Structure of Few-Body Hypernuclei

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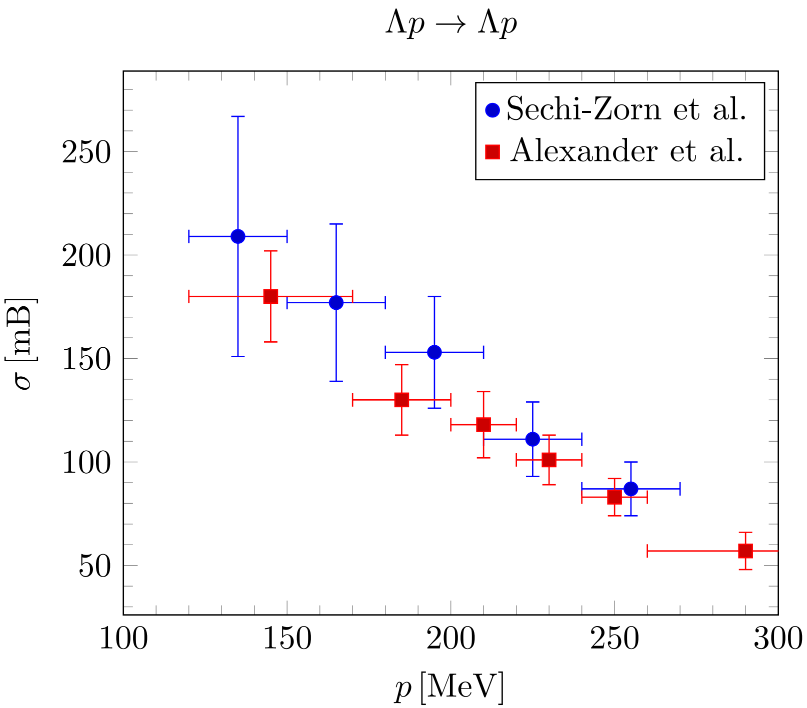
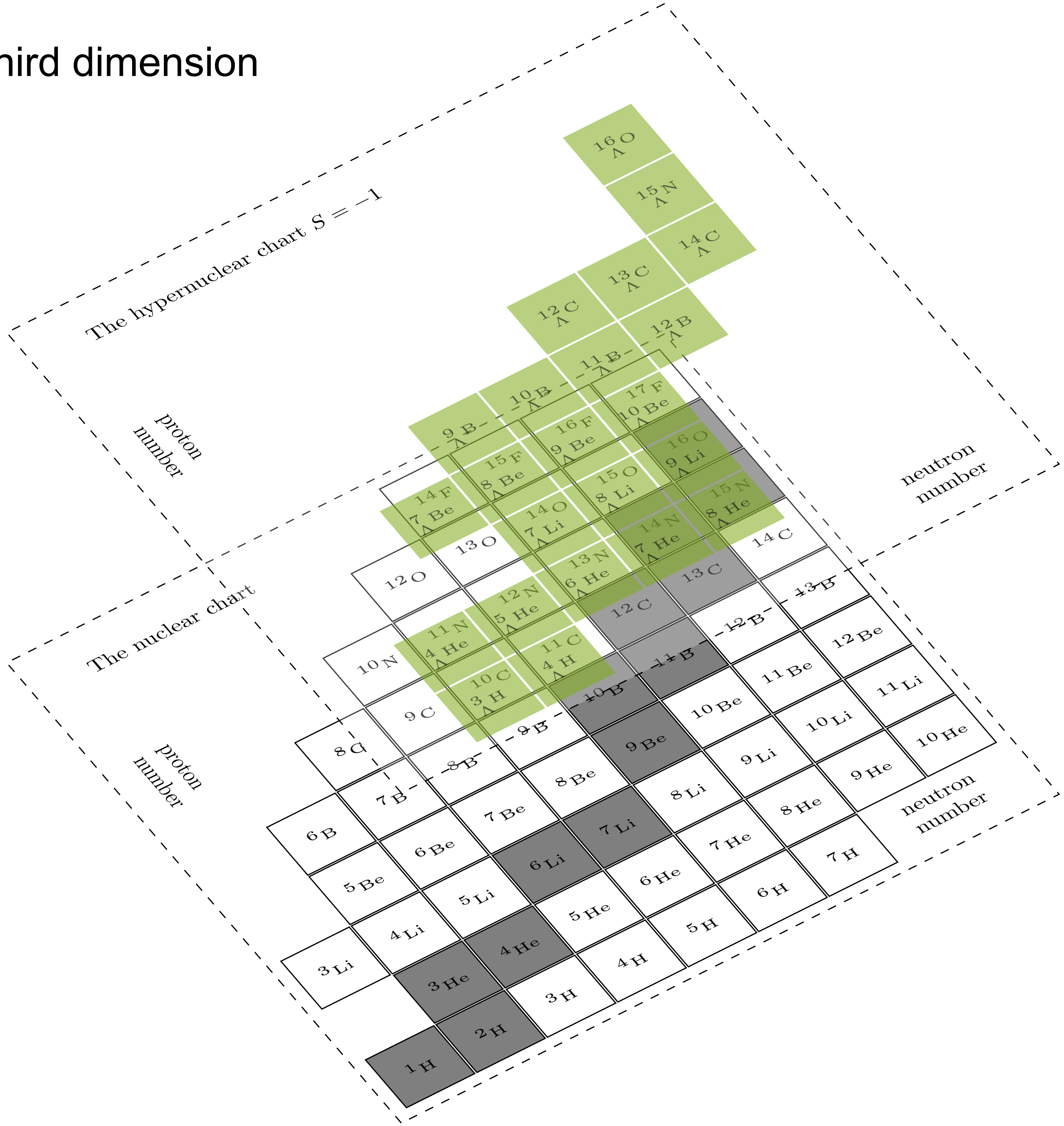
Outline

- Motivation
- Structure of Three-Body hypernuclei from pionless EFT
  - Universal Correlations
  - Lifetime of the hypertriton
- First Insights for hypernuclei from the Lattice
  - From NLEFT to (Hyper) NLEFT
  - Impurity Worldline Monte-Carlo

Hypernuclear physics in a nutshell

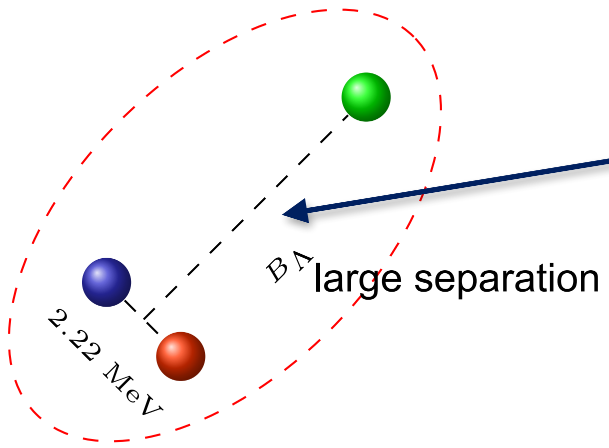
- Strangeness extents the nuclear chart to a third dimension
- Unique opportunity to study the strong force
  Without the Pauli principle
- Typical approach from nuclear physics
  does not work since two-body data is sparse





- Gateway : Three-Body Systems

The Hypertriton -Known for years still a puzzle



Emulsion:

$B_\Lambda = 0.130 \pm 0.050$ MeV Juric 1973

Heavy Ion:

$B_\Lambda = 0.406 \pm 0.120$ MeV Star 2020

$B_\Lambda = 0.102 \pm 0.063$ MeV Alice 2023

World Average:

$B_\Lambda = 0.164 \pm 0.043$ MeV Mainz 2023

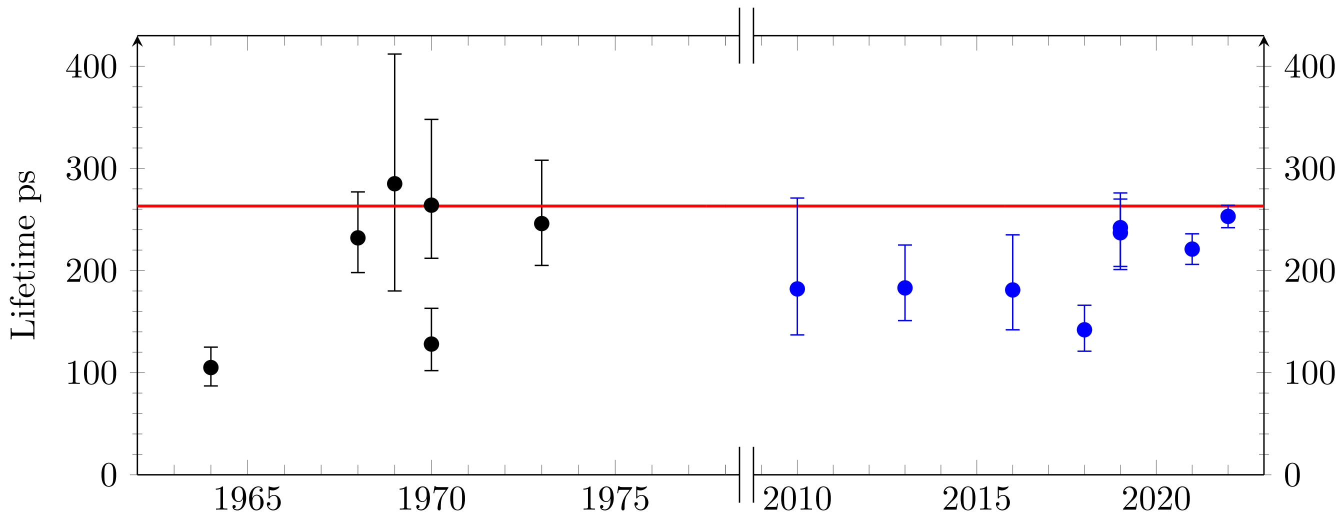
Shallow S-Wave State

$$J^P = \frac{1}{2}^+$$

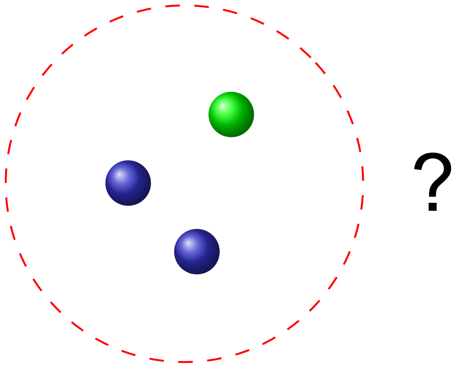
Distinguishable

$$I = 0 \Rightarrow \frac{1}{\sqrt{2}}(pn - np)\Lambda$$

large $\Lambda - d$ separation $\Rightarrow$ Λ drives the decay



Λnn-Another three-body system



Might be bound
$B_{\Lambda nn} \approx 1.1$ MeV HypHI 2013

Contradicts Hypernuclear data

Unclear Nature
Bound?
Resonance?

Shallow S-Wave State

$$J^P = \frac{1}{2}^+$$

Distinguishable

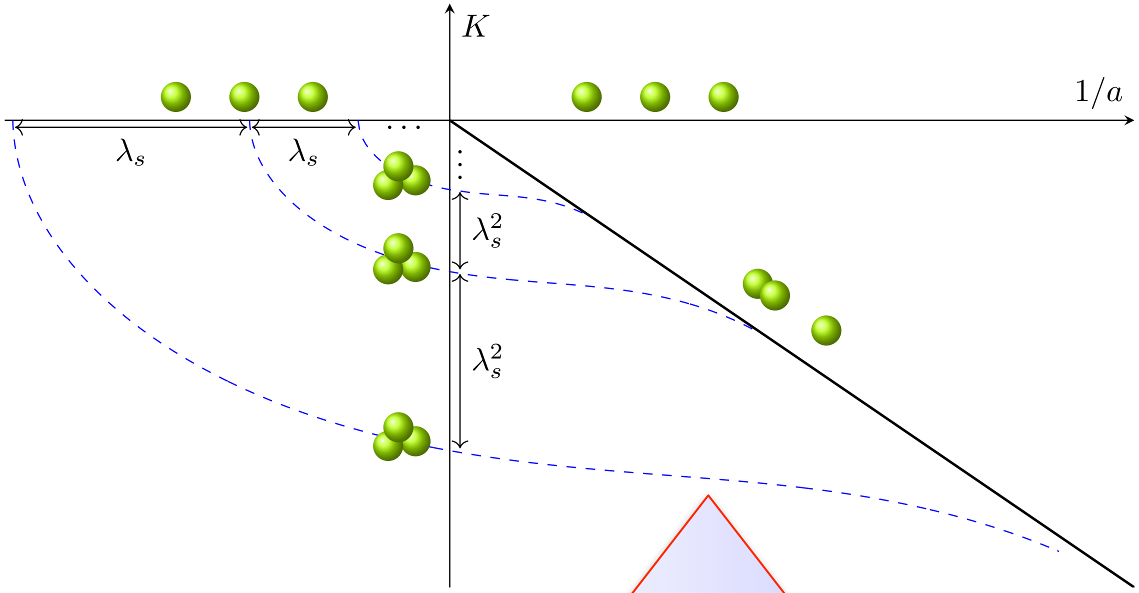
$$I = 1 \Rightarrow \Lambda nn$$

Similar to Hypertrion
Different Isospin
Channel

Exploit →

Explore Λnn and Hypertriton
Within One Theory

Theoretical Framework ⇒Pionless EFT



Physics Determined by a and Λ_*

Universal Relations Between Observables

B_Λ and $\langle r^2 \rangle$

B_Λ and τ

B_Λ and $a_{\Lambda d}$

Pionless effective field theory
Controllable Uncertainties
Systematic Improvement

Shallow S-Wave State

$$J^P = \frac{1}{2}^+$$

Distinguishable

2 Isospin Channels

Large Scattering Length

Why pionless Effective Field Theory(EFT)?

