Nuclear Lattice Effective Field Theory: Status A.D. 2024

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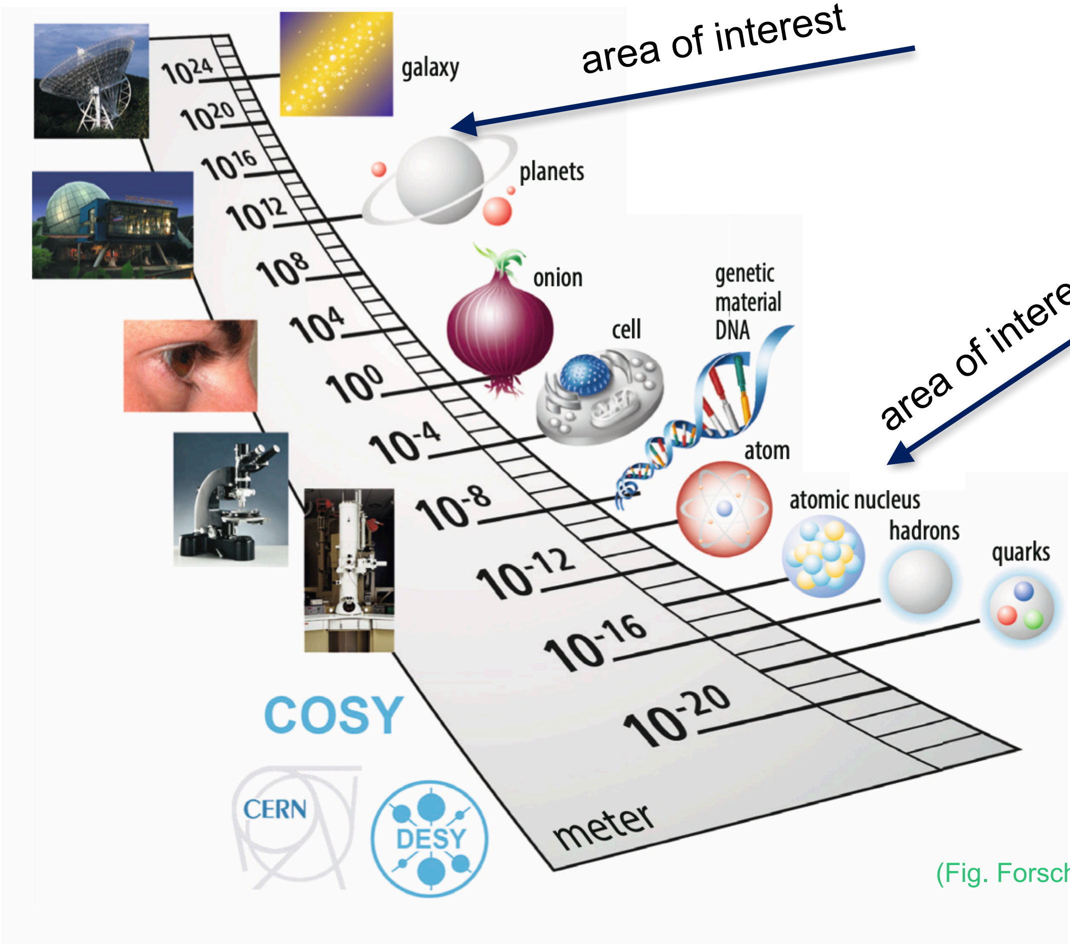
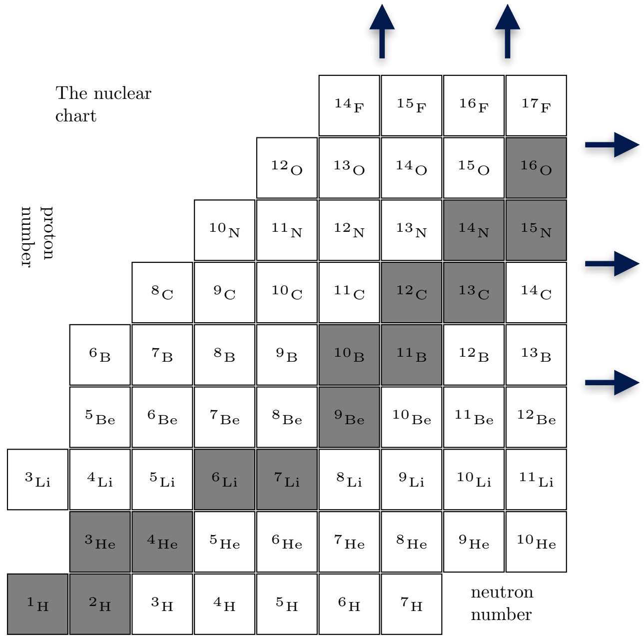
Outline

- Motivation
  - Introduction to NLEFT
  - Wavefunction Matching
  - Neutron Stars
- Summary and Outlook

What is Nuclear Lattice Effective Field Theory?

- Goal: understanding nuclear structure and reactions from small systems to stars





(Fig. Forschungszentrum Jülich)

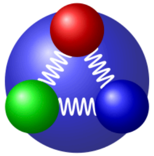
Open questions:

- -Limits of stability
- -Three-body forces
- -Cluster structures
- -Neutron stars

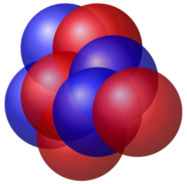
- At different scales, there are different relevant degrees of freedom
- Resolution matters: exploit this separation of scales

Dominant interactions at different scales

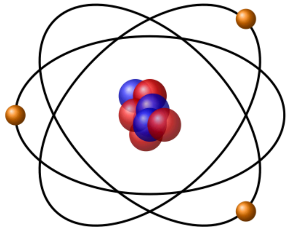
Quarks $< 10^{-16}$



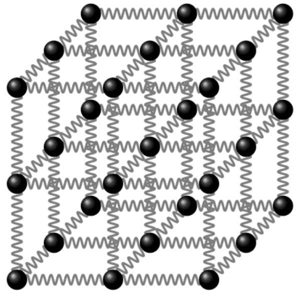
Nucleons $\sim 10^{-13}$



Nucleus $\sim 10^{-12}$



Atoms $\sim 10^{-8}$ cm



Matter

(Fig. S.Elhatisari)

strong force → remnant → nuclear force

electromagnetic force
sometimes gravity

Utilize this separation! There is no need to consider things that are not relevant at a specific scale

