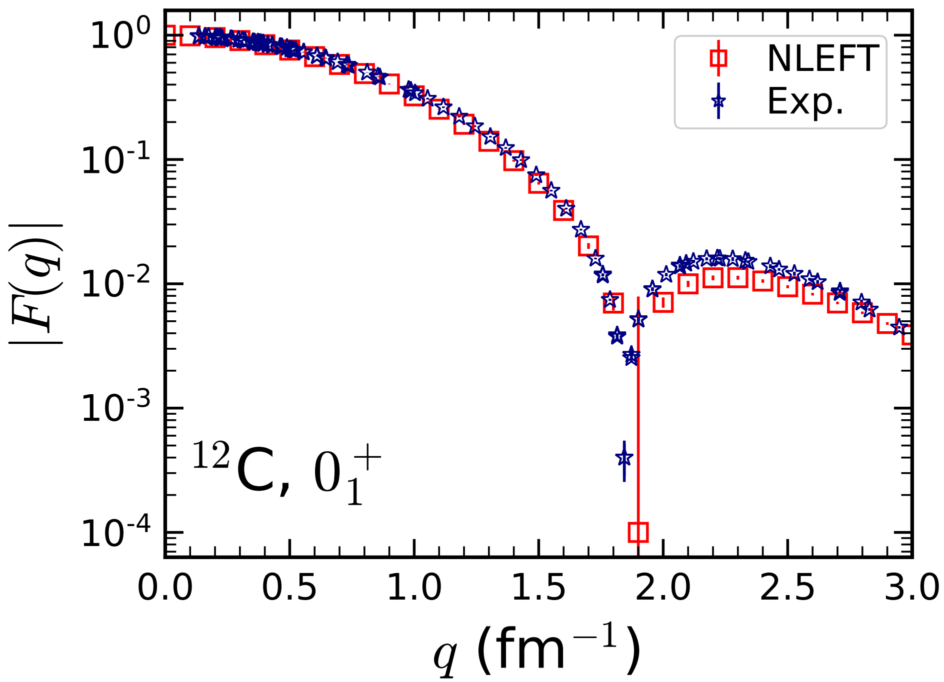


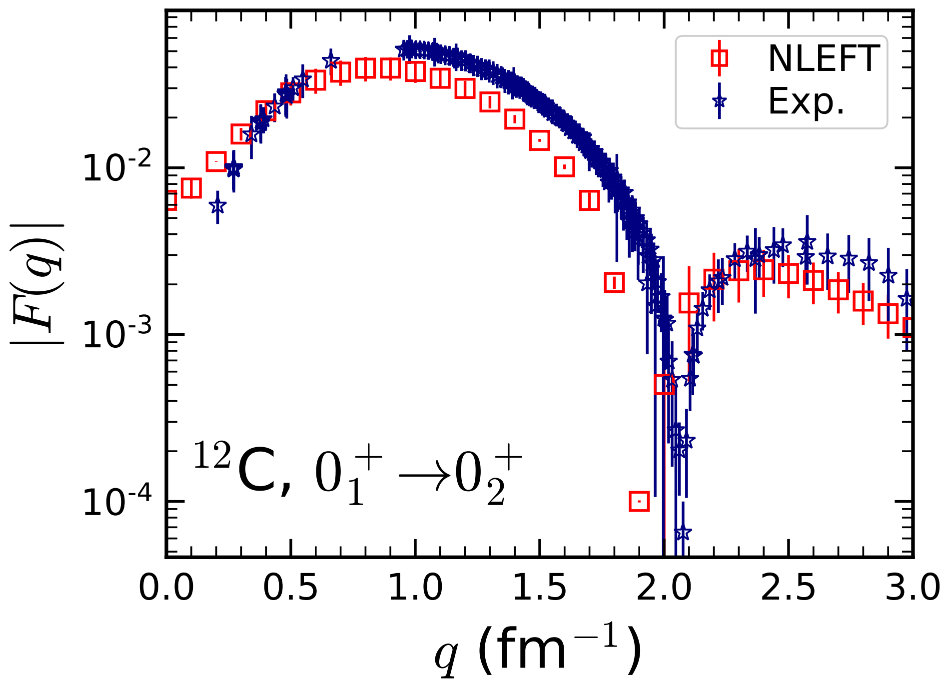
→ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

Electromagnetic properties

Shen, Elhatisari, Lähde, Lee, Lu, UGM, Nature Commun. **14** (2023) 2777

- Form factors and transition ffs [essentially parameter-free]:



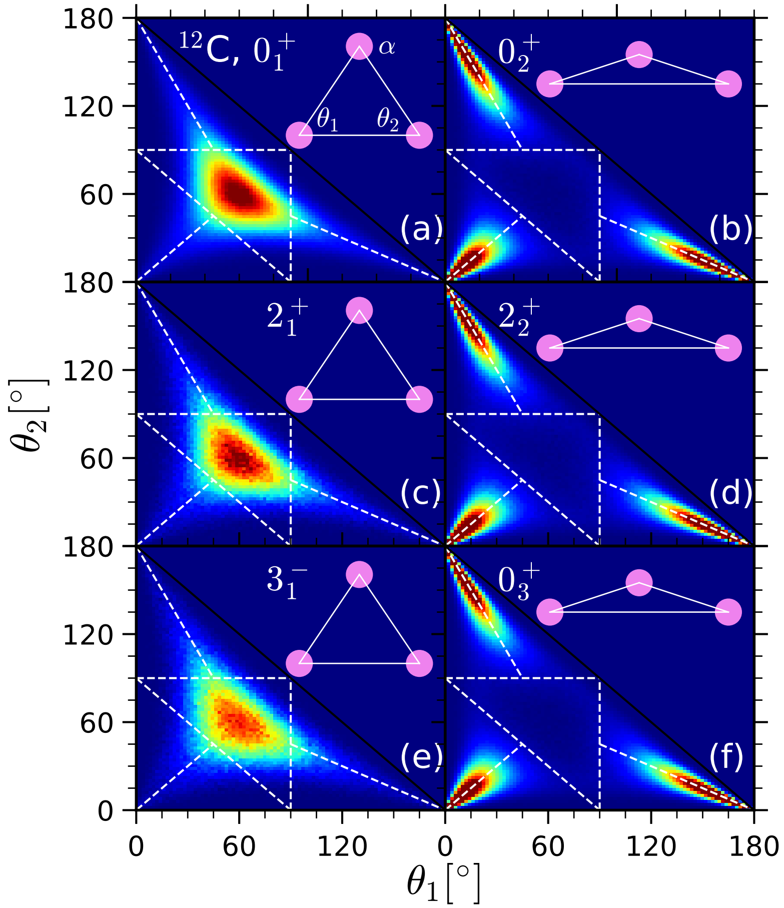


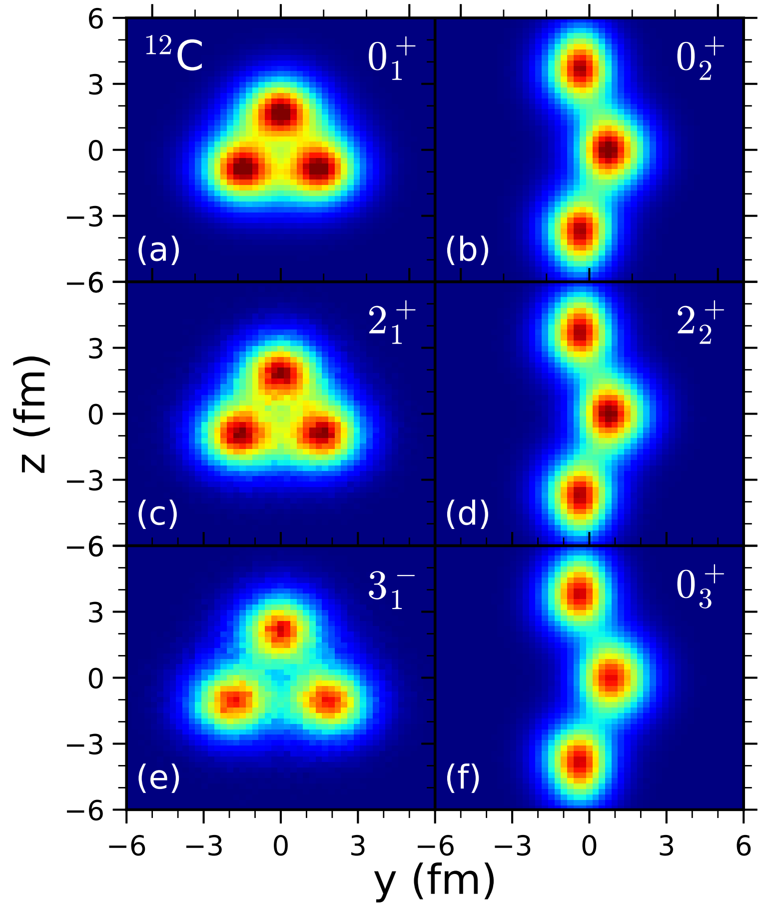
Sick, McCarthy, Nucl. Phys. A 150 (1970) 631
Strehl, Z. Phys. 234 (1970) 416
Crannell et al., Nucl. Phys. A 758 (2005) 399

Chernykh et al., Phys. Rev. Lett. 105 (2010) 022501

Emergence of geometry

- Use the pinhole algorithm to measure the distribution of α-clusters/matter:

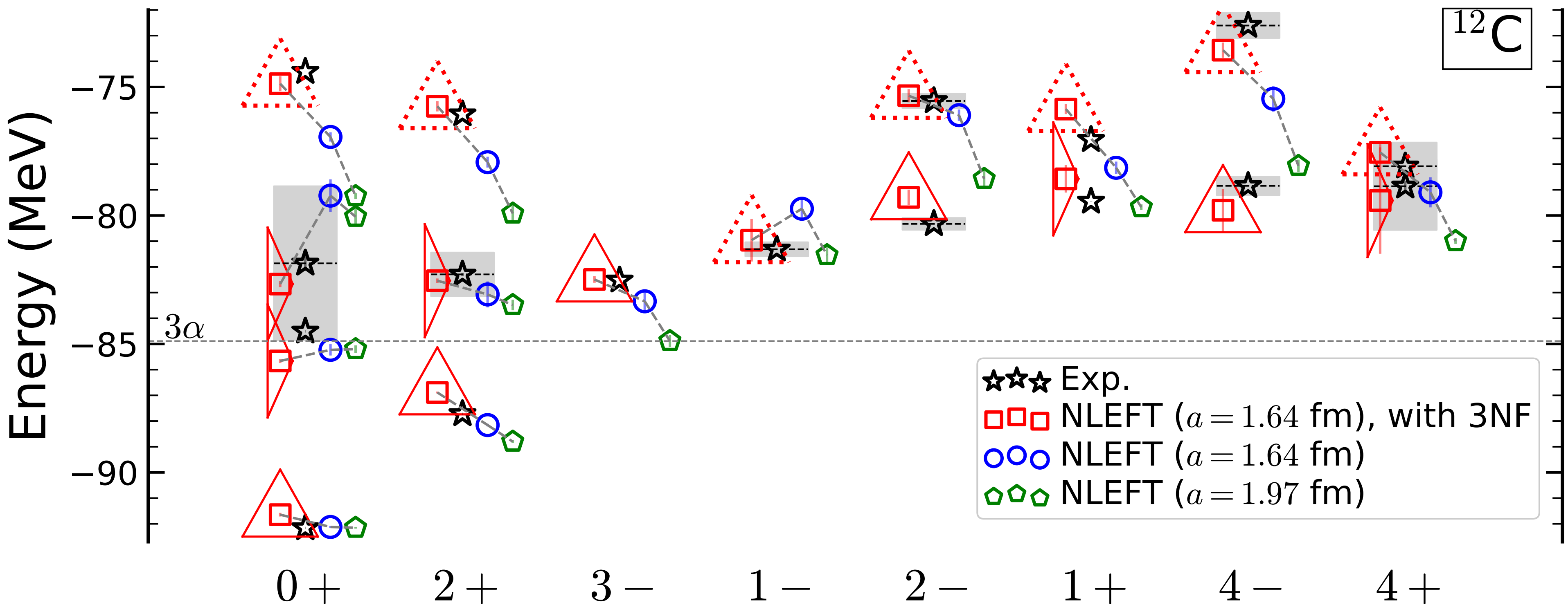




- equilateral & obstuse triangles $\rightarrow$ $\mathbf{2^+}$ states are excitations of the $\mathbf{0^+}$ states

Emergence of duality

- ^{12}C spectrum shows a cluster/shell-model duality

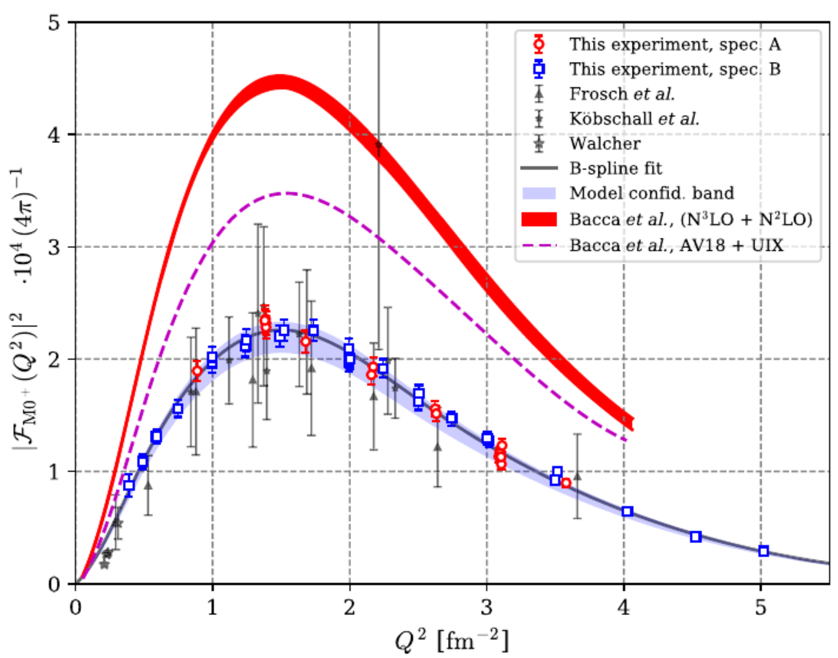


- dashed triangles: strong 1p-1h admixture in the wave function

The ^{4}He form factor puzzle

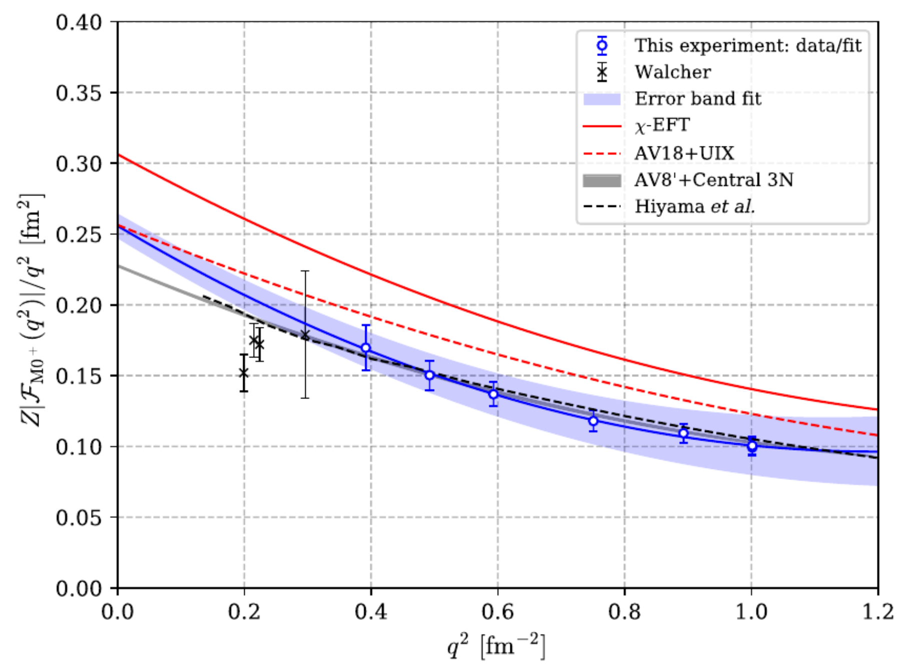
- Mainz measurements of the transition ff $F_{M0}(0_2^+ \to 0_1^+)$ appear to be in stark disagreement with *ab initio* nuclear theory Kegel et al., Phys. Rev. Lett. **130** (2023) 152502