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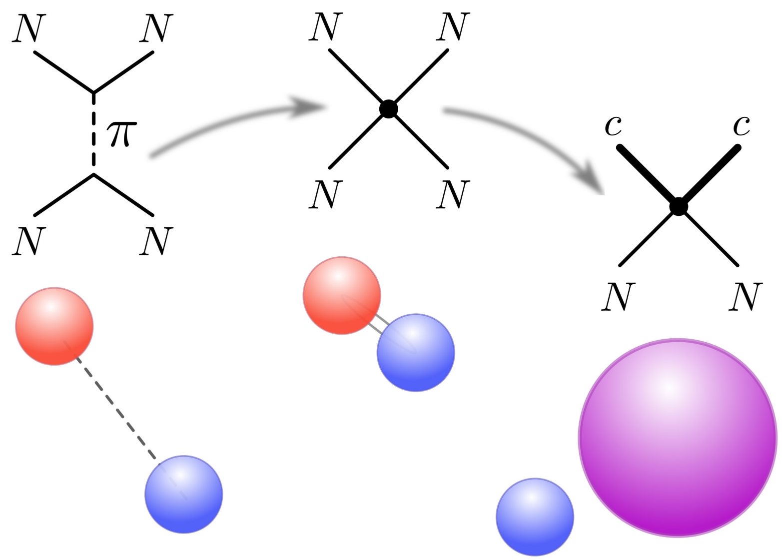
What is an EFT (in a nutshell)?

Simplifies a fundamental theory to it essential parts

Focus on the relevant degrees of freedom

Offer a systematic way to improve the theory



Picture: FB Physik TU Darmstadt

Integrate out heavy particles out of the theory

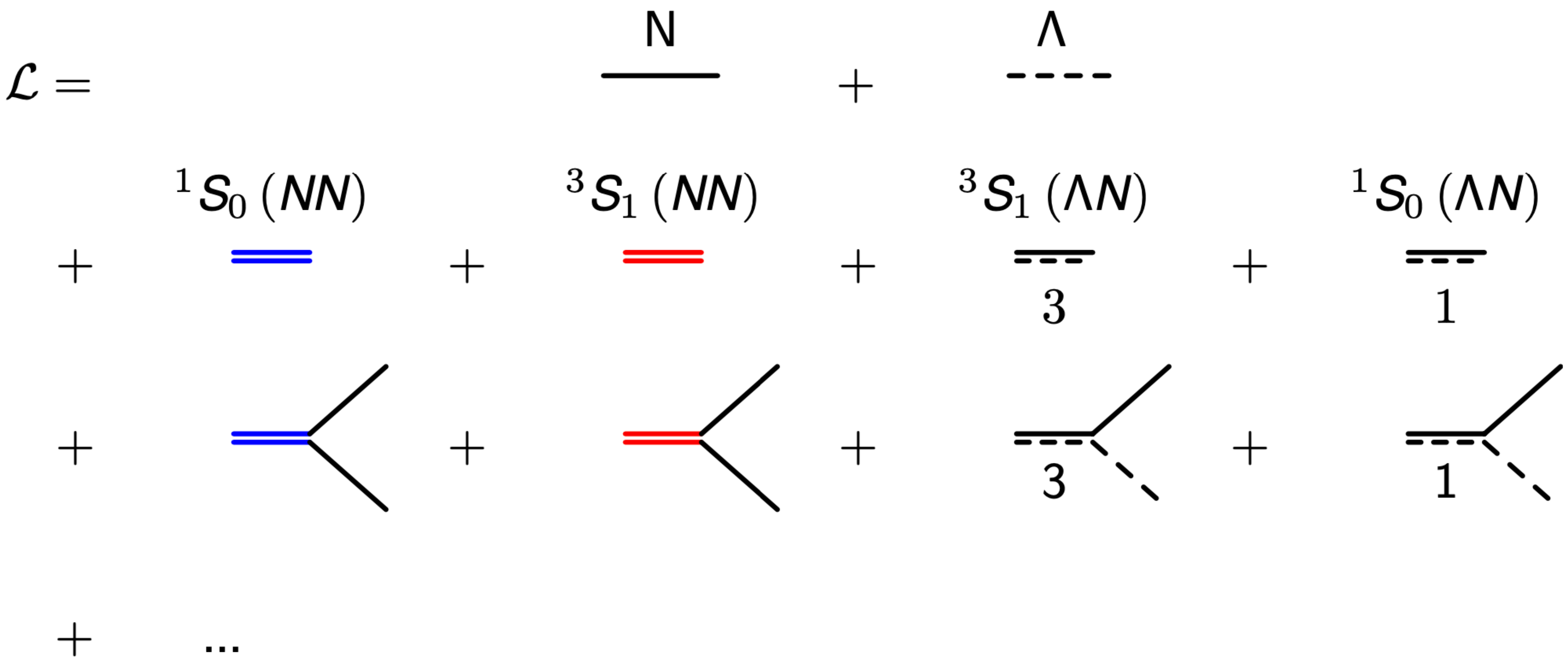
$$\frac{g^2}{m_\pi^2 - q^2} \approx \frac{g^2}{m_\pi^2} + \frac{g^2 q^2}{m_\pi^4} \qquad \xrightarrow{\gamma \sim q \ll m_\pi}$$

No explict $\Lambda \Leftrightarrow \Sigma$
But Three-Body-Force

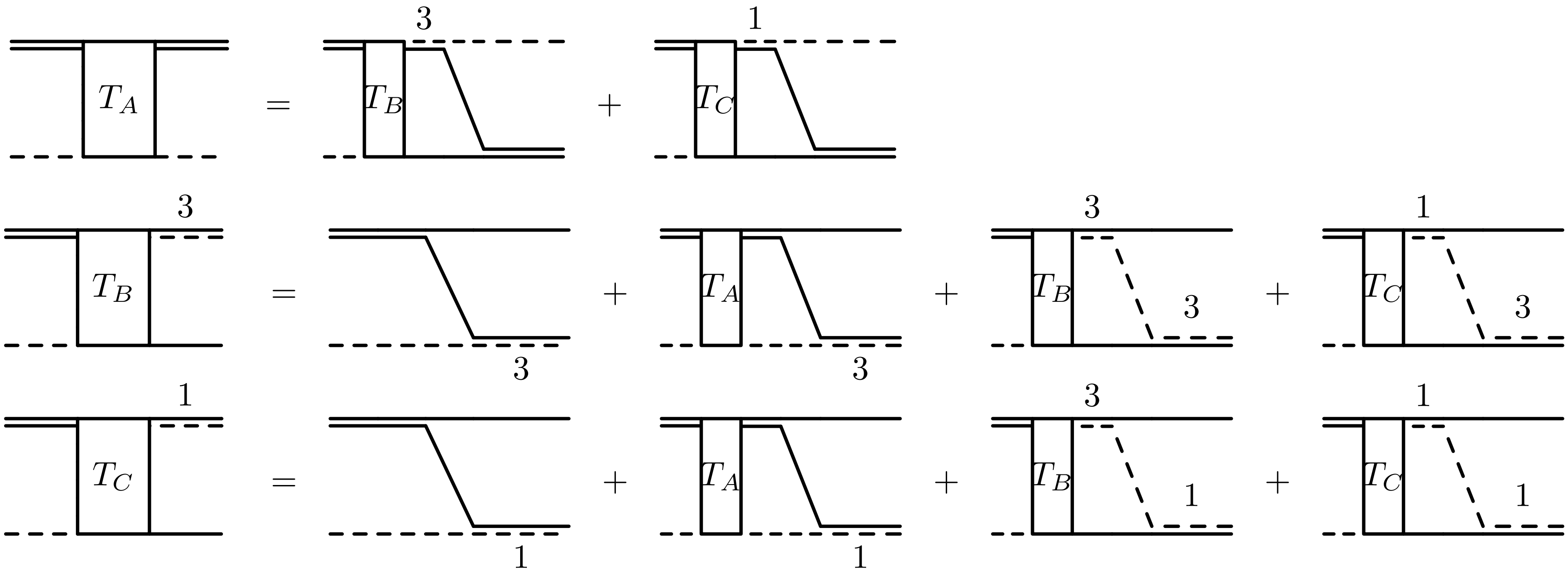
$^3S_1(NN) + \Lambda$ (Hypertriton)
$^1S_0(NN) + \Lambda$ (Λnn)

$^3S_1(\Lambda N) + N$ (Both)
$^1S_0(\Lambda N) + N$ (Both)

Three-body Hypernuclear Lagrangian

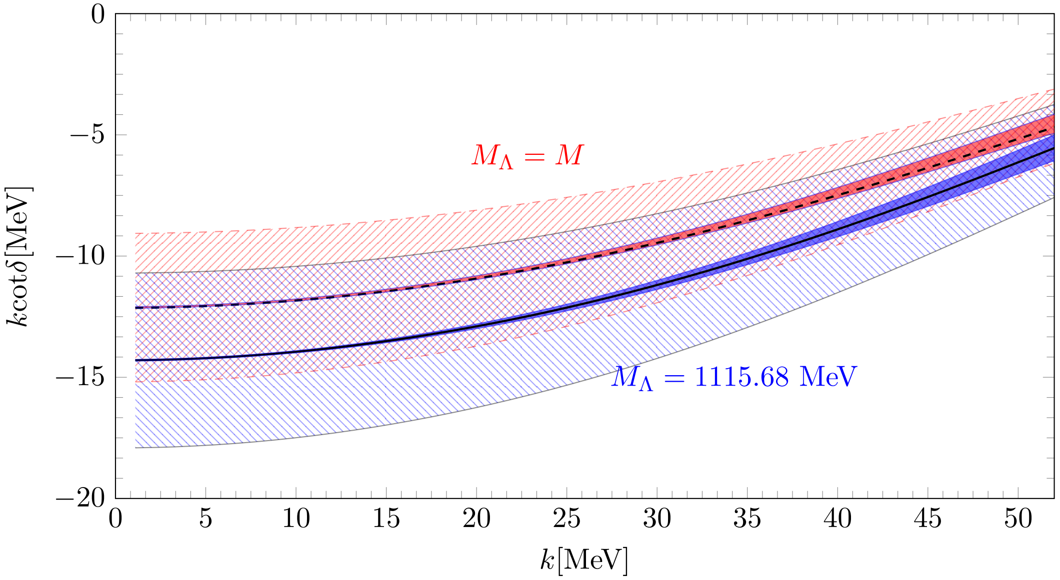


Integral equations



Integral equations are form invariant for both isospin channels

The Phillips line for the Hypertriton



Use chiral EFT inputs for ΛN interaction

Phase shift are however independent of details of the interaction

→ Shallowness of the hypertriton

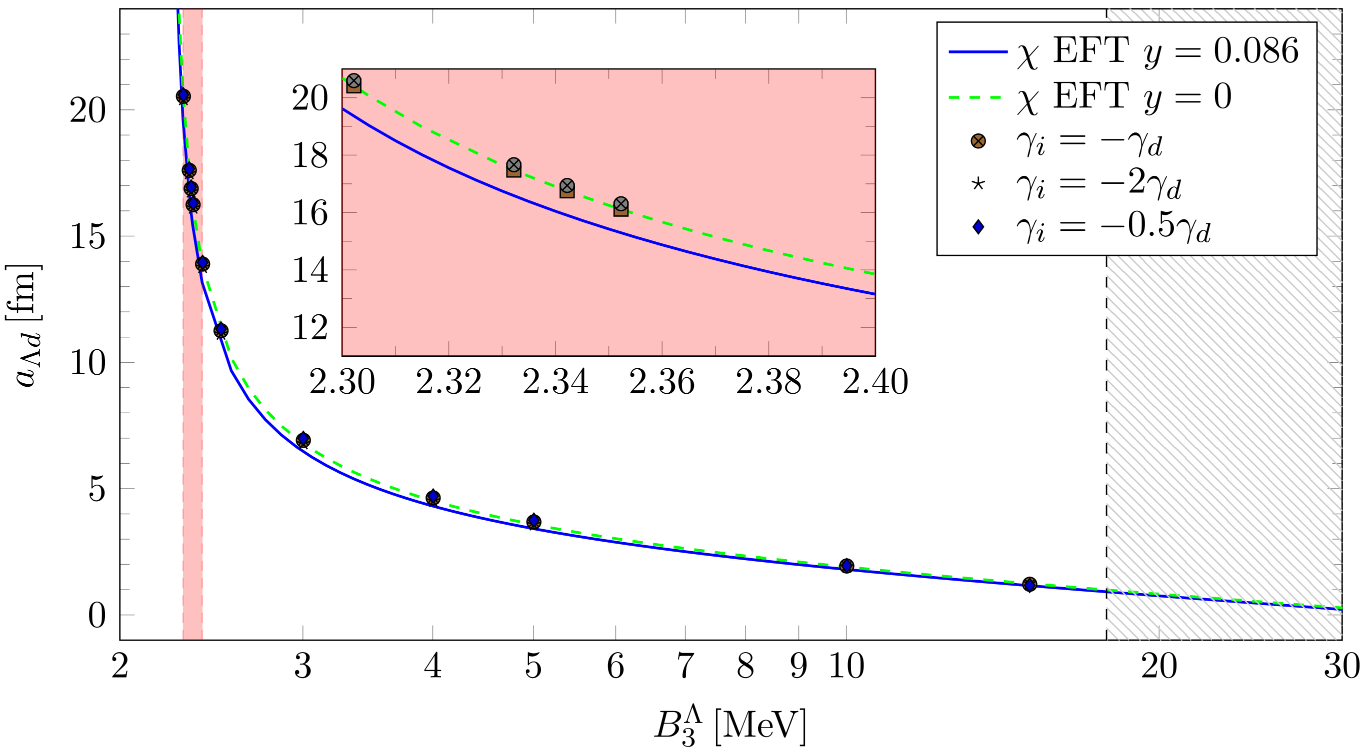
$a_{\Lambda d} = 15.4^{+4.3}_{-2.3}$ fm $\quad r_{\Lambda d} \approx 1.3$ fm