Chiral Nuclear Effective Field Theory

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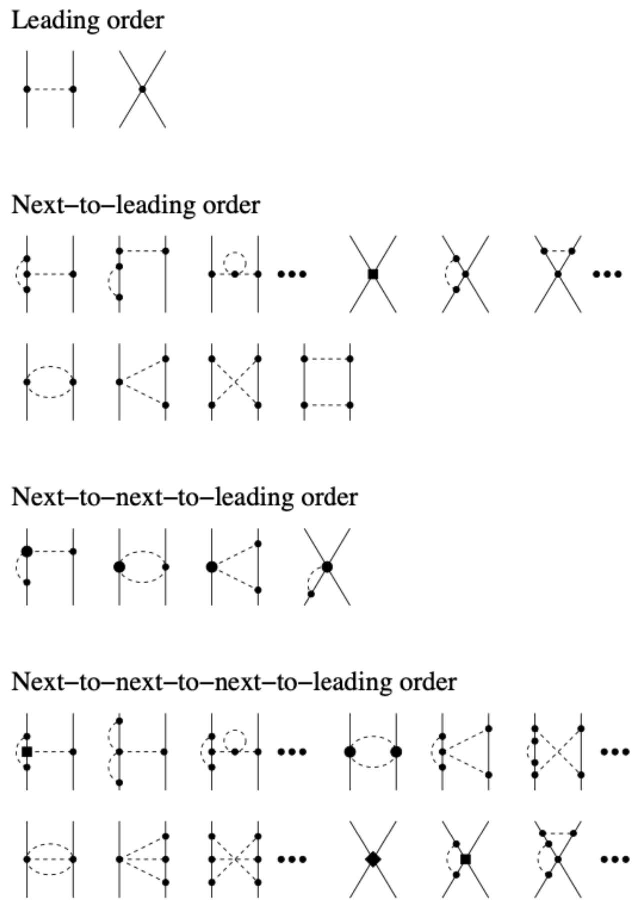
- Pick the correct scale : $\sim 10^{-15}$ m $=$ 1 fermi
- Pick the relevant degrees of freedom : protons and neutrons and pions

This is not Lattice QCD, no quarks or gluons!

construct effective field theory

- Low energy chiral effective field theory of QCD
  - No model, systematic improvement is possible due to a hierarchy of forces
  - Systematic error analysis possible
  - Typical nuclear systems are far away from breakdown scale

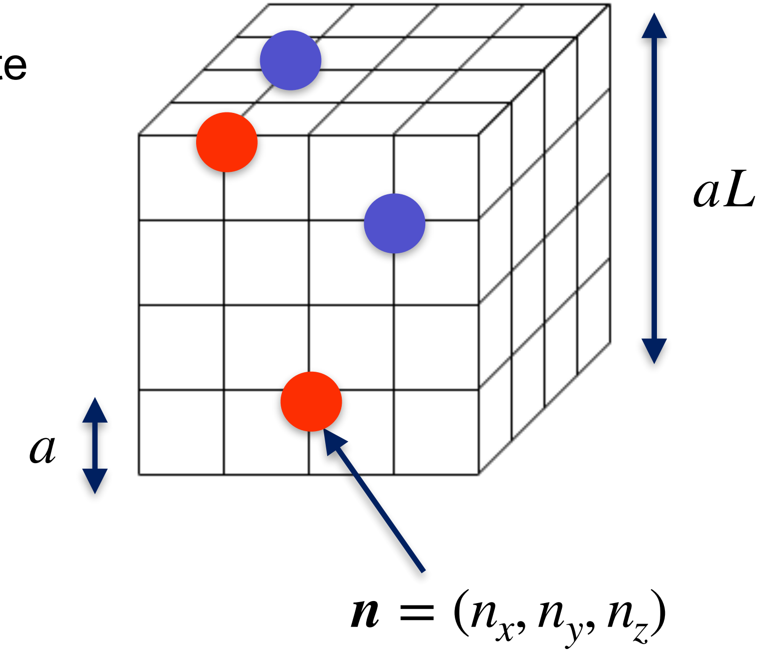
combine with lattice methods



(Epelbaum et al. RevModPhys.81.1773)