- Monopole transition ff



[calculations from 2013]

- low-momentum expansion



⇒ **A low-energy puzzle for nuclear forces?**

Ab initio calculation of the ^{4}He transition form factor

UGM, Shen, Elhatisari, Lee, Phys. Rev. Lett. **132** (2024) 062501 [2309.01558 [nucl-th]]

- Use the essential elements action, **all parameters fixed!**

- Calculate the transition ff and its low-energy expansion form the transition density

$$\rho_{\rm tr}(r) = \langle 0_1^+|\hat\rho(\vec r)|0_2^+\rangle$$

$$F(q) = \frac{4\pi}{Z}\int_0^\infty \rho_{\rm tr}(r) j_0(qr) r^2 dr = \frac{1}{Z}\sum_{\lambda=1}^\infty \frac{(-1)^\lambda}{(2\lambda+1)!} q^{2\lambda}\langle r^{2\lambda}\rangle_{\rm tr}$$

$$\frac{Z|F(q^2)|}{q^2} = \frac{1}{6}\langle r^2\rangle_{\rm tr}\left[1 - \frac{q^2}{20}\mathcal{R}^2_{\rm tr} + \mathcal{O}(q^4)\right]$$

$$\mathcal{R}^2_{\rm tr} = \langle r^4\rangle_{\rm tr}/\langle r^2\rangle_{\rm tr}$$

- The first excited state sits in the continuum & close to the ^3H-p threshold

  ↪ use large volumes $L = 10, 11, 12$ or $L = 13.2$ fm, 14.5 fm, 15.7 fm

  ↪ the lattice spacing is fixed to $a = 1.32$ fm, corresponding $\Lambda = \pi/a = 465$ MeV

The first excited state

- 3 coupled channels with $\mathbf{0^+}$ q.n's $\longrightarrow$ accelerates convergence as $L_t \to \infty$
- Shell-model wave functions (4 nucleons in $1s_{1/2}$, twice 3 in $1s_{1/2}$ and 1 in $2s_{1/2}$)

L [fm]	$E(0_1^+)$ [MeV]	$E(0_2^+)$ [MeV]	ΔE [MeV]
$\mathbf{13.2}$	$\mathbf{-28.32(3)}$	$\mathbf{-8.37(14)}$	$\mathbf{0.28(14)}$
$\mathbf{14.5}$	$\mathbf{-28.30(3)}$	$\mathbf{-8.02(14)}$	$\mathbf{0.42(14)}$
$\mathbf{15.7}$	$\mathbf{-28.30(3)}$	$\mathbf{-7.96(9)}$	$\mathbf{0.40(9)}$

$\hookrightarrow$ statistical and large-L_t errors

$\hookrightarrow$ agreement w/ experiment: $E(0_1^+) = 28.3$ MeV, $\Delta E = 0.4$ MeV

$\hookrightarrow$ ΔE consistent w/ no-core Gamov shell model (no 3NFs)

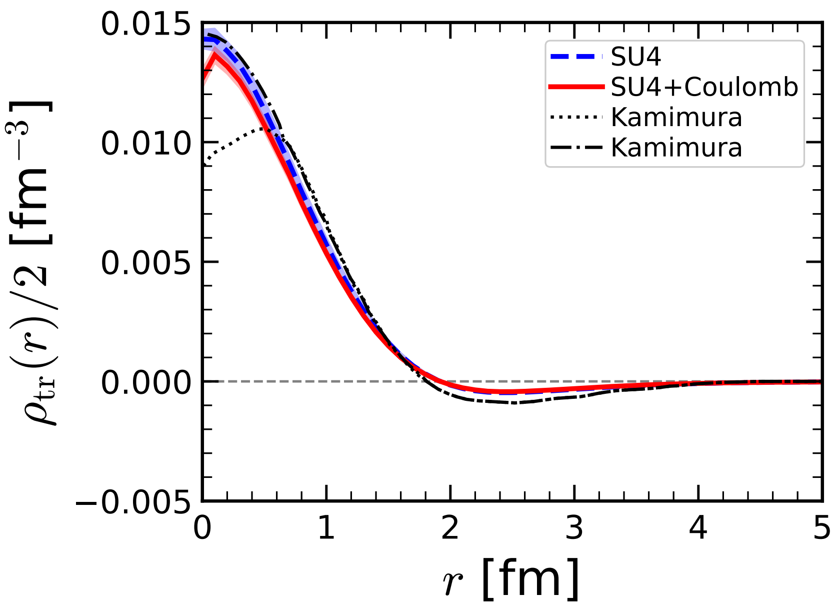
Michel, Nazarewicz, Ploszajczak, Phys. Rev. Lett. **131** (2023) 242502

$\hookrightarrow$ consistent w/ the Efimov tetramer analysis $\Delta E = 0.38(2)$ MeV

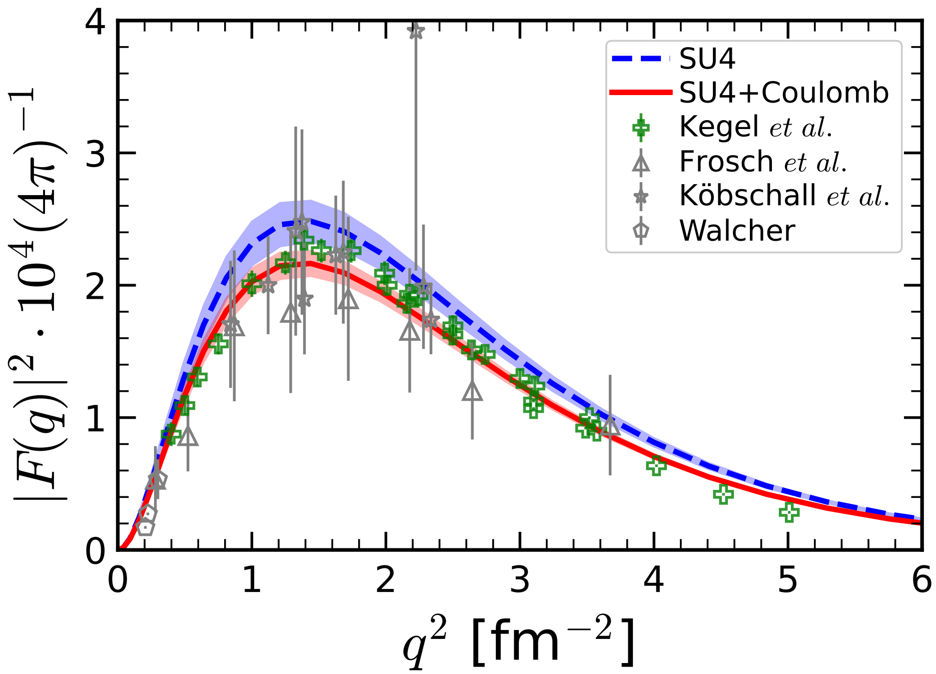
von Stecher, D'Incao, Greene, Nat. Phys. **5** (2009) 417; Hammer, Platter, EPJA **32** (2007) 113

The transition form factor

- Transition charge density



- Transition form factor



↪ agrees with the reconstructed one from Kamimura PTEP **2023** (2023) 071D01

↪ very small central depletion (no zero)

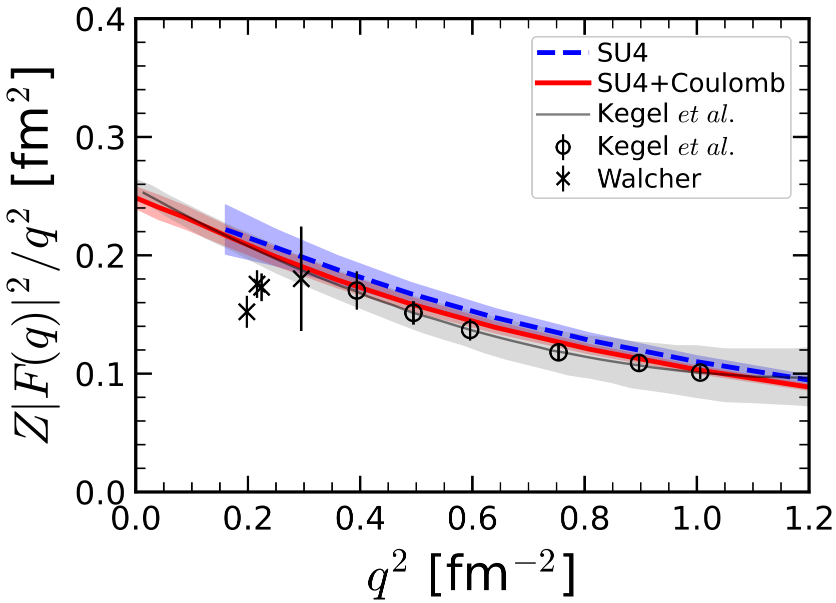
↪ excellent description of the data

↪ Coulomb required plus smaller uncertainty (improved signal)

↪ 3NFs important!

The transition form factor II

- Small momentum expansion



	$\langle r^2\rangle_{tr}$ [fm^2]	$\mathcal{R}_{tr}$ [fm]
Experiment	1.53 ± 0.05	4.56 ± 0.15
Th (AV8'+ centr. 3N)*	1.36 ± 0.01	4.01 ± 0.05
Th (AV18 + UIX)	1.54 ± 0.01	3.77 ± 0.08
Th (NLEFT)	1.49 ± 0.01	4.00 ± 0.04

*Hiyama, Gibson, Kamimura, PRC **70** (2004) 031001

$\hookrightarrow$ Also consistent description of the low-energy data

$\hookrightarrow$ **No puzzle** to the nuclear forces!

$\hookrightarrow$ Can be improved using N3LO action + wave function matching $\rightarrow$ outlook

Elhatisari et al., Nature **630** (2024) 59

The minimal nuclear interaction: Extension to hypernuclei

Strangeness in neutron matter

- Challenges:
  - no more approximate SU(4) symmetry, but SU(6)
    $\hookrightarrow$ ΛN scattering parameters still ok — Bour, UGM (2009) unpublished
  - very few scattering data, need hypernuclei as benchmark
  - a cornucopia of new three-baryon forces
    Petschauer, Kaiser, Haidenbauer, UGM, Weise, Phys. Rev. C **93** (2016) 014001
  - must deal with densities as large as $6\rho_0 \to$ tremendous sign oscillations
    $\hookrightarrow$ never attempted before in any lattice calculation

- First pathway:
  - use the minimal nuclear interactions with Λ's added
  - new auxiliary field ansatz: — Tong, Elhatisari, UGM, Sci. Bull. **70** (2025) 825

$$\tilde{\phi} = \tilde{\rho} + \frac{c_{N\Lambda}}{c_{NN}}\,\tilde{\xi}$$

$\tilde{\rho}$ = smeared nucleon density
$\tilde{\xi}$ = smeared hyperon (Λ) density

The minimal interaction with strangeness I

Tong, Elhatisari, UGM, Sci. Bull. **70** (2025) 825

- Baryon-baryon interaction (consider nucleons and Λ's plus non-local smearing):

$$V_{\Lambda N} = c_{N\Lambda} \sum_{\vec{n}} \tilde{\rho}(\vec{n})\tilde{\xi}(\vec{n}) + c_{\Lambda\Lambda} \frac{1}{2} \sum_{\vec{n}} \left[\tilde{\xi}(\vec{n})\right]^2$$

$$\tilde{\rho}(\vec{n}) = \sum_{i,j=0,1} \tilde{a}^{\dagger}_{i,j}(\vec{n})\,\tilde{a}_{i,j}(\vec{n}) + s_{\rm L} \sum_{|\vec{n}-\vec{n}'|^2=1} \sum_{i,j=0,1} \tilde{a}^{\dagger}_{i,j}(\vec{n}')\,\tilde{a}_{i,j}(\vec{n}')$$

$$\tilde{\xi}(\vec{n}) = \sum_{i=0,1} \tilde{b}^{\dagger}_{i}(\vec{n})\,\tilde{b}_{i}(\vec{n}) + s_{\rm L} \sum_{|\vec{n}-\vec{n}'|^2=1} \sum_{i=0,1} \tilde{b}^{\dagger}_{i}(\vec{n}')\,\tilde{b}_{i}(\vec{n}')$$

- Three-baryon forces (consider nucleons and Λ's, no non-local smearing $\rightarrow$ repulsion):

Petschauer, Kaiser, Haidenbauer, UGM, Weise, Phys. Rev. C **93** (2016) 014001

$$V_{NN\Lambda} = c_{NN\Lambda} \frac{1}{2} \sum_{\vec{n}} [\rho(\vec{n})]^2\, \xi(\vec{n})\,, \quad V_{N\Lambda\Lambda} = c_{N\Lambda\Lambda} \frac{1}{2} \sum_{\vec{n}} \rho(\vec{n})\, [\xi(\vec{n})]^2$$

$\hookrightarrow$ must determine 4 LECs! [smearing parameters from the nucleon sector]