Strong dependence on B_Λ

Independent of the Λ pole position

Universal relation :

$B_\Lambda \Leftrightarrow a_{\Lambda d}$



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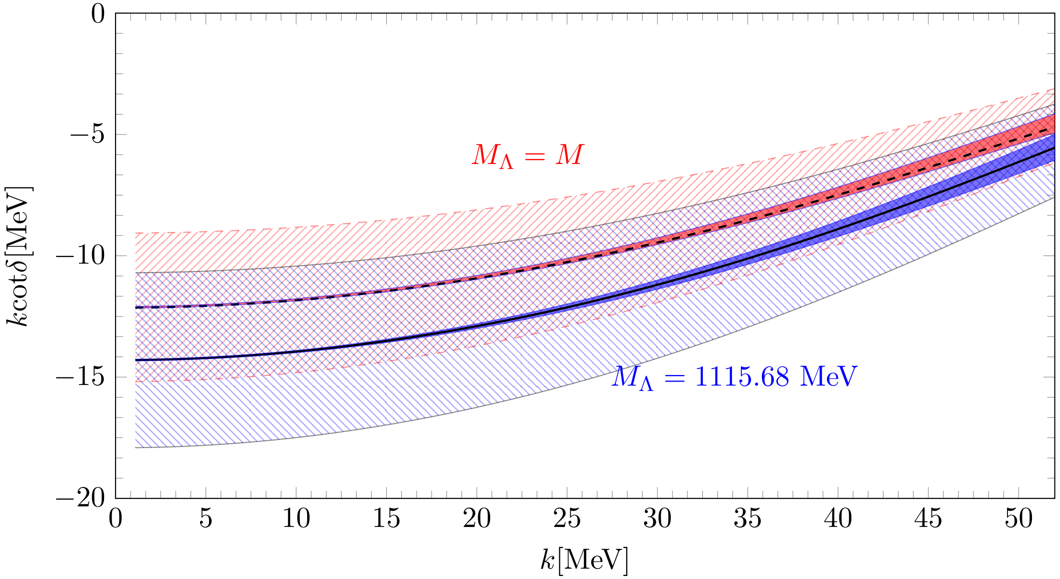

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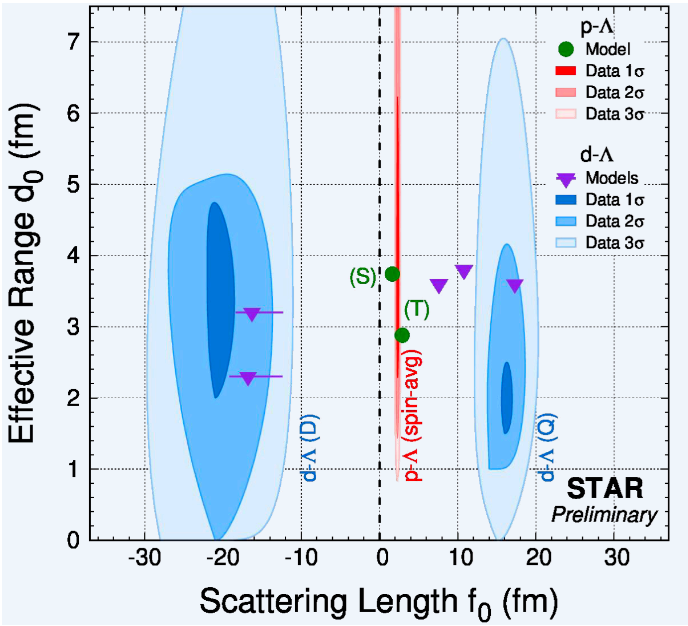
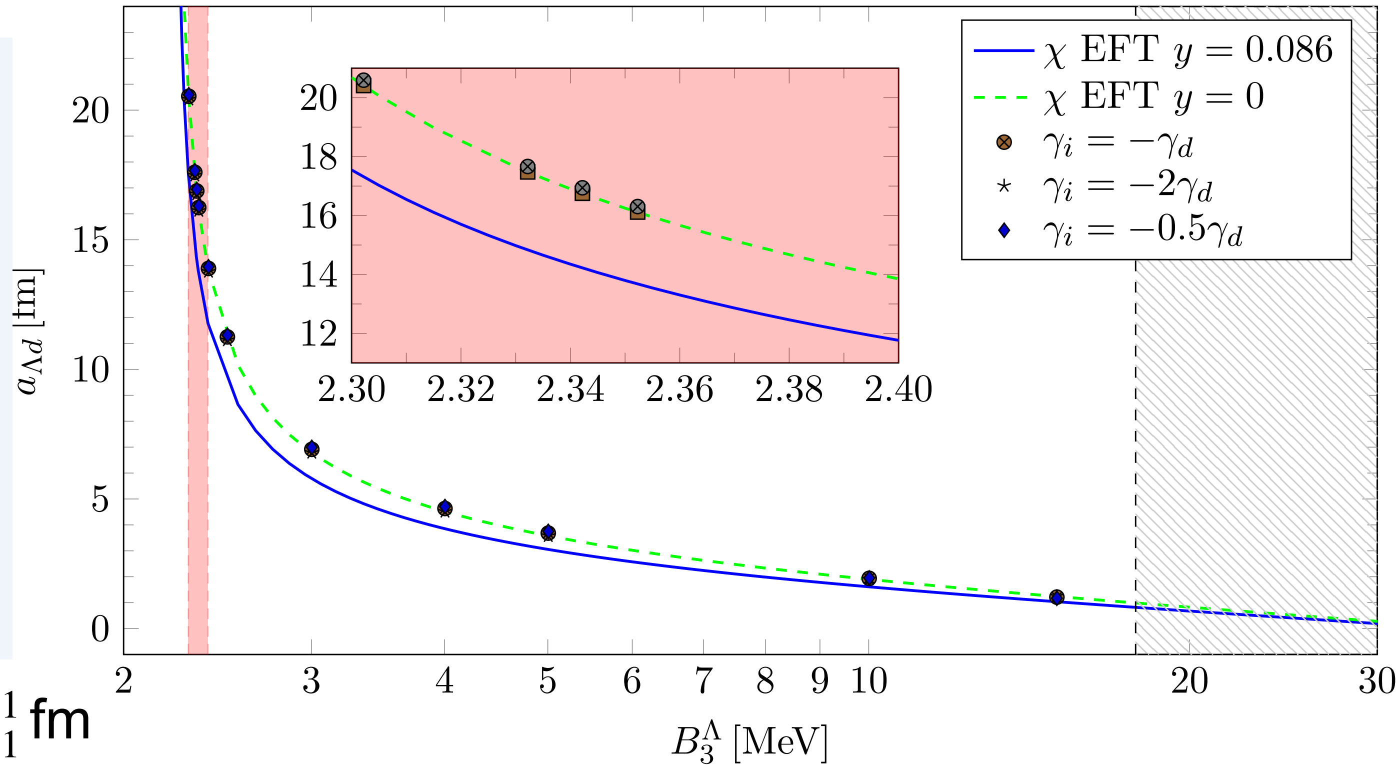
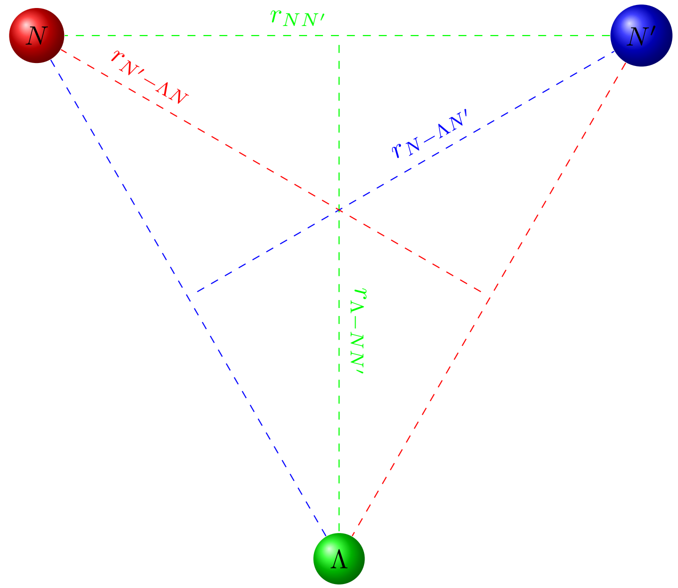




$a_{\Lambda d} = 16^{+2}_{-1}$ fm $\quad r_{\Lambda d} = 2^{+1}_{-1}$ fm

Matter radii for the Hypertriton

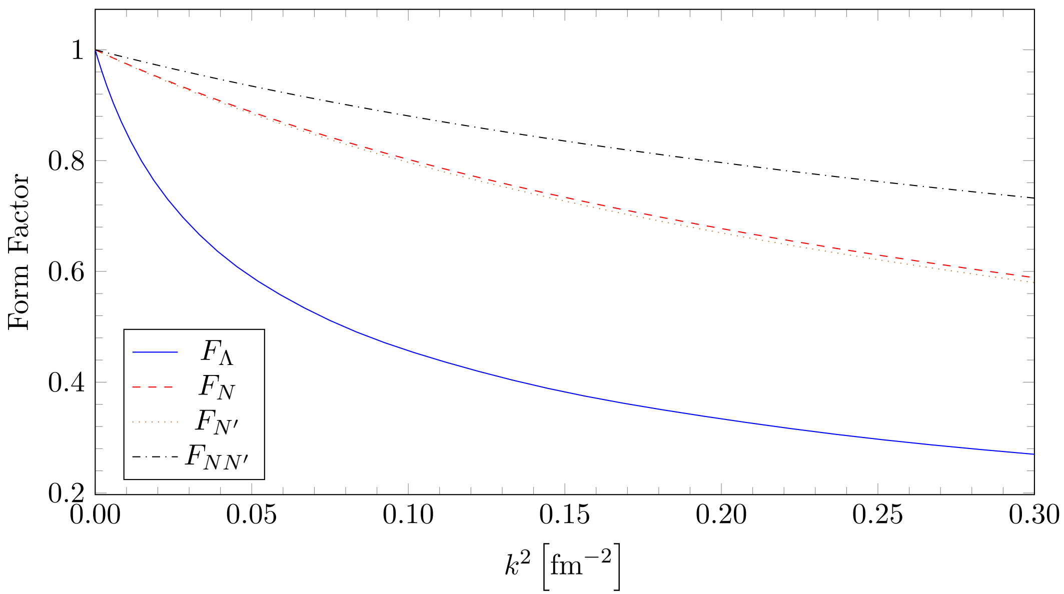


Calculation of form factors out of the Three-body wave functions

$$F_i(\boldsymbol{k}^2) = \int \mathrm{d}^3 p \int \mathrm{d}^3 q \, \psi_i(p,q) \psi_i(p, |q-k|)$$

