NLEFT calculations of nuclei near the dripline

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Nuclear Lattice EFT Collaboration







European Research Council Established by the European Commission

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NLEFT calculations of nuclei near the dripline

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Summary & perspective

Primary Decay Mode SIMPSON E C. J Phys: Conf Ser, 2020, 1643(1): 012168 64Ni 65Ni 66Ni 67Ni 68Ni 69Ni 70Ni 71Ni 72Ni 73Ni 74Ni 75Ni 49Ni 50Ni 51Ni 52Ni 53Ni 54Ni 55Ni ⁵⁸Ni ⁵⁹Ni ⁶⁰Ni ⁶¹Ni ⁶²Ni ⁶³Ni ⁵⁶Ni 57Ni Stable 48Ni ⁷⁷Ni 78Ni n e- capture 47C0 44C0 49C0 50C0 51C0 52C0 53C0 54C0 55C0 54C0 57C0 58C0 59C0 60C0 61C0 62C0 65C0 64C0 67C0 64C0 67C0 64C0 69C0 Fission 2n 74Co 75Co 77**Co** р β+ 2p 45Fe 46Fe 47Fe 48Fe 49Fe 50Fe 51Fe 52Fe 53Fe 54Fe 55Fe 56Fe 57Fe 58Fe ⁷⁶Fe α 59Fe 60Fe 61Fe 62Fe 64Fe 65Fe 72Fe 73Fe 74Fe 75Fe ß- $2\beta +$ 3p 43Mn 44Mn 45Mn 46Mn 47Mn 48Mn 49Mn 50Mn 51Mn 52Mn 53Mn 54Mn 55Mn 56Mn 57Mn 58Mn 59Mn ¹⁰Mn ⁷¹Mn ⁷²Mn ⁷³Mn 2βe+ ⁴¹Cr ⁴²Cr ⁴³Cr ⁴⁸Cr ⁴⁹Cr ⁵⁰Cr ⁵¹Cr ⁵²Cr ⁵³Cr ⁵⁴Cr 44Cr 45Cr 55Cr 68Cr 69Cr 70Cr Long-lived ^{40}V 41V 42V47V 48V 49V 50V 51V 52V 53V 43V 45V 46V54VEstimated 37Ti 38Ti 39Ti 40Ti 41Ti ⁴²Ti ⁴³Ti ⁴⁴Ti ⁴⁵Ti ⁴⁶Ti 47Ti 48Ti 49Ti 50Ti 64Ti 65Ti 52Ti Unknown 42Sc 43Sc 44Sc 45Sc 46Sc 47Sc 48Sc 49Sc ³³Ca ³⁴Ca ³⁵Ca ³⁶Ca ³⁷Ca ⁴¹Ca ⁴²Ca ⁴³Ca ⁴⁴Ca ⁴⁵Ca ⁴⁶Ca ⁴⁷Ca ³⁸Ca ³⁹Ca ⁴⁰Ca ⁴⁸Ca 49Ca 50Ca 51Ca 52Ca ⁵⁹Ca 60Ca 61Ca ³¹K ³²K ^{33}K $^{34}\mathrm{K}$ 35K 36K 37K $^{38}\mathrm{K}$ ³⁹K ⁴⁰K ⁴¹K ⁴²K ⁴³K ⁴⁴K ⁴⁵K ⁴⁶K 47K ^{48}K 49K 50K 56K 57K29Ar 30Ar 31Ar 32Ar 33Ar 34Ar 35Ar 36Ar ³⁸Ar ³⁹Ar ⁴⁰Ar 52Ar 53Ar 54Ar ³⁷Ar ⁴¹Ar ⁴²Ar ⁴³Ar ⁴⁴Ar ⁴⁵Ar ⁴⁶Ar ⁴⁷Ar 51**Ar** 32Cl 33Cl 34Cl 35Cl 36Cl 37Cl 38Cl 39Cl 40Cl 28Cl 29Cl 30Cl 31Cl 41C1 42C1 43C1 44C1 45Cl 46CL 47CL 48C1 50Cl 51Cl 52Cl 26S 27S35S 31S 37S 32**P** 33**P** 34**P** $^{25}\mathbf{P}$ 29**P** $^{30}\mathbf{P}$ $^{31}\mathbf{P}$ 35P 36P 43**P** 42P 22Si 23Si 24Si 25Si 26Si 27Si 28Si 29Si 30Si ³¹Si ³²Si ³³Si ³⁴Si 35Si 41Si 42Si 43Si 44Si 45Si 21A1 22A1 23A1 24A1 25A1 26A1 27A1 28A1 29A1 30A1 31A1 32A1 33A1 34A1 35A1 36A1 37A1 38A1 39A1 40A1 41A1 42A1 43A1 19Mg 20Mg 21Mg 22Mg 23Mg 24Mg 24Mg 24Mg 26Mg 27Mg 28Mg 29Mg 30Mg 31Mg 32Mg 33Mg 34Mg 35Mg 36Mg 37Mg 38Mg 98Mg 40Mg 41Mg 17Na ¹⁸Na ¹⁹Na ²⁰Na ²¹Na ²²Na ²²Na ²⁴Na ²⁵Na ²⁶Na ²⁷Na ²⁸Na ²⁷Na ²⁸Na ²⁹Na ³⁰Na ³¹Na ³¹Na ³¹Na ³⁴Na ³⁵Na ³⁴Na ³⁵Na ³⁶Na ³⁷Na ³⁷Na ³⁵Na ³⁶Na ³⁷Na ³⁵Na ³⁶Na ³⁷Na ³⁶Na ³⁶ 39N2 ¹⁹Ne ²⁰Ne ²¹Ne ²²Ne ²³Ne ²⁴Ne ¹⁵Ne ¹⁶Ne ¹⁷Ne ¹⁸Ne 27Ne 28Ne 29Ne ³⁰Ne ³¹Ne ³²Ne ³³Ne ³⁴Ne ²⁶Ne ^{18}F $^{14}\mathrm{F}$ 15F 16F 17F ^{20}F 29F 21F ^{27}F ^{28}F 30F 31F 11O 12O 13O 14O15**O** ¹⁶O ¹⁹O ²⁰O ²¹O 220 230 24O 25O 26O ^{13}N ^{14}N $^{10}\mathrm{N}$ пN ^{12}N ۱N 17N ^{23}N 18N21N22N8C 11C 12C ۶C 10C15**C** ^{14}C 20C ^{22}C ۶B $^7\mathrm{B}$ 8B 9B $10\mathbf{B}$ пB $^{12}\mathbf{B}$ 13B ^{14}B 15B18B 19B 20B 17B5Be ¹⁰Be ¹¹Be ¹²Be ¹³Be ¹⁴Be ¹⁵Be ¹⁶Be ۶Be ⁷Be [§]Be 9Be ³Li ⁴Li ⁵Li ⁶Li ⁷Li ⁸Li ⁹Li ¹⁰Li ¹¹Li ¹²Li ¹³Li ³He ⁴He ⁵He ⁶He ⁷He ⁸He ⁹He 10He ιH $^{2}\mathrm{H}$ 4H ⁵H зH ۴H ^{7}H 1n