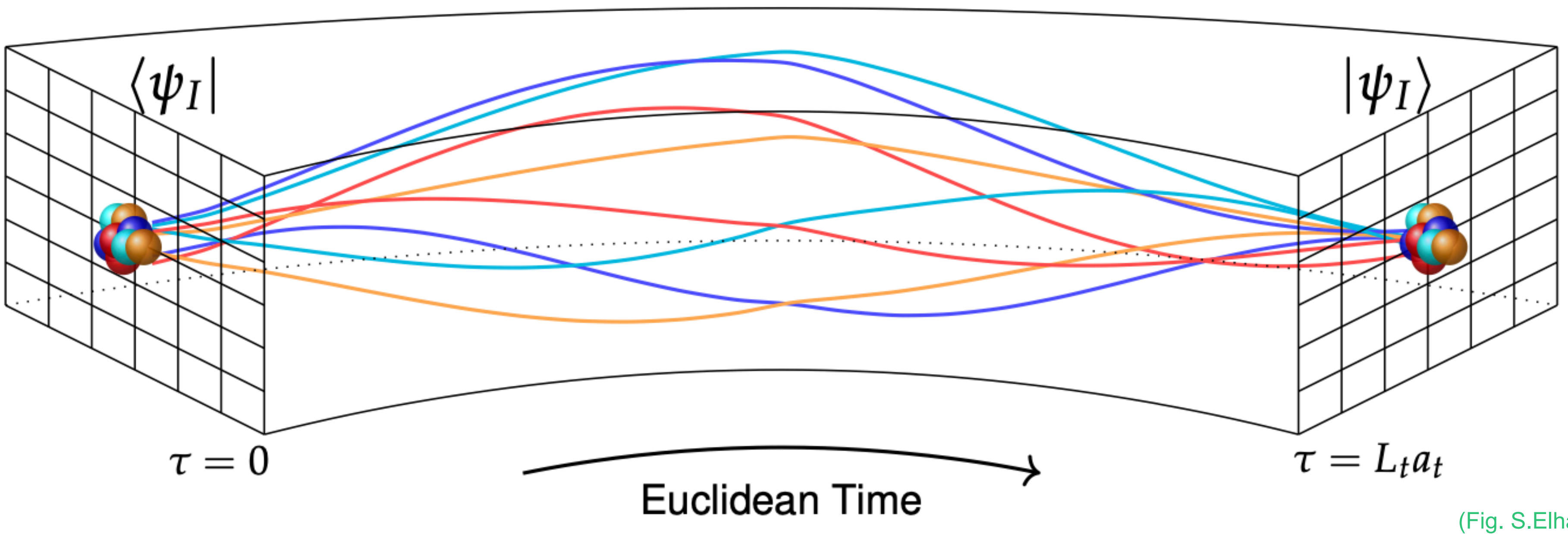
Method: Lattice Monte Carlo

- Lattice ⇒ cubic volume of size $(La)^3$ with discrete lattice site



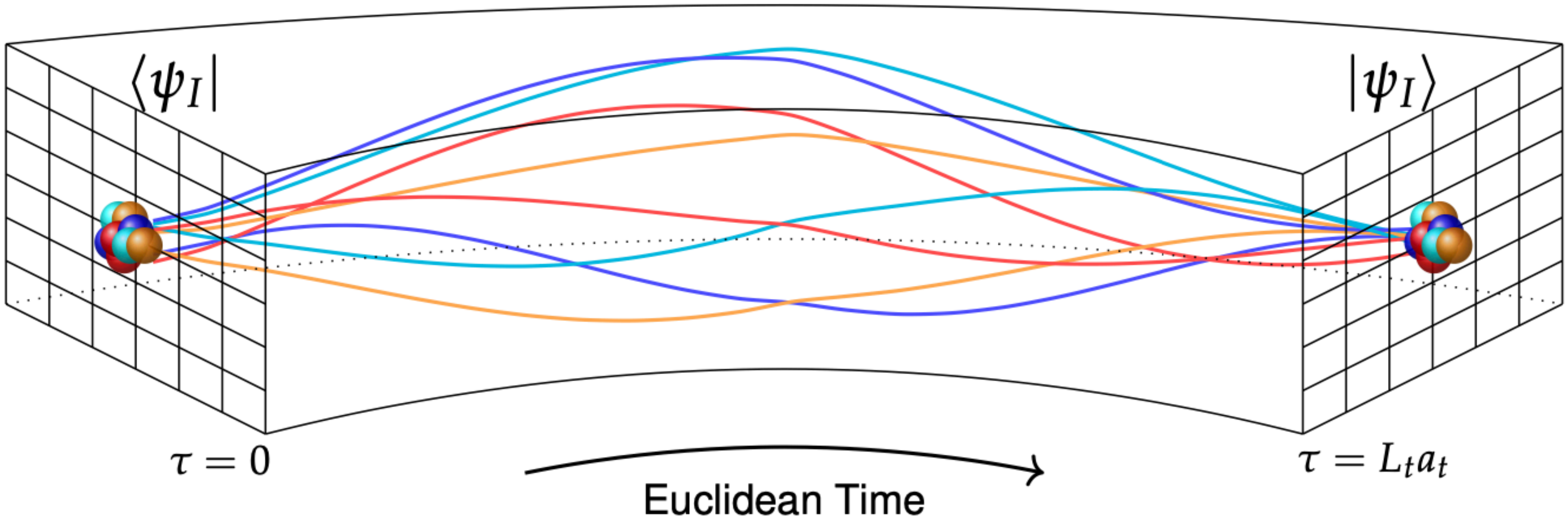
- Discretized chiral potentials, contact interactions
  one-pion exchange, Coulomb (Epelbaum et al.)

- Do Euclidean time evolution and extract i.e. energies
  as transient energy $E = -\frac{d}{d\tau}\ln(Z(\tau))$



(Fig. S.Elhatisari)

Method: Lattice Monte Carlo II



(Fig. S.Elhatisari)

- Auxiliary fields to handle many particles efficiently:
- Idea: replace interactions between nucleons with interaction of a nucleon with an auxiliary field

$$\exp(-\frac{C}{2}(N^\dagger N)^2) = \sqrt{\frac{1}{2}} \int \mathrm{d}A \exp\left[-\frac{A^2}{2} + \sqrt{C} A (N^\dagger N)\right]$$

Since nucleons only interact with an auxiliary field⇒ perfect for parallel computing

The Challenge : Sign Problem

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- Sign problem in a nutshell : makes life hard!

fermonic wave functions change sign when two fermions are interchanged. The systems are strongly interacting → highly oscillatory function, huge cancellation effects

no reasonable computation at higher orders possible

- First way out: utilize approximate SU(4) Wigner symmetry

(E. Wigner Phys.Rev 51(1937))

treats protons and neutrons on equal footing → same particle with different quantum number isospin

↓

create SU(4) invariant interactions, calculations are almost sign free

↓

some aspects of nuclear structure are well described

(S. Shen et al. Nature Commun.14 (2023))

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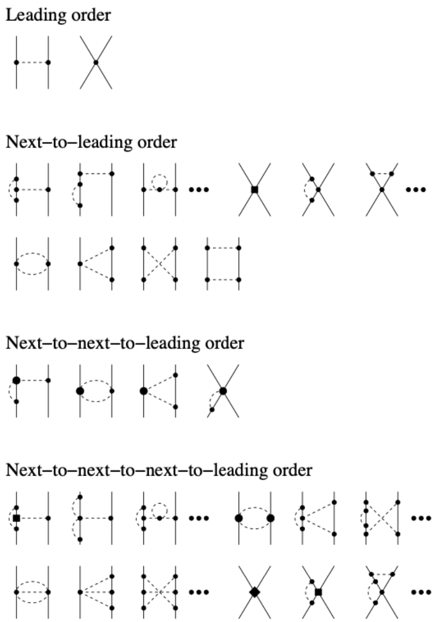
but we want and need:



Need improved perturbation theory to go beyond N2LO

Wavefunction Matching! (S.Elhatisari et al. Nature 630(2024))

- New method to solve the quantum many-body problem
- Not only applicable in our field but maybe also in yours!

starting point : H_S which has acceptable sign problem and can be simulated

How to connect?

target : H which has a severe sign problem

lowest eigenstate $|\psi_s^0\rangle$

projected and normalised :

$$|\phi_s^0\rangle = \mathscr{P}|\psi_s^0\rangle / ||\psi_s^0||$$

map $|\psi_S^0\rangle$ on $|\psi^0\rangle$

lowest eigenstate $|\psi^0\rangle$

projected and normalised :