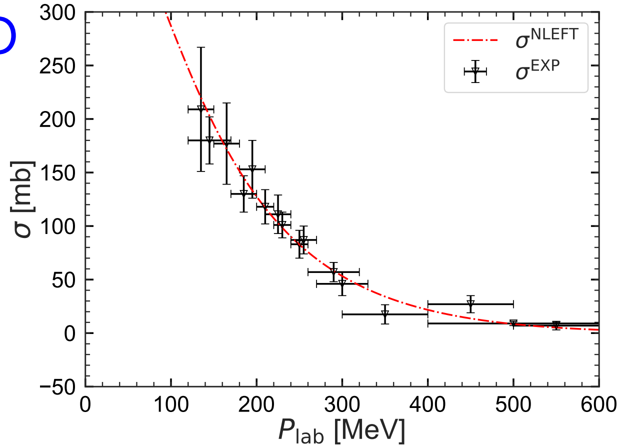
$\hookrightarrow$ consistent ΛNN & $\Lambda\Lambda N$ three-body forces included

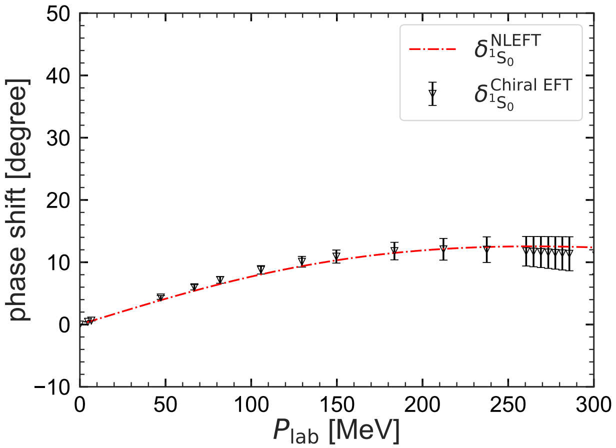
The minimal interaction with strangeness II

Tong, Elhatisari, UGM, Sci. Bull. **70** (2025) 825

- Two-body LECs from scattering data (ΛN) & chiral EFT phase shift ($\Lambda\Lambda$) at NLO

  Haidenbauer, UGM, Petschauer, Nucl. Phys. A **954** (2016) 273





- Three-body LECs from the separation energies of Λ and $\Lambda\Lambda$ hyper-nuclei:

System	NLEFT			Exp.
	HNM(I)	HNM(II)	HNM(III)	
$^5_\Lambda$He	$3.40(1)(1)$	$3.45(1)(2)$	$3.46(1)(3)$	$3.10(3)$
$^9_\Lambda$Be	$5.72(5)(4)$	$5.64(5)(3)$	$5.57(5)(3)$	$6.61(7)$
$^{13}_\Lambda$C	$10.54(17)(29)^*$	$10.09(17)(27)^*$	$9.80(17)(26)^*$	$11.80(16)$
$^6_{\Lambda\Lambda}$He	$6.91(1)(1)$	$6.91(1)(1)$	$6.91(1)(1)$	$6.91(16)$

$\hookrightarrow$ this defines our EoS of hyper-nuclear matter called **HMN(I)**

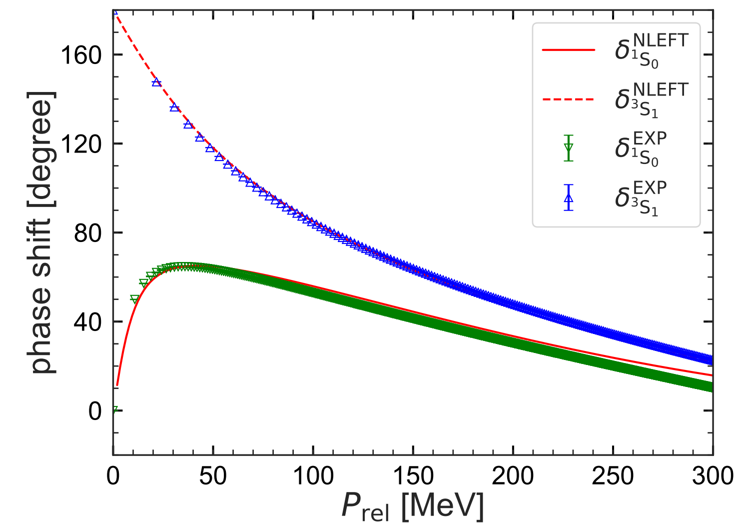
The minimal nuclear interaction: EoS & neutron star properties

Pure neutron matter

Tong, Elhatisari, UGM, Sci. Bull. **70** (2025) 825

- **Input**: S-wave phase shifts (2N)
  & symmetric nuclear matter (3N)

- Note: extension of the minimal interaction (leading SU(4) breaking)
  $\hookrightarrow$ correct scattering length a_{nn}



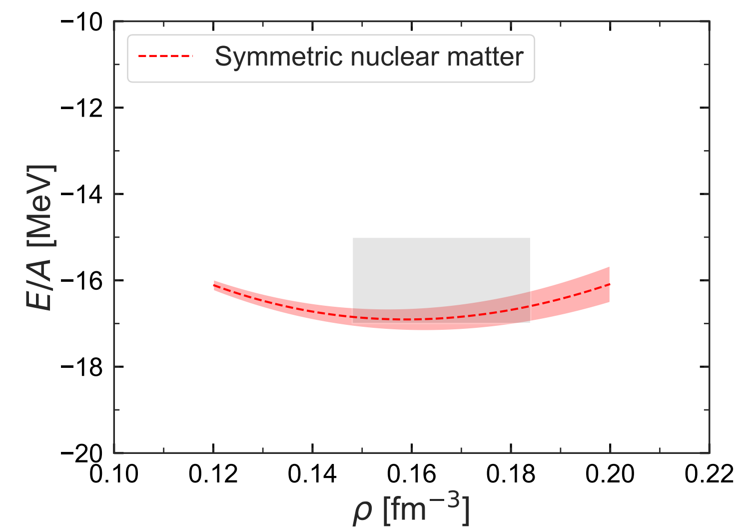
$\Rightarrow$ **Output**: Pure neutron matter (PNM) EoS

$\hookrightarrow$ up to 232 neutrons on the lattice

$\hookrightarrow$ $a = 1.1$ fm, $L = 6 \to V = 288$ fm^3

$\hookrightarrow$ $\rho_{\rm max} = 0.95$ fm$^{-3} \simeq 5.8\,\rho_0$

$\hookrightarrow$ never achieved before on the lattice!



Pure neutron matter EoS

Tong, Elhatisari, UGM, Sci. Bull. **70** (2025) 825

- Stiff EoS

  ↪ no interpolation needed!

  – comparable to the renowned APR EoS

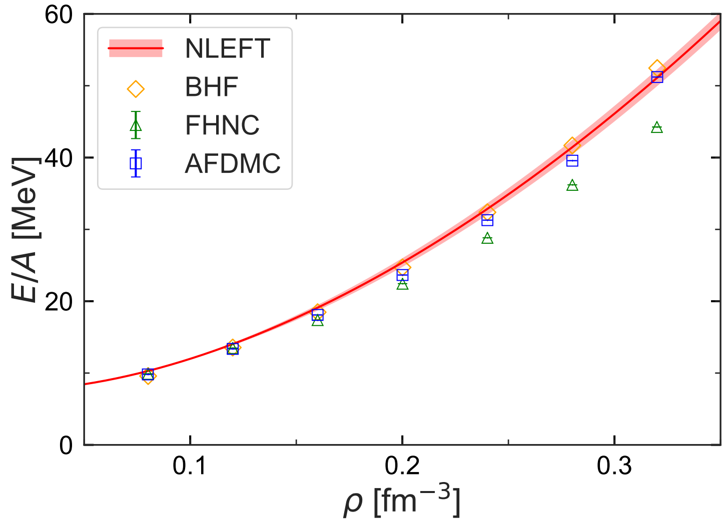
  Akmal, Pandharipande, Ravenhall, Phys. Rev. C **58** (1998) 1804

  – less stiff than the more recent AFDMC one

  Gandolfi et al., Eur. Phys. J. A **50** (2014) 10

  – stiffer than the recent FHNC EoS

  Lovato et al., Phys. Rev. C **105** (2022) 055808

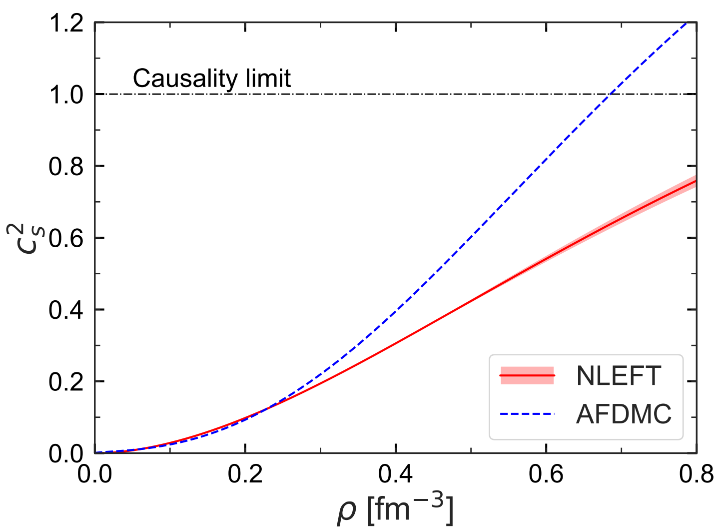


- Calculate the speed of sound c_S

  ↪ no violaton of causality in our approach

  ↪ reliable approach, no issues at higher densities

  Krastev & Sammarruca, Phys. Rev. C **74** (2006) 025808



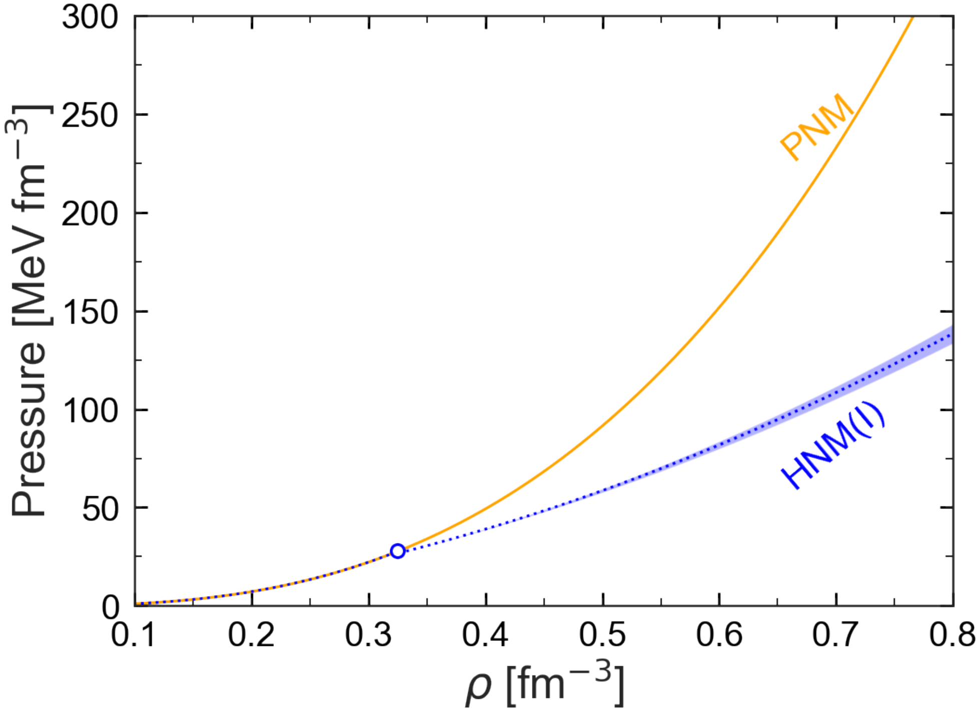
⇒ now let us look at neutron stars (up to 116 Λ's on the lattice)

Neutron star properties

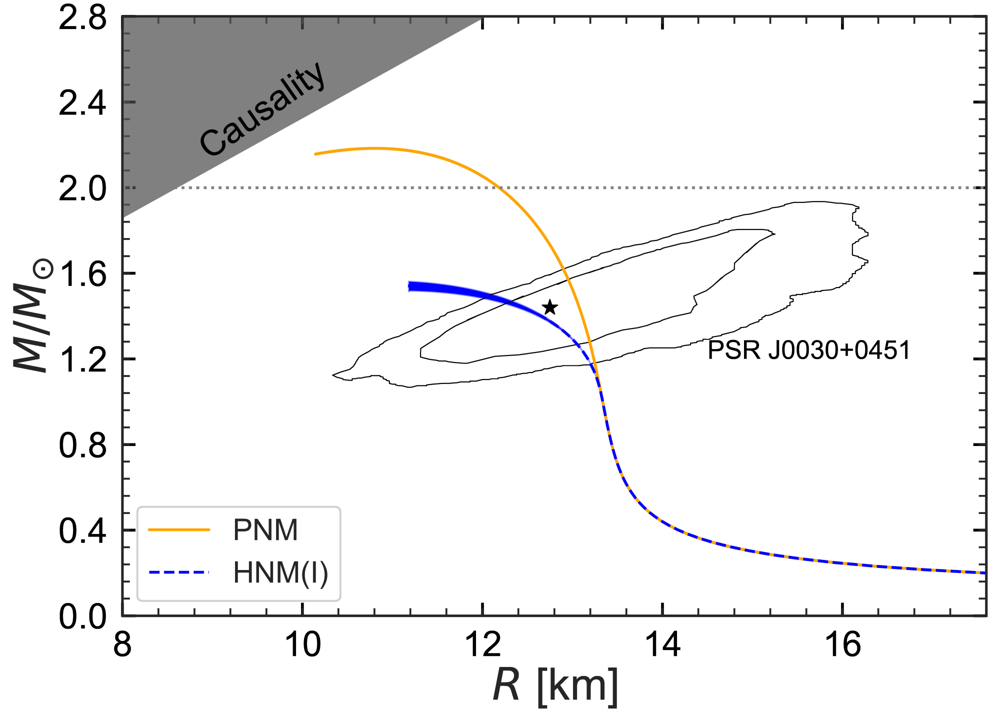
Tong, Elhatisari, UGM, Sci. Bull. **70** (2025) 825; Astrophys. J. **982** (2025) 164

- Now solve the TOV equations for the PNM and HNM(I) EoSs [idealized system]:

- EoS (PNM and HMN(I))



- Mass-radius relation



- Max. neutron star mass: $M_{\max} = 2.19(1)(1)\, M_\odot$ for PNM

$M_{\max} = 1.59(1)(1)\, M_\odot$ for HNM(I)

$\rightarrow$ need repulsion

EoS of hyper-neutron matter

Tong, Elhatisari, UGM, Sci. Bull. **70** (2025) 825; Astrophys. J. **982** (2025) 164

