



The nucleus as a quantum laboratory

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by ERC, EXOTIC

by NRW-FAIR



中国科学院
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⟨NUMERIQS⟩



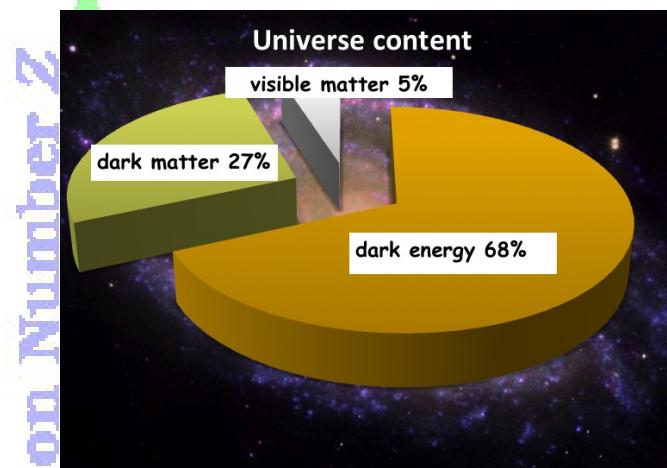
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- Introductory remarks
- Chiral EFT on a lattice
- Emergent geometry and duality in the carbon nucleus
- Towards heavy nuclei and nuclear matter in NLEFT
- *Ab initio* calculation of hyper-neutron matter
- Summary & outlook

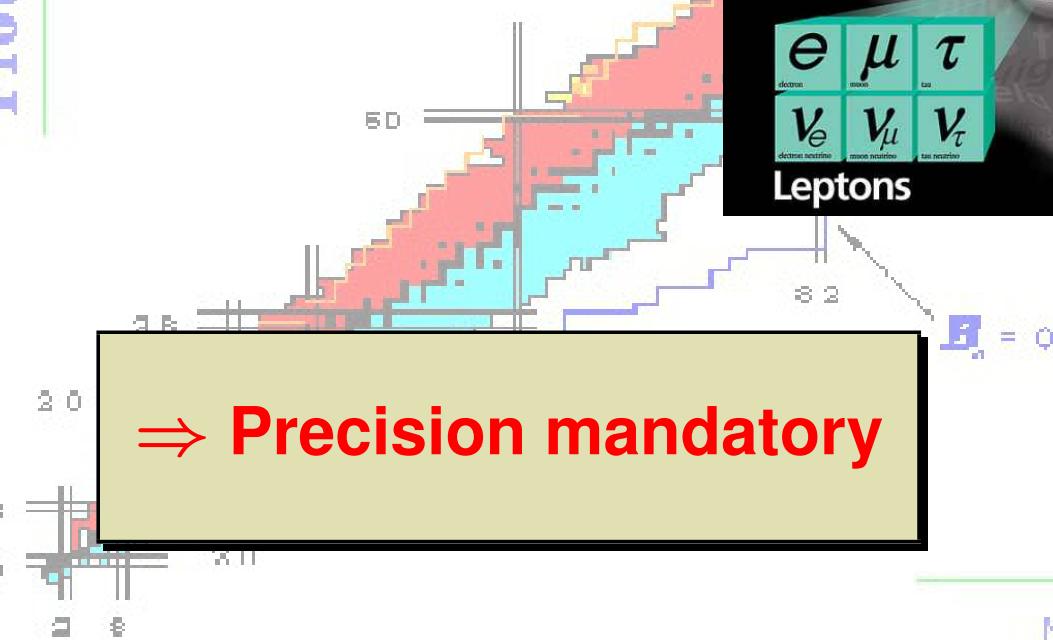
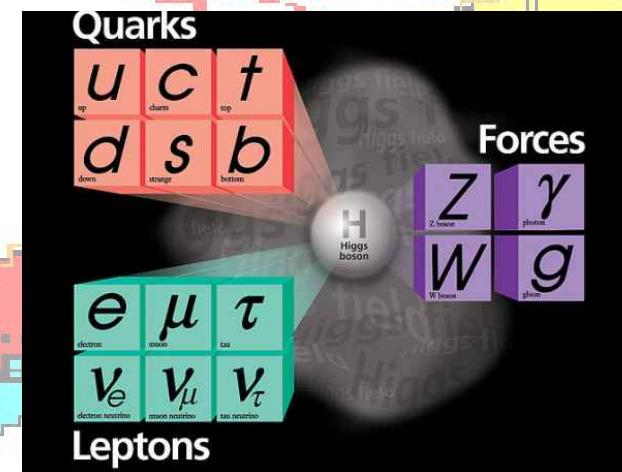
Introductory remarks

Why nuclear physics?

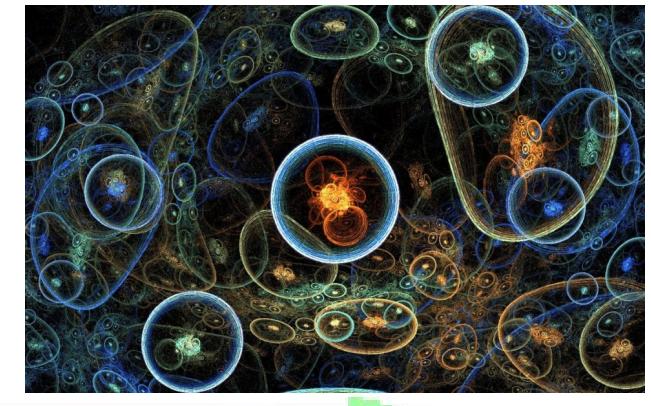
- The matter we are made off



- The last frontier of the SM



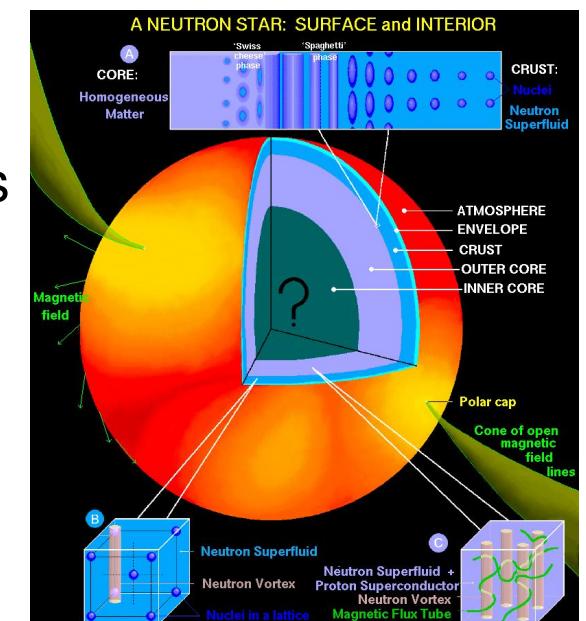
- Access to the Multiverse



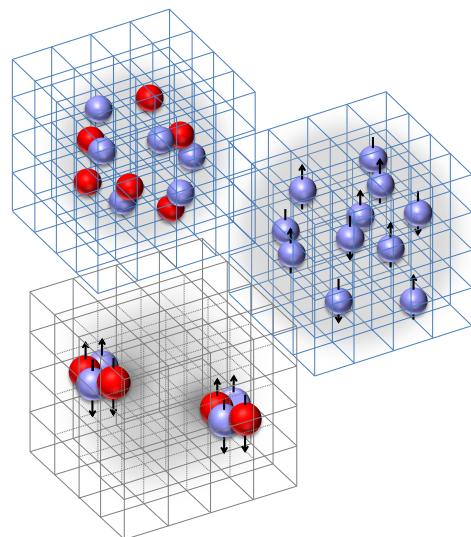
Neutron Number N

The nucleus as a quantum laboratory

- The nucleus is a challenging and fascinating many-body system
 - non-perturbative strong interactions balanced by the Coulomb force
 - many interesting phenomena: drip lines, clustering, reactions, ...
 - a plethora of few-body/many-body methods already exists
- Macroscopic nuclear matter = neutron stars
 - gained prominence again in the multi-messenger aera
 - must be able to describe these with the same methods
- I will advocate here a new quantum many-body approach
 - synthesizes chiral EFT w/ stochastic methods
 - allows to tackle nuclear structure *and* reactions
 - allows to access the multiverse



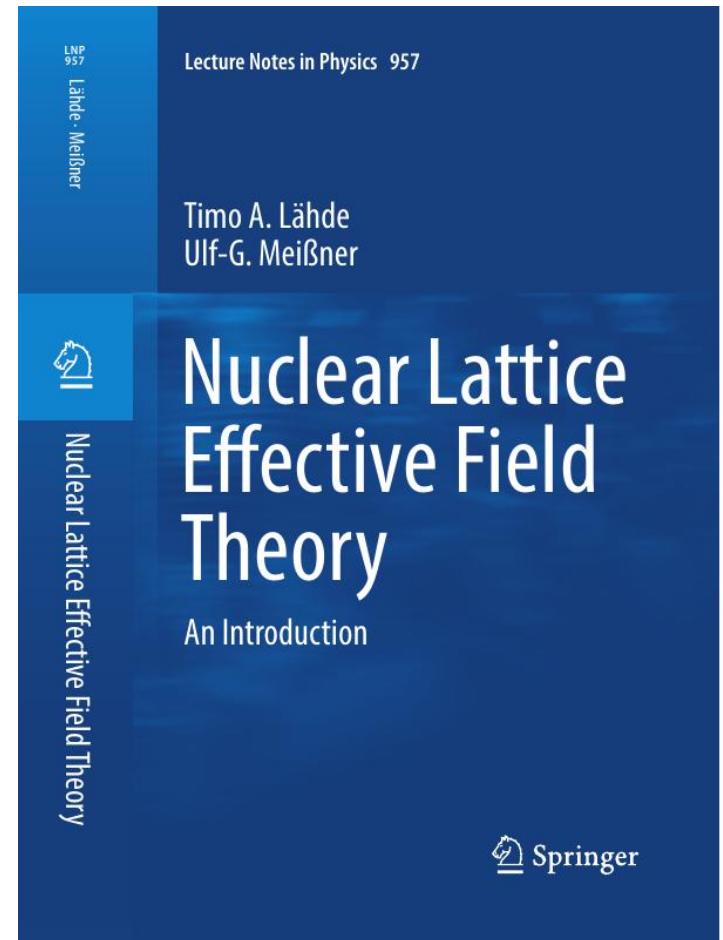
Chiral EFT on a lattice



T. Lähde & UGM

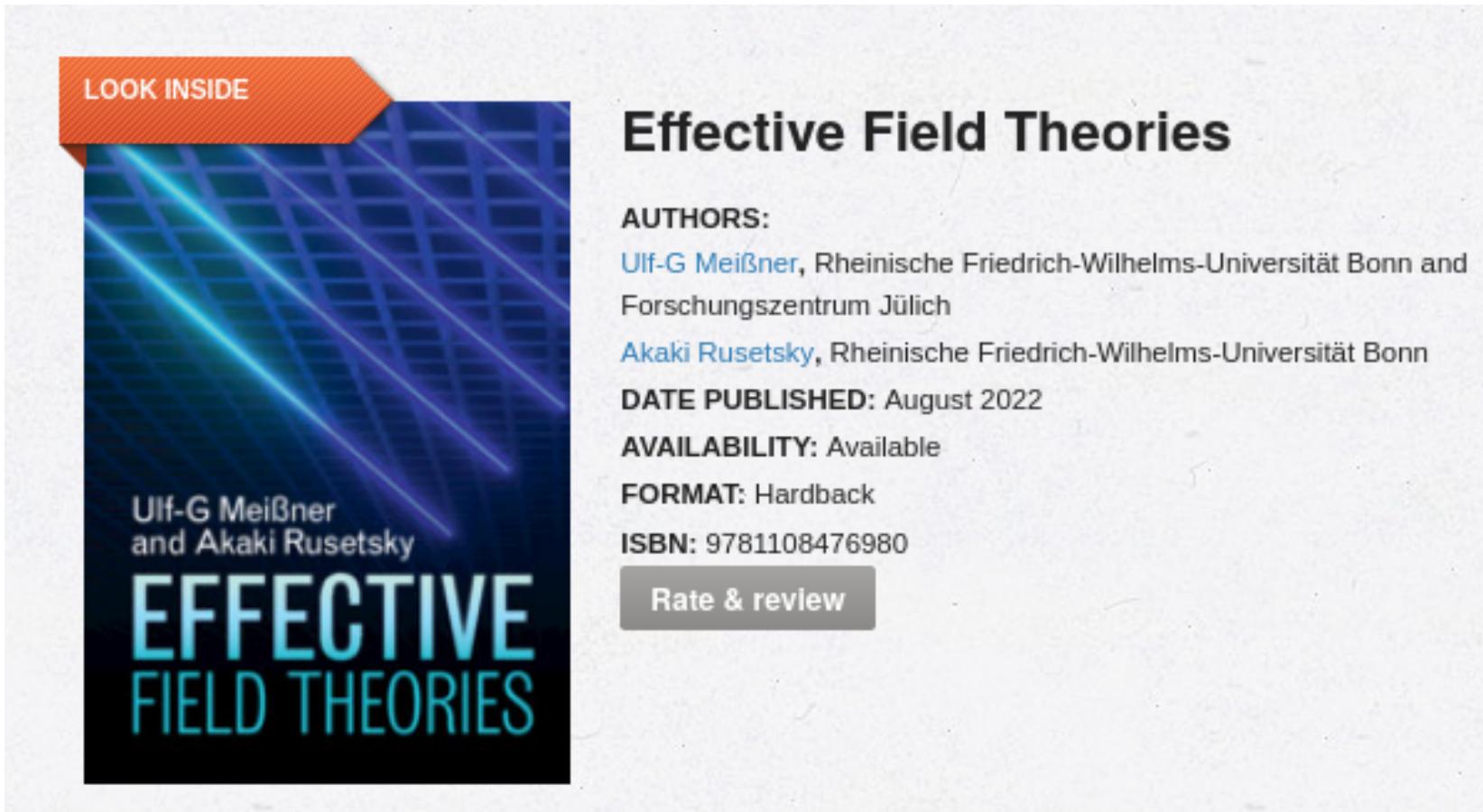
Nuclear Lattice Effective Field Theory - An Introduction

Springer Lecture Notes in Physics **957** (2019) 1 - 396



More on EFTs

- Much more details on EFTs in light quark physics:



<https://www.cambridge.org/de/academic/subjects/physics/theoretical-physics-and-mathematical-physics/effective-field-theories>

Nuclear lattice effective field theory (NLEFT)

8

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- new method to tackle the nuclear many-body problem

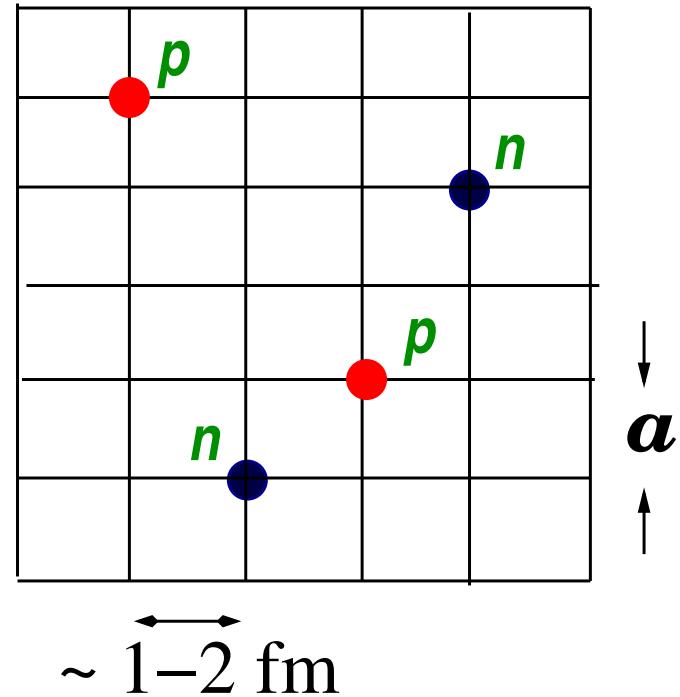
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
and contact interactions + Coulomb

→ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters

$$p_{\max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$



~ 1–2 fm

- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- physics independent of the lattice spacing for $a = 1 \dots 2 \text{ fm}$

Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

Transfer matrix method

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

with Ψ_A a Slater determinant for A free nucleons
[or a more sophisticated (correlated) initial/final state]

- Transient energy

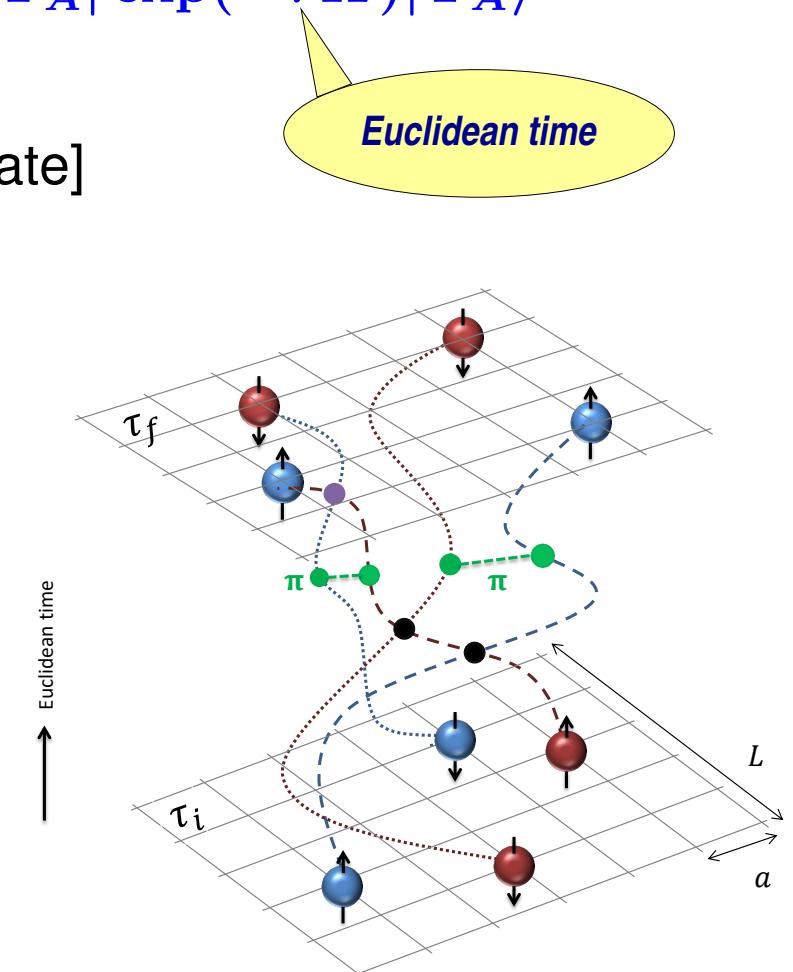
$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

→ ground state: $E_A^0 = \lim_{\tau \rightarrow \infty} E_A(\tau)$

- Exp. value of any normal–ordered operator \mathcal{O}

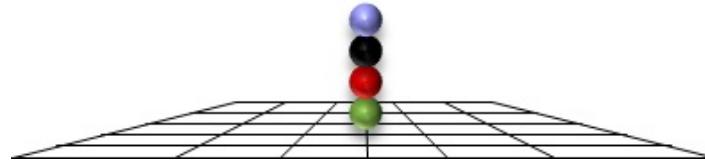
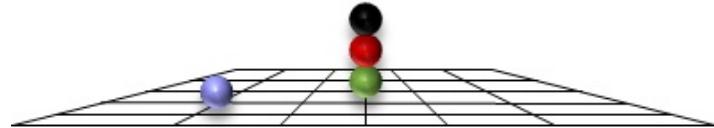
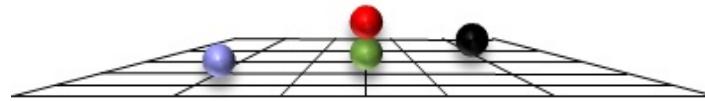
$$Z_A^\mathcal{O} = \langle \Psi_A | \exp(-\tau H/2) \mathcal{O} \exp(-\tau H/2) | \Psi_A \rangle$$

$$\lim_{\tau \rightarrow \infty} \frac{Z_A^\mathcal{O}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} | \Psi_A \rangle$$



Configurations

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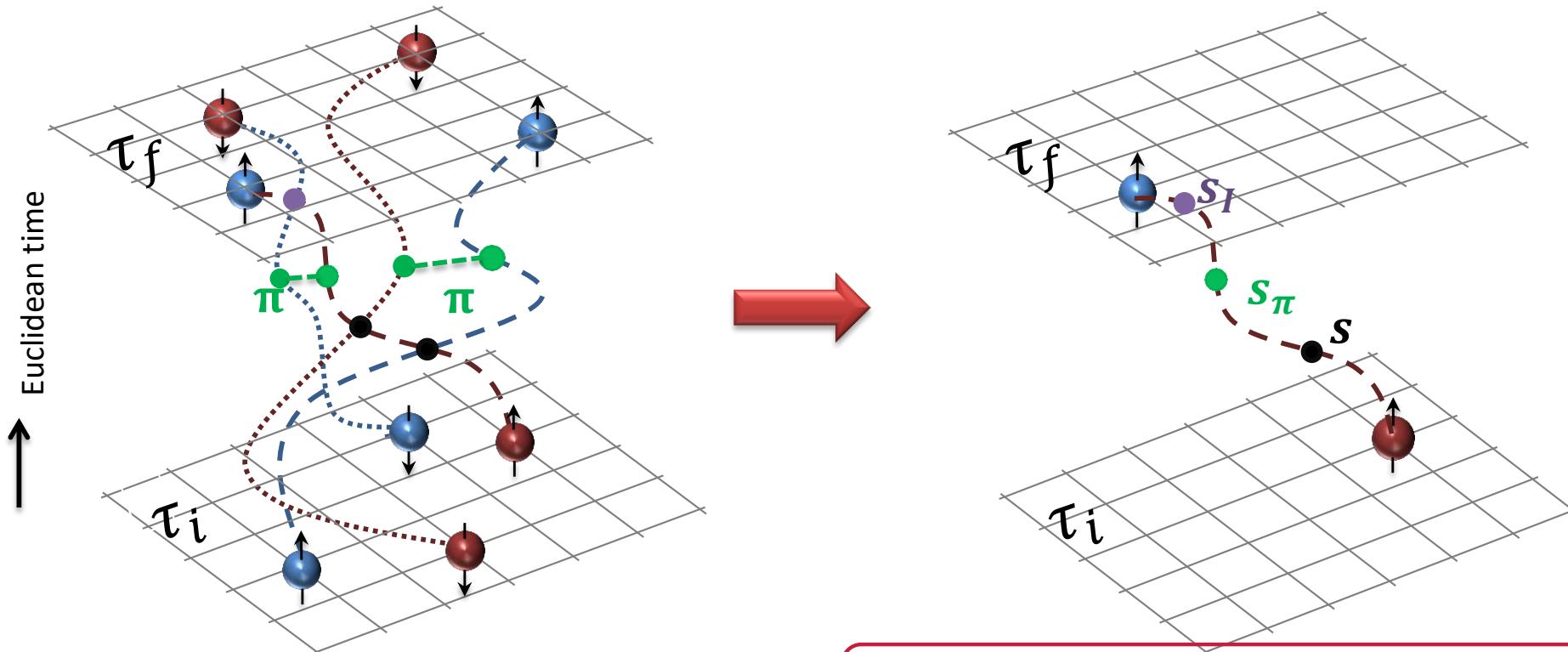


- ⇒ all *possible* configurations are sampled
- ⇒ preparation of *all possible* initial/final states
- ⇒ *clustering* emerges *naturally*

Auxiliary field method

- Represent interactions by auxiliary fields (Gaussian quadrature):

$$\exp \left[-\frac{C}{2} (N^\dagger N)^2 \right] = \sqrt{\frac{1}{2\pi}} \int ds \exp \left[-\frac{s^2}{2} + \sqrt{C} s (N^\dagger N) \right]$$



optimally suited for parallel computing!

Computational equipment

- Present = JUWELS (modular system) + FRONTIER + ...



Emergent geometry and duality in the carbon nucleus

Short reminder of Wigner SU(4) symmetry

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Wigner, Phys. Rev. C 51 (1937) 106

- If the nuclear Hamiltonian does not depend on spin and isospin, then it is obviously invariant under SU(4) transformations [really $U(4) = U(1) \times SU(4)$]:

$$N \rightarrow UN, \quad U \in SU(4), \quad N = \binom{p}{n}$$

$$N \rightarrow N + \delta N, \quad \delta N = i\epsilon_{\mu\nu}\sigma^\mu\tau^\nu N, \quad \sigma^\mu = (1, \sigma_i), \quad \tau^\mu = (1, \tau_i)$$

- LO pionless EFT: $\mathcal{L}_\pi = N^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m_N} \right) N - \frac{1}{2} (C_S(N^\dagger N)^2 + C_T(N^\dagger \vec{\sigma} N)^2)$

Mehen, Stewart, Wise, Phys. Rev. Lett. 83 (1999) 931

- Partial wave LECs: $C(^1S_0) = C_S - 3C_T, \quad C(^3S_1) = C_S + C_T$

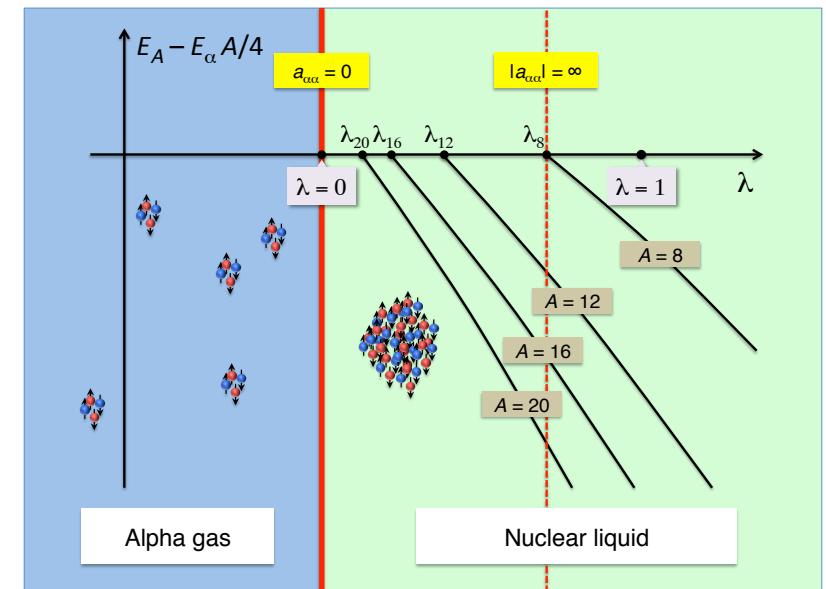
⇒ The operator $(N^\dagger N)^2$ is invariant under Wigner SU(4), but $(N^\dagger \vec{\sigma} N)^2$ is not

⇒ In the Wigner SU(4) limit, one finds: $C(^1S_0) = C(^3S_1) \rightarrow a_{np}^{S=0} = a_{np}^{S=1}$

⇒ The exact symmetry limit corresponds to a scale invariant non-relativistic system

Remarks on Wigner's SU(4) symmetry

- Wigner SU(4) spin-isospin symmetry is particularly beneficial for NLEFT
 - suppression of sign oscillations Chen, Lee, Schäfer, Phys. Rev. Lett. **93** (2004) 242302
 - provides a very much improved LO action when smearing is included Lu, Li, Elhatisari, Lee, Epelbaum, UGM, Phys. Lett. B **797** (2019) 134863
- Intimately related to α -clustering in nuclei
 - cluster states in ^{12}C like the famous Hoyle state Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501
 - nuclear physics is close to a quantum phase transition Elhatisari et al., Phys. Rev. Lett. **117** (2016) 132501



Wigner's SU(4) symmetry and the carbon spectrum

- Study of the spectrum of ^{12}C Shen, Lähde, Lee, UGM, Eur. Phys.J. A **57** (2021) 276

→ spin-orbit splittings are known to be weak

Hayes, Navratil, Vary, Phys. Rev. Lett. **91** (2003) 012502 Johnson, Phys. Rev. C **91** (2015) 034313

→ start with cluster and shell-model configurations → next slide

- Locally and non-locally smeared SU(4) invariant interaction:

$$V = \mathbf{C}_2 \sum_{\mathbf{n}', \mathbf{n}, \mathbf{n}''} : \rho_{\text{NL}}(\mathbf{n}') f_{s_L}(\mathbf{n}' - \mathbf{n}) f_{s_L}(\mathbf{n} - \mathbf{n}'') \rho_{\text{NL}}(\mathbf{n}'') :, \quad f_{s_L}(\mathbf{n}) = \begin{cases} 1, & |\mathbf{n}| = 0, \\ \mathbf{s}_L, & |\mathbf{n}| = 1, \\ 0, & \text{otherwise} \end{cases}$$

$$\rho_{\text{NL}}(\mathbf{n}) = a_{\text{NL}}^\dagger(\mathbf{n}) a_{\text{NL}}(\mathbf{n})$$

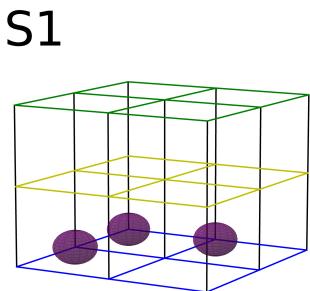
$$a_{\text{NL}}^{(\dagger)}(\mathbf{n}) = a^{(\dagger)}(\mathbf{n}) + s_{\text{NL}} \sum_{|\mathbf{n}'|=1} a^{(\dagger)}(\mathbf{n} + \mathbf{n}'), \quad s_{\text{NL}} = 0.2$$

→ only two adjustable parameters (C_2, s_L) fitted to $E_{^4\text{He}}$ & $E_{^{12}\text{C}}$

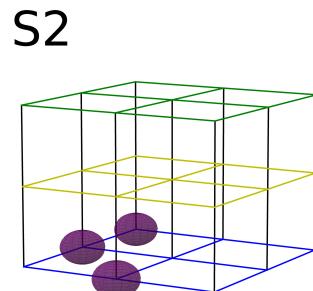
→ investigate the spectrum for $a = 1.64 \text{ fm}$ and $a = 1.97 \text{ fm}$

Configurations

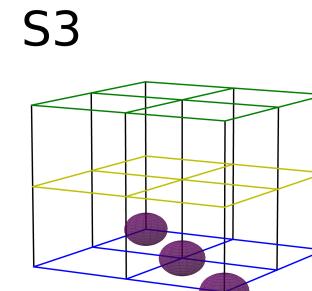
- Cluster and shell model configurations