Primary Decay Mode SIMPSON E C. J Phys: Conf Ser, 2020, 1643(1): 012168 58Ni 59Ni 60Ni 61Ni 62Ni 63Ni 64Ni 65Ni 66Ni 67Ni 68Ni 69Ni 70Ni 72Ni 72Ni 73Ni 74Ni 75Ni 49Ni 50Ni 51Ni 52Ni 53Ni 54Ni 55Ni 56Ni 57Ni Stable 48Ni ⁷⁷Ni 78Ni n e- capture 47C0 44C0 49C0 50C0 51C0 52C0 53C0 54C0 55C0 55C0 55C0 59C0 59C0 59C0 64C0 64C0 65C0 64C0 65C0 64C0 65C0 66C0 67C0 66C0 69C0 Fission 2n 74Co 75Co 77**Co** р β+ 2p 45Fe 46Fe 47Fe 48Fe 49Fe 50Fe 51Fe 52Fe 53Fe 54Fe 55Fe 56Fe 57Fe 58Fe ⁷⁶Fe α 59Fe 60Fe 61Fe 62Fe 64Fe 65Fe 73Fe 74Fe 75Fe ß- $2\beta +$ 3p 43Mn 44Mn 45Mn 46Mn 47Mn 48Mn 49Mn 50Mn 51Mn 52Mn 53Mn 54Mn 55Mn 56Mn 57Mn 58Mn 59Mn 71Mn 72Mn 73Mn 2βe+ ⁴¹Cr ⁴²Cr ⁴³Cr 49Cr 50Cr 51Cr 52Cr 53Cr 54Cr 44Cr 45Cr 55Cr 68Cr 69Cr 70Cr Long-lived ^{40}V 41V 42V48V 49V 50V 51V 52V 53V 43V 45V 46V47V54V Estimated 37Ti 38Ti ³⁹Ti ⁴⁰Ti ⁴¹Ti 42Ti 43Ti 44Ti 45Ti 46Ti 47Ti 48Ti 49Ti 50Ti 64Ti 65Ti 1Ti 52Ti Unknown ⁴²Sc ⁴³Sc ⁴⁴Sc ⁴⁵Sc 46Sc 47Sc 48Sc ³³Ca ³⁴Ca ³⁵Ca ³⁶Ca ³⁷Ca ⁴²Ca ⁴³Ca ⁴⁴Ca ⁴⁵Ca ³⁸Ca ⁴⁶Ca 47Ca 48Ca 49Ca 50Ca 51Ca ⁵⁹Ca 60Ca 61Ca ³¹K ³²K ^{33}K 36K $^{38}\mathrm{K}$ ³⁹K ⁴⁰K 41K 42K 43K 44K 45K 46K $^{34}\mathrm{K}$ 37K47K 48K 49K 50K 56K 57K29Ar 30Ar 31Ar 32Ar 33Ar 34Ar 35Ar 36Ar ³⁸Ar ³⁹Ar ⁴⁰Ar 52Ar 53Ar 54Ar ³⁷Ar ⁴¹Ar ⁴²Ar ⁴³Ar ⁴⁴Ar ⁴⁵Ar ⁴⁶Ar 51**Ar** 47Ar 48Ar 32Cl 33Cl 34Cl 35Cl 36Cl 37Cl 38Cl 39Cl 40Cl 28Cl 29Cl 30Cl 31Cl 41C1 42C1 43C1 44C1 45Cl 46C1 47C1 48C1 49C1 50Cl 51Cl 52Cl 26S 27S ^{32}S 35S 305 31S 37S 32**P** 33**P** 34**P** $^{25}\mathbf{P}$ 29**P** $^{30}\mathbf{P}$ $^{31}\mathbf{P}$ 35P 36P 42P 22Si 23Si 24Si 25Si 26Si 27Si 28Si 29Si 30Si ³¹Si ³²Si ³³Si ³⁴Si 35Si 41Si 42Si 43Si 44Si ⁴⁵Si 21AI 22AI 23AI 24AI 25AI 26AI 27AI 28AI 29AI 30AI 31AI 32AI 33AI 34AI 35AI 36AI 37AI 38A1 39A1 40A1 41A1 42A1 43A1 19Mg 20Mg 21Mg 22Mg 22Mg 24Mg 24Mg 24Mg 26Mg 27Mg 28Mg 29Mg 30Mg 31Mg 32Mg 33Mg 34Mg 35Mg 36Mg 37Mg 38Mg 39Mg 40Mg 41Mg 17Na ¹⁸Na ¹⁹Na ²⁰Na ²¹Na ²²Na ²²Na ²²Na ²⁴Na ²⁵Na ²⁶Na ²⁷Na ²⁸Na ²⁹Na ³⁰Na ³¹Na ³¹ ³⁹Na ¹⁹Ne ²⁰Ne ²¹Ne ²²Ne ²³Ne ²⁴Ne ²⁵Ne ¹⁵Ne ¹⁶Ne ¹⁷Ne ¹⁸Ne ²⁷Ne ²⁸Ne ²⁹Ne ³⁰Ne ³¹Ne ³²Ne ³³Ne ³⁴Ne ²⁶Ne ^{18}F $^{14}\mathrm{F}$ 15F 16F 17F19F ^{20}F 29F 21F ^{27}F ^{28}F 30F 31F 11O 12O 13O 14**O** 15**O** ¹⁶O ¹⁹O ²⁰O ²¹O 220 230 24O 25O 26O 270 ¹³N ¹⁴N ¹⁵N $^{10}\mathrm{N}$ пN ^{12}N ۱M 17N18N ^{22}N $^{23}\mathrm{N}$ 21N8C 11C 12C °C 10C15C ^{14}C 20C ^{22}C ۶B $^7\mathrm{B}$ 8B 9B $10\mathbf{B}$ пB $^{12}\mathbf{B}$ $^{13}\mathbf{B}$ ^{14}B 15B¹⁹B ²⁰B 17B5Be ¹²Be ¹³Be ¹⁴Be ¹⁵Be ¹⁶Be ۶Be ⁷Be [§]Be 9Be ¹⁰Be ¹¹Be ³Li ⁴Li ⁵Li ⁶Li ⁷Li ^sLi ⁹Li ¹⁰Li ¹¹Li 12Li 13Li ³He ⁴He ⁵He ⁶He ⁷He ⁸He ⁹He 10He 4H 5H 6H 7H $^{1}\mathrm{H}$ $^{3}\mathrm{H}$ ^{1}n