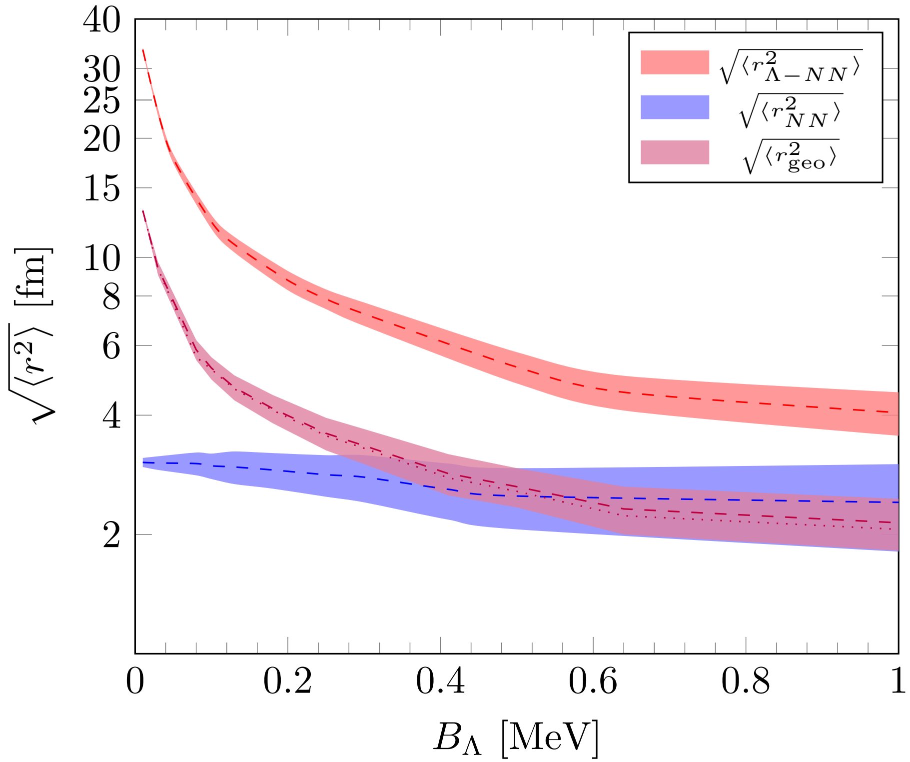
Relate different Form Factors to Different Matter Radii

$$F_i(\boldsymbol{k}^2) = 1 - \frac{1}{6}\boldsymbol{k}^2 \langle r_{i-jk} \rangle + \ldots$$



Halo structure of the hypertriton directly visible

Matter radii for the Hypertriton



Expectation from two-body calculation

$$B_2 = \frac{1}{2\mu a^2} \quad \text{and} \quad \langle r^2 \rangle = \frac{a^2}{2}$$

$$\sqrt{\langle r^2_{NN} \rangle} \approx 3.04 \, \text{fm}$$

$$\sqrt{\langle r^2_{\Lambda d} \rangle} \approx 10.34 \, \text{fm}$$

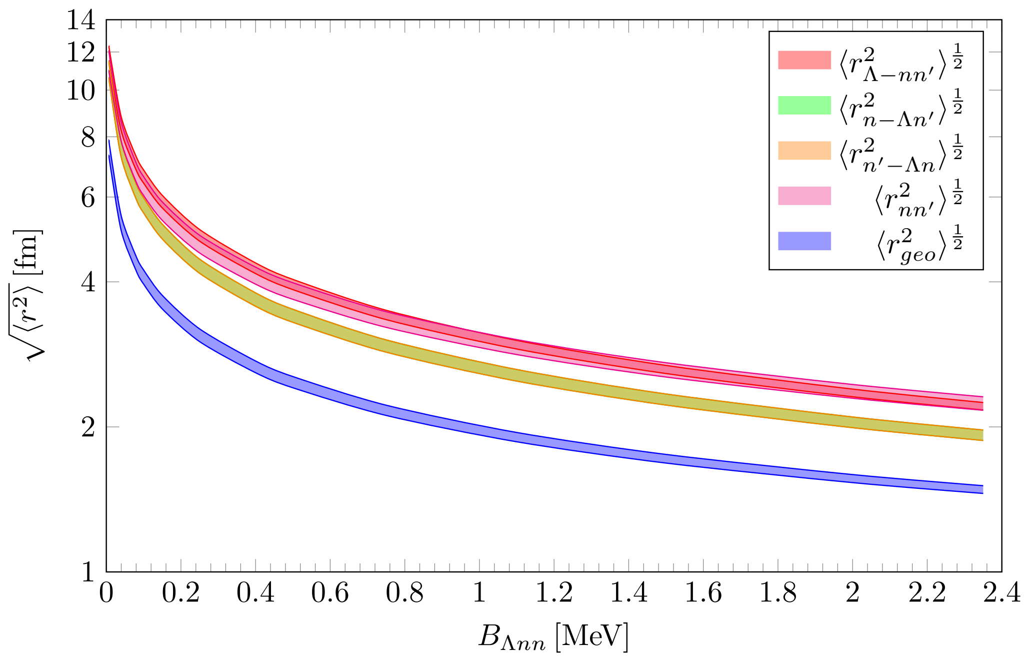
→

Universal relation between $\langle r^2 \rangle \Leftrightarrow B_\Lambda$

$\sqrt{\langle r^2_{\Lambda-NN'} \rangle}$[fm]	$\sqrt{\langle r^2_{N'-\Lambda N} \rangle}$[fm]	$\sqrt{\langle r^2_{N-N'\Lambda} \rangle}$[fm]	$\sqrt{\langle r^2_{NN'} \rangle}$[fm]	$\sqrt{\langle r^2_{geo} \rangle}$[fm]
10.79	3.96	4.02	2.96	4.66
$+3.04/-1.53$ $+0.03/-0.02$	$+0.40/-0.25$ $+0.03/-0.03$	$+0.41/-0.25$ $+0.03/-0.03$	$+0.06/-0.05$ $+0.03/-0.04$	$+1.19/-0.54$ $+0.01/-0.01$

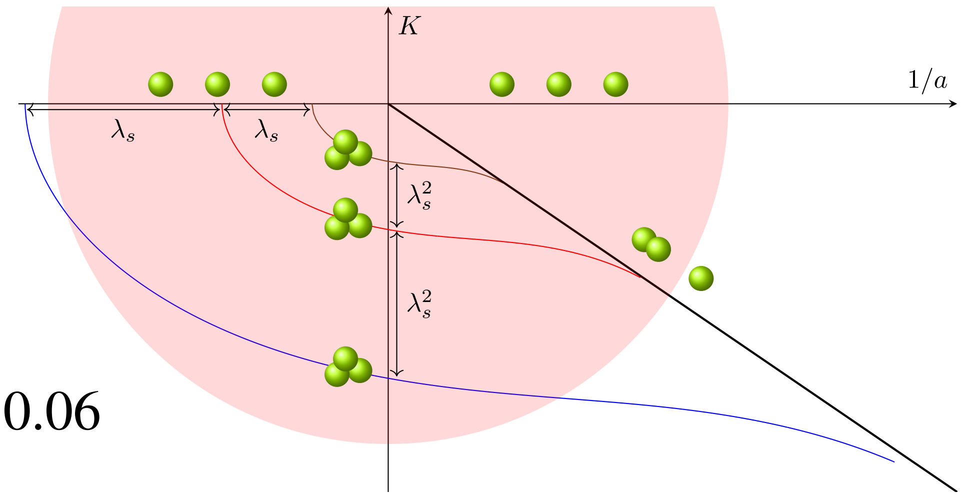
Insensitive to details of Of the ΛN Interaction

Is a Λnn physical in this theory?



Λnn is physical by construction in this theory since it exhibits the Efimov Effect

BUT! Λnn might be not within range of Applicability



$$P = \frac{\Lambda_*^{I=1,\text{breakdown}} - \Lambda_*^{I=1,\text{threshold}}}{(e^{\pi/s_0} - 1)\Lambda_*^{I=1,\text{breakdown}}} \approx 0.06$$

Hypertriton Lifetime

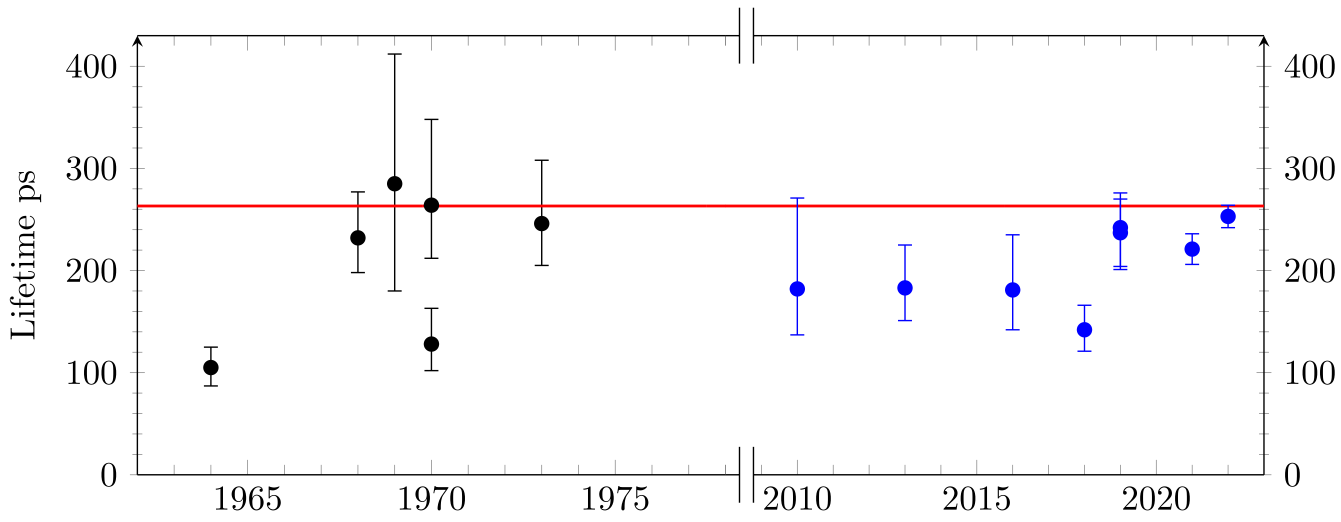
$^3_\Lambda$H $\mapsto \pi^- + ^3$ He
$^3_\Lambda$H $\mapsto \pi^- + d + p$
$^3_\Lambda$H $\mapsto \pi^- + p + n + p$

Isospin $\Delta I = \frac{1}{2}$ rule

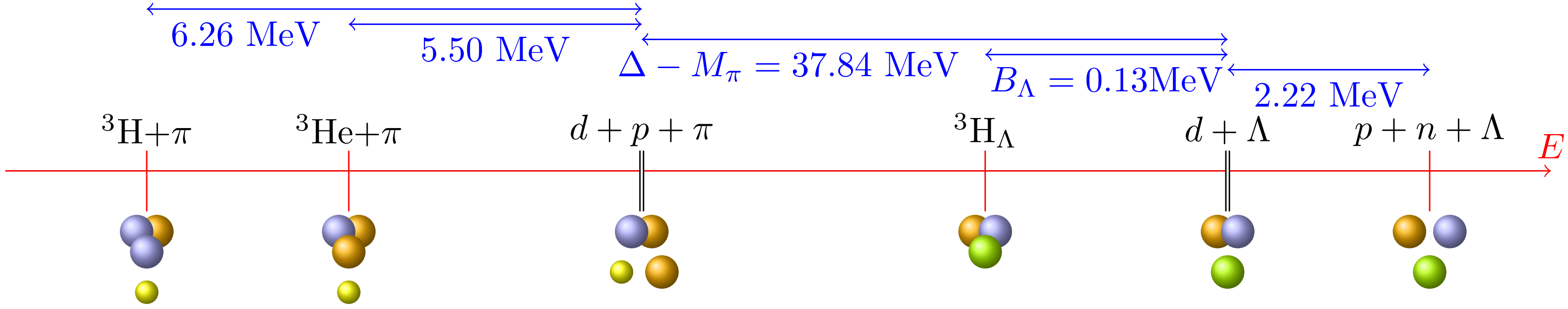
$^3_\Lambda$H $\mapsto \pi^0 + ^3$ H
$^3_\Lambda$H $\mapsto \pi^0 + d + N$
$^3_\Lambda$H $\mapsto \pi^0 + p + n + n$

Leptonic and Non-Mesonic Decays are Negligible
Deuteron Breakup suppressed by 2 order of magnitude

- Two-body picture works
- Calculate Lifetime in a Picture with a fundamental deuteron
- Focus on the B_Λ dependence