- Not surprisingly, we need more repulsion [as in the pure neutron matter case]

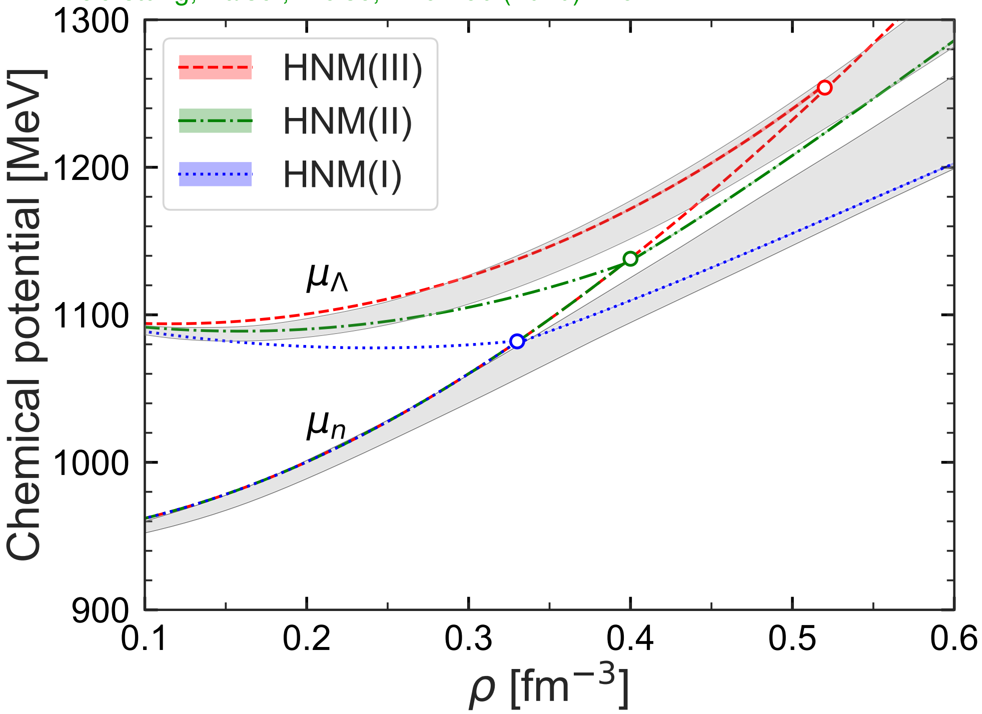
↪ this will move the threshold of $\mu_\Lambda = \mu_n$ up

↪ take $\rho_{\rm thr}$ as **data point**: $\rho_{\rm thr} = 0.4\ {\rm fm}^{-3} \to M_{\rm max} = 1.94(1)(1) M_\odot$ for HNM(II)

$\rho_{\rm thr} = 0.5\ {\rm fm}^{-3} \to M_{\rm max} = 2.17(1)(1) M_\odot$ for HNM(III)

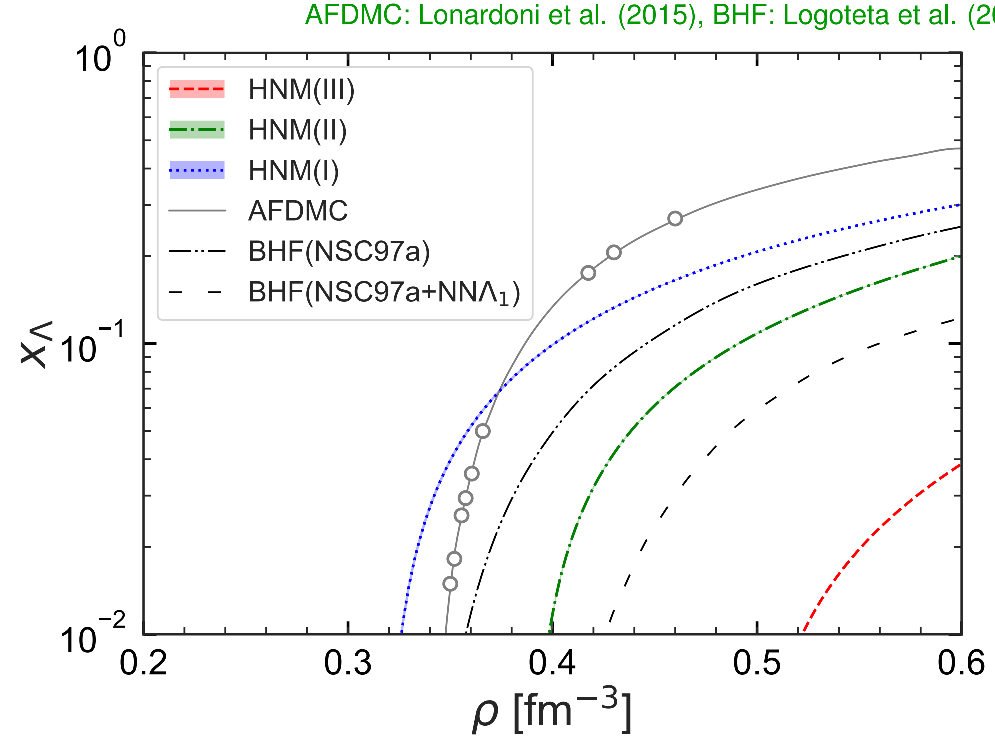
- Chemical potentials

Gerstung, Kaiser, Weise, EPJA **56** (2020) 175



- Λ fraction

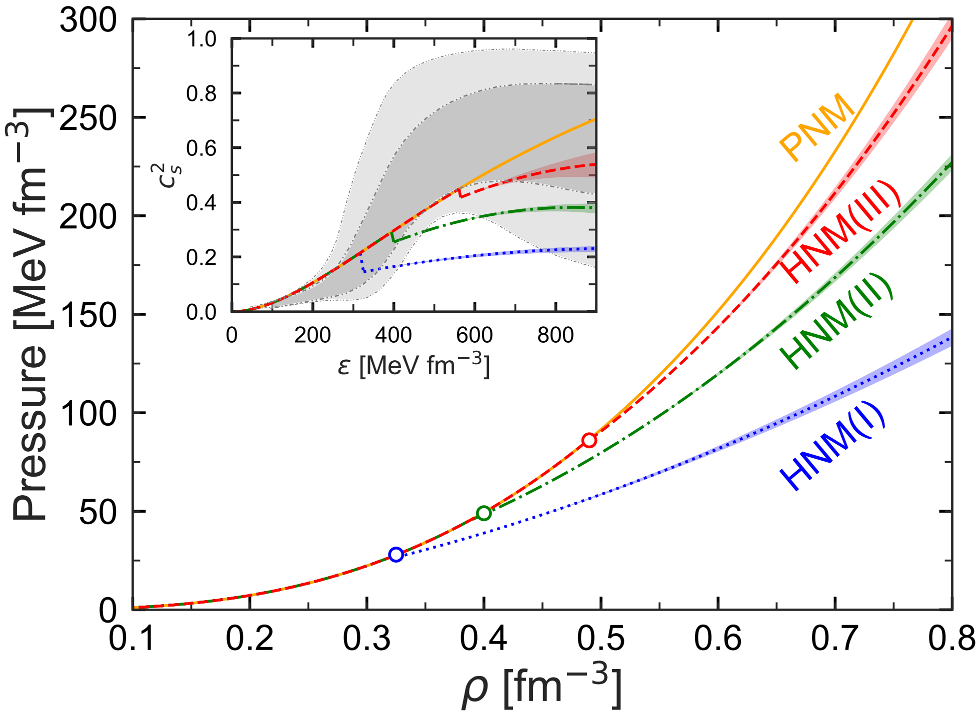
AFDMC: Lonardoni et al. (2015), BHF: Logoteta et al. (2019)



EoS of hyper-neutron matter cont'd

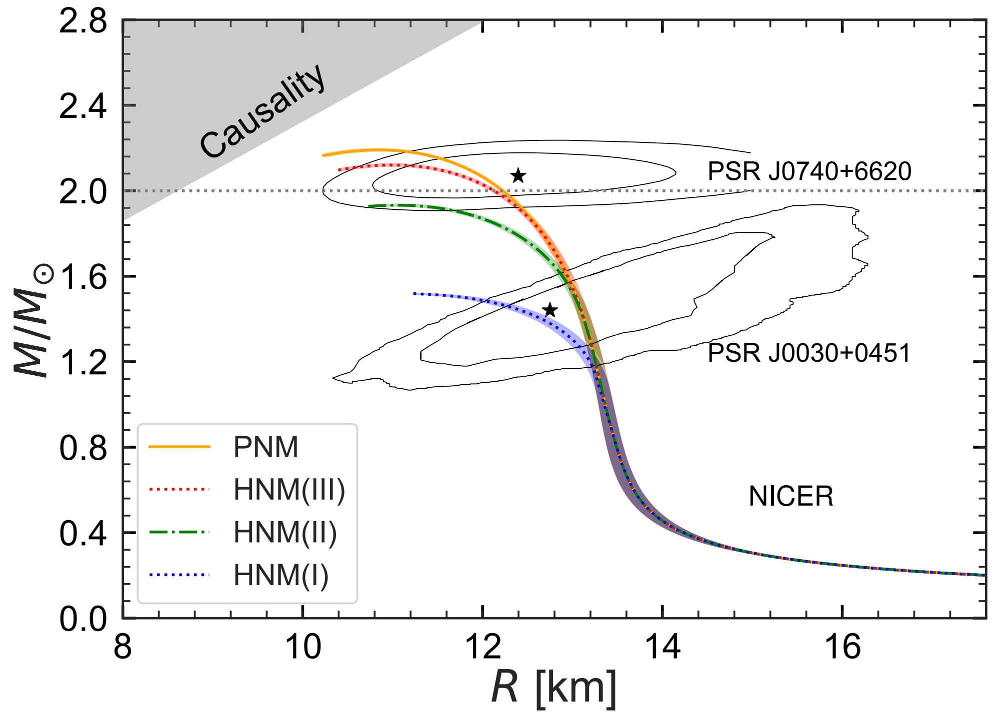
Tong, Elhatisari, UGM, Sci. Bull. **70** (2025) 825; Astrophys. J. **982** (2025) 164

- EoS & speed of sound



Brandes, Kaiser, Weise, Phys. Rev. D **107** (2023) 014011

- Mass-radius relation



NICER: Miller et al., Astrophys. J. Lett. **887** (2019) L24

$\hookrightarrow$ HNM(III) is consistent with all known constraints

$\hookrightarrow$ still a non-vanishing Λ fraction at large densities

$\hookrightarrow$ description of the hypernuclei still ok $\surd$

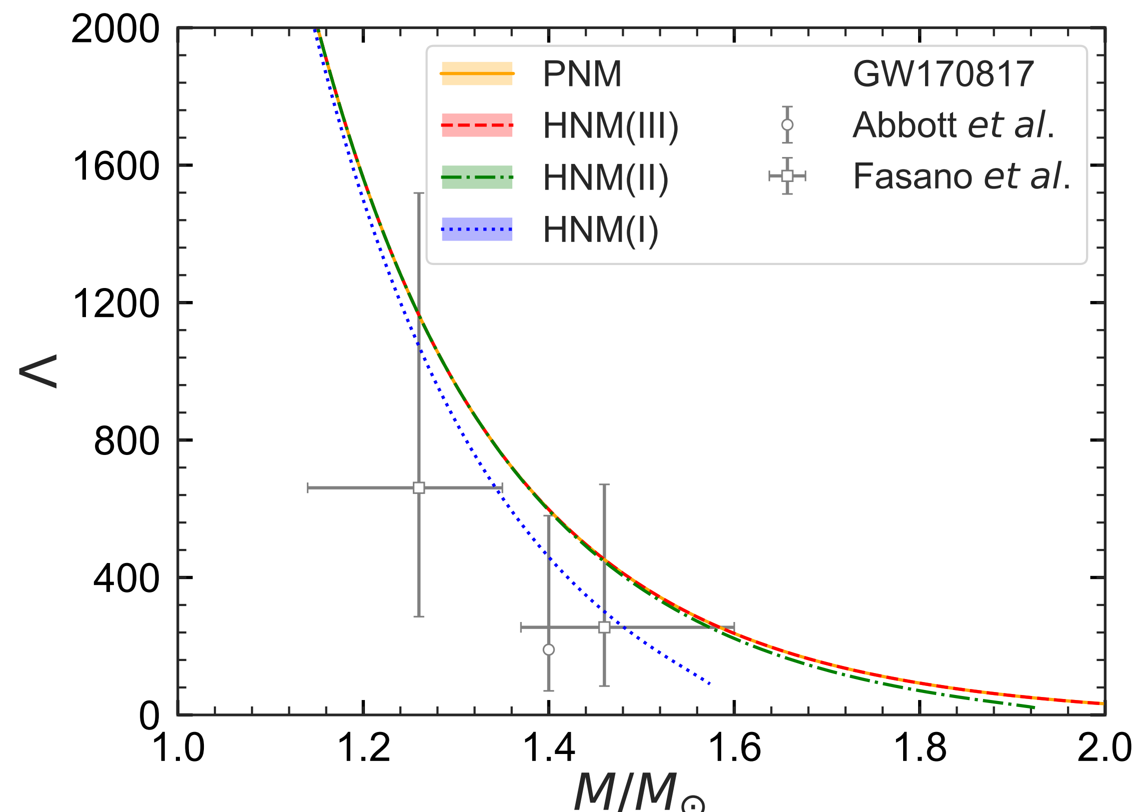
Tidal deformability

Tong, Elhatisari, UGM, Astrophys. J. **982** (2025) 164

- Another important quantity:
  tidal deformability $\Lambda_{1.4M_\odot}$
  (for a canonical neutron star)

$\hookrightarrow$ we have

$$\Lambda_{1.4M_\odot} = 597(5)(18) \text{ for PNM}$$
$$= 451(5)(31) \text{ for HNM(I)}$$
$$= 587(5)(19) \text{ for HNM(II)}$$
$$= 597(5)(18) \text{ for HNM(III)}$$



$\hookrightarrow$ consistent with the data

$\Lambda_{1.4M_\odot} < 800$ Abbott et al. (2017)

$\Lambda_{1.4M_\odot} = 190^{+300}_{-120}$ Abbott et al. (2018)

$\Lambda_{1.46M_\odot} = 255^{+416}_{-117}$ Fasano et al. (2019)

$\Lambda_{1.26M_\odot} = 661^{+858}_{-375}$ Fasano et al. (2019)