Primary Decay Mode SIMPSON E C. J Phys: Conf Ser, 2020, 1643(1): 012168 ⁶⁰Ni ⁶¹Ni 63Ni 65Ni 66Ni 67Ni 68Ni 51Ni 52Ni 53Ni 54Ni ⁵⁹Ni Stable n e- capture 55Co 56Co 57Co 58Co 59Co 60Co 61Co 62Co 63Co Fission 2n p β+ 2p 55Fe 56Fe 57Fe α ⁴⁵Fe 52Fe 53Fe ⁵⁴Fe ⁵⁸Fe ⁷⁶Fe ß. 2β + 3p ⁴³Mn ⁴⁴Mn ⁴Mn ⁵⁵Mn 2βe+ ⁴¹Cr 42Cr 43Cr 52Cr 53Cr Long-lived 41V49V 50V 52VEstimated 38Ti 40Ti 48Ti 49Ti 50Ti 47Ti Unknown 46Sc 47Sc 48Sc ⁴⁵Sc ³³Ca ³⁴Ca **Essential elements for nuclear binding** ^{42}K Ar 42Ar ³⁶Ar ³⁸Ar ³⁹Ar ⁴⁰Ar 28C 29C 34C1 35Cl 36Cl 37Cl 38Cl 39CI 40CI 0 Pion-less EFT Leading Order 33P 22Si (MeV) 21A1 22A1 23A1 24A1 25A1 26A1 27A1 28A1 29A1 30A1 -100 ¹⁹Mg ²⁰Mg ²¹Mg ²²Mg ²³Mg ²⁴Mg ²⁵Mg ²⁶Mg ²Mg ³³Mg 27Mg 28Mg Binding energy ¹⁷Na ¹⁸Na ¹⁹Na ²⁰Na ²¹Na ²²Na ²³Na ²⁴Na ²⁵Na ¹¹Na ³²Na 15Ne 16Ne 17Ne 18Ne 9Ne -200 18F 16F 15**O** 16O 190 -300 ^{13}N 8**C** Lattice 11**C** ۶B 7**B** 8**B** 9B 10**B** 12**B** Exp -400 5Be ٥Be ⁸Be $^{2}\mathrm{Be}$ ⁷Be ¹⁴Be ¹⁵Be ³Li ⁴Li ۶Li •Li ST i пLi ¹²Li 9T.i 10Li 10 20 30 40 50 0 ³He 4He 'He 'He 'He 'He 'He Mass number A 4H 5H 6H 7H $^{1}\mathrm{H}$ зH Lv, et al., Phys. Lett. B 797 (2019) 134863 1n