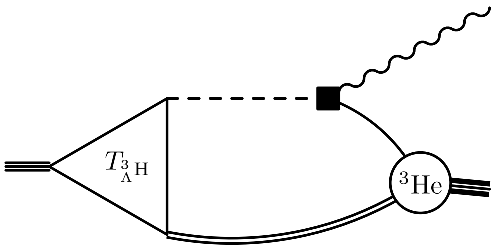


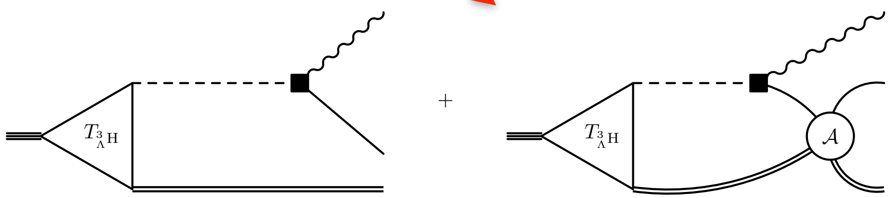
Hypertriton Lifetime



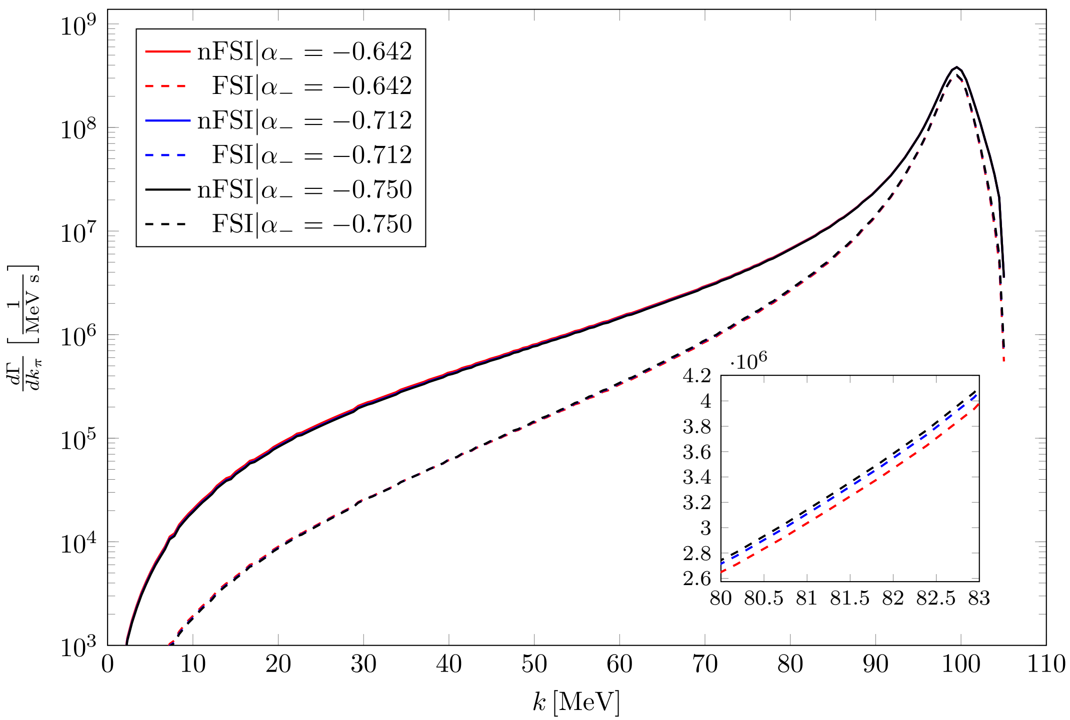
Triton/He Final state

Deuteron Final state

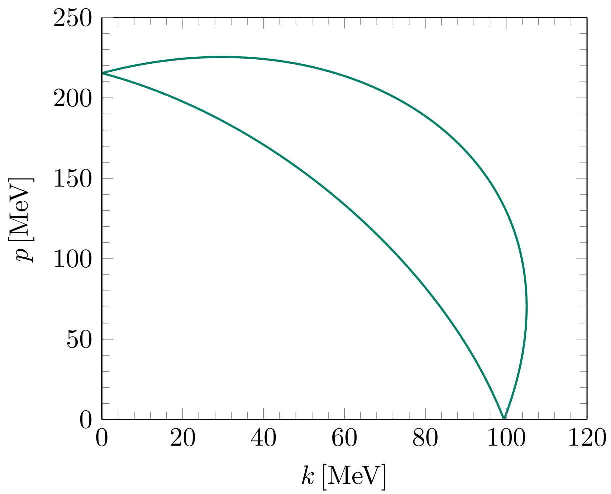


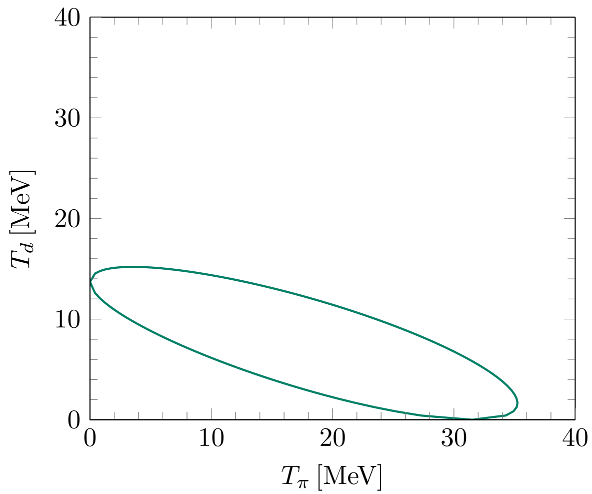


Hypertriton Lifetime

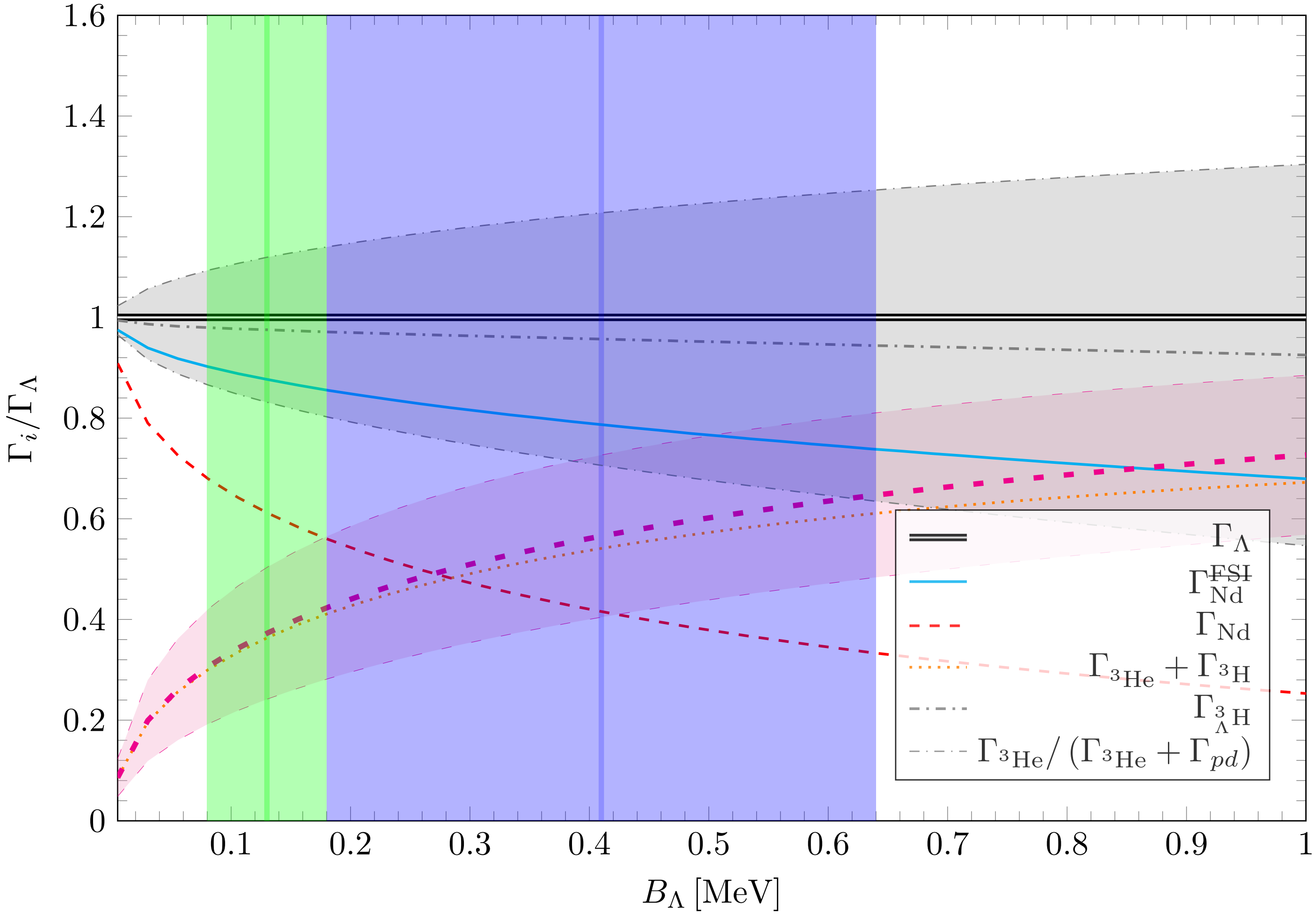


- Minor dependence on the polarisation parameter $\alpha\_$
- Main contribution from $k \sim 100$ MeV
- Final State interaction are Important





Hypertriton Lifetime



- Γ barely depend on B_Λ
- Final State interaction are important
- The Branching ratio R_3 depends strongly on B_Λ
- Star Branching ratio $0.32(5)(8)$

Emulsion data $R_3 = \Gamma_{\mathrm{He}}/(\Gamma_{\mathrm{pd}} + \Gamma_{\mathrm{He}}) = 0.3 - 0.4$

Pionic Final State Interactions

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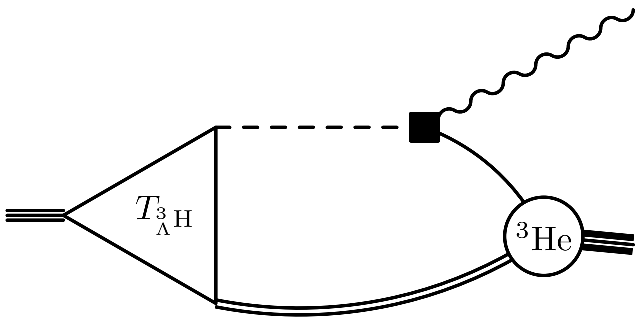
Work by Perez-Obiol And Gal suggest significant contribution from Pionic final states
Perez-Obiol 2020, Gal 2019

Different Type of calculation Only has two-body decay Channel and uses the Branching ratio as an input
Contribution : $\approx 0.15\Gamma_\Lambda$

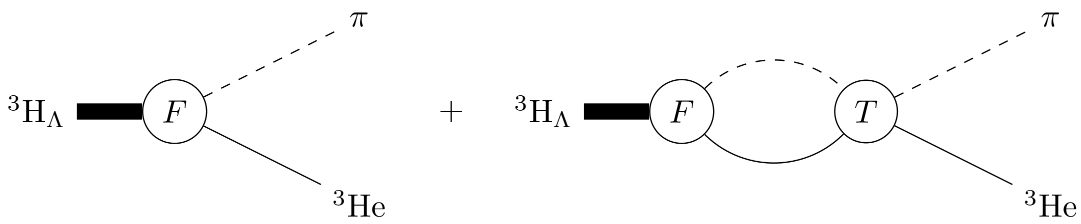
NLO Effect

Choose this channel!



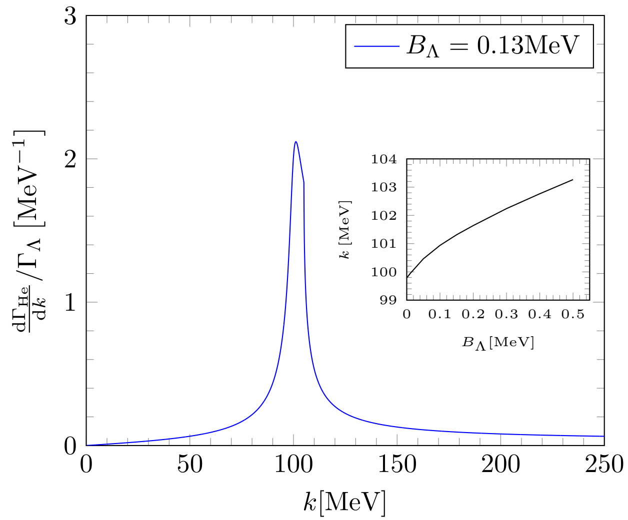
- Only two particles in FSI
- FSI is momentum locked
- Direct comparison is possible
- Not much data available

Pionic Final State Interactions



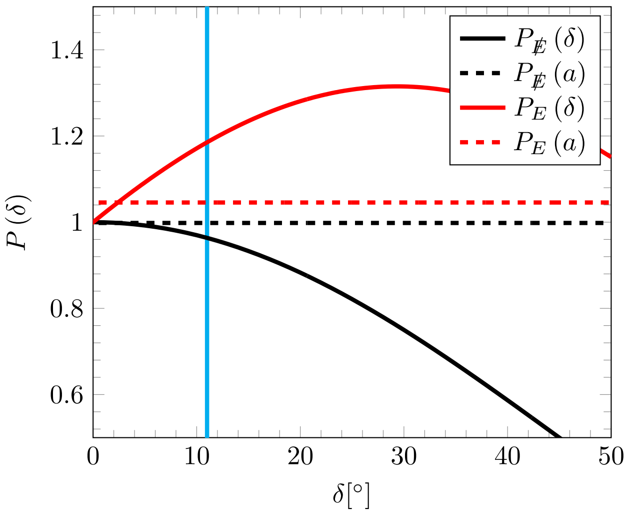
Problem! Not much known About ^{3}He – π scattering