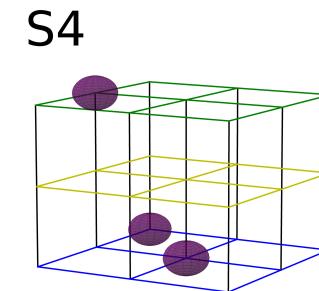
— isoscele right triangle



— “bent-arm” shape

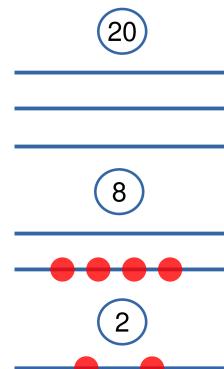


— linear diagonal chain



— acute isoscele triangle

Gaussian wave packets
 $w = 1.7 - 2.1 \text{ fm}$

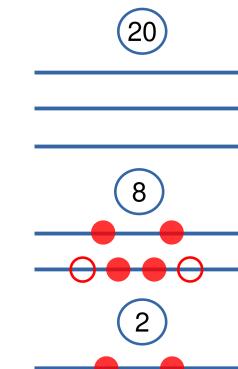


— ground state $|0\rangle$

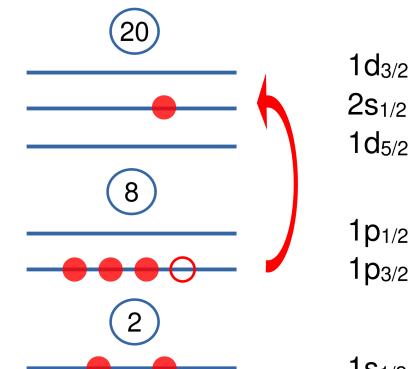
1d_{3/2}
2s_{1/2}
1d_{5/2}

1p_{1/2}
1p_{3/2}

1s_{1/2}



— 2p-2h state, $J_z = 0$

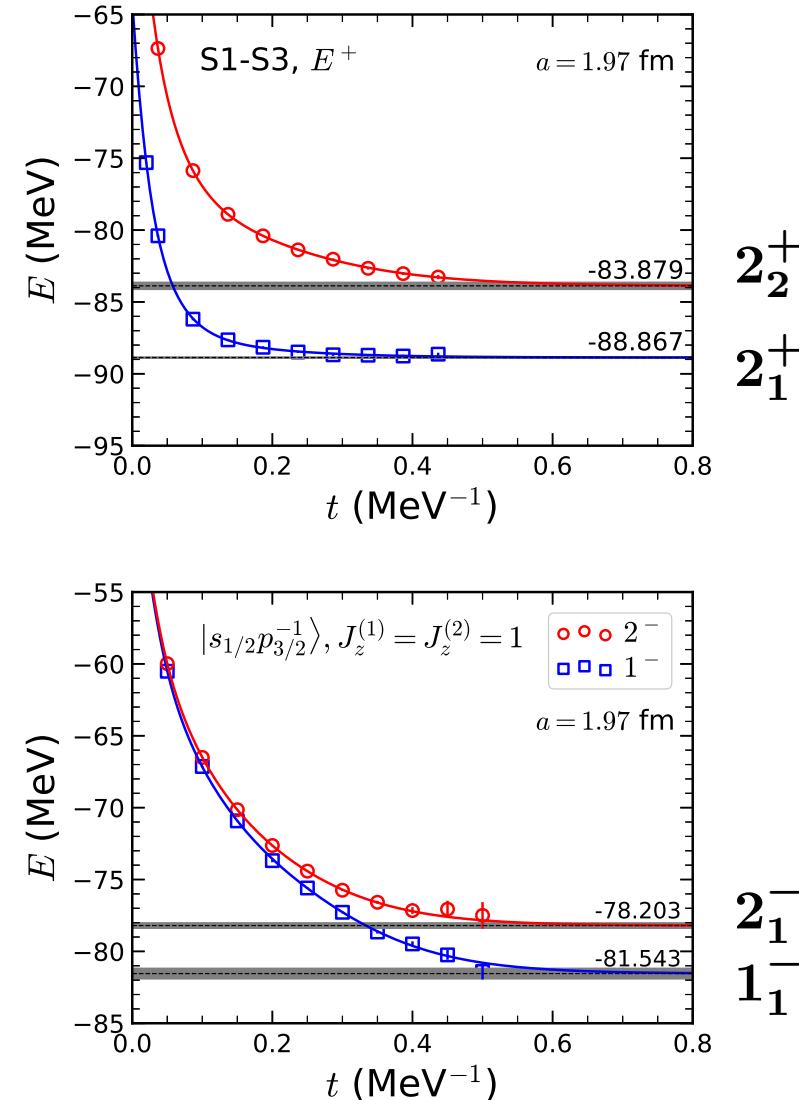
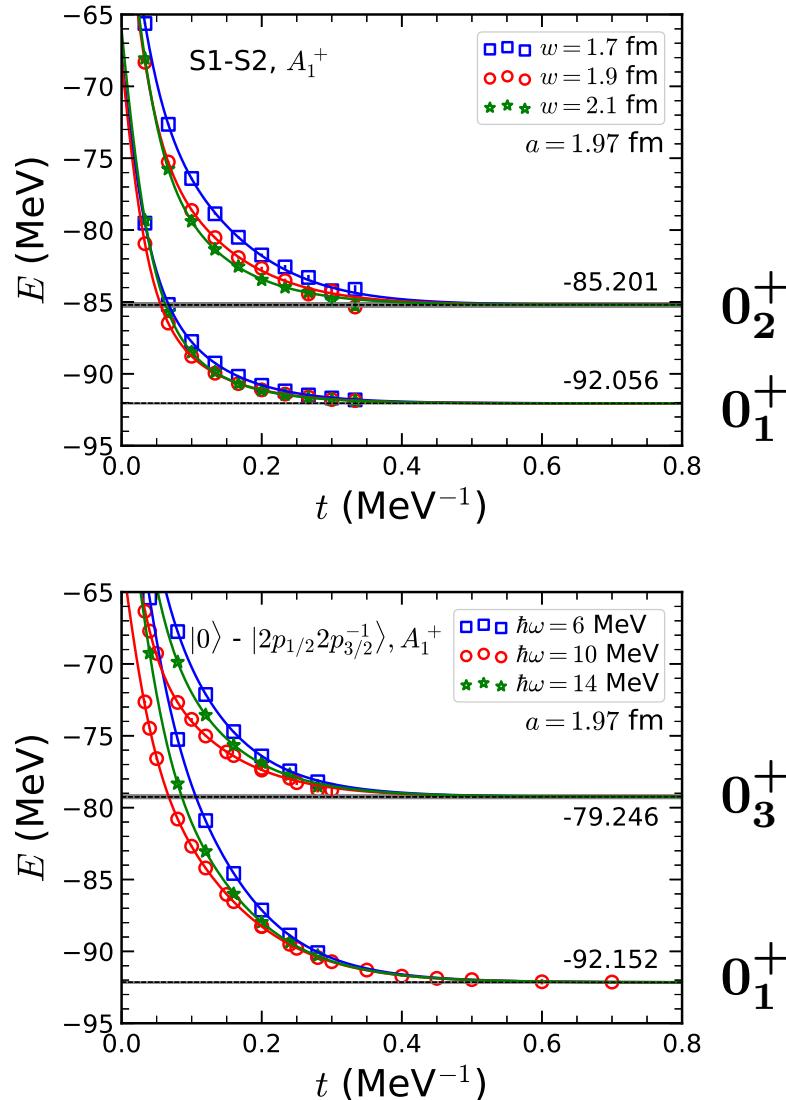


— 1p-1h state, $J_z^{(1)} = J_z^{(2)} = 1$

Transient energies

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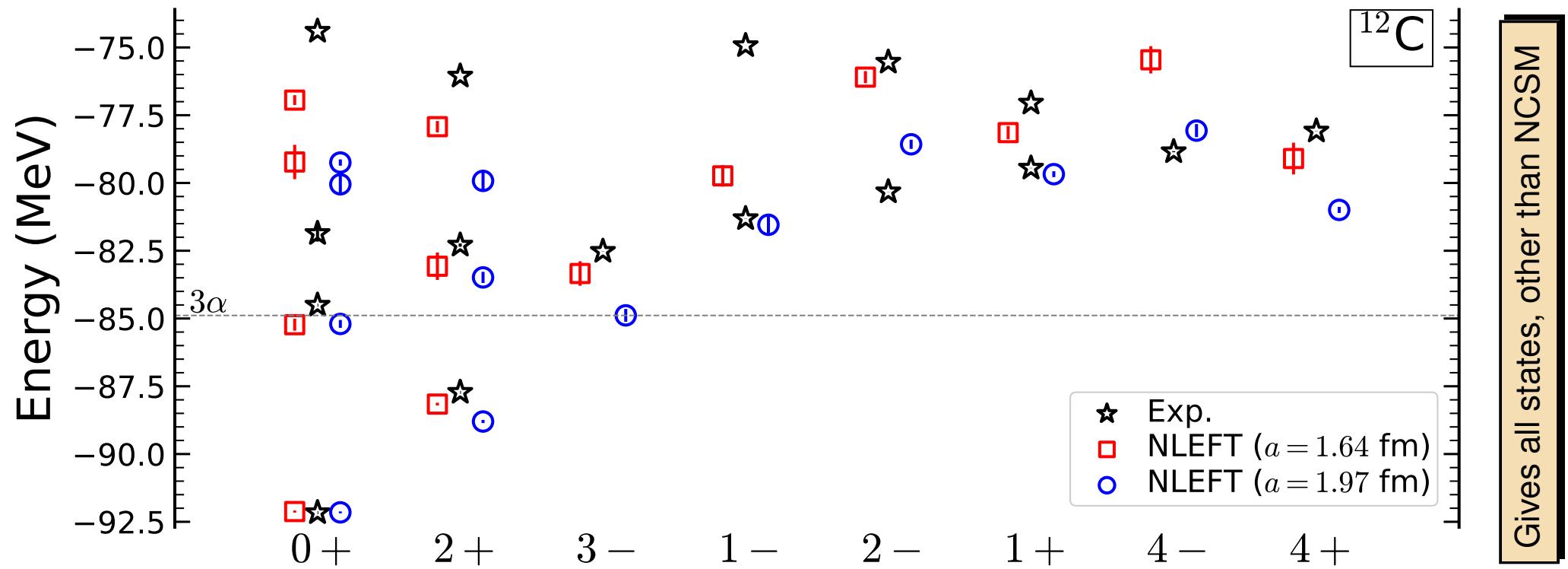
- Transient energies from cluster and shell-model configurations



Spectrum of ^{12}C

Shen, Lähde, Lee, UGM, Eur. Phys.J. A 57 (2021) 276 [arXiv:2106.04834]

- Amazingly precise description → great starting point



→ solidifies earlier NLEFT statements about the structure of the 0_2^+ and 2_2^+ states

A closer look at the spectrum of ^{12}C

Shen, Elhatisari, Lähde, Lee, Lu, UGM, Nature Commun. **14** (2023) 2777

- Include also 3NFs: $V = \frac{C_2}{2!} \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2 + \frac{C_3}{3!} \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^3$

- Fit the four parameters:

C_2, C_3 – ground state energies of ^4He and ^{12}C

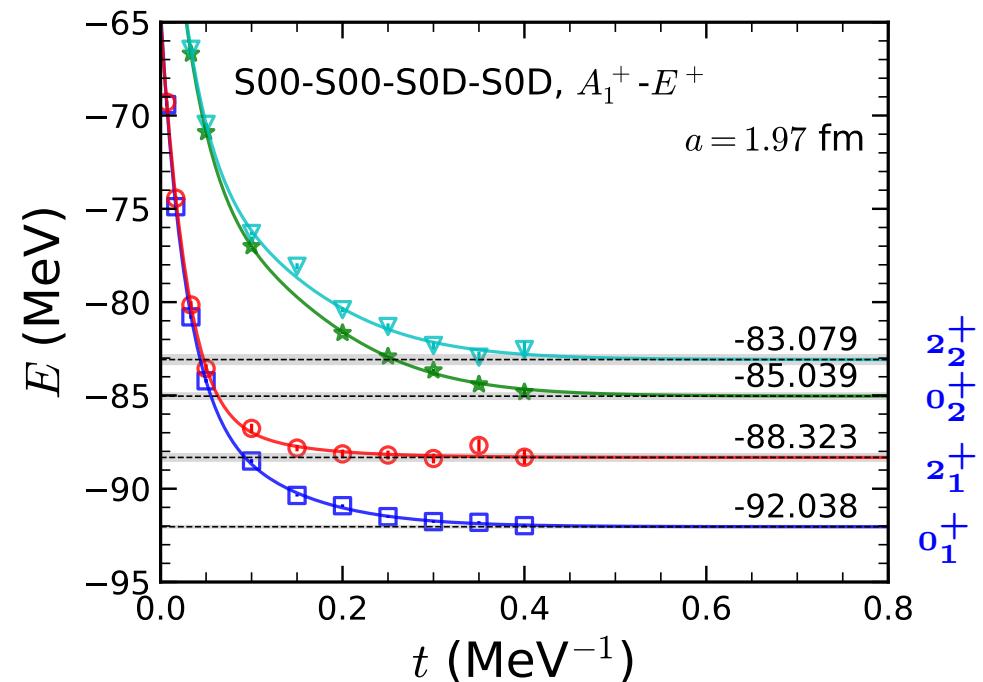
s_L – radius of ^{12}C around 2.4 fm

s_{NL} – best overall description
of the transition rates

- Calculation of em transitions

requires coupled-channel approach

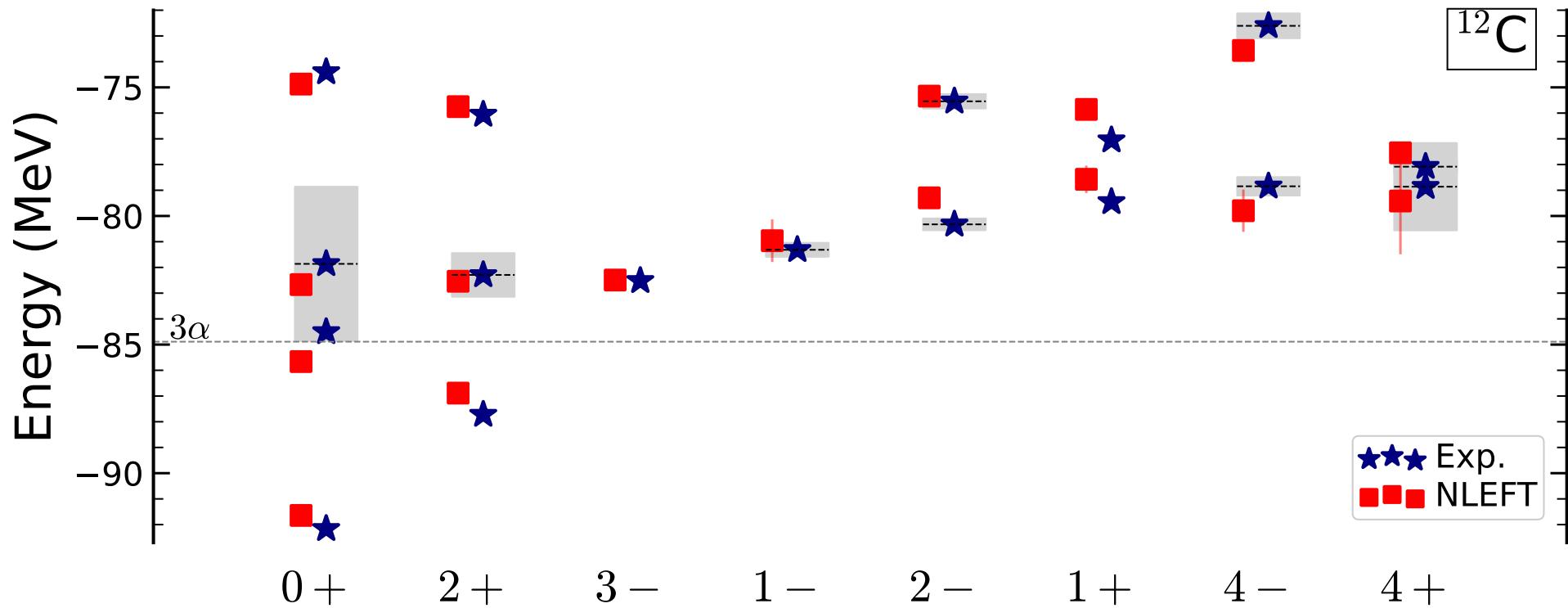
e.g. 0^+ and 2^+ states



Spectrum of ^{12}C reloaded

Shen, Elhatisari, Lähde, Lee, Lu, UGM, Nature Commun. 14 (2023) 2777

- Improved description when 3NFs are included, amazingly good



→ solidifies earlier NLEFT statements about the structure of the 0_2^+ and 2_2^+ states