describes binding energies, radii, and the EoS of neutron matter

Primary Decay Mode SIMPSON E C. J Phys: Conf Ser, 2020, 1643(1): 012168 65Ni 66Ni 67Ni 68Ni Stable 54Ni n e- capture 56Co 59Co 60Co 61Co 62Co 63Co Fission 2n 47Co 55Co 58Co р β+ 2p 55Fe 56Fe 57Fe 58Fe ⁷⁶Fe α ⁴⁵Fe 46Fe 47Fe 52Fe 53Fe ⁵⁴Fe ⁵⁹Fe ß- $2\beta +$ 3p ⁴³Mn ⁴⁴Mn 45Mn ⁴Mn ⁵⁵Mn ⁵¹Mn 52Mn 2βe+ ⁴¹Cr 42Cr 43Cr ⁵²Cr ⁵³Cr ⁵⁰Cr ¹Cr 54C Long-lived ^{40}V 41V ^{49}V 50V52V42VEstimated 37Ti 38Ti 39Ti ⁴⁰Ti 47Ti ⁴⁸Ti ⁴⁹Ti ⁵⁰Ti Ti Unknown ⁴⁵Sc 46Sc 47Sc 48Sc 44Sc ⁵⁰Sc ⁴²Ca ⁴³Ca ⁴⁴Ca ³⁴Ca ⁴⁵Ca ⁴⁶Ca 47Ca 44K 45K Wave function matching ³⁹K 40K 41K 42K 43K 29Ar 30Ar ³⁶Ar ³⁸Ar ³⁹Ar ⁴⁰Ar ¹Ar 36Cl 37Cl 38Cl 39Cl 40Cl 41Cl 28C 29C 34C1 35Cl 48C1 49C1 50Cl 51Cl 52Cl b 9 32S 35S 28Si ²⁴Mg ²⁰Ne 33**P** ⁵⁰Cr 8 ⁴⁰Ca 22Si 23Si 25Si 27**S**i 28S ³⁶Ar ⁴He ⁸Be 21A1 22A1 23A1 24A1 25A1 26A1 27A1 28Al 29Al 30A1 170 ²⁰O 18⊏ 7 ²²O ¹⁹Mg ²⁰Mg ²¹Mg ²²Mg ²³Mg ²⁴Mg ²⁵Mg ²⁶Mg ¹⁰Be ²⁷Mg ²⁸Mg ²⁹Mg ¹²C₀; 24O 17Na 18Na 19Na (MeV) ²⁰Na ²¹Na ²²Na ²³Na ²⁴Na ²⁵Na 26NIa ${}^{12}C_{2^+}$ 7Li 6 ⁶Li ${}^{12}C_{0_2^+}$ 15Ne 16Ne 17Ne 18Ne ⁹Ne ²¹Ne ²²Ne **.** A 16F 18F 17F BA 5 P 11**O** 12**O** 13**O** 14**O** 15**O** 16O 180 190 200 210 220 230 240 250 ⁶He 12N ^{13}N 4 Experiment 8**C** 11C N3LO (fit) ЗH ۶B $^7\mathrm{B}$ 8**B** 9B 10**B** ШB 12B13**B** 4B 3 😰 ³He (Prediction) ⁵Be ٥Be ⁸Be ¹⁰Be ¹¹Be $^{2}\mathrm{Be}$ ⁷Be ³Be ¹⁴Be ¹⁵Be ³Li ⁴Li ۶Li 2 ^sLi 9Li пLi ¹²Li 10Li 16 17 18 20 22 24 28 32 36 40 50 58 3 6 7 8 9 10 11 12 13 14 15 4 ³He 4He ⁵He ⁶He ⁷He ⁸He ⁹He А ¹H ²H $^{3}\mathrm{H}$ 4H ⁵H ۴H $^{7}\mathrm{H}$ S. Elhatisari et al., Nature 630 (2024) 59 ¹n

Precisely describes binding energies, radii, and nuclear-matter saturation at N³LO