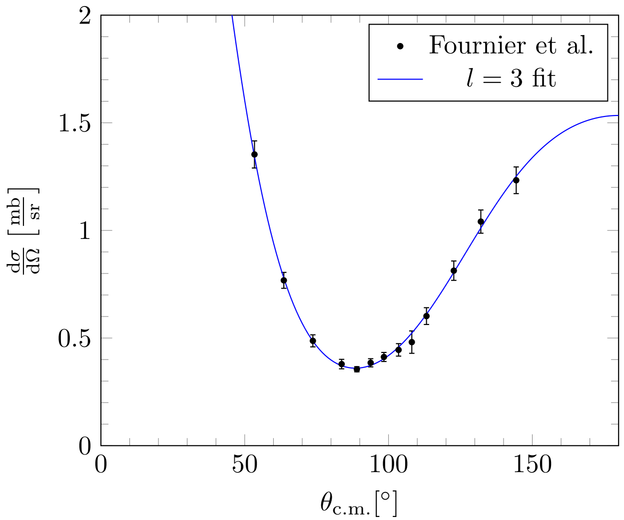
Only evaluate at the dominating momentum $\bar{k}$



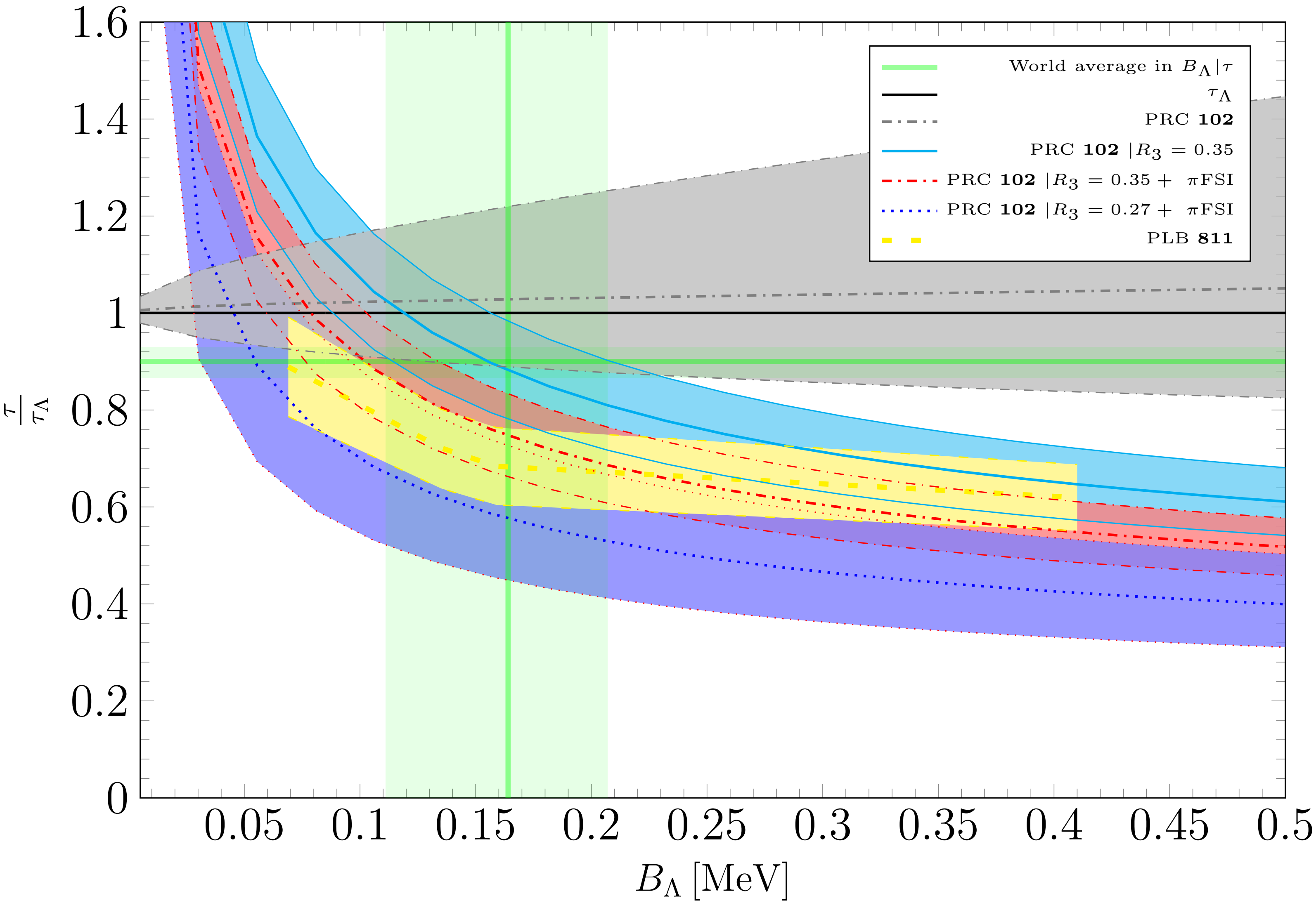
$$M = F^2(k)\frac{(k\cot\delta + \bar{k})^2}{(k\cot\delta)^2 + k^2} \equiv F^2(k)P_E(\delta)$$

Typical momentum depends only weak on B_3





Pionic Final State Interactions



→ Universal relation between $\tau \Leftrightarrow B_\Lambda$

Intermediate Summary

Three-body-hypernuclei are important to understand physics beyond the u- and d-quark sector

Pionless EFT results consistent for large interparticle distance from 2-body Estimate

Results for the hypertriton lifetime with a fundamental deuteron including The full three-body phase space

Branching ratio favours small binding energy

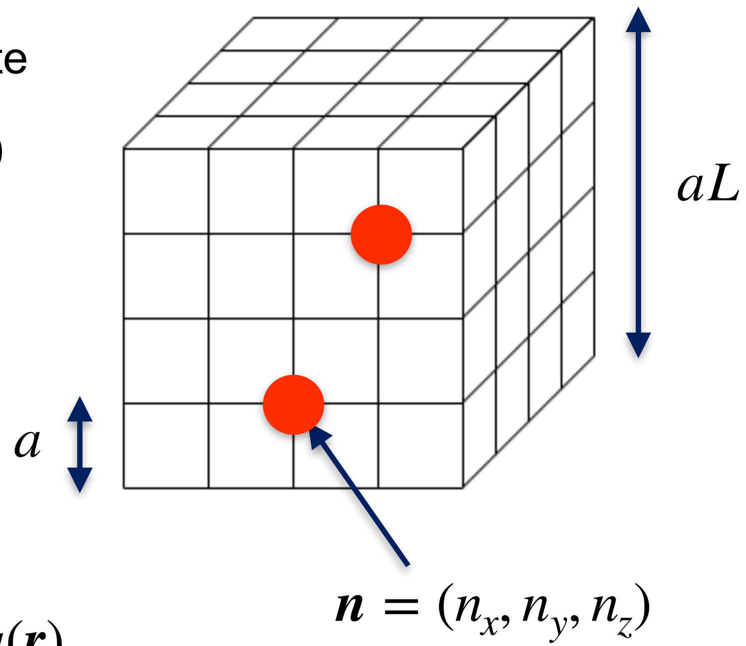
Important to combine different observables: binding energy, lifetime and branching ratios!

→ Change Gears! Now Nuclear Lattice EFT

Method: Lattice Monte Carlo

- Lattice ⇒ Cubic Volume of size $(La)^3$ with discrete lattice site
  (a = lattice spacing, serves as UV cutoff for the EFT $\Lambda = \frac{\pi}{a}$)

- We need to make our Hamiltonian discrete.



Example: Spin ↑ particle(s)

$$H = \frac{1}{2m}\int \mathrm{d}^3r\, \nabla a^\dagger(\boldsymbol{r}) \cdot \nabla a(\boldsymbol{r}) = -\frac{1}{2m}\int \mathrm{d}^3r\, a^\dagger(\boldsymbol{r}) \cdot \nabla^2 a(\boldsymbol{r})$$

⇒

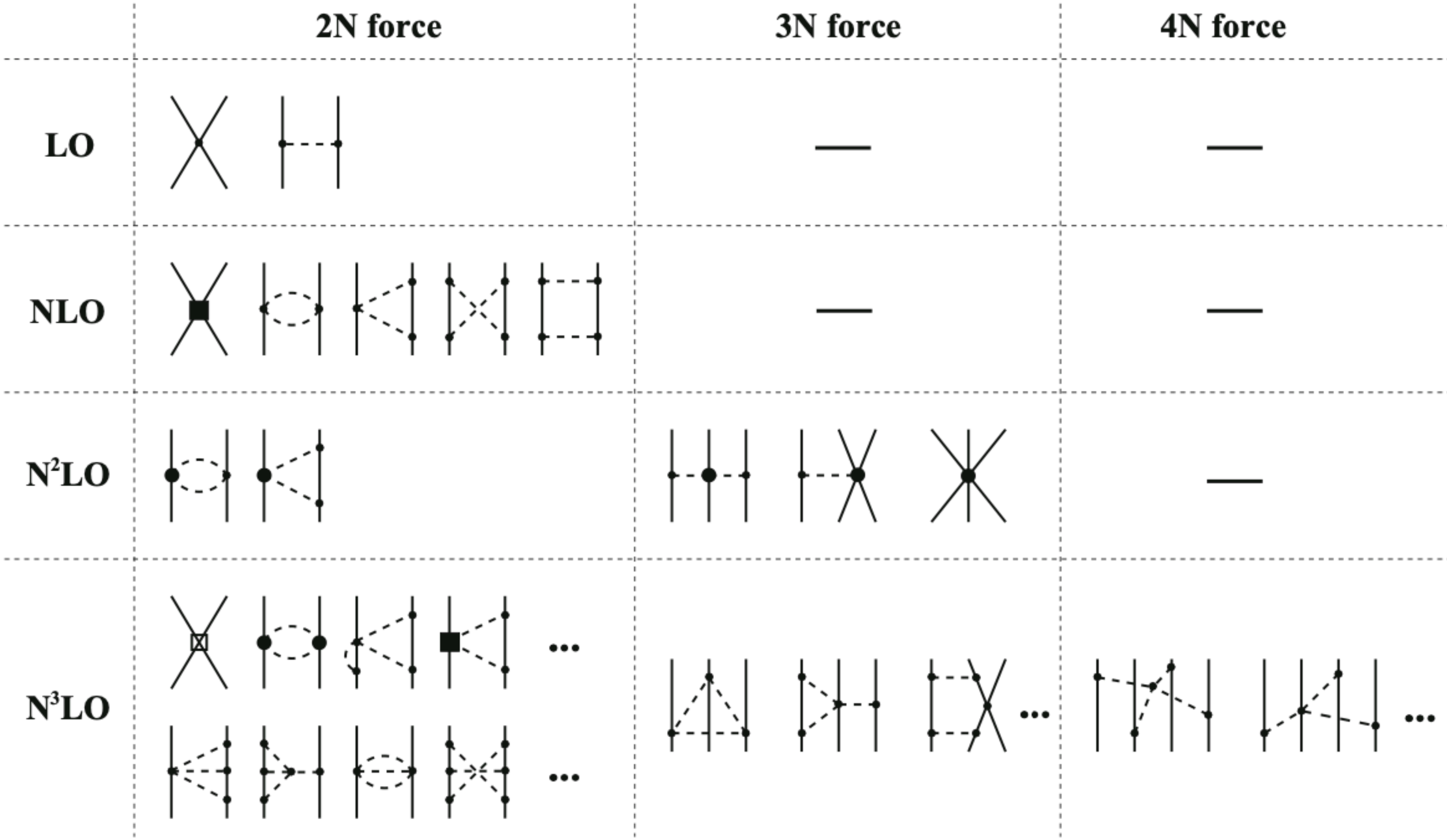
Nearest neighbours

$$H_L = \frac{3}{\tilde{m}}\sum_n a_i^\dagger(\boldsymbol{n})a_i(\boldsymbol{n}) - \frac{1}{2\tilde{m}}\sum_n\sum_{l=1}^{3}\left[a_i^\dagger(\boldsymbol{n})a_i(\boldsymbol{n}+\hat{\boldsymbol{e}}_l) + a_i^\dagger(\boldsymbol{n})a_i(\boldsymbol{n}-\hat{\boldsymbol{e}}_l)\right]$$

simplest version, many more possible, do the same with the potential

Nuclear Lattice Effective Field Theory

- Different Interaction: Chiral EFT



Epelbaum

Method: Lattice Monte Carlo III

- For a general Operator $\mathcal{O}$, the expectation value in path integral formalism is given

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}s \; \mathcal{O}[s] \exp(-S_E[s, \beta])$$

$$\langle \mathcal{O} \rangle = \approx \frac{\sum_s \mathcal{D}s \; \mathcal{O}[s] \exp(-S_E[s])}{\sum_s \exp(-S_E[s])}$$ $\propto$ complex phase $\Rightarrow$ sign proplem