Electromagnetic properties

Shen, Elhatisari, Lähde, Lee, Lu, UGM, Nature Commun. **14** (2023) 2777

- Radii (be aware of excited states), quadrupole moments & transition rates

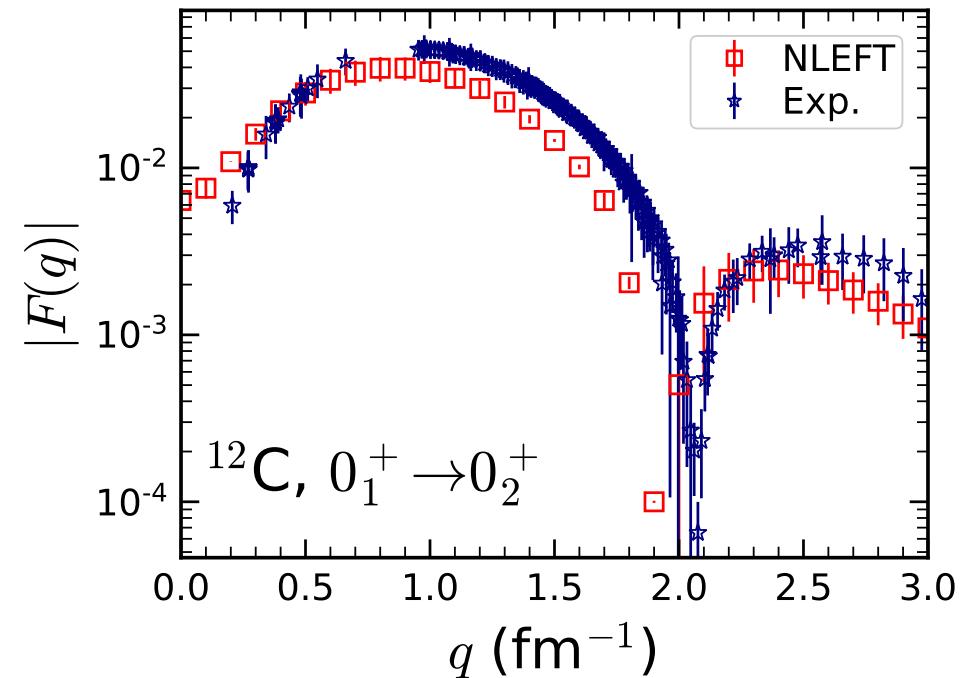
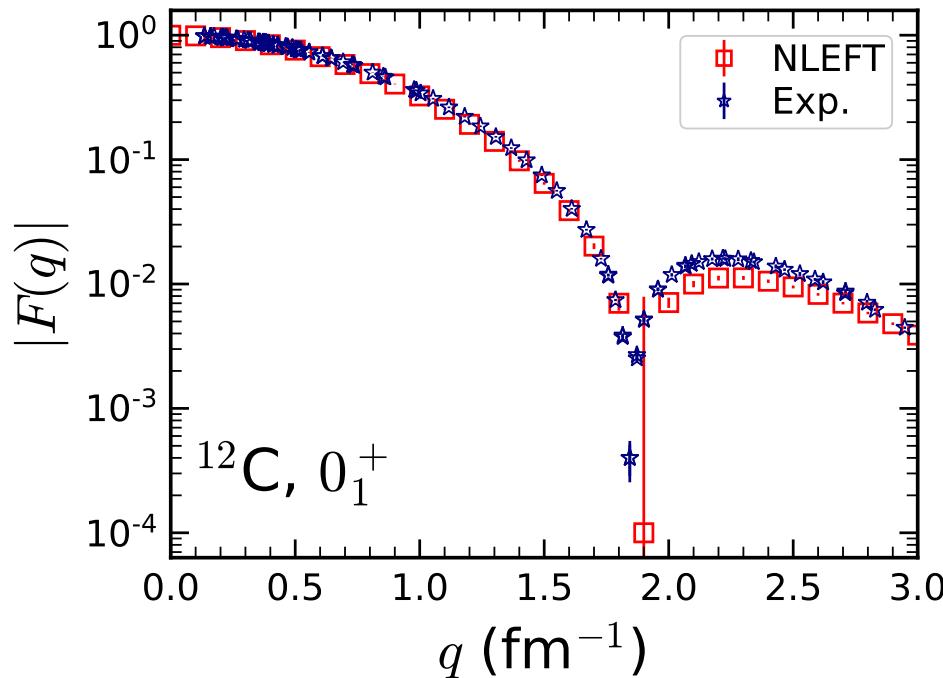
| | NLEFT | FMD | α cluster | BEC | RXMC | Exp. |
|-------------------|----------------|------|------------------|------|------|----------------|
| $r_c(0_1^+)$ [fm] | 2.53(1) | 2.53 | 2.54 | 2.53 | 2.65 | 2.47(2) |
| $r(0_2^+)$ [fm] | 3.45(2) | 3.38 | 3.71 | 3.83 | 4.00 | — |
| $r(0_3^+)$ [fm] | 3.47(1) | 4.62 | 4.75 | — | 4.80 | — |
| $r(2_1^+)$ [fm] | 2.42(1) | 2.50 | 2.37 | 2.38 | — | — |
| $r(2_2^+)$ [fm] | 3.30(1) | 4.43 | 4.43 | — | — | — |

| | NLEFT | FMD | α cluster | NCSM | Exp. |
|---|----------------|-----|------------------|--------|-----------------|
| $Q(2_1^+)$ [$e \text{ fm}^2$] | 6.8(3) | — | — | 6.3(3) | 8.1(2.3) |
| $Q(2_2^+)$ [$e \text{ fm}^2$] | —35(1) | — | — | — | — |
| $M(E0, 0_1^+ \rightarrow 0_2^+)$ [$e \text{ fm}^2$] | 4.8(3) | 6.5 | 6.5 | — | 5.4(2) |
| $M(E0, 0_1^+ \rightarrow 0_3^+)$ [$e \text{ fm}^2$] | 0.4(3) | — | — | — | — |
| $M(E0, 0_2^+ \rightarrow 0_3^+)$ [$e \text{ fm}^2$] | 7.4(4) | — | — | — | — |
| $B(E2, 2_1^+ \rightarrow 0_1^+)$ [$e^2 \text{ fm}^4$] | 11.4(1) | 8.7 | 9.2 | 8.7(9) | 7.9(4) |
| $B(E2, 2_1^+ \rightarrow 0_2^+)$ [$e^2 \text{ fm}^4$] | 2.5(2) | 3.8 | 0.8 | — | 2.6(4) |

Electromagnetic properties cont'd

Shen, Elhatisari, Lähde, Lee, Lu, UGM, Nature Commun. **14** (2023) 2777

- Form factors and transition ffs [essentially parameter-free]:



Sick, McCarthy, Nucl. Phys. A 150 (1970) 631

Strehl, Z. Phys. 234 (1970) 416

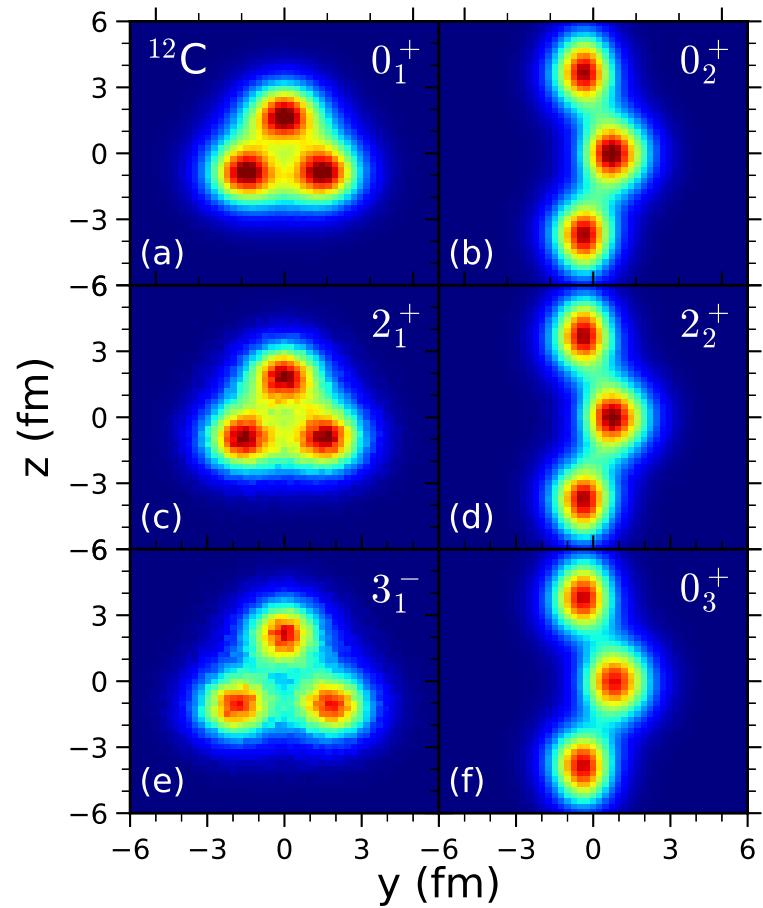
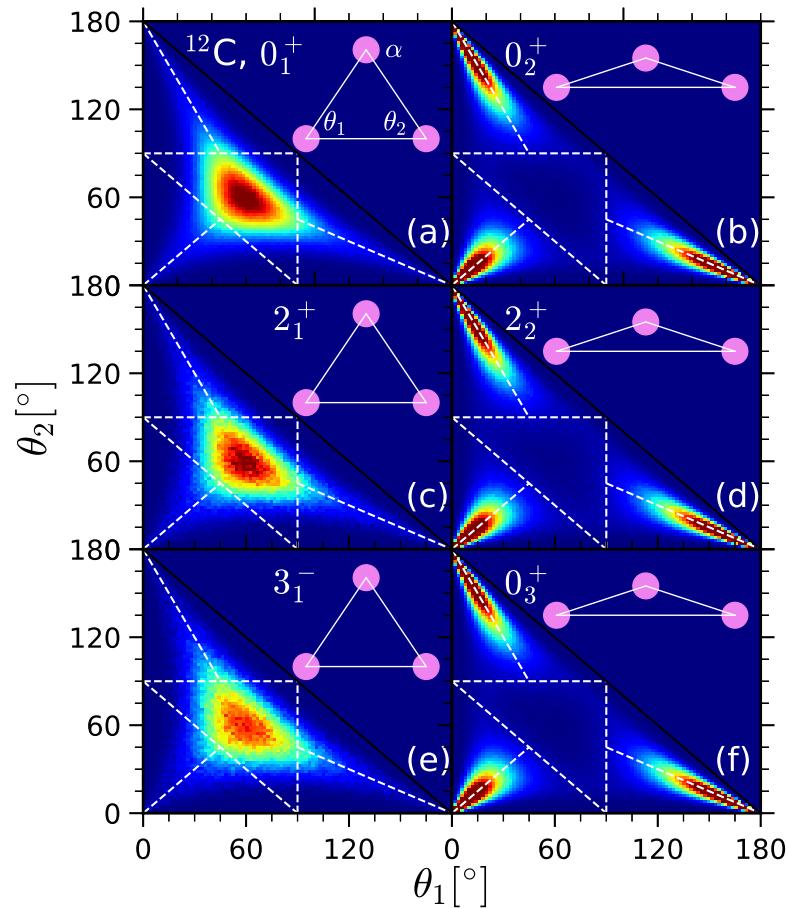
Crannell et al., Nucl. Phys. A 758 (2005) 399

Chernykh et al., Phys. Rev. Lett. 105 (2010) 022501

Emergence of geometry

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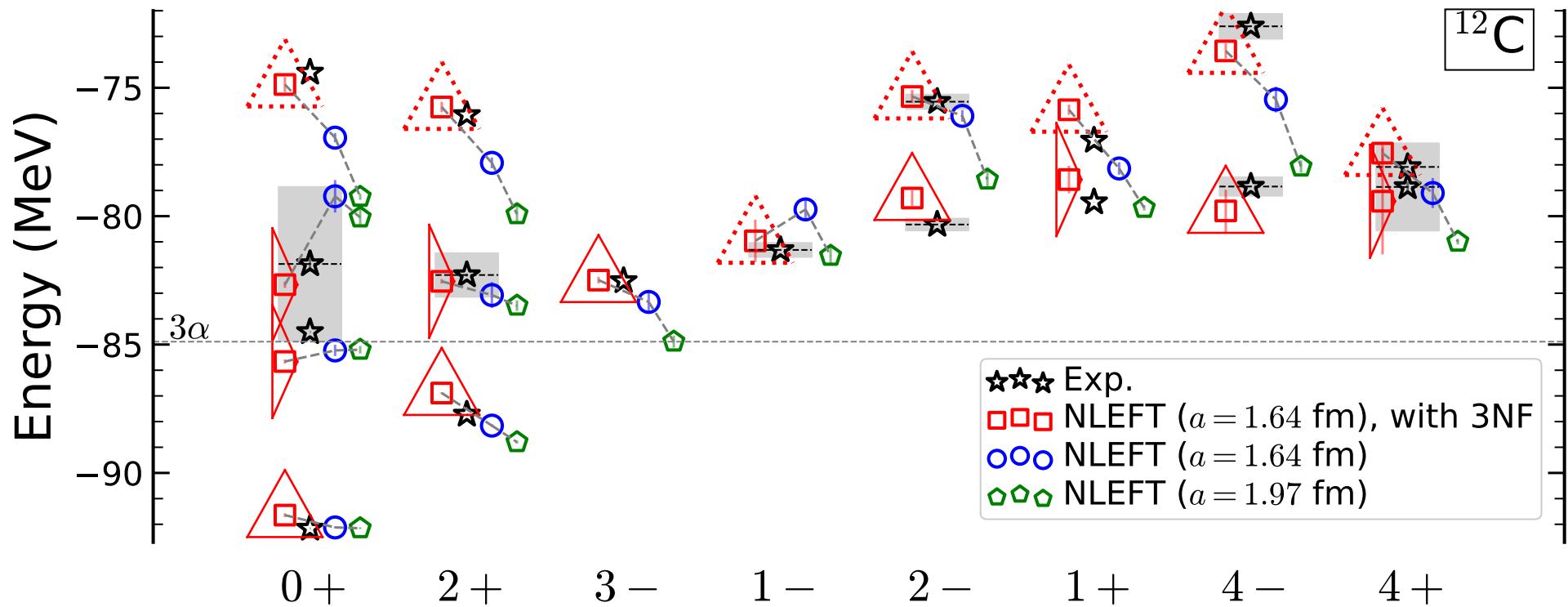
- Use the pinhole algorithm to measure the distribution of α -clusters/matter:



- equilateral & obtuse triangles \rightarrow 2^+ states are excitations of the 0^+ states

Emergence of duality

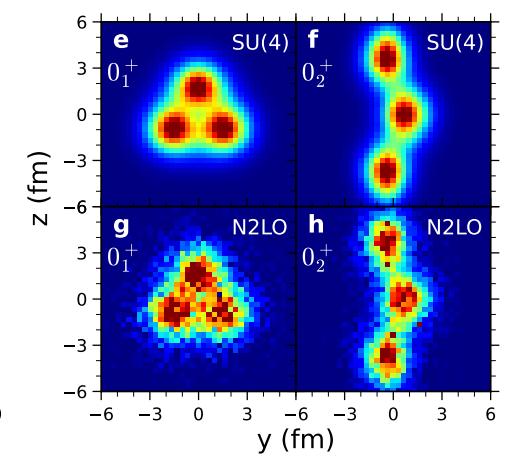
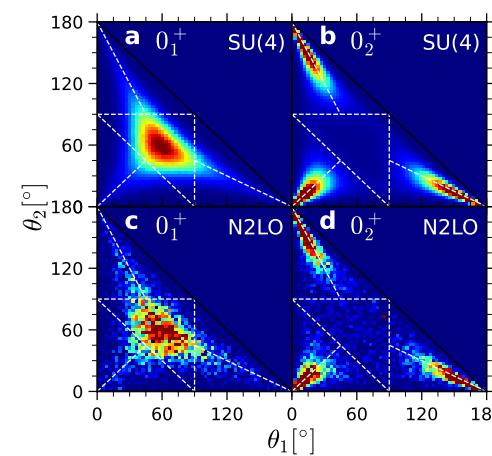
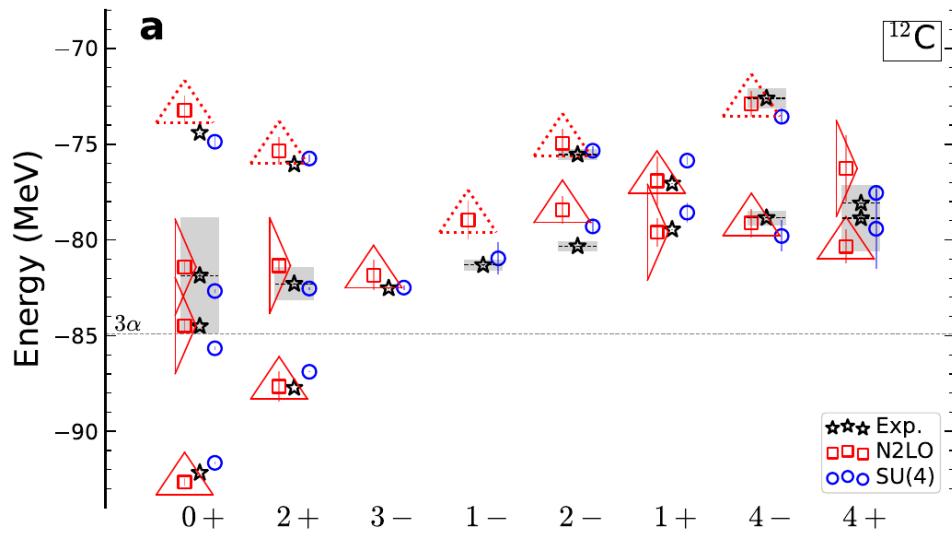
- ^{12}C spectrum shows a cluster/shell-model duality



- dashed triangles: strong 1p-1h admixture in the wave function

Sanity check

- Repeat the calculations w/ the time-honored N2LO chiral interaction
 - ↪ better NN phase shifts than the SU(4) interaction
 - ↪ but calculations are much more difficult (sign problem)



- spectrum as before (good agreement w/ data)
- density distributions as before (more noisy, stronger sign problem)

Towards heavy nuclei and nuclear matter in NLEFT

Towards heavy nuclei in NLEFT

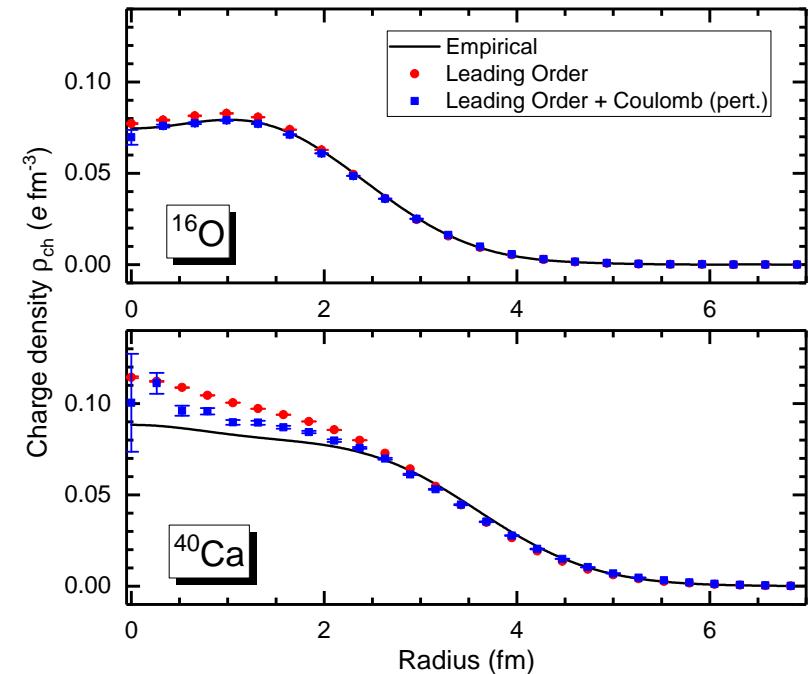
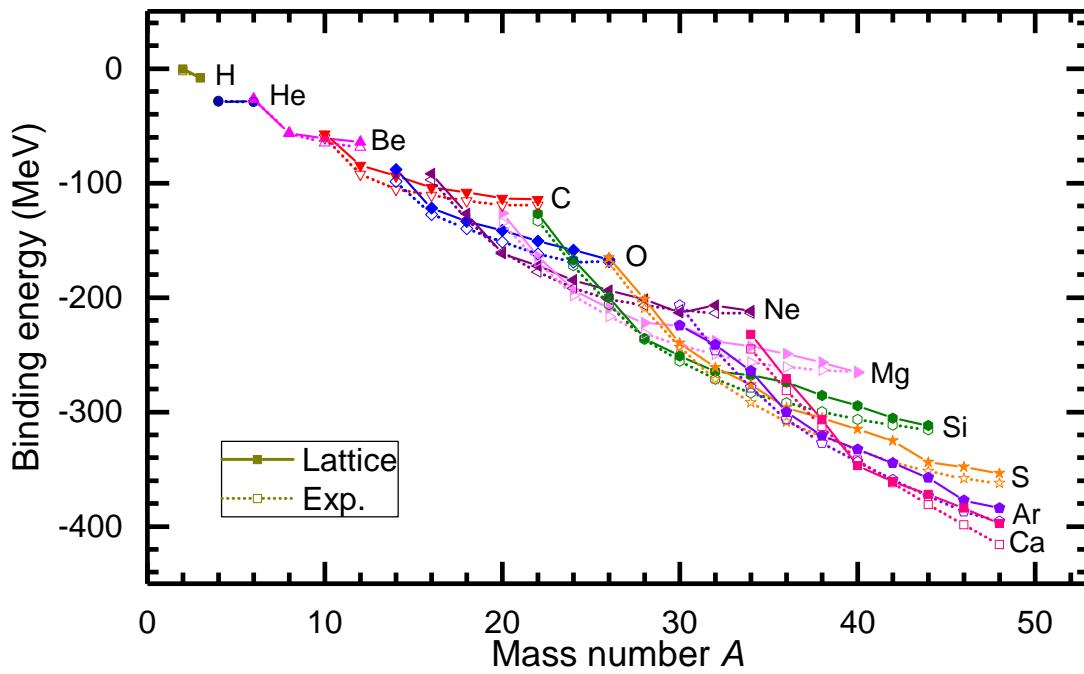
- Two step procedure:
 - 1) Further improve the LO action
 - minimize the sign oscillations
 - minimize the higher-body forces
 - gain an understanding of the essentials of nuclear binding
 - essentially done ✓ → next slide
 - 2) Work out the corrections to N3LO
 - first on the level of the NN interaction ✓
 - new important technique: **wave function matching** ✓
 - second for the spectra/radii/... of nuclei (first results) ✓
 - third for nuclear reactions (nuclear astrophysics)

Essential elements of nuclear binding

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Lu, Li, Elhatisari, Lee, Epelbaum, UGM, Phys. Lett. B 797 (2019) 134863

- LO smeared SU(4) symmetric action with 2NFs and 3NFs:



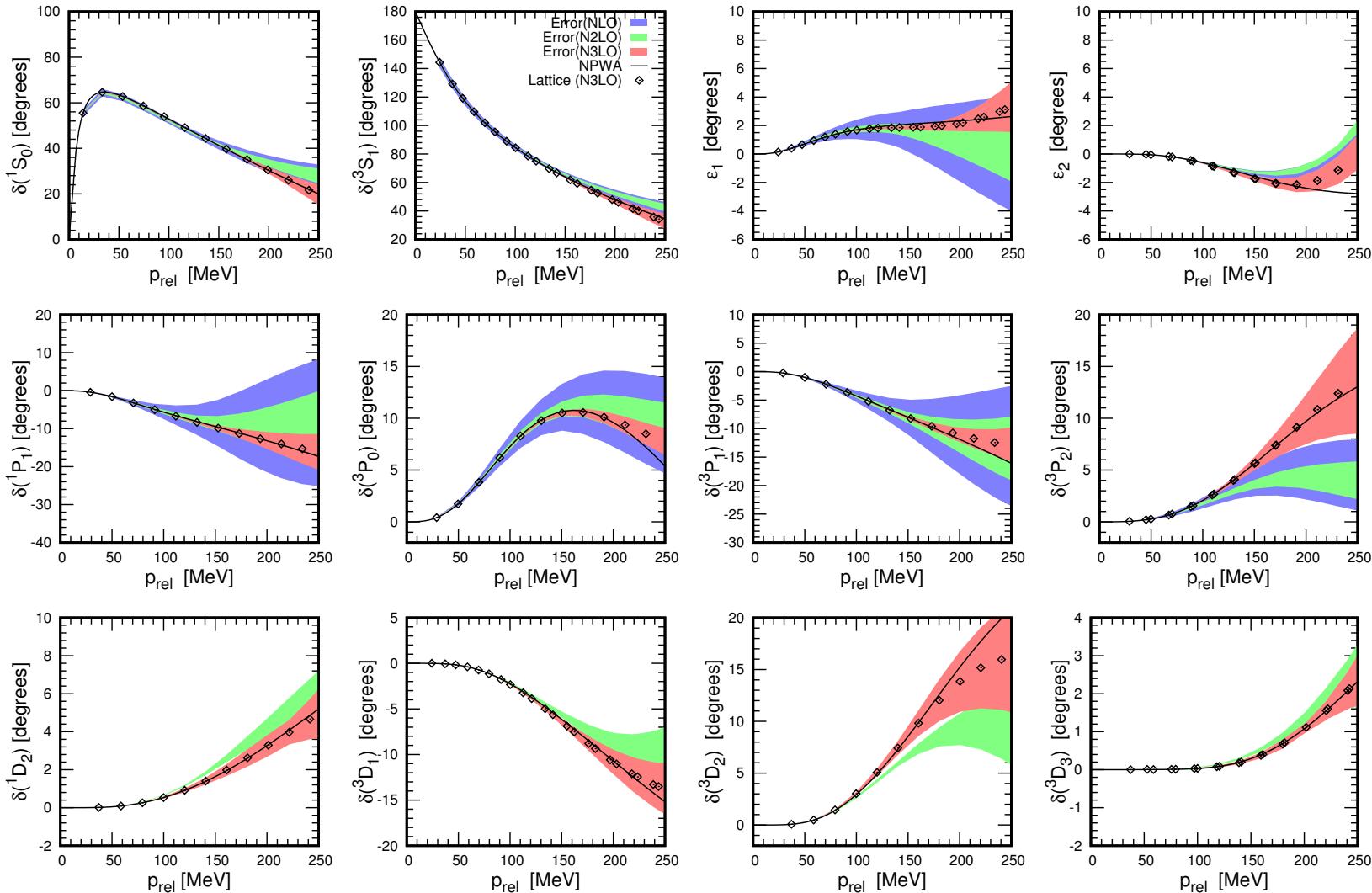
- Masses of 88 nuclei up to $A = 48$, deviation $< 4\%$ (except ^{12}C)
- Charge radii deviate by at most 5% (expect 3H)
- Neutron matter EoS also consistent w/ other calculations (APR, GCR, ...)