Primary Decay Mode SIMPSON E C. J Phys: Conf Ser, 2020, 1643(1): 012168 ⁶⁰Ni ⁶¹Ni ⁶²Ni 63Ni 64Ni 65Ni 66Ni 67Ni 68Ni 69Ni ⁴⁹Ni ⁵⁰Ni ⁵¹Ni ⁵²Ni 53Ni 54Ni 55Ni ⁵⁹Ni 70Ni 71Ni 72Ni Stable 56Ni 57Ni 73Ni 74Ni 75Ni n e- capture 49Co 50Co 51Co 52Co 53Co 54Co 55Co 56Co 57Co 58Co ⁵⁹Co 60Co 61Co 62Co 63Co 64Co 65Co Fission 2n 47Co ⁴⁸Co p β+ 2p 55Fe 56Fe 57Fe 58Fe α ⁴⁵Fe 46Fe 47Fe 48Fe 49Fe 50Fe 51Fe 52Fe 53Fe 54Fe ⁵⁹Fe ⁷⁴Fe ⁷⁵Fe ⁷⁶Fe ß- $2\beta +$ 3p ⁴³Mn ⁴⁴Mn ⁴⁵Mn 50Mn 51Mn 52Mn ⁵³Mn ⁴Mn ⁵⁵Mn ⁵⁶Mn ⁵⁷Mn ⁷¹Mn ⁷²Mn ⁷³Mn 2βe+ 41Cr 42Cr 43Cr ⁵⁰Cr ⁵¹Cr ⁵²Cr ⁵³Cr 54Cr 68Cr 69Cr 70Cr Long-lived ^{40}V 41V ^{49}V 42V48V50V 52VEstimated 37Ti 38Ti 39Ti ⁴⁰Ti 41Ti ⁴³Ti 44Ti 45Ti ⁴⁶Ti 47Ti ⁴⁸Ti ⁴⁹Ti 2Ti 64Ti 65Ti Unknown 43Sc 44Sc 45Sc 42Sc46Sc 47Sc 48Sc ⁴²Ca ⁴³Ca ⁴⁴Ca ³³Ca ³⁴Ca ⁴⁵Ca ⁴⁶Ca 47Ca ⁹Ca ²⁰⁸Pb 40K ^{44}K 45K ³¹K ³²K 33K 34K 37K 38K 39K 41K 42K ^{43}K 46K 48K ²⁹Ar ³⁰Ar ³¹Ar 34Ar 35Ai ³⁶Ar ³⁷Ar ³⁸Ar ³⁹Ar ⁴⁰Ar ¹Ar ⁴²Ar ⁴³Ar 44Ar 45Ar ⁴⁶Ar 53Ar 54Ar 52Ar (this work) 28Cl 29Cl 30Cl 32C1 33Cl 34Cl 35Cl 36Cl 37Cl 38Cl 39Cl 40C1 50Cl 51Cl 52Cl 26S 35S 315 36S 31P 33P 34**P** ¹³²Sn 22Si 23Si 24Si 25Si 27S 28Si 29Si 31Si 32Si 33Si 44Si 21A1 22A1 23A1 24A1 25A1 26A1 27A1 28A1 29A1 30A1 31A1 32A1 33Al ³⁴Al ³⁹Al ⁴⁰Al ⁴¹Al ⁴²Al ⁴³Al 35Al 36Al 37Al 38A1 2020 ¹⁰⁰Sn ¹⁹Mg ²⁰Mg ²¹Mg ²²Mg ²³Mg ²⁴Mg ²⁵Mg ²⁶Mg ²⁷Mg ²⁸Mg ²⁹Mg ³⁰Mg ³¹Mg 32Mg 33Mg 34Mg 35Mg 36Mg 37Mg 38Mg 39Mg 40Mg 41Mg ¹⁷Na ¹⁸Na ¹⁹Na ²⁰Na ²¹Na ²²Na ²³Na ²⁴Na ²⁵Na ²⁶Na 29Na 30Na 31Na 32Na 33Na 34Na ³⁵Na ³⁷Na 27Na28Na 2018 ⁷⁸Ni ¹⁵Ne ¹⁶Ne ¹⁷Ne ¹⁸Ne ¹⁹Ne ²⁰Ne ²¹Ne ²²Ne ²⁸Ne ²⁹Ne 0Ne ³¹Ne ³²Ne ³³Ne ^{15}F 16F 17F ^{18}F 29F 20F 27F $^{30}\mathbf{F}$ ^{31}F 82 2016 2016 🛄 2015 11**O** 12**O** 13**O** 14**O** 15**O** 17O 18O19O 20O 210 220 230 240 250 201 $^{13}\mathbf{N}$ 126 ^{10}N пN ^{12}N log(*R*_{TOP500} Nucleons, 50 G 8C ۶**C** 10C пC 20 ^{22}C 2000 2820 82 ۶B $^{7}\mathbf{B}$ 8B ۶B 10**B** пB 12B13**B** ^{4}B $^{20}\mathbf{B}$ 15**B** 17**B** 19B 5Be 6Be ⁷Be [§]Be ¹⁰Be ¹¹Be ¹²Be ¹³Be ¹⁴Be ¹⁵Be ⁹Be 50 ³Li 4Li 5Li Li ^sLi пLi ¹²Li ηLi 9Li ¹⁰Li 13Li Ъ ³He 4He ⁵He ⁶He ⁷He ⁸He ⁹He 28 20 ιH 4H ۶H ٥H Protons, 2 Neutrons, N B.S. Hu et al., Nat. Phys. 18, 1196–1200 (2022)