$$|\phi^0\rangle = \mathscr{P}|\psi^0\rangle / ||\psi^0||$$

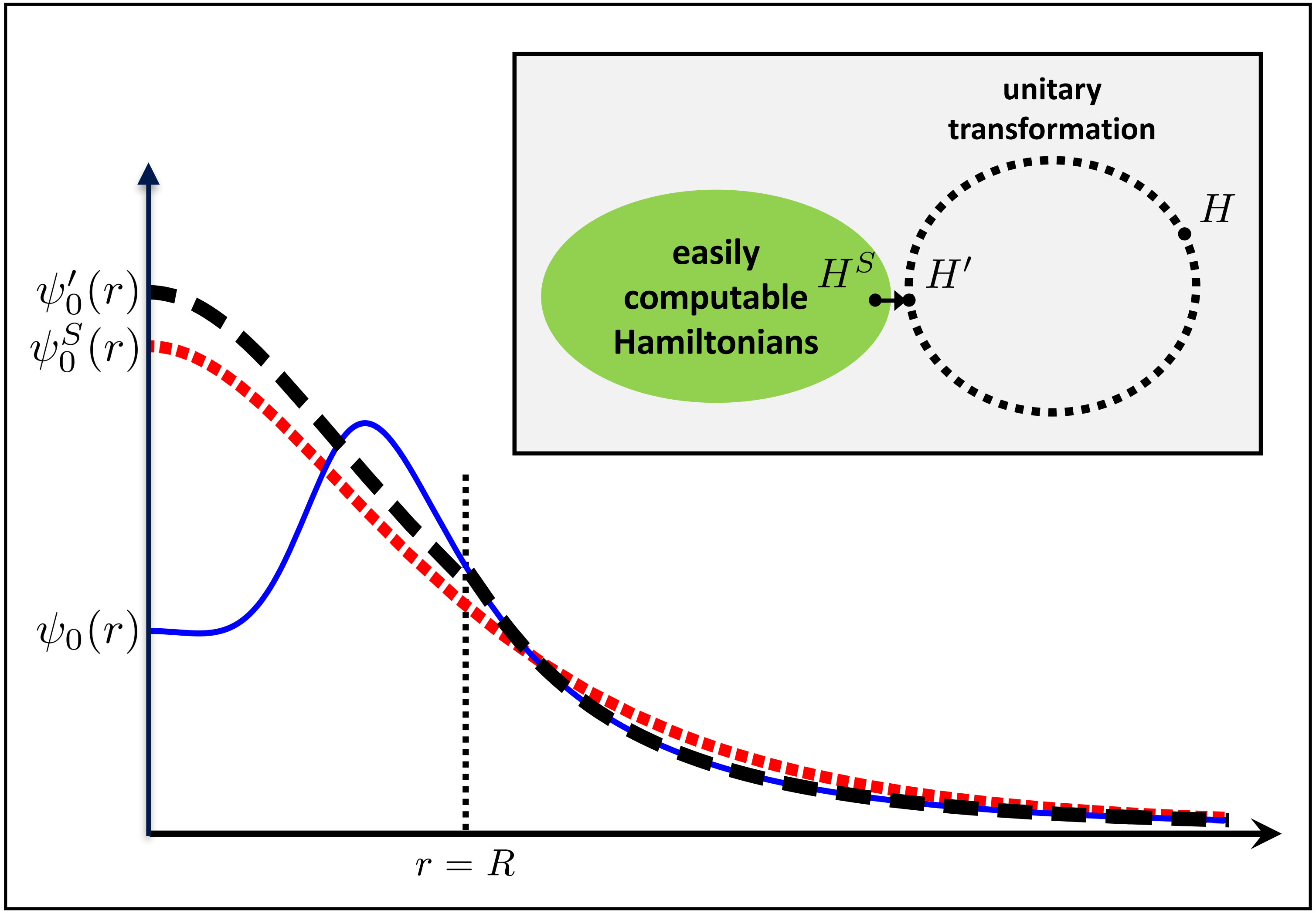
$$H' = U^\dagger H U$$

$$U = \begin{cases} 1 & r > R \\ |\phi^\perp\rangle\langle\phi_S^\perp| & r < R \end{cases}$$

Unitary trafo with finite range

perturbation theory now works from H_S to H'

Wavefunction Matching II !

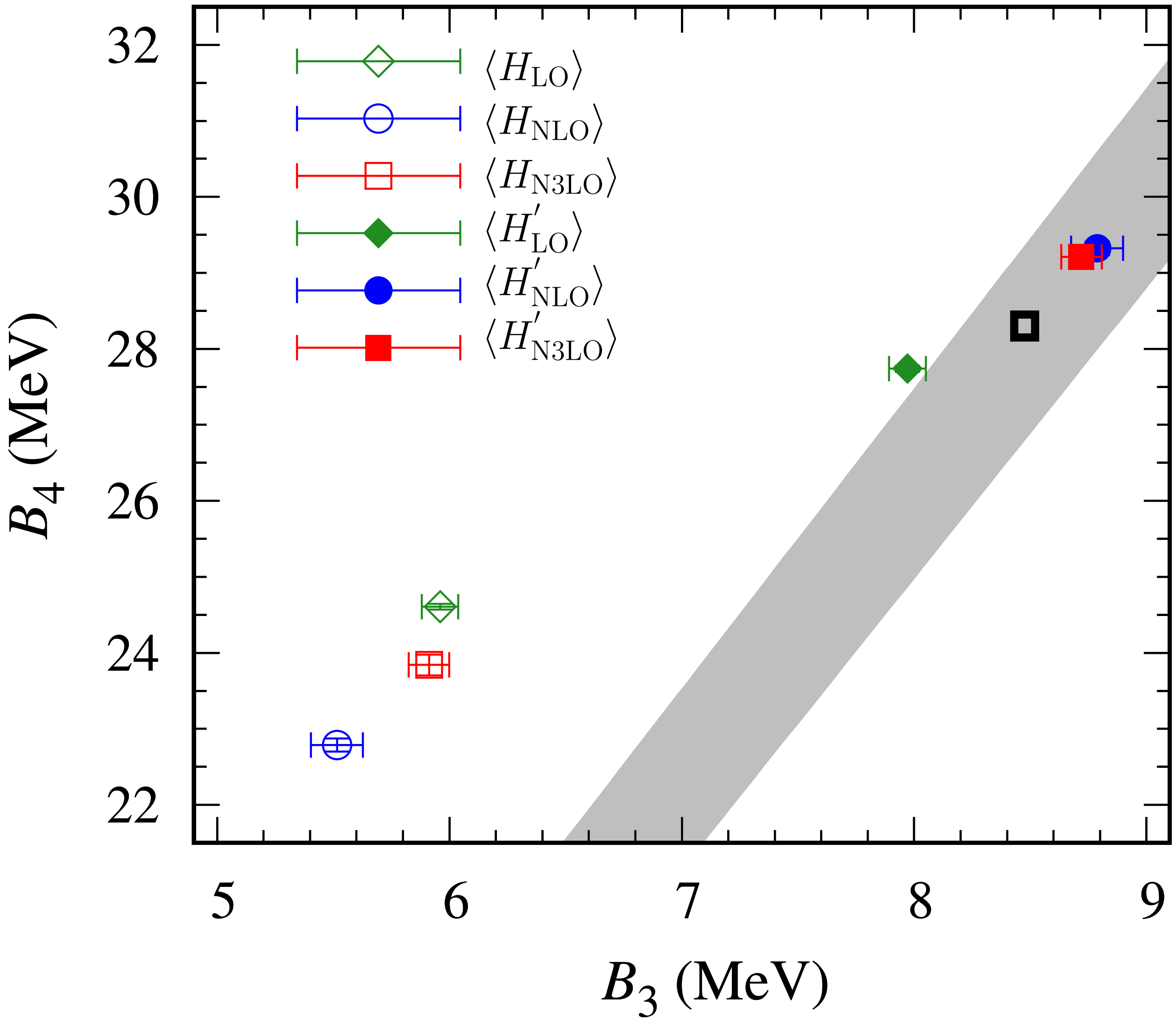


- Use H^s for non-perturbative part and do perturbation theory to H'
- Direct use of H', no reconstruction of higher order forces needed!