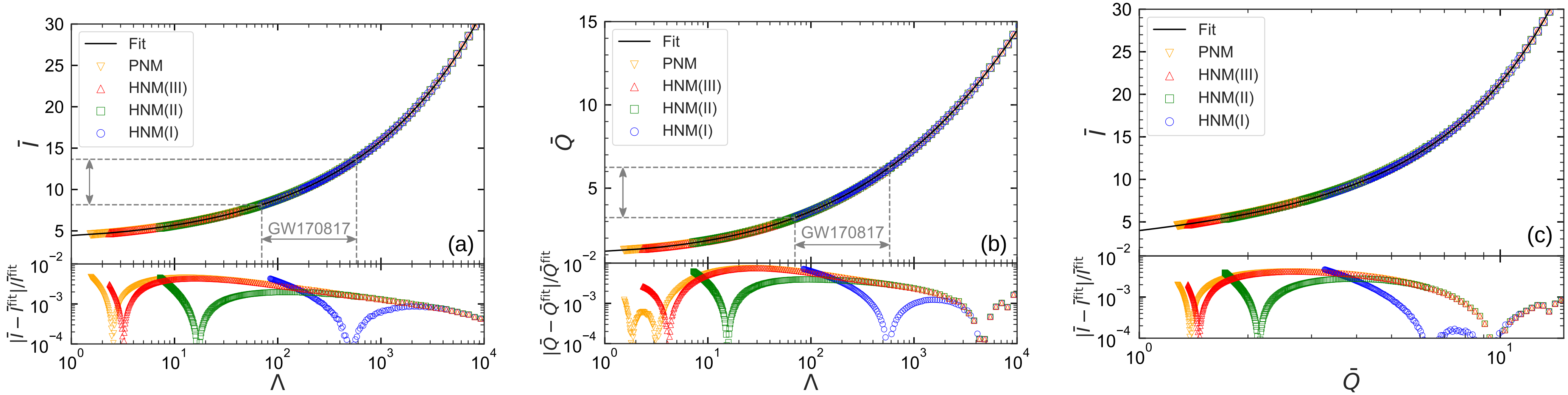
I-Love-Q relations

Tong, Elhatisari, UGM, Astrophys. J. **982** (2025) 164

- These I-Love-Q relations where found earlier by analyzing a large number of neutron star EoSs w/ and w/o hyperons (slow rotation approximation)

Yagi and Younes (2013,2017); Sedrakian (2023), Li et al. (2023)

- Our *ab initio* calculations give $[\bar{I} = I/M^3, \bar{Q} = -QM/(I\Omega)^2]$



$\hookrightarrow$ underlying cause remains to be understood

$\hookrightarrow$ valuable tool to extract the moment of inertia of a (canonical) neutron star

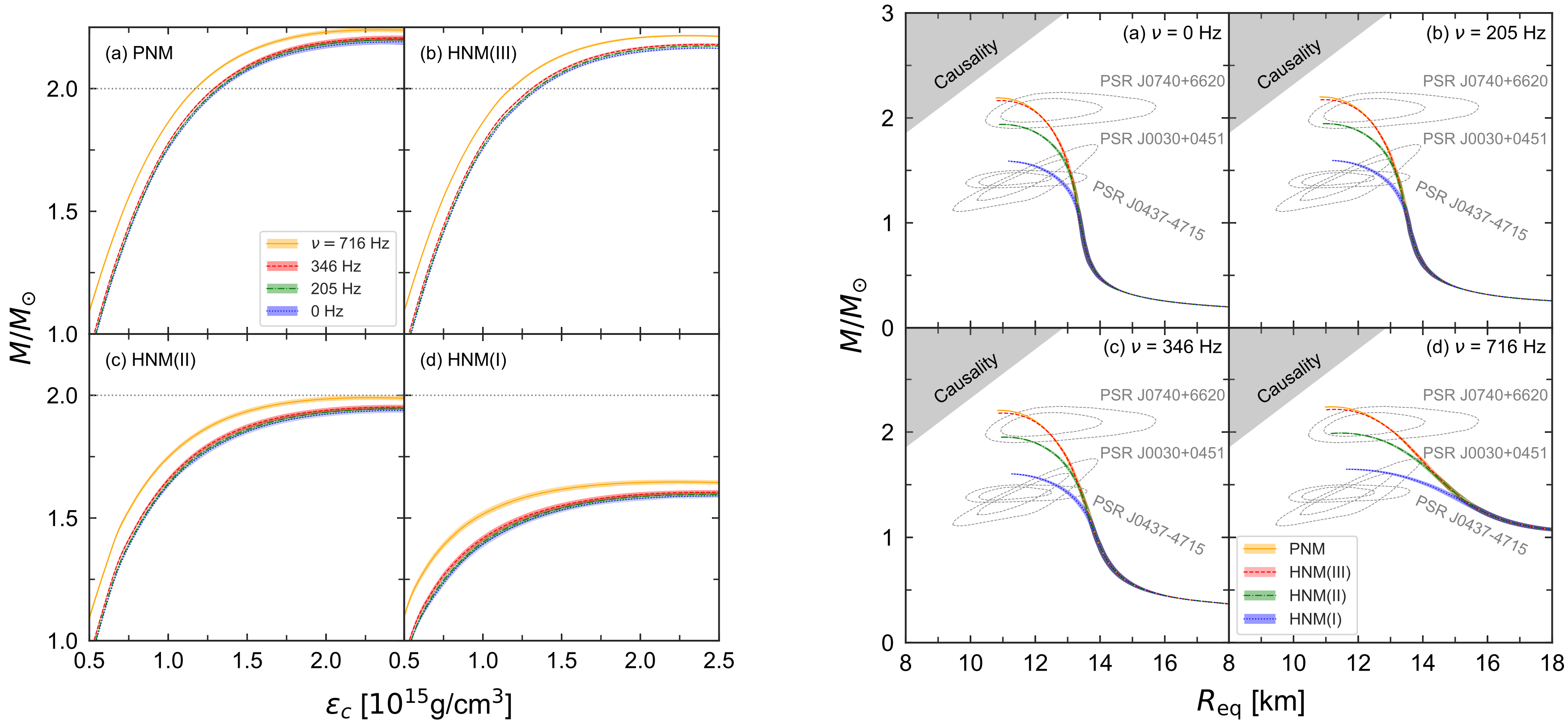
Abbott et al. (2018); Landry & Kumar (2018); Wang et al. (2022)

EoS for rotating neutron stars

Tong, Elhatisari, UGM, Astrophys. J. **982** (2025) 164

- Also rapidly rotating neutron stars observed: $\nu = 205, 346, 716$ Hz

Vinciguerra et al., Ap. J. **961** (2024) 62; Salmi et al., Ap. J. **974** (2024) 294; Hessels et al., Science **311** (2006) 1901



$\hookrightarrow$ impact of centrifugal forces visible

$\hookrightarrow$ mass-radius relations mostly consistent with the data

Hypernuclear matter in β equilibrium

- From the idealized systems to "real" neutron stars (n, p, Λ, e, μ)

$\hookrightarrow$ equilibrium conditions: $\boxed{\mu_n - \mu_p = \mu_e\,, \quad \mu_e = \mu_\mu\,, \quad \mu_n = \mu_\Lambda}$

$\hookrightarrow$ charge neutrality condition: $\boxed{\rho_p = \rho_e + \rho_\mu}$

- this leads to the following set of equations:

$$\frac{\partial E_{\rm HMN}}{\partial N_p} - \frac{\partial E_{\rm HNM}}{\partial N_n} + (m_p - m_n) + (3\pi^2\rho_e)^{1/3} = 0\,,$$

$$m_\mu + \frac{(3\pi^2)^{5/3}}{6\pi^2 m_\mu}\rho_\mu^{2/3} - (3\pi^2\rho_e)^{1/3} = 0\,,$$

$$\frac{\partial E_{\rm HMN}}{\partial N_n} - \frac{\partial E_{\rm HNM}}{\partial N_\Lambda} + m_n - m_\Lambda = 0\,,$$

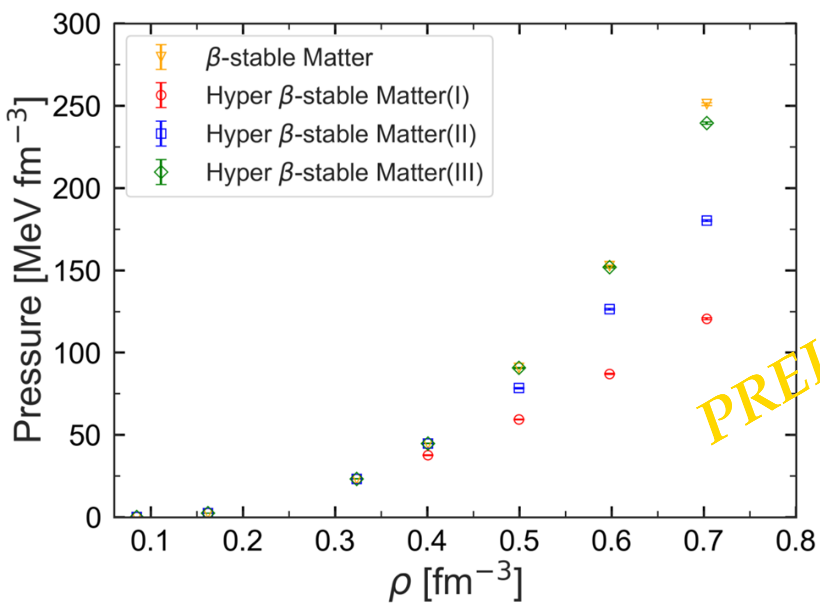
$$\rho_p - \rho_e - \rho_\mu = 0\,.$$

$\hookrightarrow$ tremendous increase in compute resources compared to the idealized case (factor of 30 at high densities)

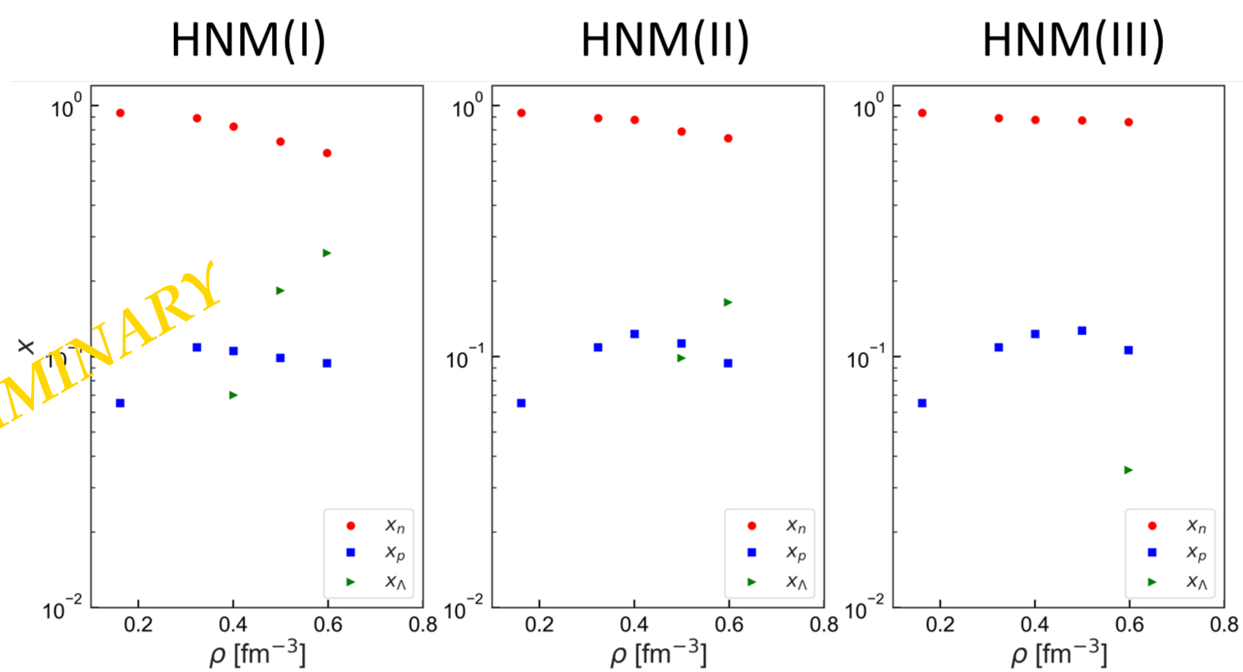
Hypernuclear matter in β equilibrium: First results

Tong, Elhatsari, UGM, *in preparation*

- EoS



- particle fractions



$\hookrightarrow$ EoS for β-stable matter and hyper β-stable matter (III) almost identical

$\hookrightarrow$ the proton fraction never exceeds $\simeq 10\%$

$\hookrightarrow$ similar to other recent results

Logoteta et al., Eur. Phys. J. A **55**:207 (2019); Vidana et al., Eur. Phys. J. A **61**:59 (2025)

Summary & outlook

- Nuclear lattice simulations: a new quantum many-body approach

  $\rightarrow$ based on the successful continuum nuclear chiral EFT

  $\rightarrow$ a number of highly visible results already obtained

- Recent developments

  $\rightarrow$ highly improved LO action based on SU(4)

  $\hookrightarrow$ a number of interesting application (^{12}C, ^{4}He,...)