NLEFT framework

Chiral force

Discretized in lattice space a ~ 1.32 fm np phase shifts to N³LO

Many-body method

Auxiliary field Quantum Monte Carlo

Full sets of many-body correlations Get states from imaginary time projection

 $|\Psi_{g.s.}
angle \propto \lim_{ au o \infty} \exp(- au H) |\Psi_A
angle$ Wave function Matching

 H_s (contact, regularized OPE) Mapping the unitarily transformed H'_x to H_s

Perturbative method in QMC

$$\langle \mathcal{O} \rangle = \frac{\langle \Psi | \mathcal{O} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \frac{\langle \Psi^{(0)} + \Psi^{(1)} + \dots | \mathcal{O} | \Psi^{(0)} + \Psi^{(1)} + \dots \rangle}{\langle \Psi^{(0)} + \Psi^{(1)} + \dots | \Psi^{(0)} + \Psi^{(1)} + \dots \rangle}$$

Lv et al., Phys. Lett. B 797 (2019) 134863; Phys. Rev. Lett. 128, 242501 (2022)



S. Elhatisari *et al.*, Nature 630 (2024) 59 Li *et al.*, Phys. Rev. C 98 (2018) 044002; Phys. Rev. C 99 (2019) 064001



Pinhole method

$$p = \frac{|\langle \Psi_f | M(s_{L_t}) \cdots M(s_{L_t/2+1}) \rho_{i_A, j_A}(\mathbf{n}_A) M(s_{L_t/2}) \cdots M(s_1) | \Psi_i \rangle|}{\sum |\langle \Psi_f | M(s_{L_t}) \cdots M(s_{L_t/2+1}) \rho_{i_A, j_A}(\mathbf{n}_A) M(s_{L_t/2}) \cdots M(s_1) | \Psi_i \rangle|}$$



Coordinate space

Configuration space

Insert A-body density operator sample |r,i,j> Translation e^{iPRn} |r> orth.|r> No rotational invariance (irrep projection) Insert A-body HO basis in m-scheme sample $|nljm\rangle$ with constraints on P_{tot} and M_{tot} Spherical basis Breaks translational invariance

S. Elhatisari et al., Phys. Rev. Lett. 119, 222505, S. Zhang et al., arXiv:2411.17462

Combining density correlations with any observable of physical interest, we are able to probe the underlying structure of the nuclei. 12/28

Why ²²Si?

Doubly closed shell nucleus at the dripline Shell closure in competition with proximity to the dripline

- Mass measurement of ²²Al suggested a halo structure
- Isospin asymmetry in the ²²Si/²²O Gamow-Teller transition
- ΔR_{ch} of ³⁶⁻³⁸Ca impacted by nuclear superfluidity and weak binding
 - S. E. Campbell et al., PRL 132, 152501 (2024), J. Lee et al., PRL 125, 192503 (2020), A. J. Miller et al., Nat. Phys. 15, 432-436 (2019)

Mass, excited states, radii yet to be determined

- High-precision mass measurement at IMP (ongoing)
- Invariant mass of excited states at FRIB (ongoing)
- Radii? $\Delta R_{ch} \propto |N-Z| \times L$ imposes constraint on the slope L of the symmetry energy

K. König et al., Phys. Rev. Lett. 132, 162502 (2024)

Theoretical studies

- Diverse S_{2p} predictions from various many-body methods

Doubly closed shell nucleus at the dripline



Theoretical studies

S. R. Stroberg, et al., Annu. Rev. Nucl. Part. Sci. 69, 307 (2019), 1902.06154

- Diverse S_{2p} predictions from various many-body methods

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Theoretical studies

S. R. Stroberg, et al., Annu. Rev. Nucl. Part. Sci. 69, 307 (2019), 1902.06154

- Diverse S_{2p} predictions from various many-body methods

- 2+ states evolution insufficient to determine the N = 8 shell closure J. G. Li *et al.*, PLB 846, 138197 (2023)
 - Charge radii questions magicity at N = 32 of Ca and K isotopes A. Koszorús, et al. Nat. Phys. 17, 439–443 (2021)
 F. Wienholtz *et al.*, Nature 498, 346–349 (2013), R. F. Garcia Ruiz *et al.*, Nat. Phys. 12,594–598 (2016)

Explore this question from the perspective of nucleon distributions in ²²Si



S. Zhang et al., arXiv:2411.17462



τ from 0.3 MeV⁻¹ to 0.4 MeV⁻¹

g.s. energies change by less than 0.36 MeV ²²Si consistently more bound than ²⁰Mg

S. Zhang et al., arXiv:2411.17462

R_{ch}	$\Delta R_{ m ch}(^{22}{ m Si}^{-20}{ m Mg})$	$\Delta R_{\rm ch}(^{22}{\rm Si}-^{22}{ m O})$
H_{χ} [24] 3.277 (19)	0.070 (31)	0.361 (32)
H_{χ} [19] 3.196 (17)	0.070 (28)	0.370 (29)

Experiment: $\Delta R_{\rm ch}(^{28}{\rm Si}^{-26}{\rm Mg}) = 0.088 \text{ fm}$ $\Delta R_{\rm ch}(^{30}{\rm Si}^{-28}{\rm Mg}) = 0.064 \text{ fm}$ $\Delta R_{\rm ch}(^{32}{\rm Si}^{-30}{\rm Mg}) = 0.042 \text{ fm}$

D. T. Yordanov *et al.*, PRL 108, 042504 (2012) K. König *et al.*, PRL 132, 162502 (2024)

No significantly changes in shell structure from proton-rich to neutron-rich side.