This is no renormalization group transformation

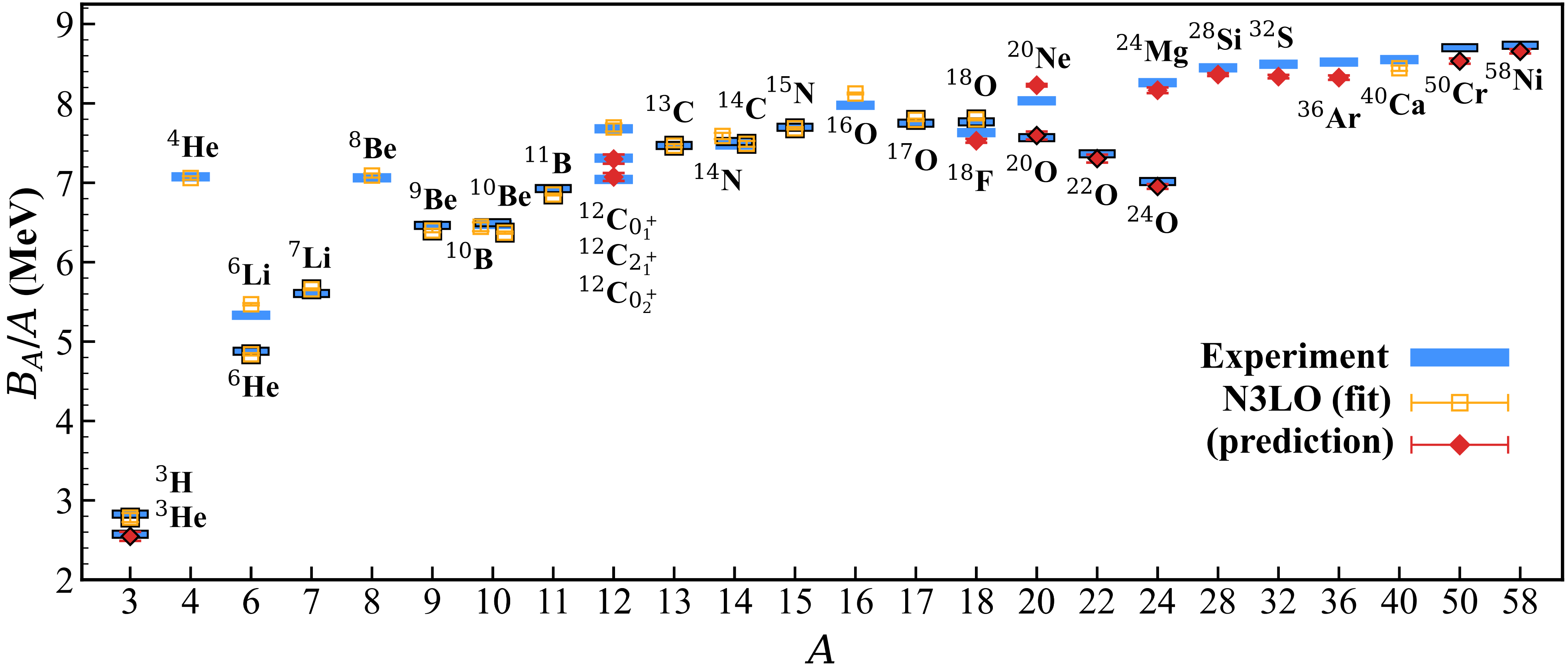
Does it Work in Practice? (Tjon Band)

- Universal correlation between 3- and 4-body binding energy (L.Platter et al. PLB 607(2005))
- Tjon band is reproduced using Wavefunction Matching