NN interaction at N3LO

30

Li et al., Phys. Rev. C **98** (2018) 044002; Phys. Rev. C **99** (2019) 064001

- np phase shifts including uncertainties for $a = 1.32$ fm (cf. Nijmegen PWA)



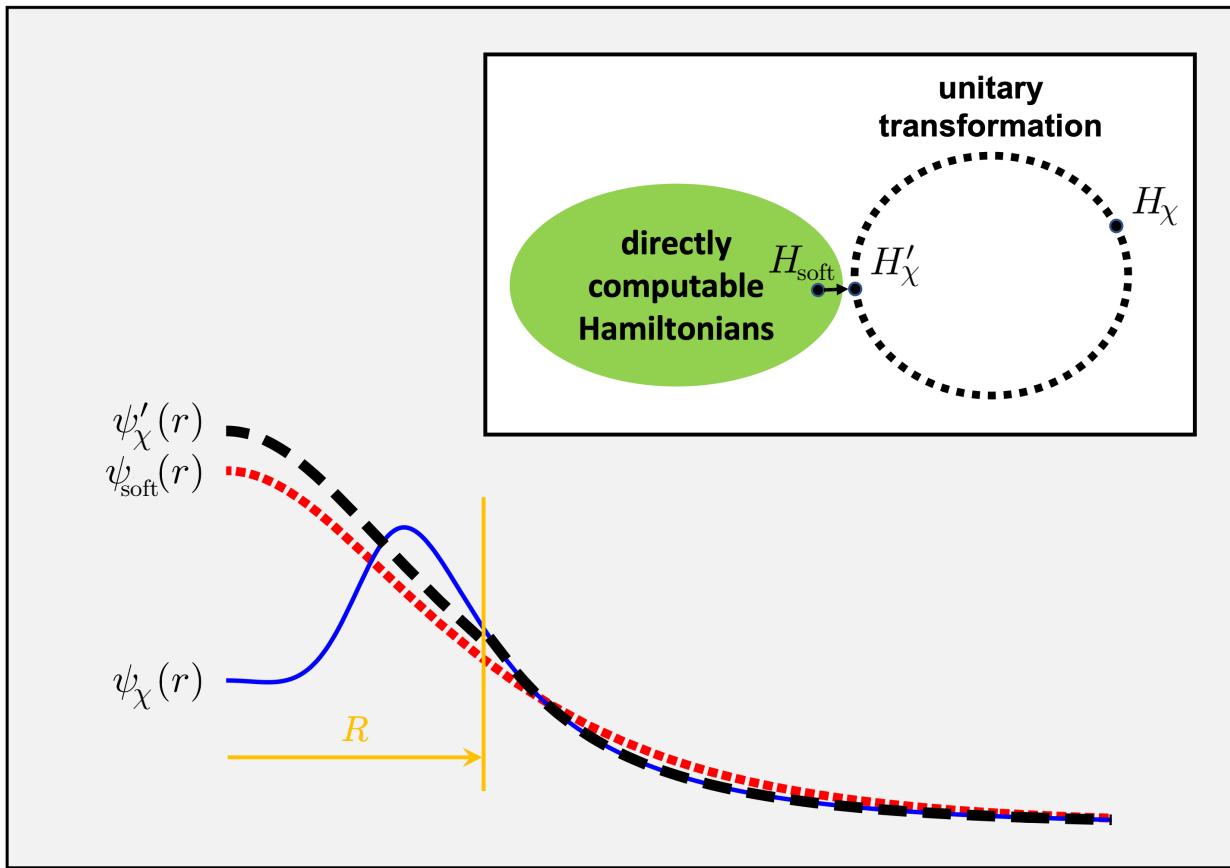
uncertainty estimates à la Epelbaum, Krebs, UGM,
Eur. Phys. J. A **51** (2015) 53

Wave function matching I

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Elhatisari et al., acc. for publication in ... [arXiv:2210.17488 [nucl-th]]

- Graphical representation of w.f. matching



- W.F. matching is a “Hamiltonian translator”: eigenenergies from H_1 but w.f. from $H_2 = U^\dagger H_1 U$

Wave function matching II

Elhatisari et al., acc. for publication in ... [arXiv:2210.17488 [nucl-th]]

- \mathbf{H}_{soft} has tolerable sign oscillations, good for many-body observables
- \mathbf{H}_χ has severe sign oscillations, derived from the underlying theory
→ can we find a unitary trafo, that creates a chiral \mathbf{H}_χ that is pert. th'y friendly?

$$\mathbf{H}'_\chi = \mathbf{U}^\dagger \mathbf{H}_\chi \mathbf{U}$$

- Let $|\psi_{\text{soft}}^0\rangle$ be the lowest eigenstate of \mathbf{H}_{soft}
- Let $|\psi_\chi^0\rangle$ be the lowest eigenstate of \mathbf{H}_χ
- Let $|\phi_{\text{soft}}\rangle$ be the projected and normalized lowest eigenstate of \mathbf{H}_{soft}

$$|\phi_{\text{soft}}\rangle = \mathcal{P} |\psi_{\text{soft}}^0\rangle / ||\psi_{\text{soft}}^0\rangle||$$

- Let $|\phi_\chi\rangle$ be the projected and normalized lowest eigenstate of \mathbf{H}_χ

$$|\phi_\chi\rangle = \mathcal{P} |\psi_\chi^0\rangle / ||\psi_\chi^0\rangle||$$

$$\hookrightarrow U_{R',R} = \theta(r - R)\delta_{R',R} + \theta(R' - r)\theta(R - r)|\phi_\chi^\perp\rangle\langle\phi_{\text{soft}}^\perp|$$

Wave function matching III

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Elhatisari et al., acc. for publication in ... [arXiv:2210.17488 [nucl-th]], L. Bovermann, PhD thesis

- W.F. matching for the light nuclei

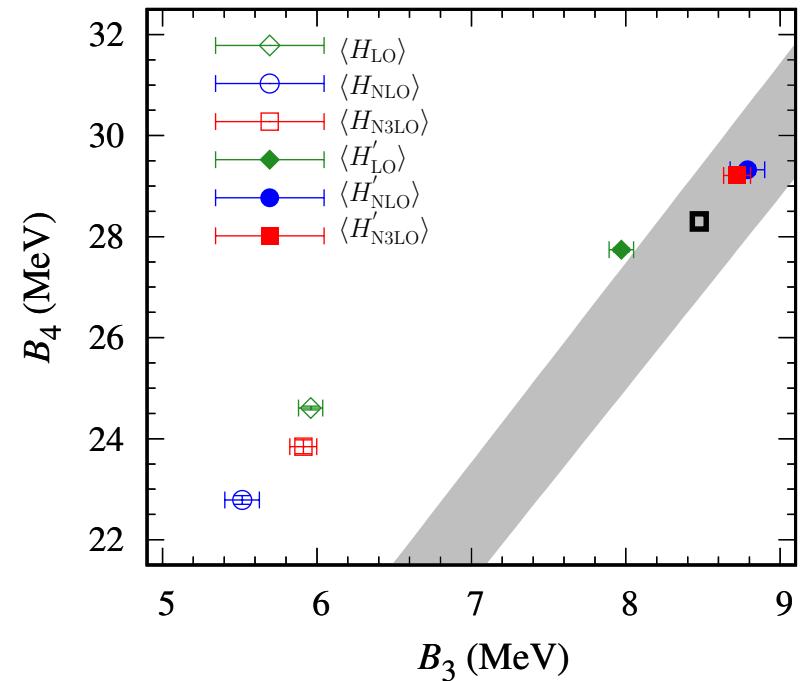
| Nucleus | B_{LO} [MeV] | $B_{\text{N}3\text{LO}}$ [MeV] | Exp. [MeV] |
|--|-----------------------|--------------------------------|--------------|
| $E_{\chi,d}$ | 1.79 | 2.21 | 2.22 |
| $\langle \psi_{\text{soft}}^0 H_{\chi,d} \psi_{\text{soft}}^0 \rangle$ | 0.45 | 0.62 | |
| $\langle \psi_{\text{soft}}^0 H'_{\chi,d} \psi_{\text{soft}}^0 \rangle$ | 1.65 | 2.01 | |
| $\langle \psi_{\text{soft}}^0 H_{\chi,t} \psi_{\text{soft}}^0 \rangle$ | 5.96(8) | 5.91(9) | 8.48 |
| $\langle \psi_{\text{soft}}^0 H'_{\chi,t} \psi_{\text{soft}}^0 \rangle$ | 7.97(8) | 8.72(9) | |
| $\langle \psi_{\text{soft}}^0 H_{\chi,\alpha} \psi_{\text{soft}}^0 \rangle$ | 24.61(4) | 23.84(14) | 28.30 |
| $\langle \psi_{\text{soft}}^0 H'_{\chi,\alpha} \psi_{\text{soft}}^0 \rangle$ | 27.74(4) | 29.21(14) | |

- reasonable accuracy for the light nuclei

- Tjon-band recovered with H'_{χ}

Platter, Hammer, UGM, Phys. Lett. B **607** (2005) 254

→ now let us go to larger nuclei....

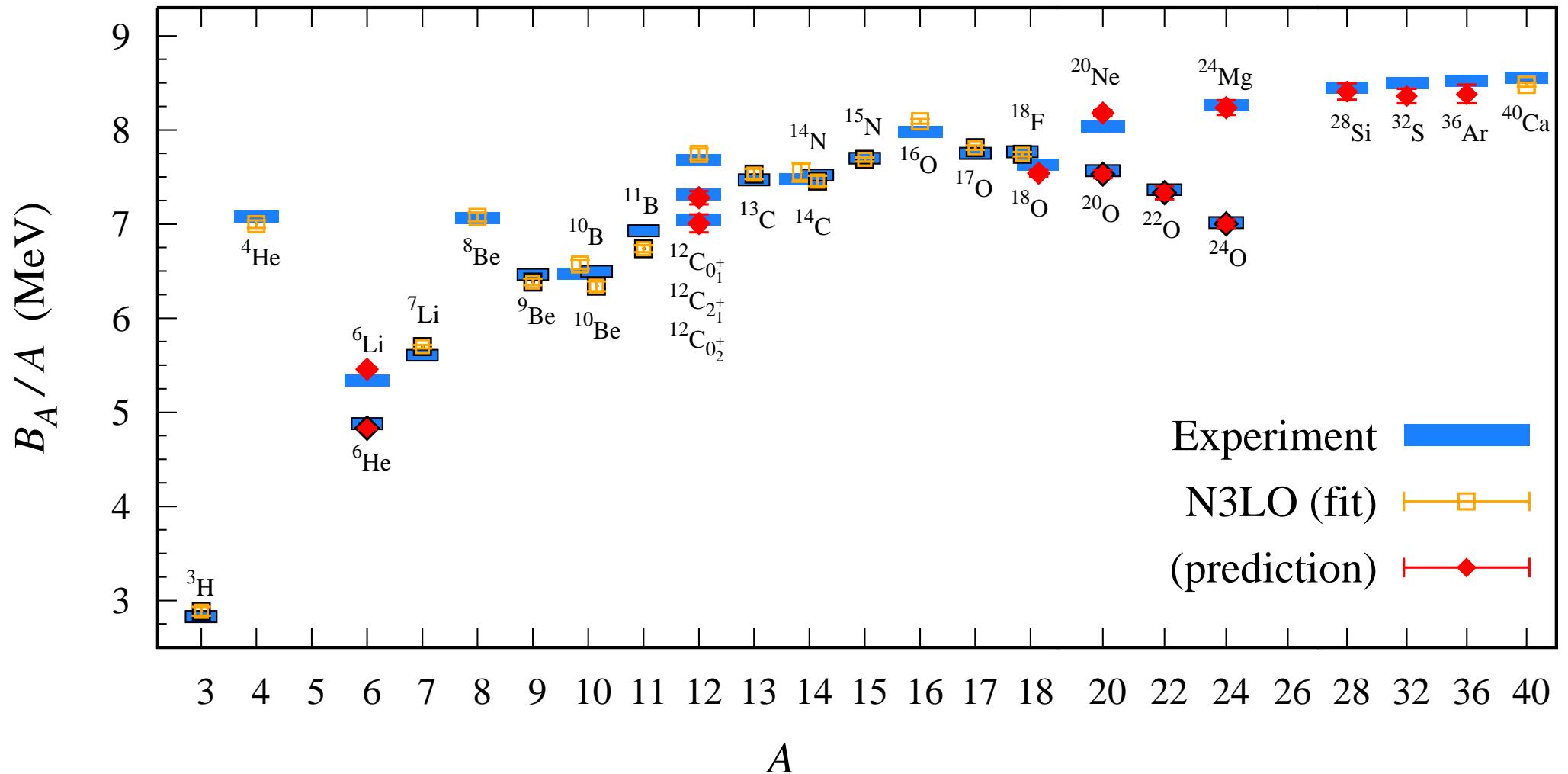


Nuclei at N3LO

34

- Binding energies of nuclei for $a = 1.32 \text{ fm}$: Determining the 3NFs

Elhatisari et al., acc. for publication in ... [arXiv:2210.17488 [nucl-th]]

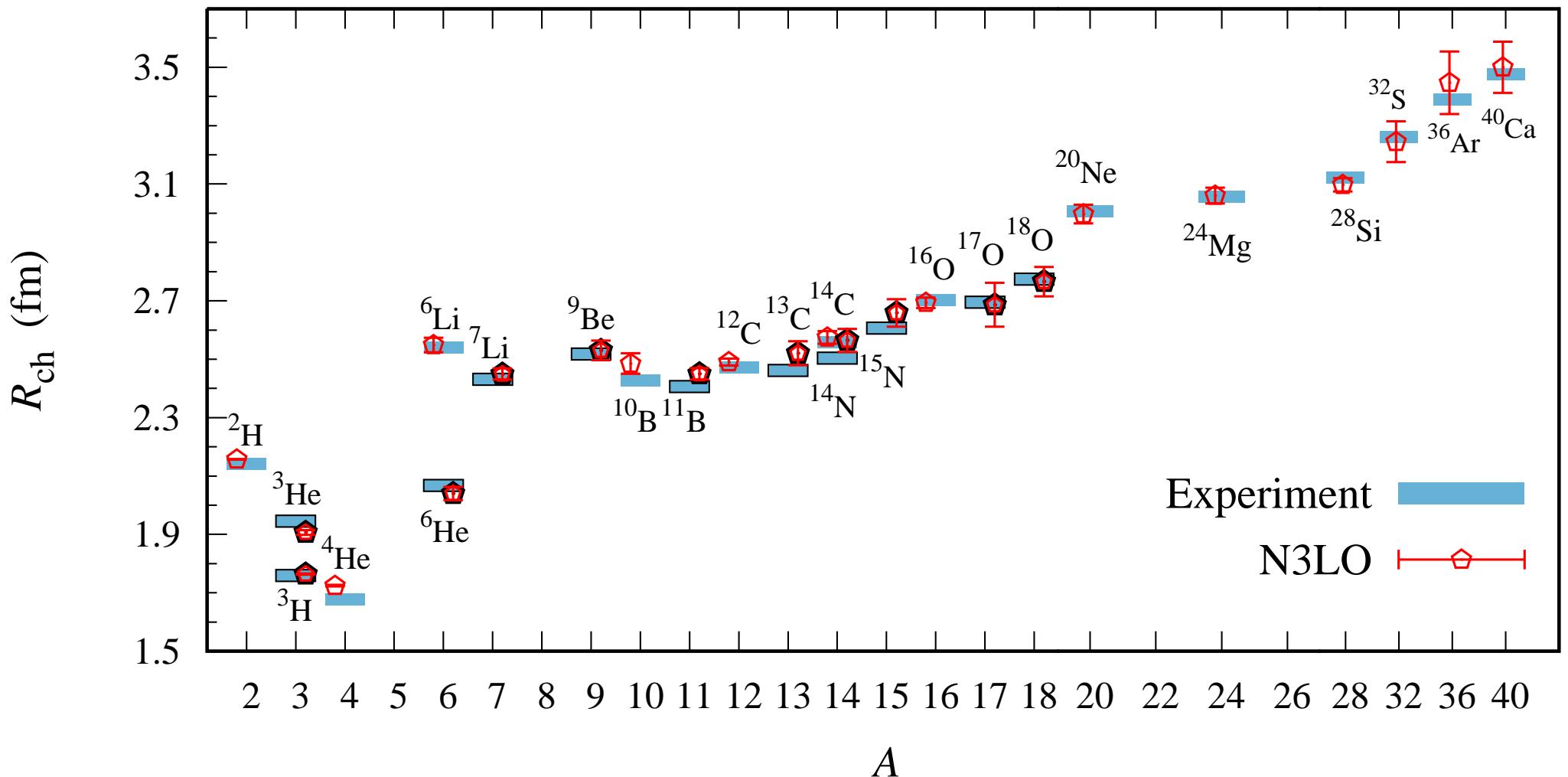


Charge radii at N3LO

35

- Prediction: Charge radii ($a = 1.32$ fm, statistical errors can be reduced)

Elhatisari et al., acc. for publication in ... [arXiv:2210.17488 [nucl-th]]