  $\hookrightarrow$ neutron matter EoS at high denstities w/ $\boldsymbol{\Lambda}$'s (neutron stars)

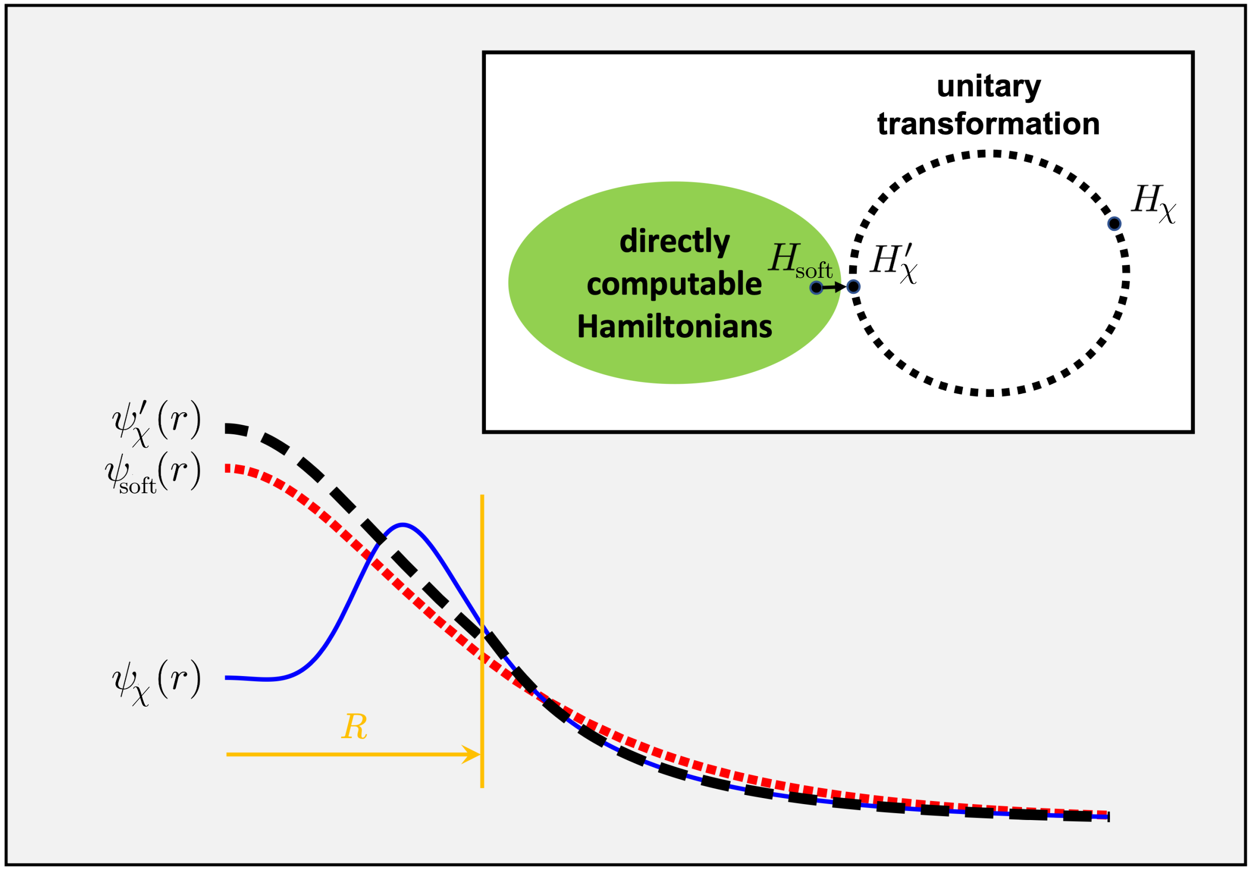
- Precision calculations at N3LO using wave function matching $\rightarrow$ a few slides

$\hookrightarrow$ stay tuned!

Wave function matching

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]]

- Graphical representation of w.f. matching

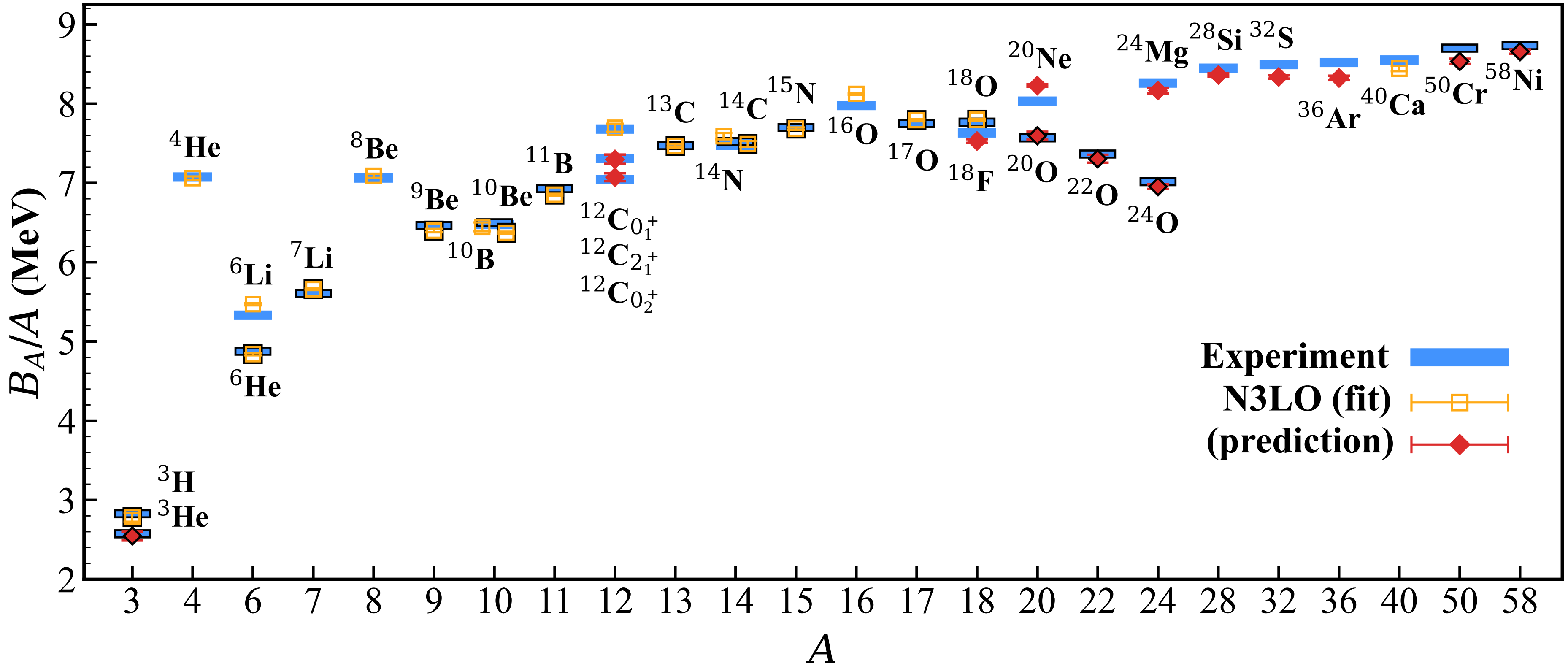


- W.F. matching is a “Hamiltonian translator”:
  eigenenergies from H_1 but w.f. from $H_2 = U^\dagger H_1 U$

Nuclei at N3LO

- Binding energies of nuclei for $a = 1.32$ fm: Determining the 3NF LECs

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]

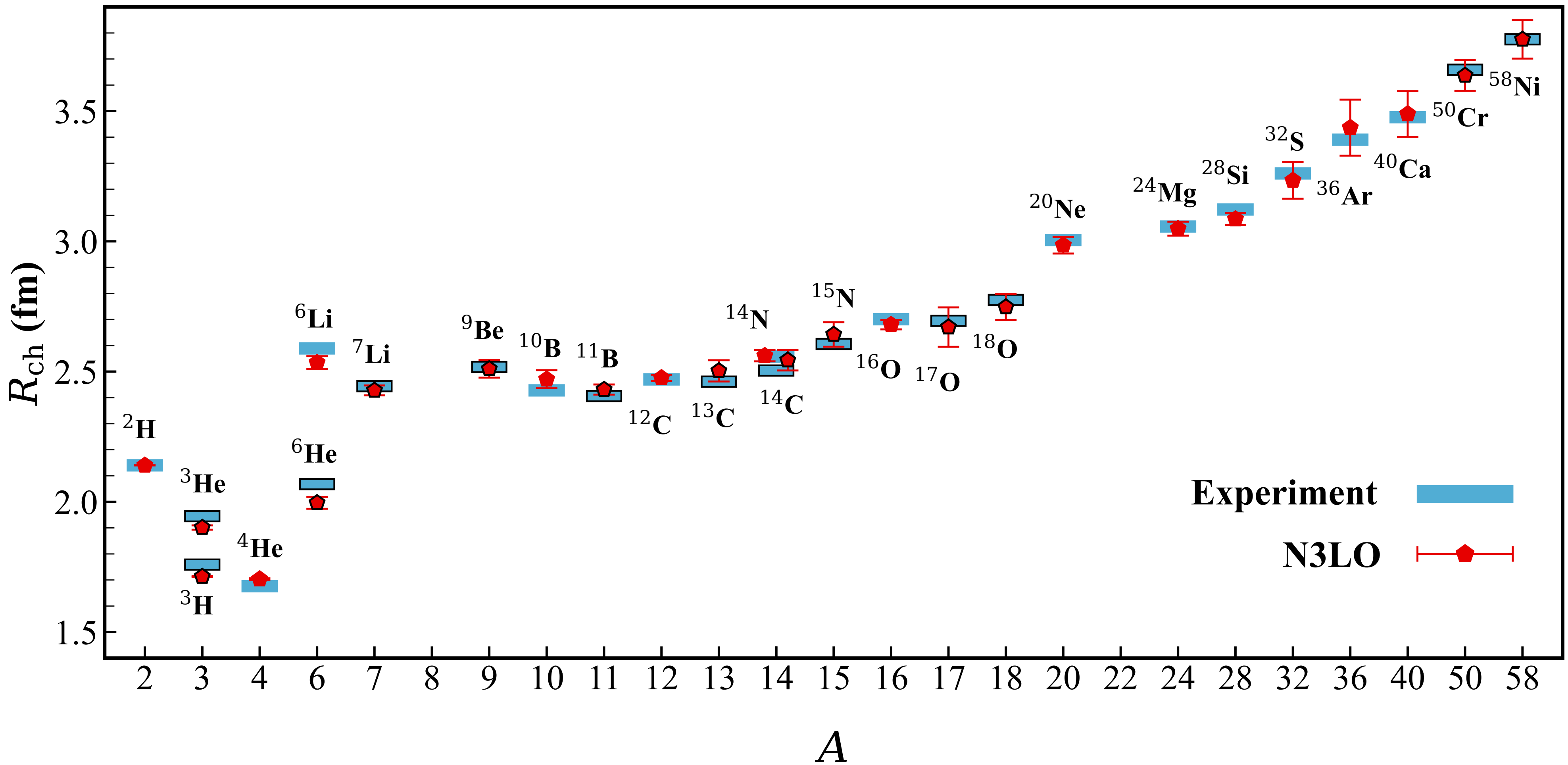


→ excellent starting point for precision studies

Prediction: Charge radii at N3LO

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]]

- Charge radii ($a = 1.32$ fm, statistical errors can be reduced)

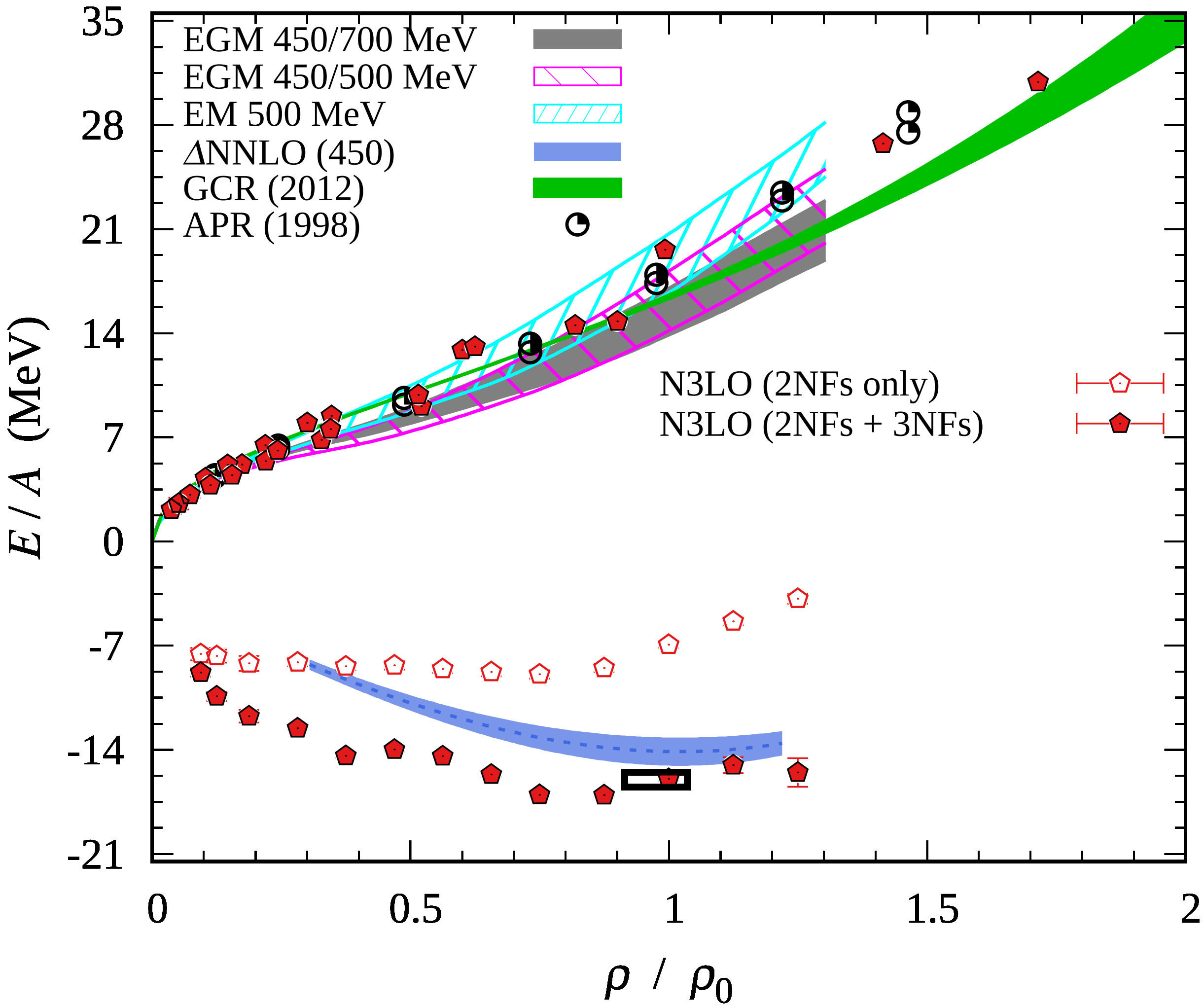


$\hookrightarrow$ no radius problem!