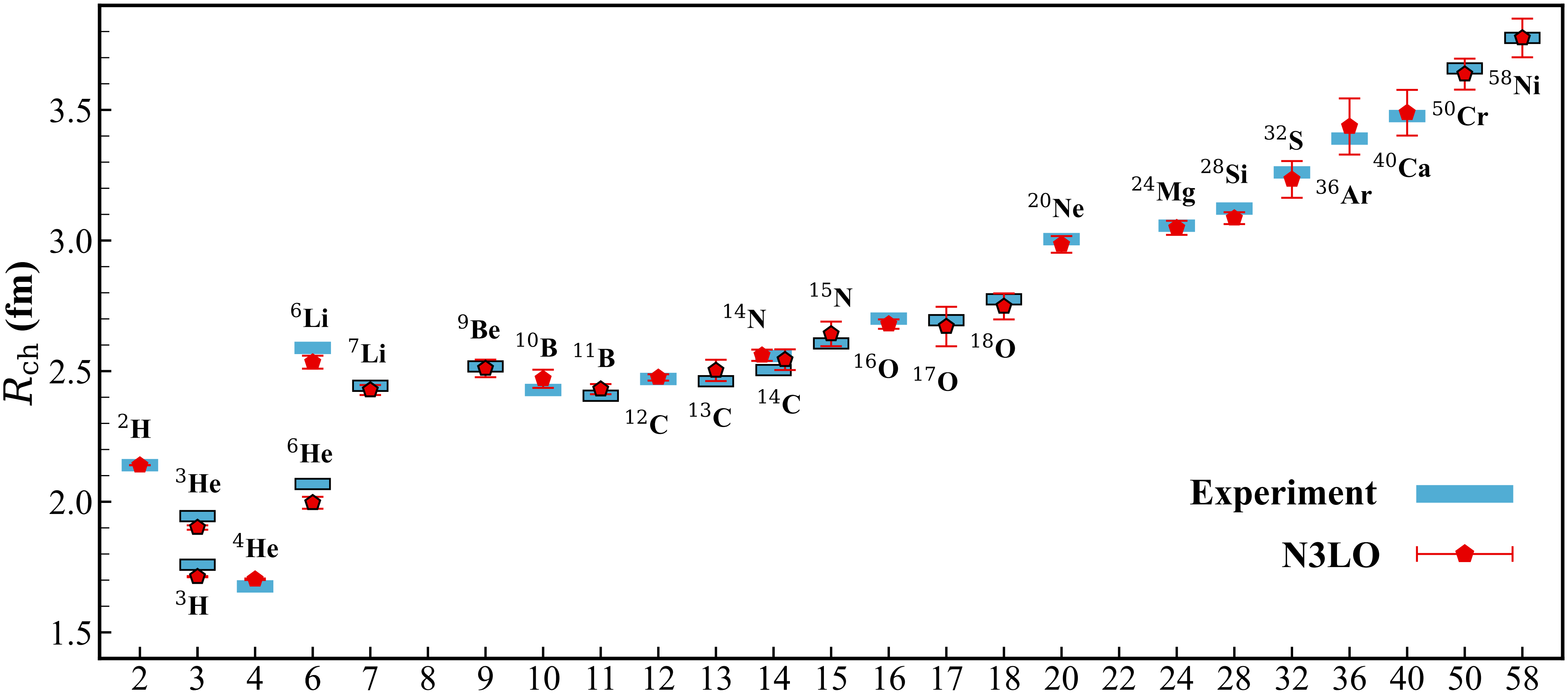
Binding Energy up to Medium Mass Nuclei

- Fix 3NF forces using history matching, systematic errors accessible
- Ground and exited states are well reproduced over a large excerpt of the nuclear chart
- $a = 1.32$ fm



Predictions for Charge Radii

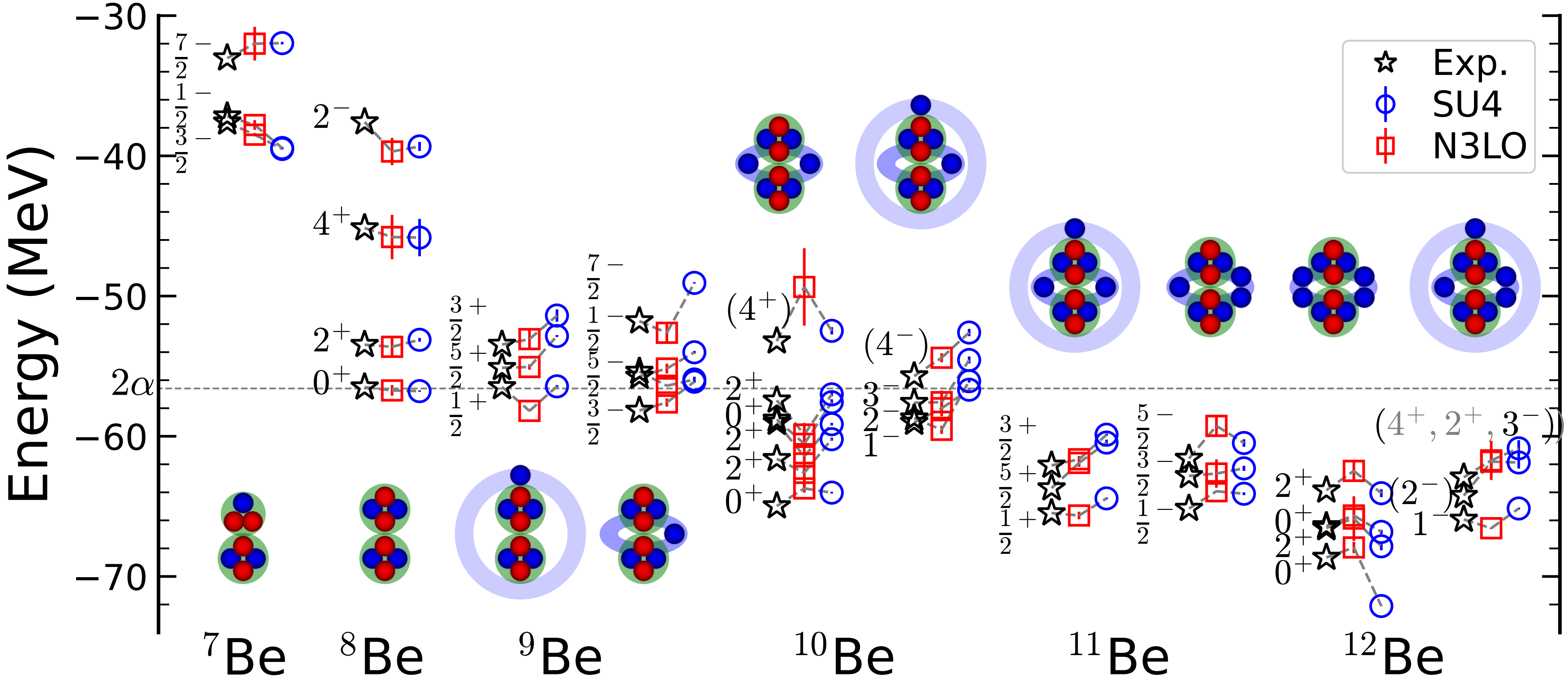
- Charge radii are spot on!
- No fitting to charge radii, everything is a prediction!
- $a = 1.32$ fm, statistical errors can be reduced



In Detail Study of Beryllium Isotopes

- Very different structures can be described within one calculation

- SU(4) interaction works quite well
- N3LO improves the result further



(S. Shen et al. arxiv: 2411.14935 (2024))

A very brief Introduction to Neutron Stars

- Compact remnants of stars at the end of their life
- Typical quantities:

$$r \sim 10 \text{ km}$$

$$M \sim (1-2)M_\odot$$

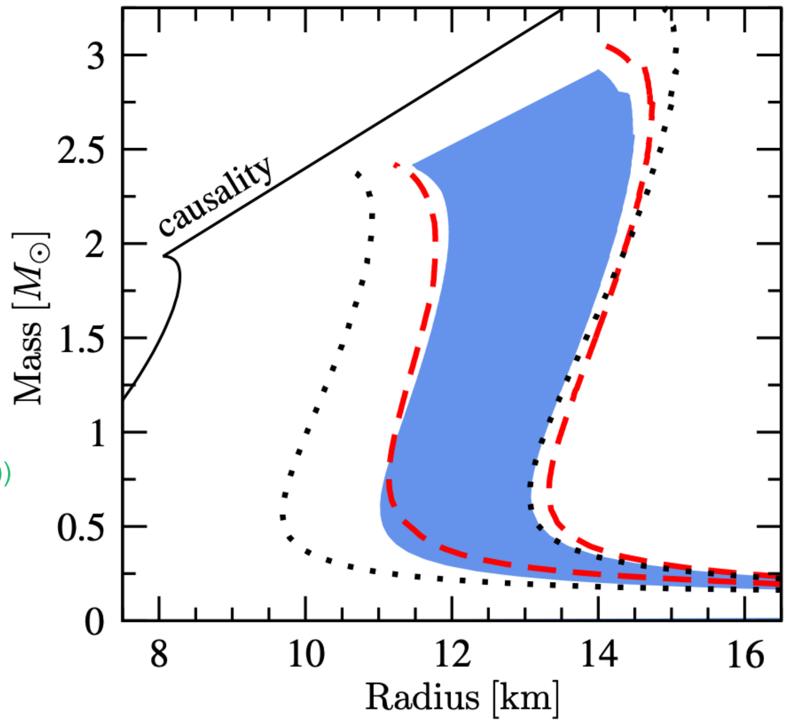
super dense objects,
ρ several time saturation density in the core