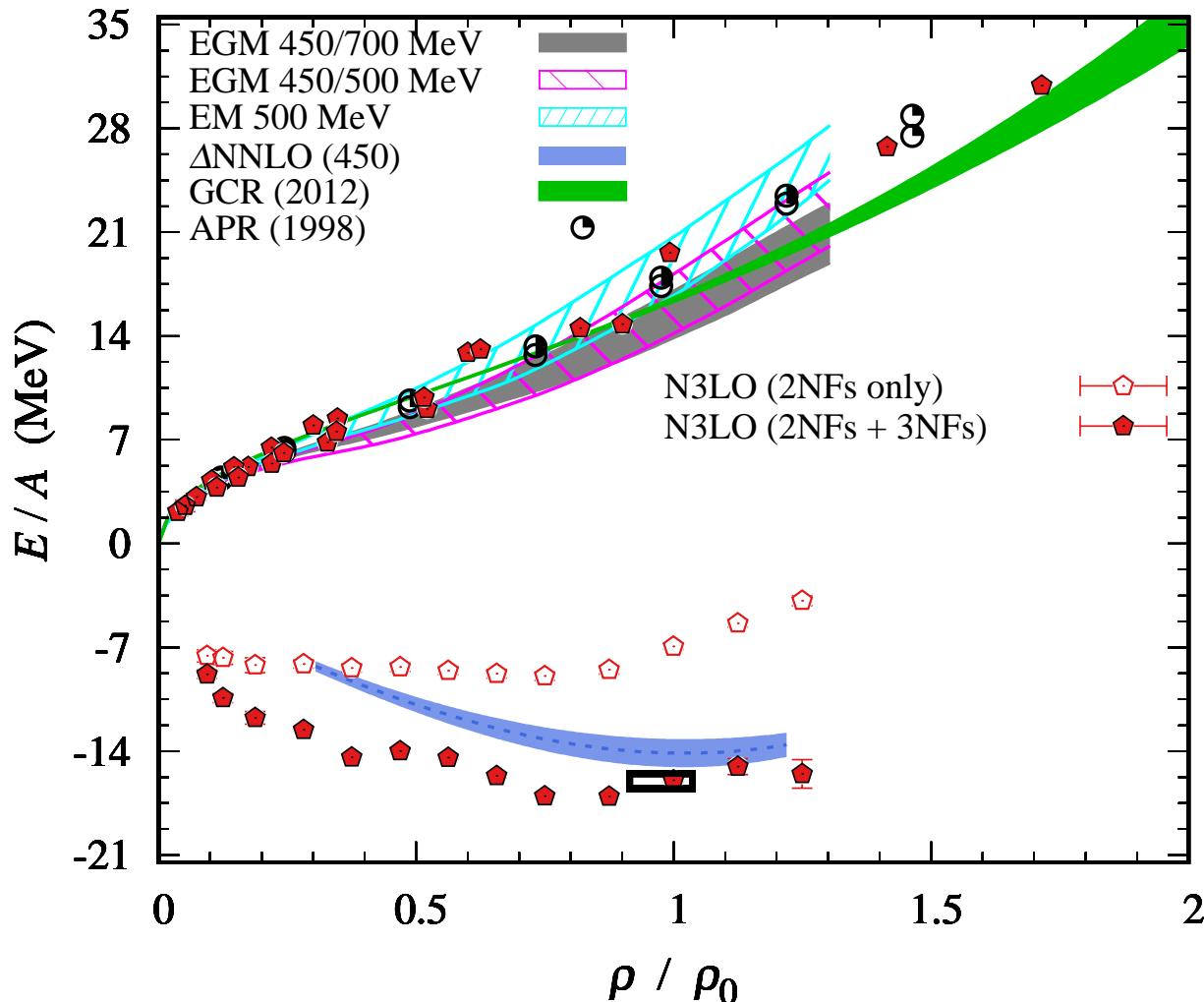


Neutron & nuclear matter at N3LO

36

- Prediction: EoS of pure neutron matter & nuclear matter ($a = 1.32 \text{ fm}$)

Elhatisari et al., acc. for publication in ... [arXiv:2210.17488 [nucl-th]]



→ can be improved using twisted b.c.'s

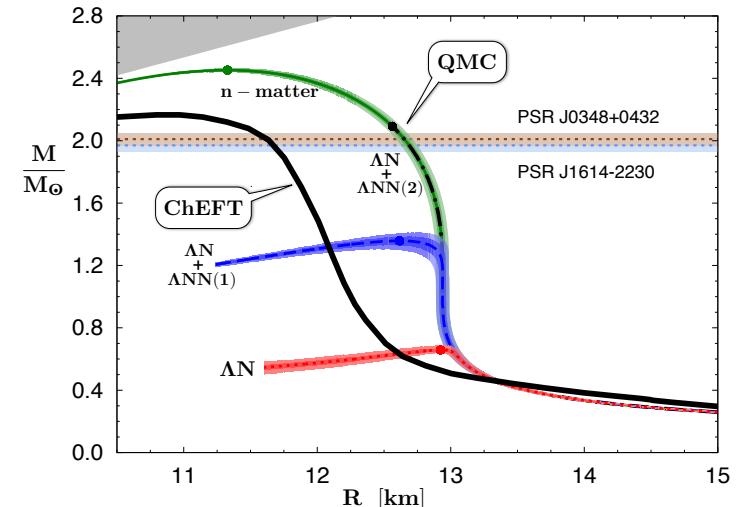
Ab initio calculation of hyper-neutron matter

Towards hyper-neutron matter

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- Densities in the interior of neutron stars
 - up to $5 \cdot \rho_0$ [$\rho_0 = 0.17 \text{ fm}^{-3}$]
 - possible appearance of hyperons
 - “hyperon puzzle”
 - many possible solutions
(3-body forces, BSM physics, modified gravity)
 - Neutron matter EoS plays an important role in **multimessenger astronomy** [gravitational waves]
- Can we address this topic w/ NLEFT? If so, how?
 - large densities require a small lattice spacing
 - need to extend the minimal nuclear interaction to such densities
 - need to extend the minimal nuclear interaction to the strangeness sector

Tong, Elhatisari, UGM, in progress



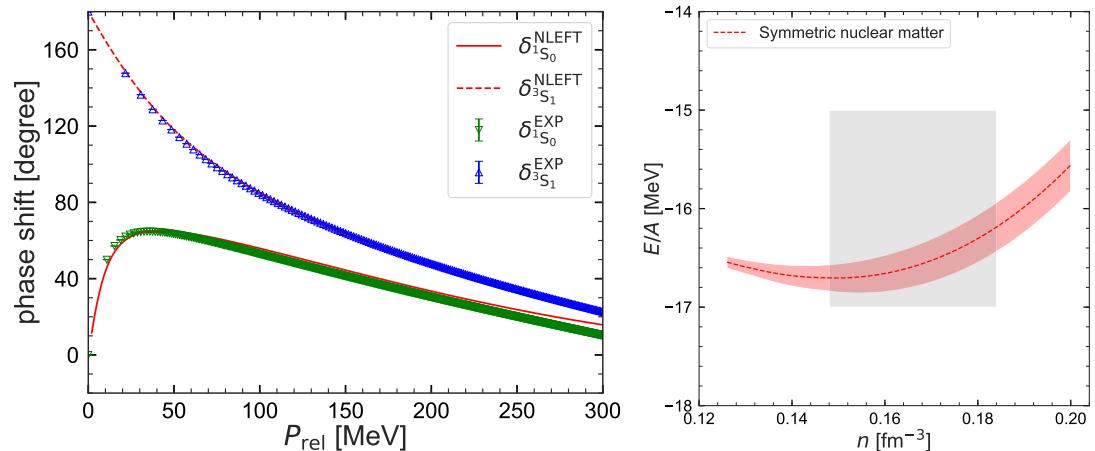
@ W. Weise

Pure neutron matter

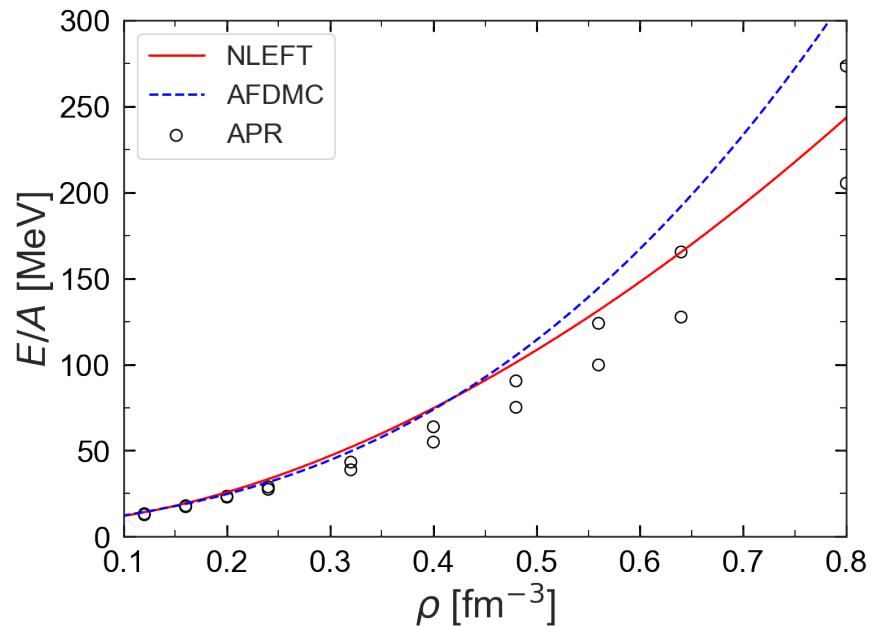
39

- Input: S-wave phase shifts (2N)
& symmetric nuclear matter (3N)

Tong, Elhatisari, UGM, in progress



⇒ Output: Pure neutron matter (PNM) EoS



- comparable to the renowned APR EoS
Akmal, Pandharipande, Ravenhall, Phys. Rev. C **58** (1998) 1804
- less stiff than the recent AFDMC one
Gandolfi et al., Eur. Phys. J. A **50** (2014) 10
- work out consequences for
neutron stars based on this PNM EoS

The minimal interaction with strangeness I

40

Tong, Elhatisari, UGM, in progress

- Baryon-baryon interaction (consider nucleons and Λ 's plus non-local smearing):

$$V_{\Lambda N} = \textcolor{red}{c_{N\Lambda}} \sum_{\vec{n}} \tilde{\rho}(\vec{n}) \tilde{\xi}(\vec{n}) + \textcolor{red}{c_{\Lambda\Lambda}} \frac{1}{2} \sum_{\vec{n}} [\tilde{\xi}(\vec{n})]^2$$

$$\tilde{\rho}(\vec{n}) = \sum_{i,j=0,1} \tilde{a}_{i,j}^\dagger(\vec{n}) \tilde{a}_{i,j}(\vec{n}) + s_L \sum_{|\vec{n}-\vec{n}'|^2=1} \sum_{i,j=0,1} \tilde{a}_{i,j}^\dagger(\vec{n}') \tilde{a}_{i,j}(\vec{n}')$$

$$\tilde{\xi}(\vec{n}) = \sum_{i=0,1} \tilde{b}_i^\dagger(\vec{n}) \tilde{b}_i(\vec{n}) + s_L \sum_{|\vec{n}-\vec{n}'|^2=1} \sum_{i=0,1} \tilde{b}_i^\dagger(\vec{n}') \tilde{b}_i(\vec{n}')$$

- Three-baryon forces (consider nucleons and Λ 's, no non-local smearing):

Peschauer, Kaiser, Haidenbauer, UGM, Weise, Phys. Rev. C 93 (2016) 014001

$$V_{NN\Lambda} = \textcolor{red}{c_{NN\Lambda}} \frac{1}{2} \sum_{\vec{n}} [\rho(\vec{n})]^2 \xi(\vec{n}) , \quad V_{N\Lambda\Lambda} = \textcolor{red}{c_{N\Lambda\Lambda}} \frac{1}{2} \sum_{\vec{n}} \rho(\vec{n}) [\xi(\vec{n})]^2$$

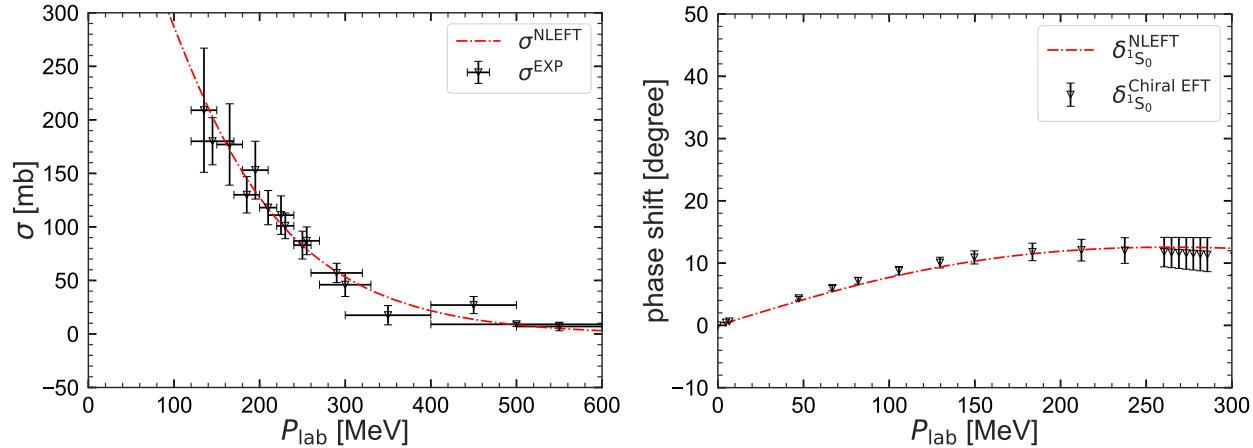
- must determine 4 LECs! [smearing parameters from the nucleon sector]
- first time that the $\Lambda\Lambda N$ three-body force is included

The minimal interaction with strangeness II

41

Tong, Elhatisari, UGM, in progress

- Two-body LECs from scattering data (ΛN)
& chiral EFT phase shift ($\Lambda\Lambda$)



- Three-body LECs from hyper-nuclei (separation energies):

| Nucleus | NLEFT [MeV] | Exp. [MeV] |
|------------------------------------|------------------|------------------|
| $^5_\Lambda \text{He}$ | 3.10(9) | 3.10(3) |
| $^9_\Lambda \text{Be}$ | 6.64(13) | 6.61(7) |
| $^{13}_\Lambda \text{C}$ | 11.71(14) | 11.80(16) |
| $^6_{\Lambda\Lambda} \text{He}$ | 6.96(9) | 6.91(16) |
| $^{10}_{\Lambda\Lambda} \text{Be}$ | 14.35(13) | 14.70(40) |