 ΔR_{ch} of ²²Si-²²O show good agreement with two chiral NN+3N forces

Lattice simulations:

Consistent with CC prediction within 3σ confidence ΔR_{ch} [CC] = 0.429 (21)

Relatively small, a suppression of the slope L?



Nucleon distributions

Nucleon ordering operators:

S. Zhang et al., arXiv:2411.17462



First and second layers of ⁴He share identical Gaussian widths

⁵He&⁶He outer layers exhibit significant spatial extension, unlike inner layers

Reveals arrangement and localization, linking to the shell closure to some extent

Nucleon distributions

Spatial arrangement of protons in the N = 8 isotones



The extra 2*p* in ²²Si are likely positioned near the regions corresponding to the 9th and 11th layers

Outer layer in ²²Si shows a similar broadening pattern to the inner layers and exhibits spatial localization characteristics

Nucleon distributions



The extra 2*p* in ²²Si are likely positioned near the regions corresponding to the 9th and 11th layers

Outer layer in ²²Si shows a similar broadening pattern to the inner layers and exhibits spatial localization characteristics

Spatial localizations in Coordinate space linked to shell closure features in SM languages

Presence of shell closure at Z = 14 and N = 8 in ²²Si

Occupation numbers via Pinhole method with HO basis

States	\mathbf{n}_{π}	$\mathbf{n}_{ u}$
$0s_{1/2}[1/2]$	0.935 (132)	1.009 (123)
$0s_{1/2}[-1/2]$	0.970 (135)	0.981 (124)
$0p_{3/2}[3/2]$	0.670 (151)	0.931 (135)
$0p_{3/2}[1/2]$	0.965 (164)	0.805 (123)
$0p_{3/2}[-1/2]$	0.945 (155)	0.901 (139)
$0p_{3/2}[-3/2]$	0.751 (148)	0.911 (137)
$0p_{1/2}[1/2]$	0.901 (131)	0.877 (135)
$0p_{1/2}[-1/2]$	1.049 (161)	0.938 (147)
$0d_{5/2}[5/2]$	1.018 (205)	
$0d_{5/2}[3/2]$	1.056 (229)	
$0d_{5/2}[1/2]$	0.980 (217)	
$0d_{5/2}[-1/2]$	1.106 (223)	
$0d_{5/2}[-3/2]$	0.875 (209)	
$0d_{5/2}[-5/2]$	1.063 (204)	
$1s_{1/2}[1/2]$	0.141 (173)	
$1s_{1/2}[-1/2]$	0.160 (150)	
$0d_{3/2}[3/2]$	-0.127 (169)	
$0d_{3/2}[1/2]$	-0.003 (162)	
$0d_{3/2}[-1/2]$	0.202 (156)	
$0d_{3/2}[-3/2]$	0.145 (143)	

Occupation number $<<1 \rightarrow$ **particle-hole excitations**

Op orbitals of proton:

 $0p_{3/2}[3/2] = 0.979$ (6) with H_s $0p_{3/2}[-3/2] = 0.982$ (7) with H_s fully occupied w/o perturbation corrections

Outer $0d_{5/2}$ orbitals:

fully occupied \rightarrow suggests limited particle-hole excitation

Statistical Considerations:

Lower-than-expected occupation for $0p_{3/2}$ negative value for $0d_{3/2}[3/2]$ Sign problem with N³LO & Insufficient statistics due to full many-body correlations

Upper limits considering errors:

 $\begin{array}{l} 0p_{3/2}[\ \ 3/2] \leq 0.821 \\ 0p_{3/2}[-3/2] \leq 0.899 \\ 0d_{3/2}[\ 3/2] \sim 0 \end{array}$

Occupation numbers via Pinhole method with HO basis

States	\mathbf{n}_{π}	$\mathbf{n}_{ u}$
$0s_{1/2}[1/2]$	0.935 (132)	1.009 (123)
$0s_{1/2}[-1/2]$	0.970 (135)	0.981 (124)
$0p_{3/2}[3/2]$	0.670 (151)	0.931 (135)
$0p_{3/2}[1/2]$	0.965 (164)	0.805 (123)
$0p_{3/2}[-1/2]$	0.945 (155)	0.901 (139)
$0p_{3/2}[-3/2]$	0.751 (148)	0.911 (137)
$0p_{1/2}[1/2]$	0.901 (131)	0.877 (135)
$0p_{1/2}[-1/2]$	1.049 (161)	0.938 (147)
$0d_{5/2}[5/2]$	1.018 (205)	
$0d_{5/2}[3/2]$	1.056 (229)	
$0d_{5/2}[1/2]$	0.980 (217)	
$0d_{5/2}[-1/2]$	1.106 (223)	
$0d_{5/2}[-3/2]$	0.875 (209)	
$0d_{5/2}[-5/2]$	1.063 (204)	
$1s_{1/2}[1/2]$	0.141 (173)	
$1s_{1/2}[-1/2]$	0.160 (150)	
$0d_{3/2}[3/2]$	-0.127 (169)	
$0d_{3/2}[1/2]$	-0.003 (162)	
$0d_{3/2}[-1/2]$	0.202 (156)	
$0d_{3/2}[-3/2]$	0.145 (143)	