Open questions:

-density in the centre
-equation of state (EoS)
-inclusion of other baryons

limited by Tolman-Oppenheimer-Volkoff equations

(R.Tolman Phys. Rev. 55(1939))
(J.Oppenheimer, G. Volkoff Phys. Rev. 55(1939))

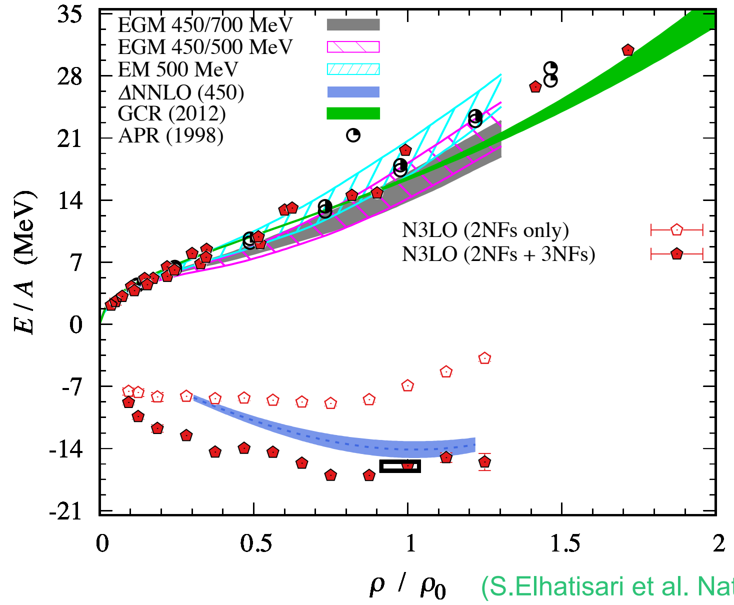


(Hebeler et al. Astrophys.J. 773(2013))

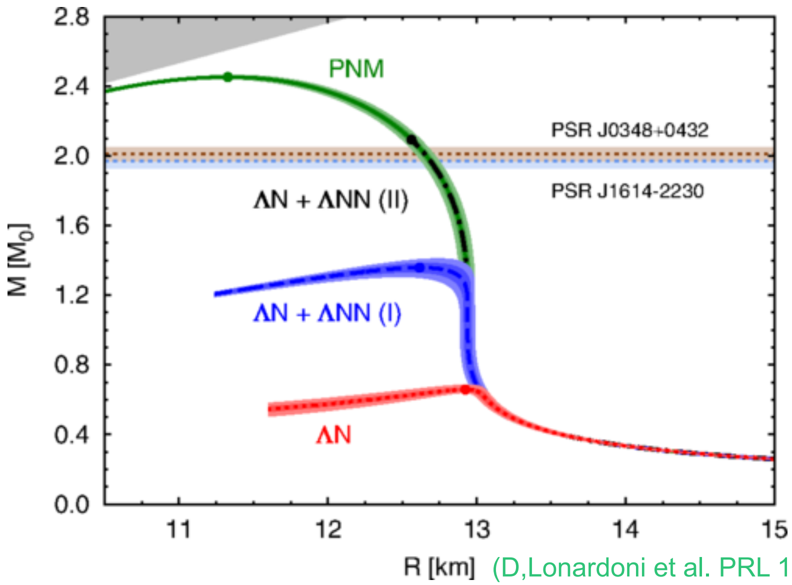
Mass-Radius relation, if EoS correct, every neutron star falls on this line

extract this from NLEFT, check inclusions of other baryons as well

EoS of pure neutron and symmetric nuclear matter



(S.Elhatisari et al. Nature 630(2024))



(D,Lonardoni et al. PRL 114(2015))

addition of hyperons, typically softens the equation of states

neutrons stars are no longer supported

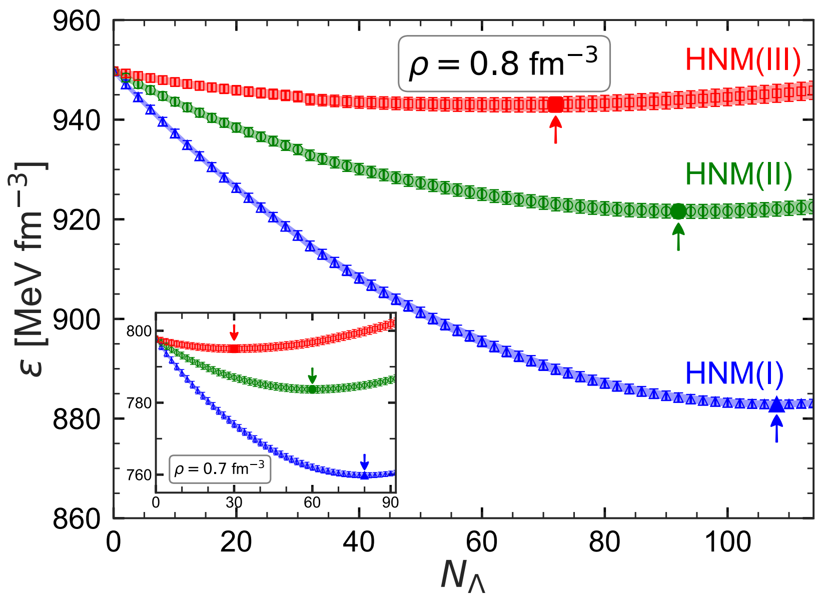
naiv assumptions make it reasonable for hyperons to appear at such densities

Hyperon Puzzle

resolved by three-body forces

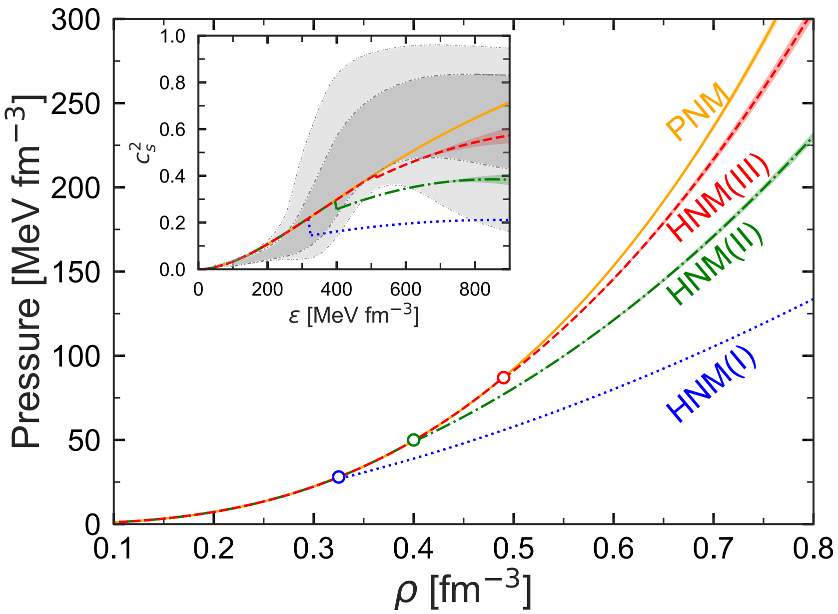
no ab initio calculation with large amount of hyperons