→ this defines our EoS of hyper-nuclear matter called **HMN(I)**

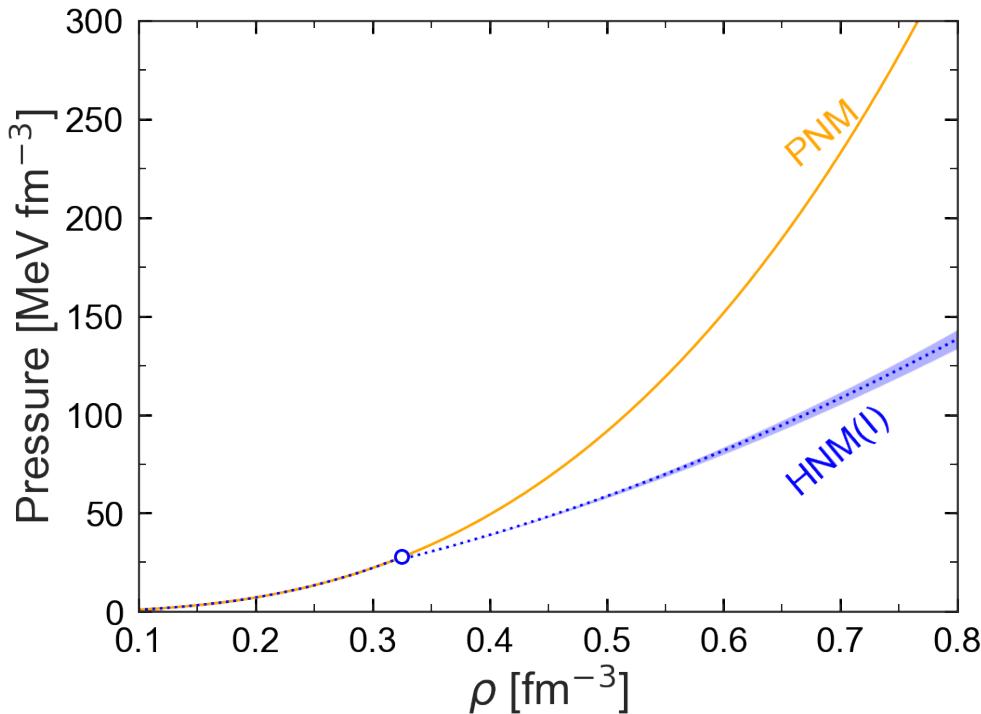
Neutron star properties

42

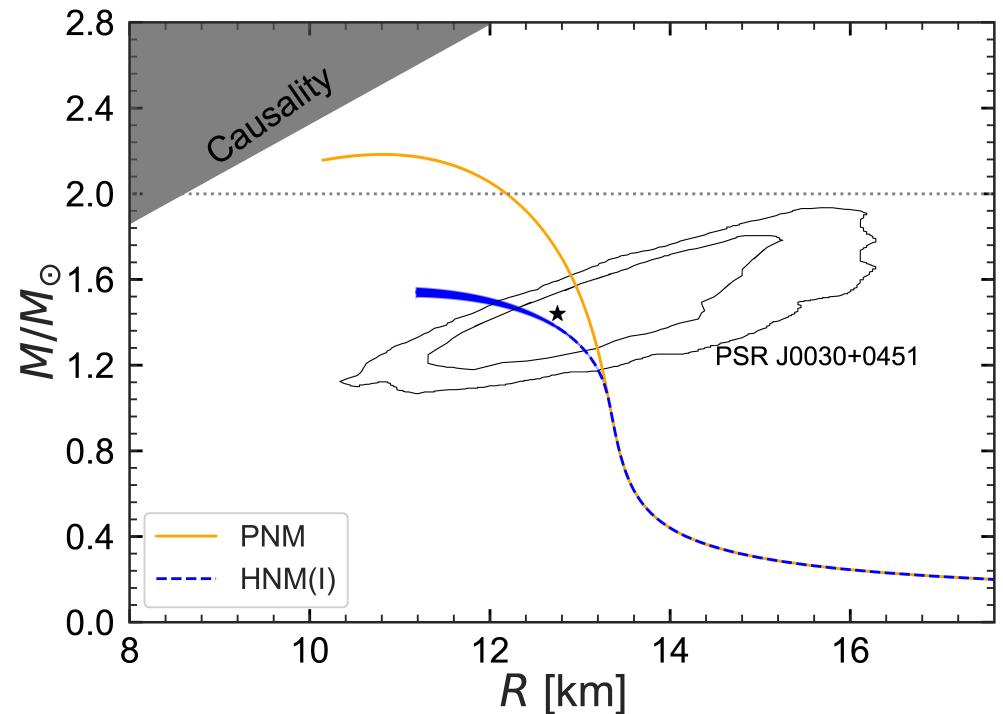
Tong, Elhatisari, UGM, in progress

- Now solve the TOV equations for the PNM and HNM(I) EoSs:

- EoS (PNM and HNM(I))



- Mass-radius relation



- Maximum neutron star mass: $M_{\max} = 2.18(1) M_\odot$ for PNM
 $M_{\max} = 1.54(2) M_\odot$ for HNM(I) \rightarrow need repulsion

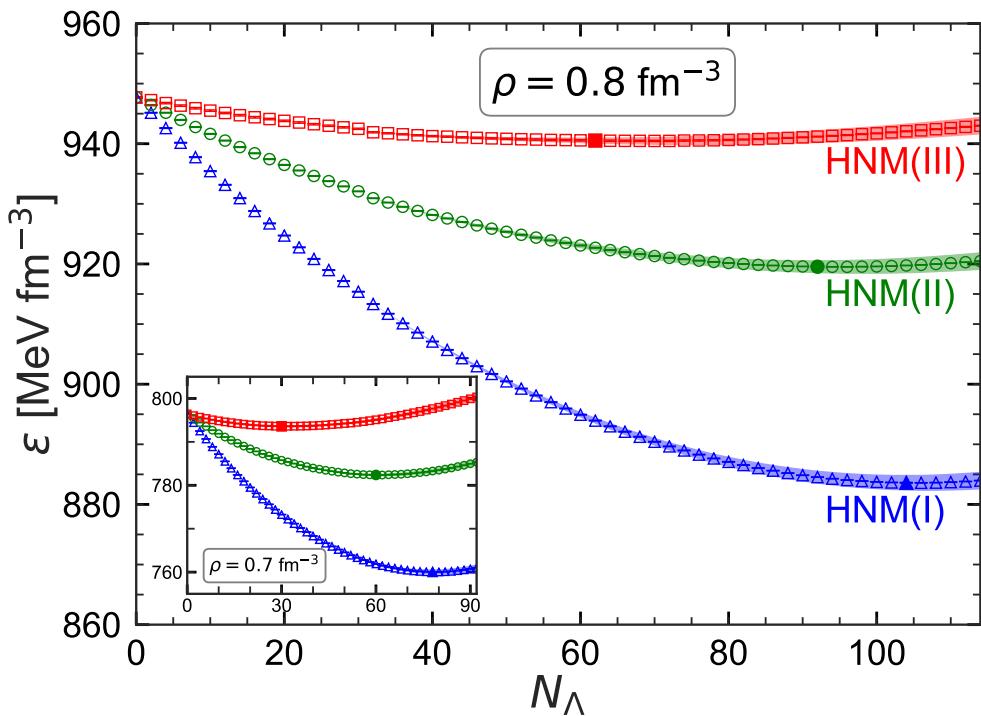
EoS of hyper-neutron matter

43

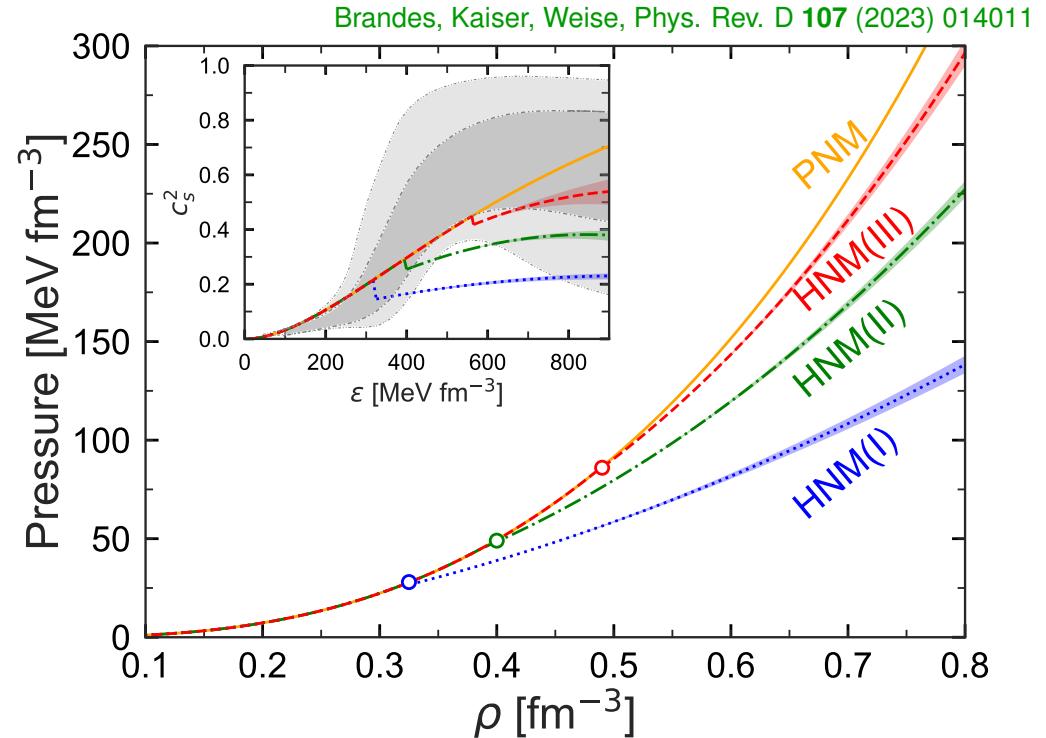
Tong, Elhatisari, UGM, in progress

- Not surprisingly, we need more repulsion [as in the pure neutron matter case]
 - this will move the threshold of $\mu_\Lambda = \mu_n$ up
 - take M_{\max} as data point: $M_{\max} = 1.9M_\odot$ for HNM(II)
 $M_{\max} = 2.1M_\odot$ for HNM(III)

- Energy density for N_Λ



- EoS & speed of sound



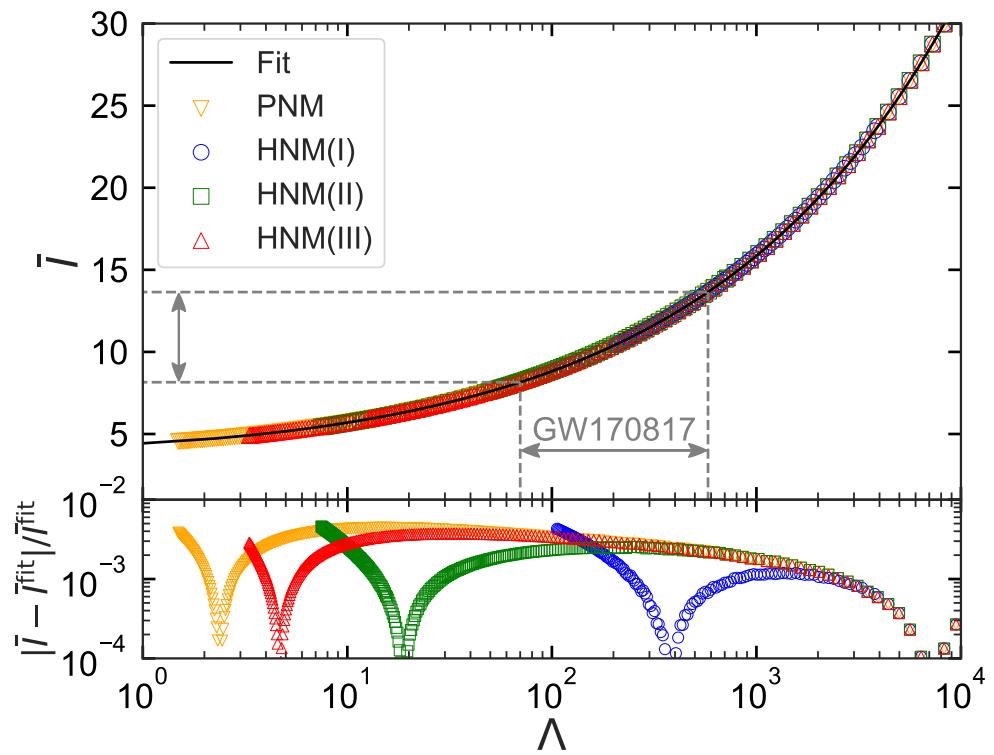
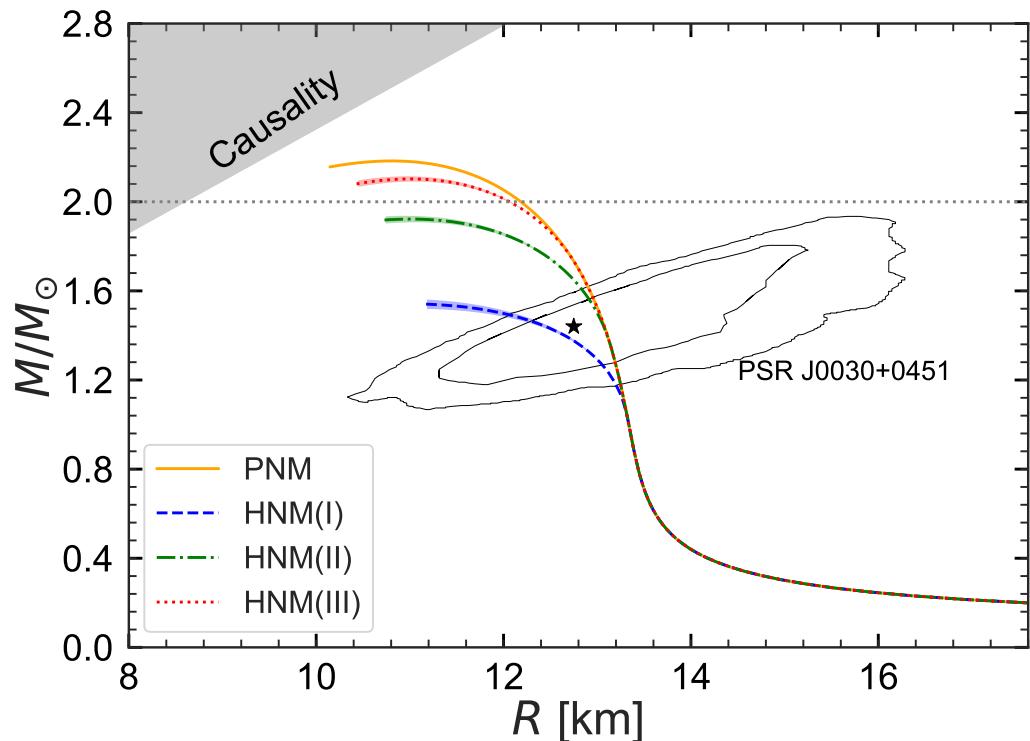
Neutron star properties

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Tong, Elhatisari, UGM, in progress

- Mass-radius relation and I -Love relation:

Yagi, Yunes, Science 341 (2013) 365



GW170817: Abbott et al., Phys. Rev. Lett. 121 (2018) 161101

- All EoSs consistent with the NICER result

Miller et al., Astrophys. J. Lett. 887 (2019) L24

- $\bar{I} = I/M^3$ mom. of inertia
- Λ = tidal deformability
- First *ab initio* calc. of this univ. relation

Summary & outlook

- Nuclear lattice simulations: a new quantum many-body approach
 - based on the successful continuum nuclear chiral EFT
 - a number of highly visible results already obtained
- Recent developments
 - minimal nuclear interaction & applications
 - chiral interaction at 3NF, first promising results
 - extension to hyper-nuclei & EoS in neutron stars

⇒ stayed tuned for many new results!