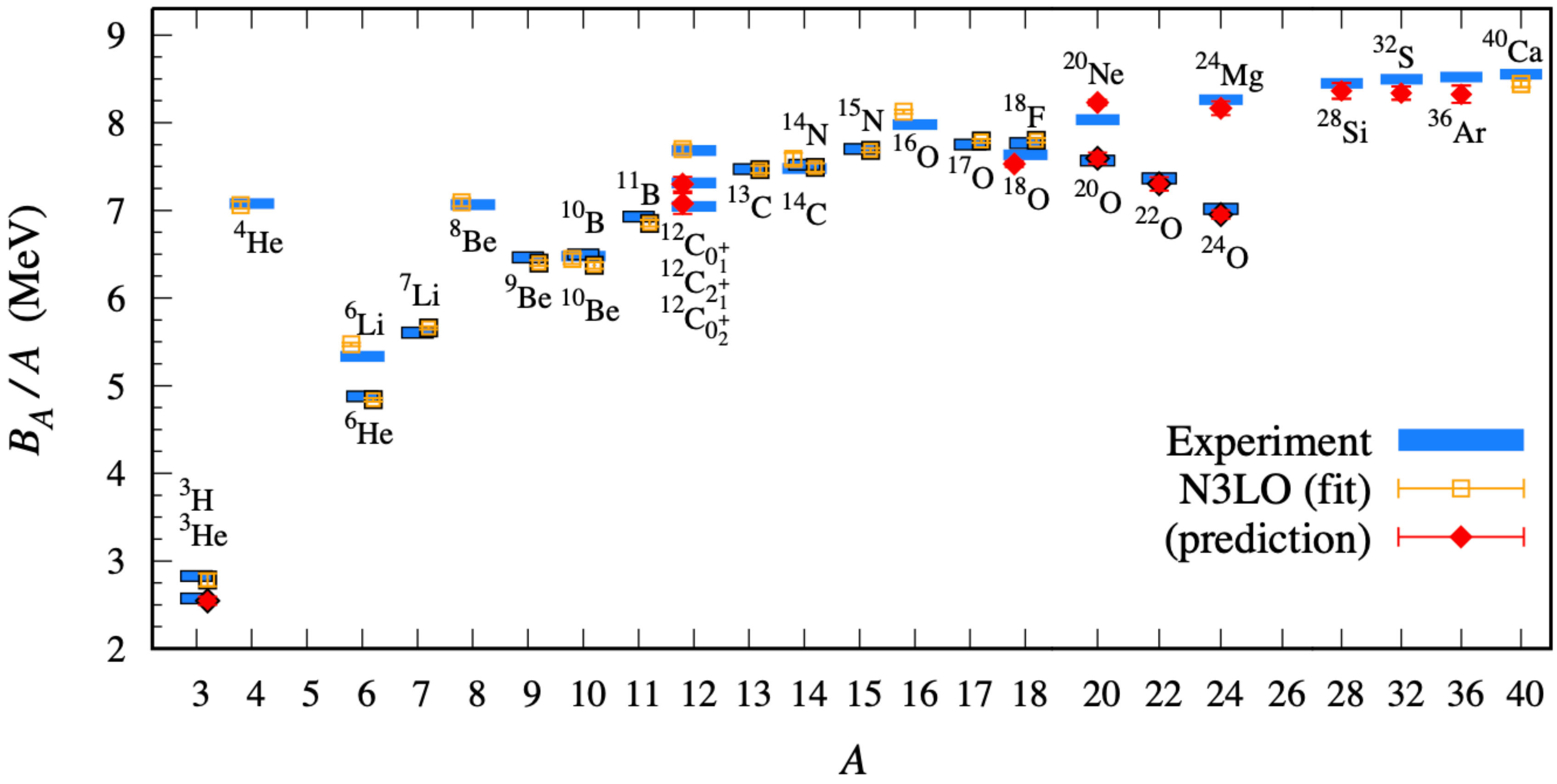
Metropolis Accept/Reject sampling with respect to the action
(Importance Samling, Markov chains …)

- Auxiliary Fields to handle many particles efficiently:
- Idea: Replace Interactions between nucleons with Interaction of a nucleon with an auxiliary field

$$\exp(-\frac{C}{2}(N^\dagger N)^2) = \sqrt{\frac{1}{2}} \int \mathrm{d}A \exp\left[-\frac{A^2}{2} + \sqrt{C} A (N^\dagger N)\right]$$

Since Nucleons only interact with an auxiliary field $\Rightarrow$ Perfect for parallel computing

Hypernuclear Lattice Effective Field Theory



Elhatisari et al. 2022

Idea:
Combine $N^3LO \quad \chi EFT(NN)$
with hypernuclear interactions

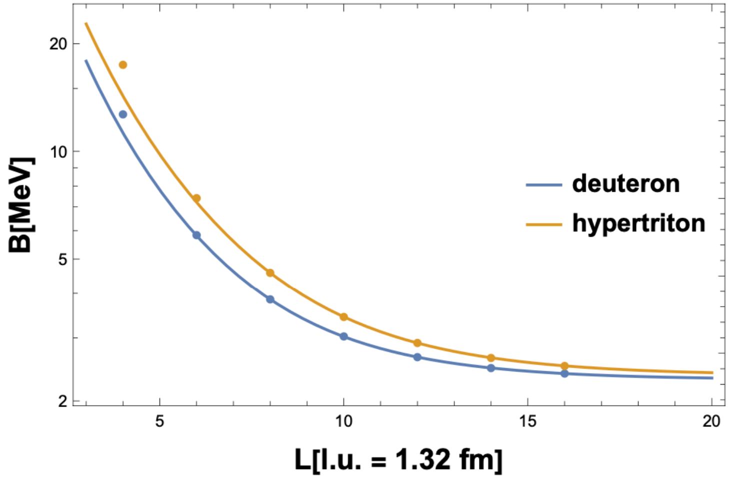
→

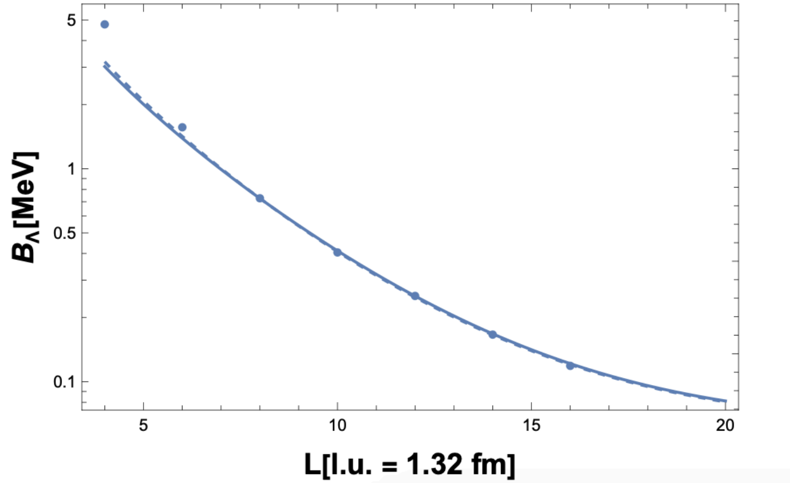
Explore Hypernuclei on the Lattice
2 options , new challenges

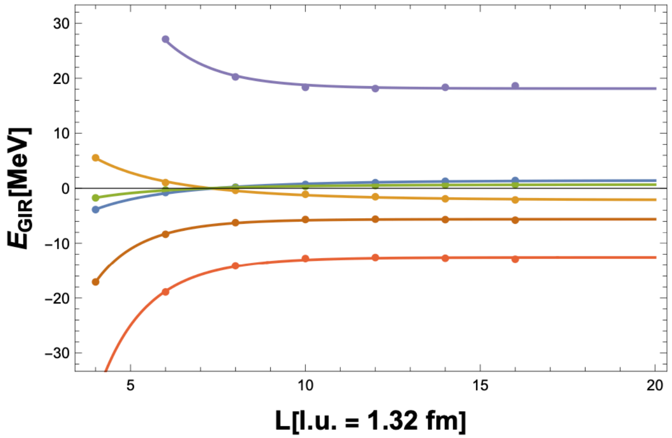
Option 1: Auxiliary field Monte Carlo

Idea: Use the same Method as for Nuclei, one particle more

Mass imbalance not too bad $\Delta E \approx 80$ MeV → Sign problem could be fine







Shallow Hypertriton ⇒ L dependence
Typical nuclear Box: L=10,12

$$c_{^1S_0} = -1.40 \cdot 10^{-7}\,\text{MeV}^{-2}$$
$$c_{^3S_1} = -1.06 \cdot 10^{-7}\,\text{MeV}^{-2}$$

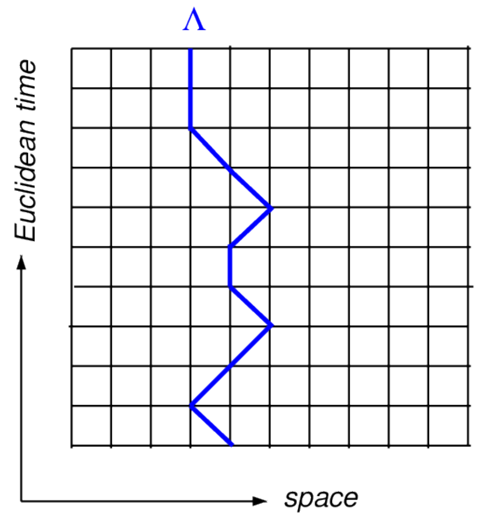
Option 2: Worldline Monte Carlo

AFMC does not converge as good as in a pure nuclear matter simulation

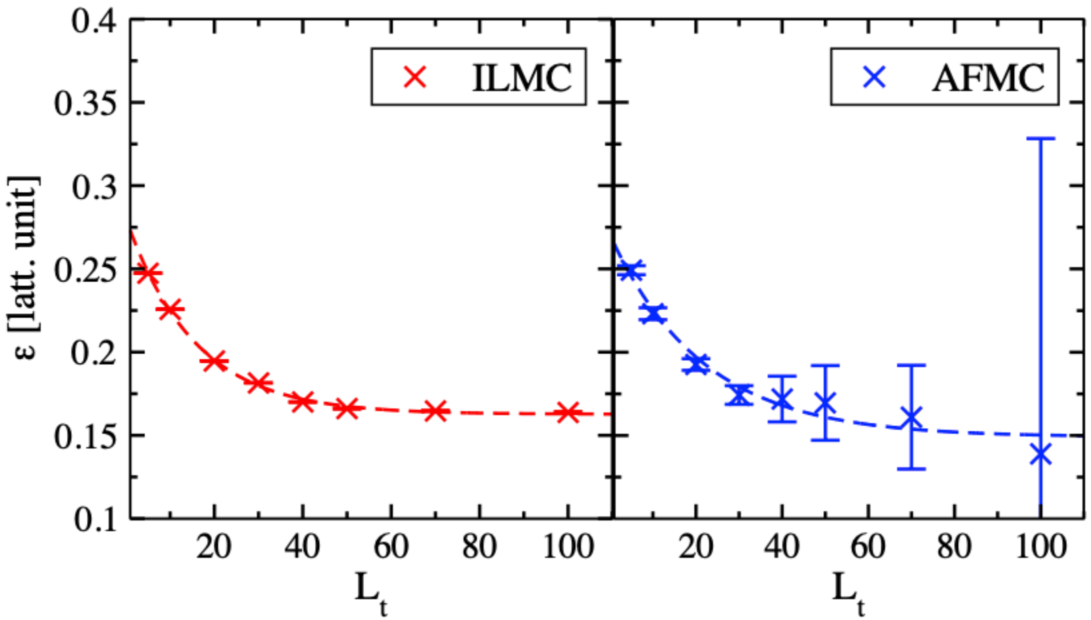
Need to develop a method that threats this impurities more efficient

Treat Impurity as worldline:

(S.Bour, D.Lee, H.-W. Hammer, U.-G. Meißner)



(D. Frame, T. A. Lähde, D. Lee, U.-G. Meißner)



We however want to study systems with more impurities !! $^{6}He_{\Lambda\Lambda}$

$$\hat{H}_0 = \frac{1}{2m} \sum_{s=\uparrow_a,\uparrow_b,\downarrow} \int \mathrm{d}^3\boldsymbol{r}\, \nabla a_s^\dagger(\boldsymbol{r}) \nabla a_s(\boldsymbol{r})$$

← Kinetic Energy Term

$$\hat{H}_I = C_{II} \int \mathrm{d}^3\boldsymbol{r}\, \hat{\rho}_{\uparrow_b}(\boldsymbol{r})\hat{\rho}_{\uparrow_a}(\boldsymbol{r}) + C_{IB} \int \mathrm{d}^3\boldsymbol{r} \left[\hat{\rho}_{\uparrow_a}(\boldsymbol{r})\hat{\rho}_{\downarrow}(\boldsymbol{r}) + \hat{\rho}_{\uparrow_b}(\boldsymbol{r})\hat{\rho}_{\downarrow}(\boldsymbol{r})\right]$$