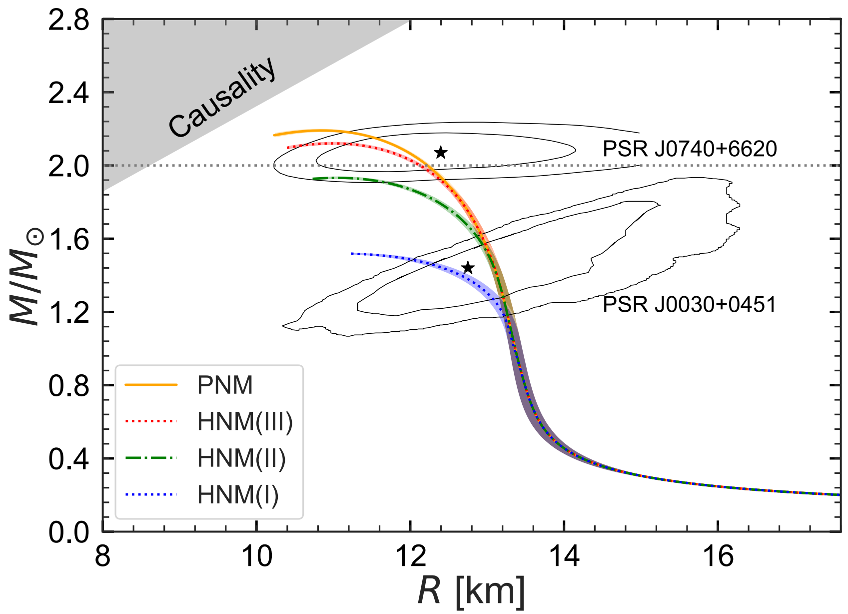
Hypernuclear matter from NLEFT



calculation with ~ 240 particles and up $50\,\%$ hyperons

we explore a large set of different configurations up to 5 times saturation density





(H.Tong et al. Sci. Bull. (in press) arixiv:2405.01887(2024))

Summary

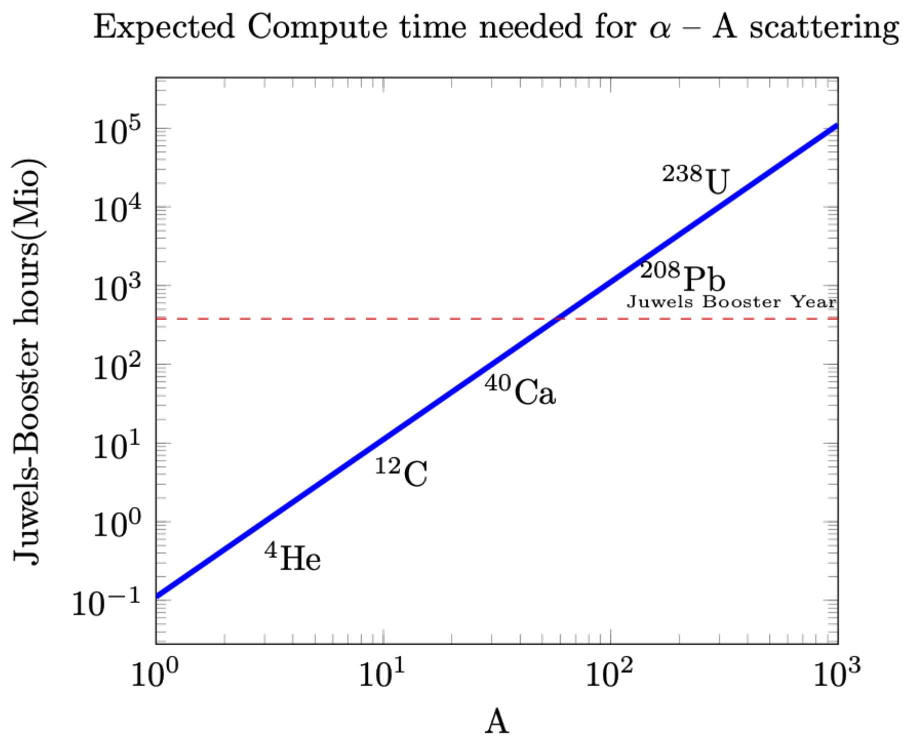
- Give you a first insight in Nuclear Lattice Effective Field Theory
    - Relevant degrees of freedoms are neutrons and protons
    - Combine effective field theory with lattice methods
- Presented a new approach to solve the quantum many-body problem:
    - Wavefunction Matching
    - Makes a whole set of new calculations available
    - High precision consistent calculation of matter radii and binding energies
- Showed how to connect ~ 200 particles with 10^{69} at the example of neutron stars
    - First ab initio calculation using a significant number of Λs
    - EoS including Λ particles which is consistent with nature

Outlook

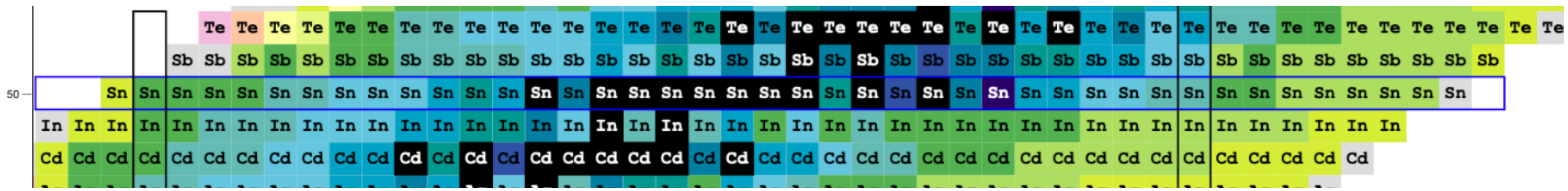
- JUPITER changes the game:

$^{12}C(\alpha,\gamma)^{16}O$ is now in reach

critical to understand the carbon/oxygen ratio in the universe



the Sn line:



(Nudat3(2025))

typical benchmark to go to even heavier nuclei → different structures accessible