Prediction: Neutron & nuclear matter at N3LO

Elhatisari et al., Nature **630** (2024) 59 [arXiv:2210.17488 [nucl-th]]

- EoS of pure neutron matter & nuclear matter ($a = 1.32$ fm)



$\hookrightarrow$ can be improved using twisted b.c.'s

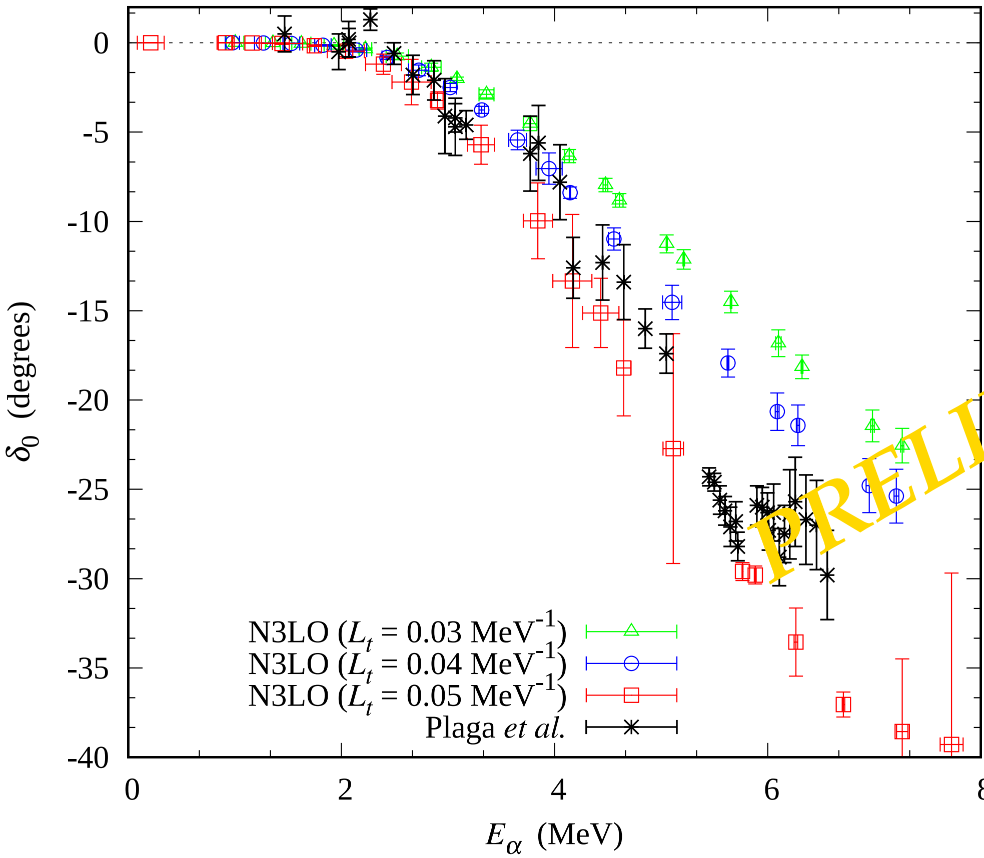
Scattering: Alpha-carbon scattering at N3LO

Elhatisari, Hildenbrand, UGM, ... NLEFT, in progress

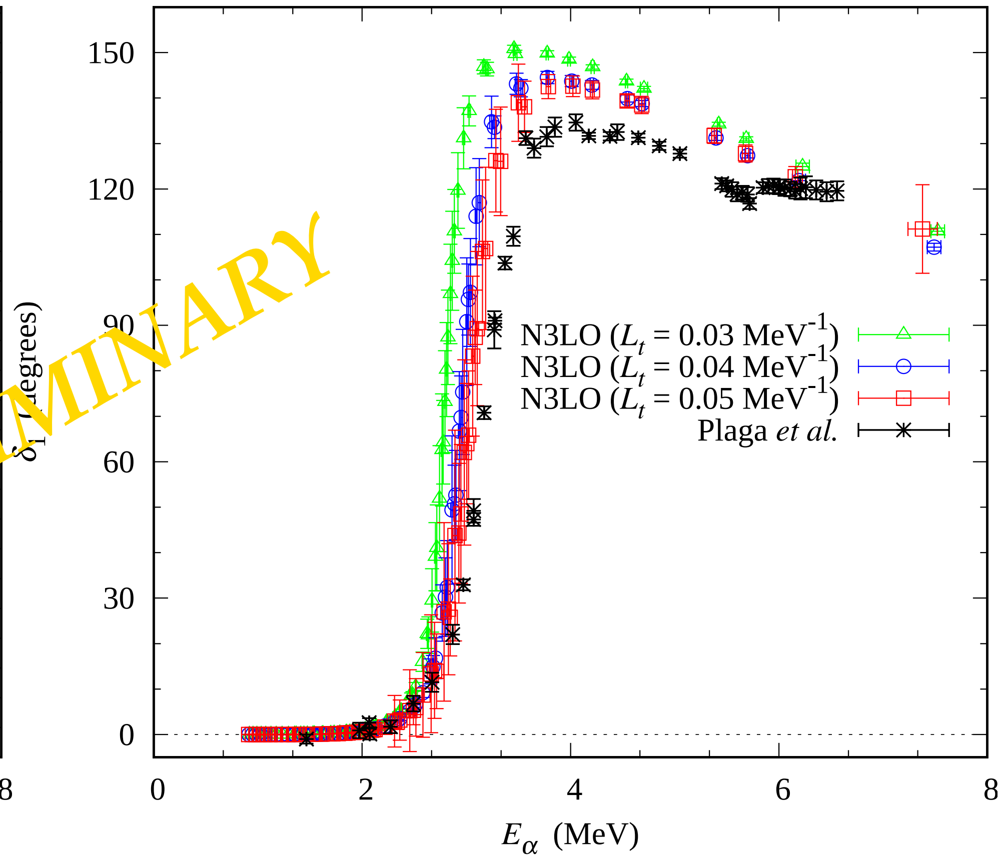
- Use the APM, first step for the holy grail of nuclear astrophysics $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$

  $\hookrightarrow$ different Euclidean times & different initial states

- S-wave



- P-wave



Plaga et al., Nucl. Phys. A **465** (1987) 291

clear resoanance signal!

SPARES

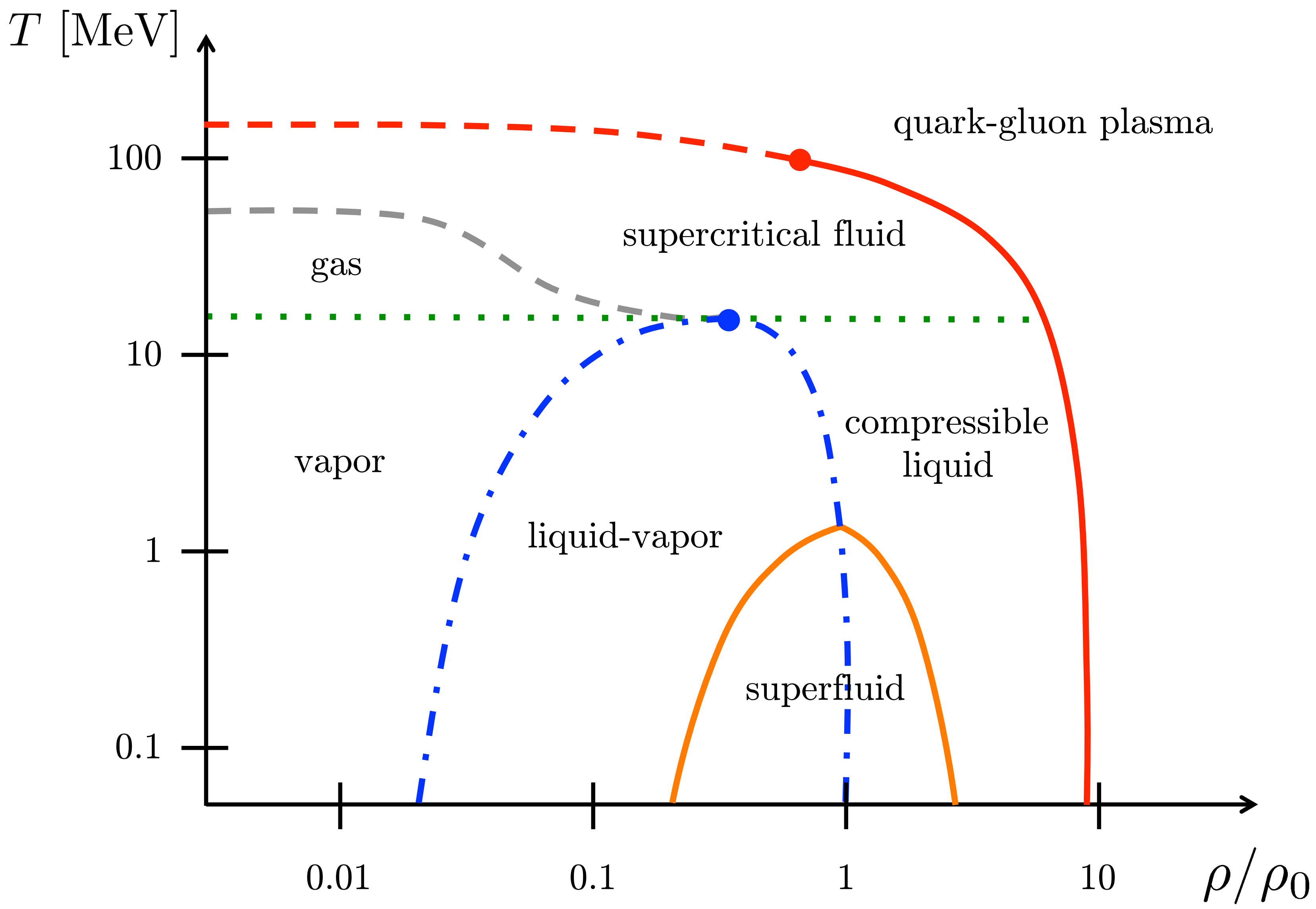
Ab Initio Nuclear Thermodynamics

B. N. Lu, N. Li, S. Elhatisari, D. Lee, J. Drut, T. Lähde, E. Epelbaum, UGM,
Phys. Rev. Lett. **125** (2020) 192502 [arXiv:1912.05105]

Phase diagram of strongly interacting matter

- Sketch of the phase diagram of strongly interacting matter

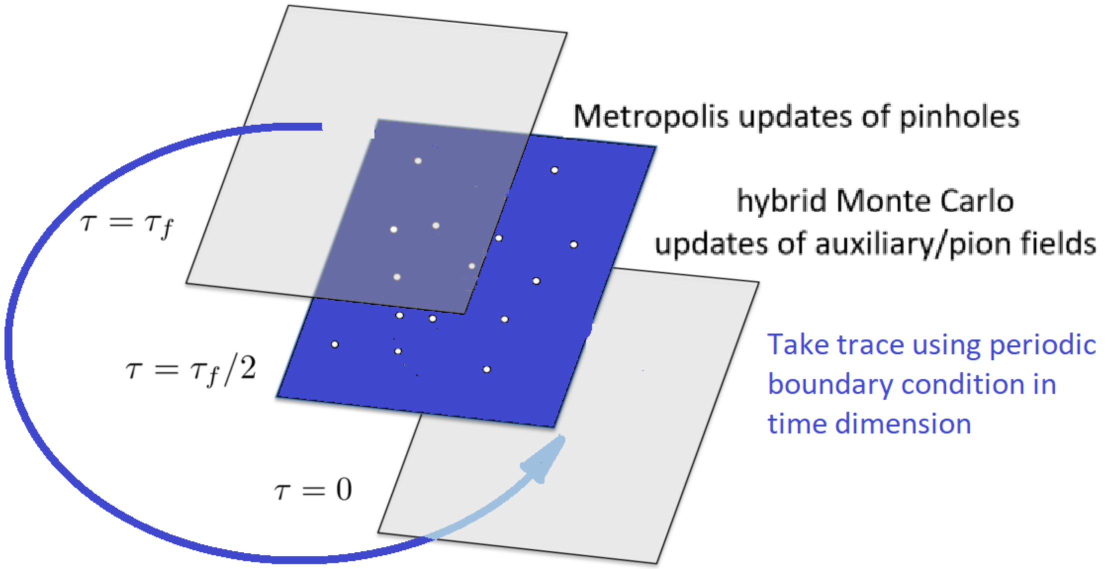
Fig. courtesy B.-N. Lu



Pinhole trace algorithm (PTA)

- The pinhole states span the whole A-body Hilbert space

- The canonical partition function can be expressed using pinholes:



$$Z_A = \mathrm{Tr}_A \left[\exp(-\beta H)\right] \,,\ \beta = 1/T$$

$$= \sum_{n_1,\cdots,n_A} \int \mathcal{D}s\mathcal{D}\pi \langle n_1,\cdots,n_A|\exp[-\beta H(s,\pi)]|n_1,\cdots,n_A\rangle$$

- allows to study: liquid-gas phase transition $\to$ this talk
  thermodynamics of finite nuclei
  thermal dissociation of hot nuclei
  cluster yields of dissociating nuclei

New paradigm for nuclear thermodynamics

- The PTA allows for simulations with fixed neutron & proton numbers at non-zero T

$\hookrightarrow$ thousands to millions times faster than existing codes using the grand-canonical ensemble ($t_{\rm CPU} \sim VN^2$ vs. $t_{\rm CPU} \sim V^3N^2$)

- Only a mild sign problem $\to$ pinholes are dynamically driven to form pairs

- Typical simulation parameters:

up to $N = 144$ nucleons in volumes $L^3 = 4^3, 5^3, 6^3$

$\hookrightarrow$ densities from 0.008 fm^{-3} ... 0.20 fm^{-3}

$a = 1.32$ fm $\to$ $\Lambda = \pi/a = 470$ MeV , $a_t \simeq 0.1$ fm

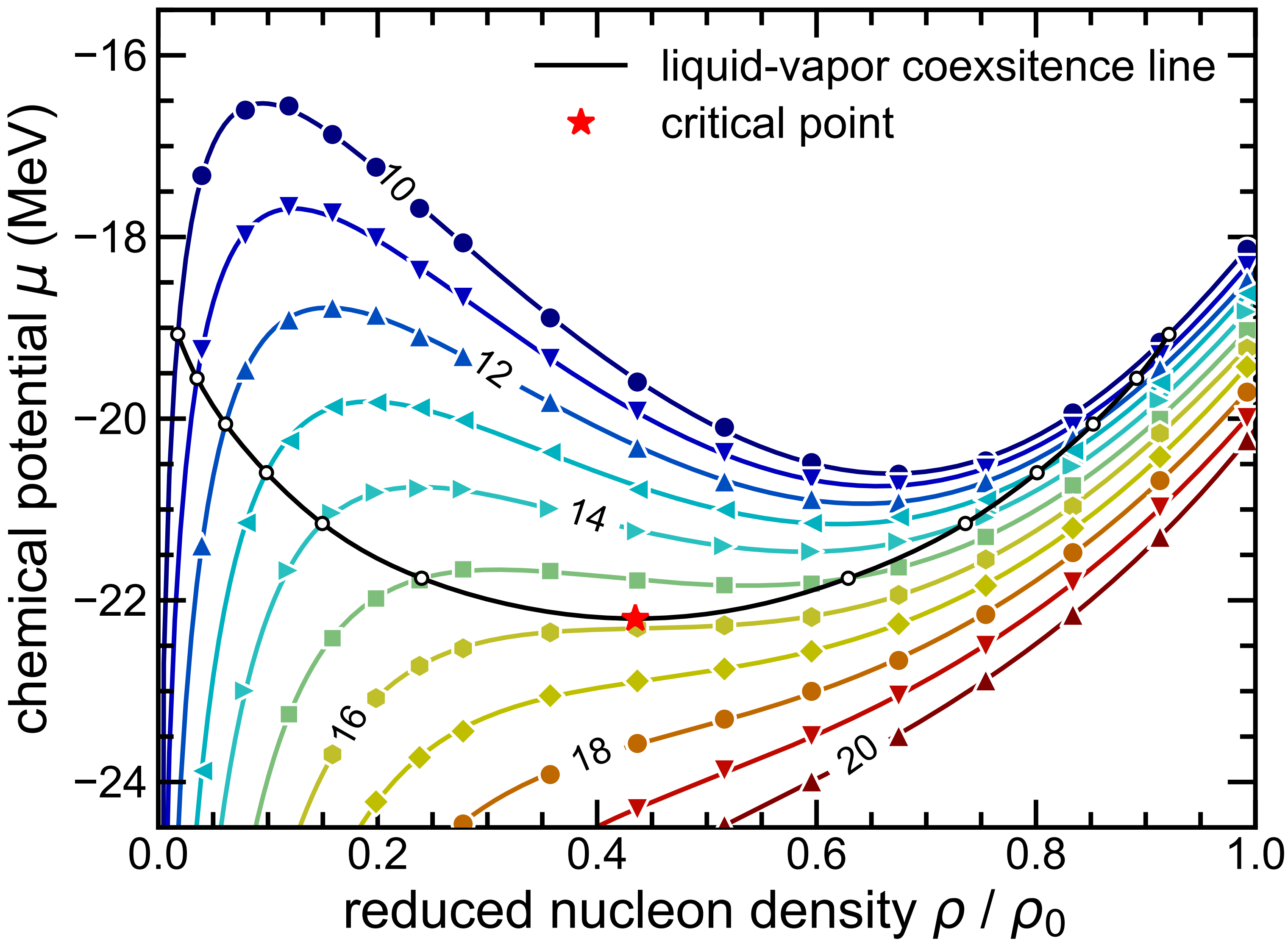
consider $T = 10\ldots 20$ MeV

- use twisted bc's, average over twist angles $\to$ acceleration to the td limit

- very favorable scaling for generating config's: $\Delta t \sim N^2 L^3$

Chemical potential

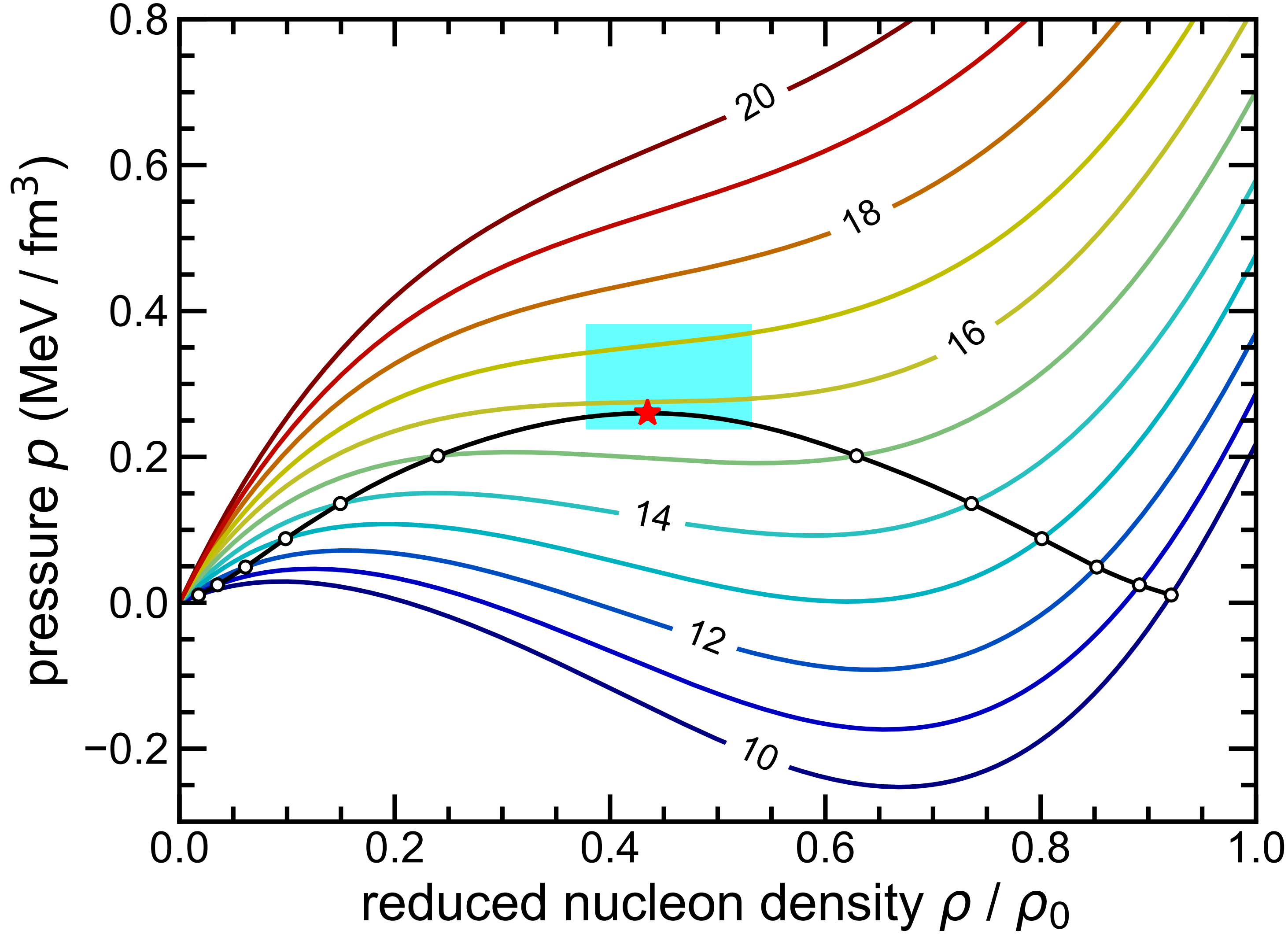
- Calculated from the free energy: $\boldsymbol{\mu = (F(N+1) - F(N-1))/2}$



at very low densities like the ideal gas $\mu \propto \log(\rho)$

Equation of state

- Calculated by integrating: $dP = \rho\, d\mu$
- Crtitical point: $T_c = 15.8(1.6)$ MeV, $P_c = 0.26(3)$ MeV/fm^3, $\rho_c = 0.089(18)$ fm^{-3}



Experiment: $T_c = 15.0(3)$ MeV, $P_c = 0.31(7)$ MeV/fm^3, $\rho_c = 0.06(2)$ fm^{-3}

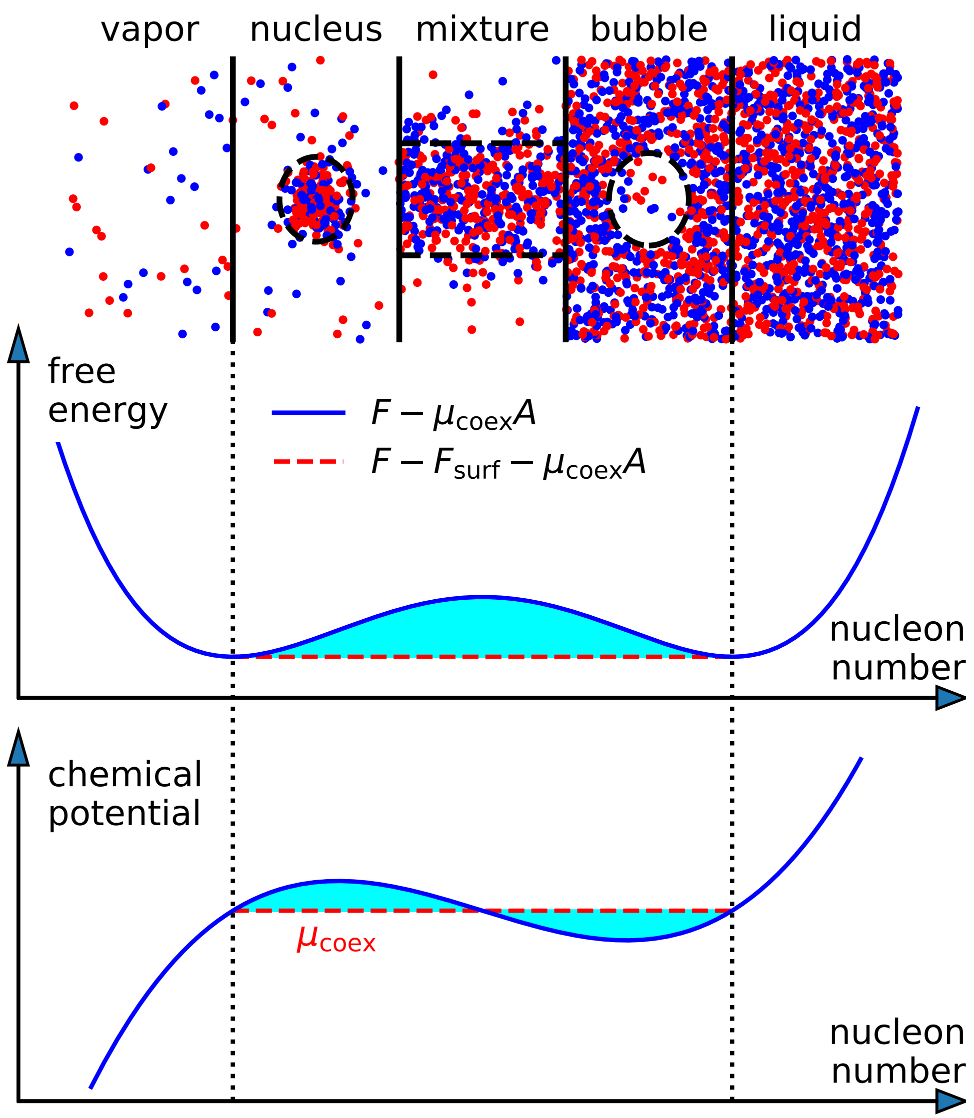
Vapor-liquid phase transition

- Vapor-liquid phase transition
  in a finite volume V & $T < T_c$

- the most probable configuration
  for different nucleon number A

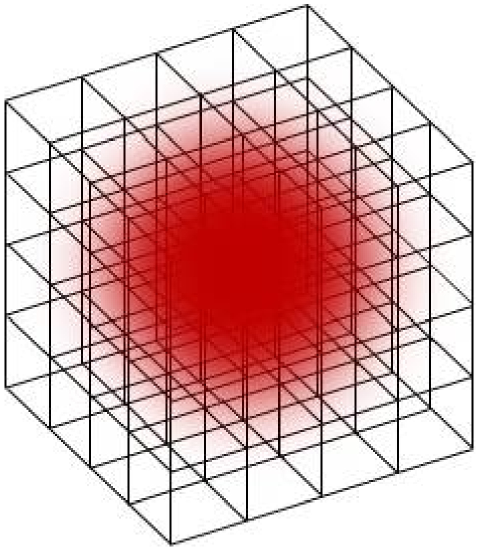
- the free energy

- chemical potential $\mu = \partial F / \partial A$



CENTER-of-MASS PROBLEM

- AFQMC calculations involve states that are superpositions of many different center-of-mass (com) positions



$$Z_A(\tau) = \langle \Psi_A(\tau) | \Psi_A(\tau) \rangle$$

$$|\Psi_A(\tau)\rangle = \exp(-H\tau/2)|\Psi_A\rangle$$

- but: translational invariance requires summation over all transitions

$$Z_A(\tau) = \sum_{i_{\rm com}, j_{\rm com}} \langle \Psi_A(\tau, i_{\rm com}) | \Psi_A(\tau, j_{\rm com}) \rangle \, , \;\; {\rm com} = {\rm mod}((i_{\rm com} - j_{\rm com}), L)$$

$i_{\rm com}$ $(j_{\rm com})$ = position of the center-of-mass in the final (initial) state

$\rightarrow$ density distributions of nucleons can not be computed directly, only moments

$\rightarrow$ need to overcome this deficieny

PINHOLE ALGORITHM

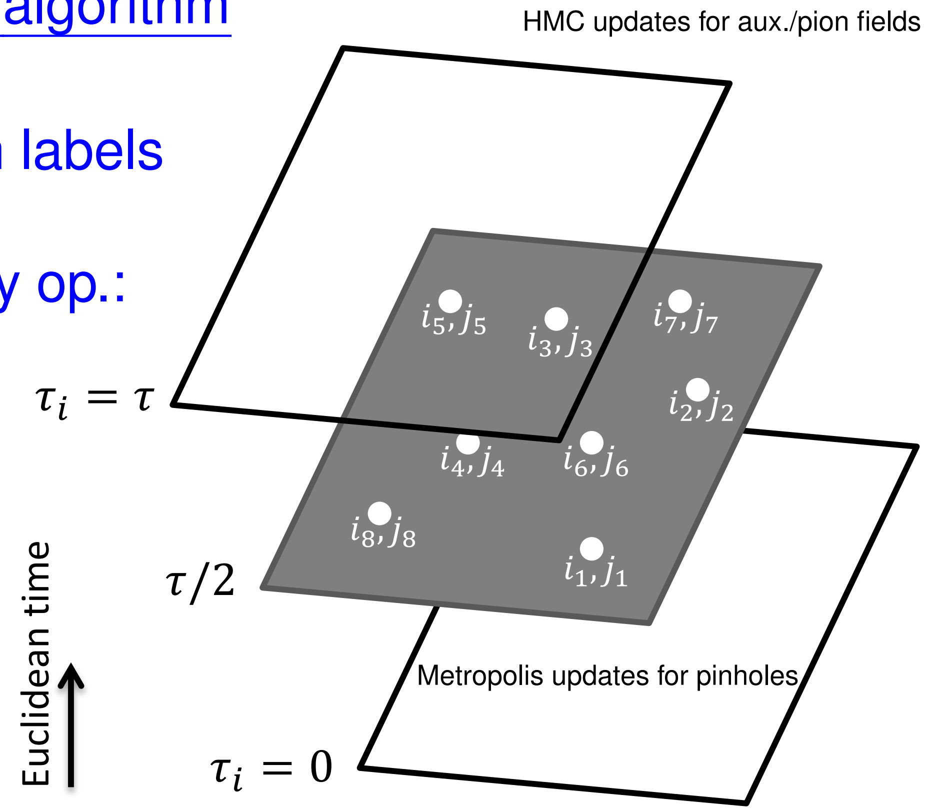
- Solution to the CM-problem:
  track the individual nucleons using the pinhole algorithm

- Insert a screen with pinholes with spin & isospin labels that allows nucleons with corresponding spin & isospin to pass = insertion of the A-body density op.:

$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots\mathbf{n}_A)$$
$$= :\rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$

- MC sampling of the amplitude:

$$A_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A,L_t)$$
$$= \langle\Psi_A(\tau/2)|\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A)|\Psi_A(\tau/2)\rangle$$



- Allows to measure proton and neutron distributions

- Resolution scale $\sim a/A$ as cm position $\mathbf{r}_{\rm cm}$ is an integer $\mathbf{n}_{\rm cm}$ times a/A