← Contact Interactions

Worldline - Worldline Interaction

Worldline - Background Interaction

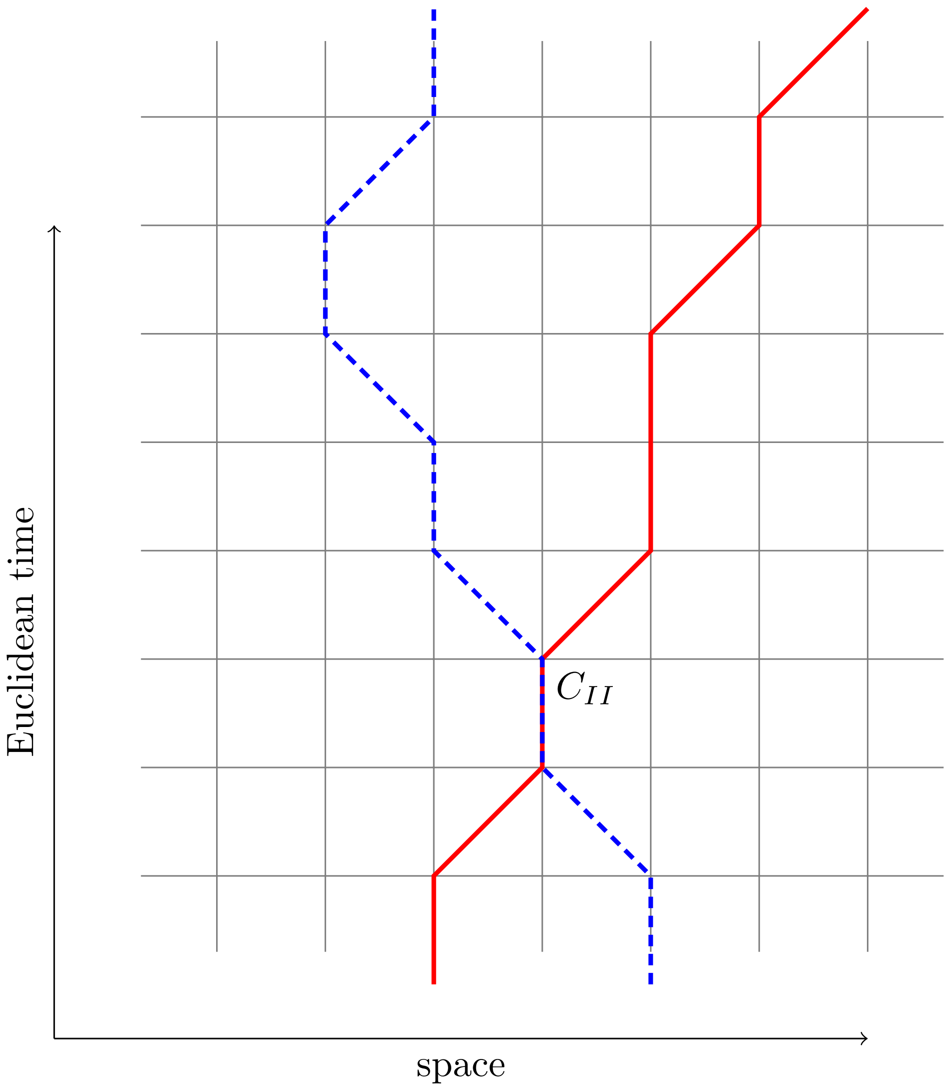
Idea: Integrate out the impurities from the lattice action :

$$\langle \chi_{n_{t+1}}^{\downarrow}, \chi_{n_{t+1}}^{\uparrow_a}, \chi_{n_{t+1}}^{\uparrow_b} | \hat{M} | \chi_{n_t}^{\downarrow}, \chi_{n_t}^{\uparrow_a}, \chi_{n_t}^{\uparrow_b} \rangle \Rightarrow \langle \chi_{n_{t+1}}^{\downarrow} | \hat{\tilde{M}} | \chi_{n_t}^{\downarrow} \rangle$$

With any state in occupation number basis is given by:

$$|\chi_{n_t}^{\downarrow}, \chi_{n_t}^{\uparrow_a}, \chi_{n_t}^{\uparrow_b}\rangle = \prod_{\boldsymbol{n}} \left[a_{\downarrow}^\dagger(\boldsymbol{n})\right]^{\chi_{n_t}^{\downarrow}(\boldsymbol{n})} \left[a_{\uparrow_a}^\dagger(\boldsymbol{n})\right]^{\chi_{n_t}^{\uparrow_a}(\boldsymbol{n})} \left[a_{\uparrow_b}^\dagger(\boldsymbol{n})\right]^{\chi_{n_t}^{\uparrow_b}(\boldsymbol{n})} |0\rangle$$

What can happen?



- both worldline hop

$$\overline{M}^{\boldsymbol{n}\pm\hat{l},\boldsymbol{n}}_{\boldsymbol{n}'\pm\hat{l}',\boldsymbol{n}'} = W_h^2 : e^{-\alpha H_0^{\downarrow}} :$$

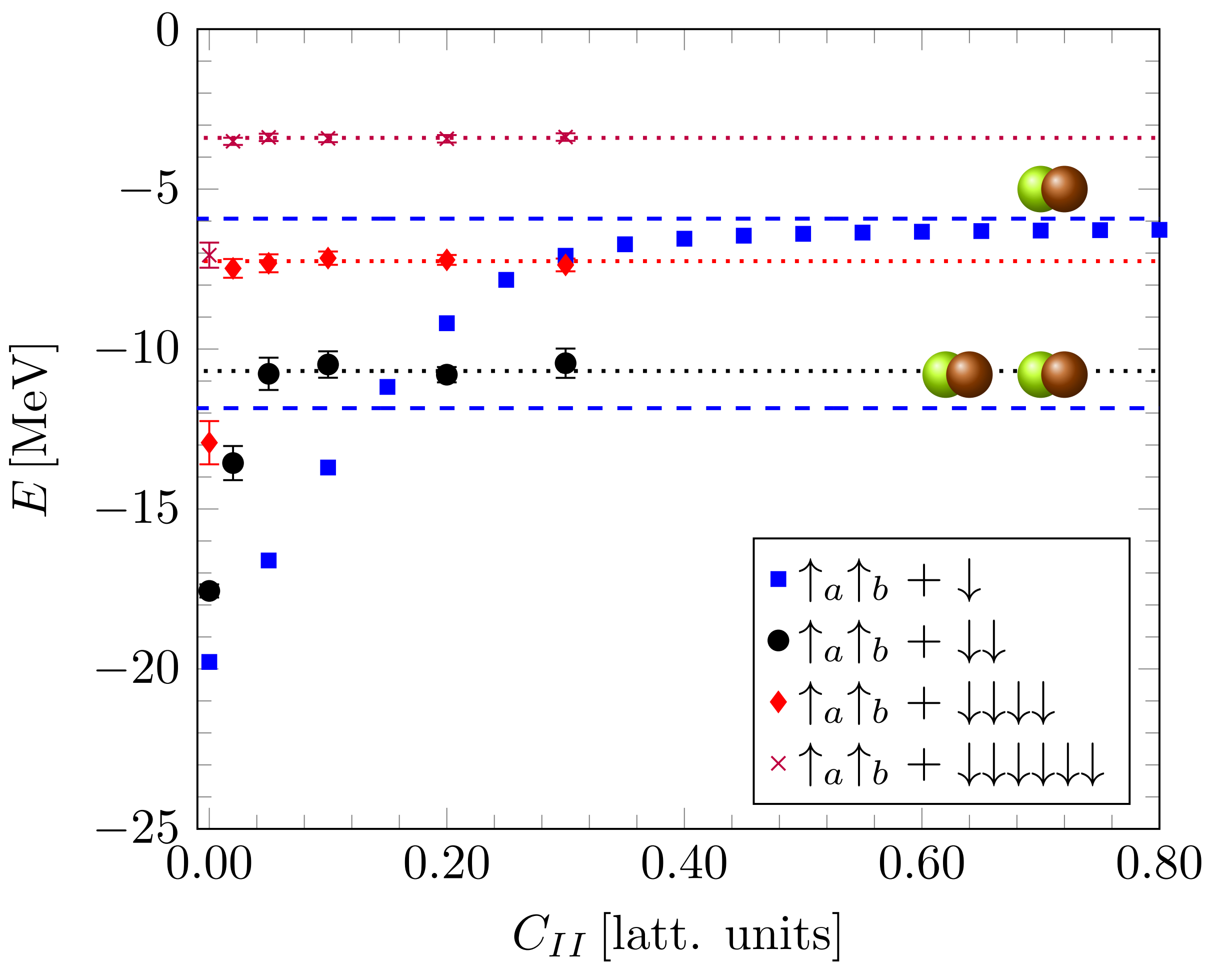
- one worldline hops, one stays

$$\overline{M}^{\boldsymbol{n}\pm\hat{l},\boldsymbol{n}}_{\boldsymbol{n}',\boldsymbol{n}'} = W_h W_s : e^{-\alpha H_0^{\downarrow} - \frac{\alpha C_{IB}\,\rho_{\downarrow}(\boldsymbol{n}')}{W_s}} :$$

- both worldlines stay

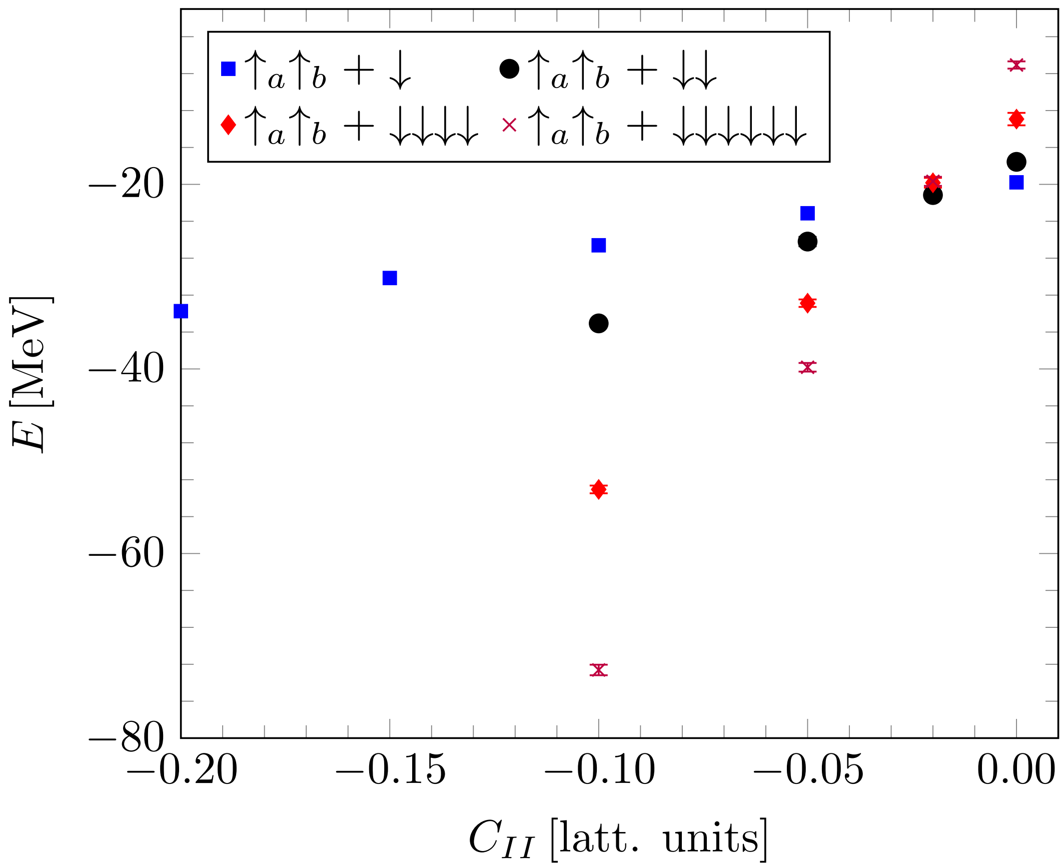
$$\overline{M}^{\boldsymbol{n},\boldsymbol{n}}_{\boldsymbol{n}',\boldsymbol{n}'} = W_s^2 : e^{-\alpha H_0^{\downarrow}} \exp\left[\frac{-\delta_{\boldsymbol{n},\boldsymbol{n}'}\alpha C_{II}}{W_s^2} - \frac{\alpha C_{IB}\rho_{\downarrow}(\boldsymbol{n})}{W_s} - \frac{\alpha C_{IB}\,\rho_{\downarrow}(\boldsymbol{n}')}{W_s} + \mathcal{O}(\alpha^2)\right] :$$

Results: Attractive Impurity-Background Interaction
Repulsive Impurity-Impurity interaction



- Impurity-Background interaction chosen to be attractive $a \sim 3$ fm
- Trimer stays bound even for very repulsive C_{II}
- The four particle bound state however consists out of two dimers
- Further particles fill up the fermi sea of the box and do not contribute to the binding

Results: Attractive Impurity-Background Interaction
Attractive Impurity-Impurity interaction



- Around $C_{II} \sim -0.02$ the four particle system is deeper bound than the 3-body system
- Higher-particle systems show a similar behaviour at the same point
- Indication of a rich phase structure

Summary and Outlook

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Implementation of Hypernuclear physics on the Lattice is in progress

Two options : AFMC and Worldline+AFMC

Offers another approach to hypernuclear physics with precise interactions

Three-body-hypernuclei are important to understand physics beyond the u- and d-quark sector

Pionless EFT results consistent for large interparticle distance from 2-body Estimate

Results for the hypertriton lifetime with a fundamental deuteron including The full three-body phase space

Branching ratio favours small binding energy

Important to combine different observables: binding energy, lifetime and branching ratios!