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 $0p_{3/2}[3/2] \le 0.821$ $0p_{3/2}[-3/2] \le 0.899$ $0d_{3/2}[3/2] \sim 0$

Occu. numbers support the existence of shell closure (as indicated by the 2⁺ and spatial localizations) rather than a genuine destruction of the shell closure

Providing pathways for future comparisons with configuration space many-body methods 22/28

S. Zhang *et al.*, arXiv:2411.17462

Observation of 0_2^+ in ⁸He with the condensatelike $\alpha + 2n + 2n$ cluster

E = 6.66 (6) MeV

 $M(IS0) = 11^{+1.8}_{-2.3} \text{ fm}^2$

Green-solid ovals ($\psi_{nn} \sim 90^{\circ}$):

Enhancement at $\theta_{nn} \sim 0^{\circ}$

significant nn correlations



Z. H. Yang, et. al., PhysRevLett.131.242501 (2023)

Compared to a pure four-neutron system, multi-neutron correlation may be enhanced when 4*n* are weakly bound or unbound around a core.

Uncertainty Analysis:

- Volume Effects: L = 14
- Chiral forces: nuclear force parameterizations via history matching

Intrinsic structure

- *A*-body density with full correlations
- Cluster identification α, 2n, t

Multi-neutron correlations

nn correlation analysis

Three-body systems (Jacobi basis, via pinhole coordinate)

 $\alpha + 2n + 2n$ t + 2n + 2n

2n - 2n correlation

Three-body correlation functions:

 $\alpha + 2n + 2n / t + 2n + 2n$



Uncertainty Analysis:

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nn correlation analysis

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Multi-neutron correlations

nn correlation analysis

Three-body systems (Jacobi basis, via pinhole coordinate)

$$\alpha$$
 + 2*n* + 2*n*

$$t + 2n + 2n$$

2n - 2n correlation

Three-body correlation functions:



T-type

$$\vec{A} = n^1(x,y,z) - CoM(x,y,z)$$

 $\vec{B} = n^2(x,y,z) - CoM(x,y,z)$
 $\vec{C} = n^1(x,y,z) - n^2(x,y,z)$
 $\theta = \frac{\arccos(\vec{A} \cdot \vec{B})}{|\vec{A}||\vec{B}|} \quad \psi = \pi - \theta/2 - \frac{\arccos(\vec{A} \cdot \vec{C})}{|\vec{A}||\vec{C}|}$



Uncertainty Analysis:

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Multi-neutron correlations

nn correlation analysis

Three-body systems (Jacobi basis, via pinhole coordinate)

 α + 2*n* + 2*n*

t+2n+2n

2*n* - 2*n* correlation

Three-body correlation functions:

$$\vec{A} = \vec{V}_{1} = \vec{C}$$

1-type

$$\vec{A} = n^{1}(x,y,z) - CoM(x,y,z)$$

 $\vec{B} = n^{2}(x,y,z) - CoM(x,y,z)$
 $\vec{C} = n^{1}(x,y,z) - n^{2}(x,y,z)$
 $\theta = \frac{\arccos(\vec{A} \cdot \vec{B})}{|\vec{A}||\vec{B}|} \quad \psi = \pi - \theta/2 - \frac{\arccos(\vec{A} \cdot \vec{C})}{|\vec{A}||\vec{C}|}$
If $\theta_{1}, \theta_{2} \sim 0; \psi_{1}, \psi_{2} \sim \pi/2$
 $d_{1}(x,y,z) = [n^{1}(x,y,z) + n^{2}(x,y,z)]/2$

 $\alpha(x,y,z)$

$$\rho_3(r_{12}, r_{13}, r_{23}) = \binom{A}{3} \left\langle \prod_{i < j=1}^3 \delta(|\mathbf{r}_i - \mathbf{r}_j| - r_{ij}) \right\rangle$$

Summary & perspective

Proton-rich²²Si

Calculations agree with existing data and predict ²²Si as a proton-dripline nucleus, along with its 2⁺ state, radius, and spatial properties

Using nucleon ordering operators, we reveal nucleon spatial arrangement and localization, linked to shell closure features

Introducing a novel pinhole method with HO basis, offering new perspectives into a more comprehensive understanding of nuclear structure

nn correlations in light nuclei

Energies, nucleon density of ⁸He and ⁷H

Cluster identification, such as α , 2n

Further developments

Comparison our ²²Si results with upcoming experimental data

Uncertainty quantification in the energies of ⁸He and ⁷H, multi-neutron correlations

Extension to heavy nuclei with the benefits of GPU resource

