Big Bang Nucleosynthesis and Deuteron-Deuteron reactions

Frontiers in Nuclear Lattice EFT: From Ab Initio Nuclear Structure to Reactions

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Motivation

- Fundamental constants: show up in every discipline of science
- We know them to precisions given units of parts per 10^9 [1]

permeability of free space	μ_0	$4\pi \times 10^{-7}$ N A^{-2} = 12.566 370 614 ... $\times 10^{-7}$ N A^{-2}	exact
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297\,352\,5664(17)\times 10^{-3} = 1/137.035\,999\,139(31)^\dagger$	0.23, 0.23
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.817\,940\,3227(19)\times 10^{-15}$ m	0.68
$(e^-$ Compton wavelength$)/2\pi$	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e \alpha^{-1}$	$3.861\,592\,6764(18)\times 10^{-13}$ m	0.45
Stefan-Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	$5.670\,367(13)\times 10^{-8}$ W m^{-2} K^{-4}	2300
Fermi coupling constant**	$G_F/(\hbar c)^3$	$1.166\,378\,7(6)\times 10^{-5}$ GeV^{-2}	510
weak-mixing angle	$\sin^2\hat{\theta}(M_Z)$ $(\overline{\text{MS}})$	$0.231\,22(4)^{\dagger\dagger}$	1.7×10^5
$W^\pm$ boson mass	m_W	$80.379(12)$ GeV/c^2	1.5×10^5

- Some theories predict changes in these constants over cosmological time scales

How fine-tuned is our universe?[2]

- How can we test this? $\Rightarrow$ Laboratory: Big Bang Nucleosynthesis (BBN)[3]

[1] PDG: Workman et al., 2022, [2] Dirac, 1973 and many others, [3] Olive, Steigman, and Walker, 2000; Iocco et al., 2009; Cyburt et al., 2016; Pitrou et al., 2018a

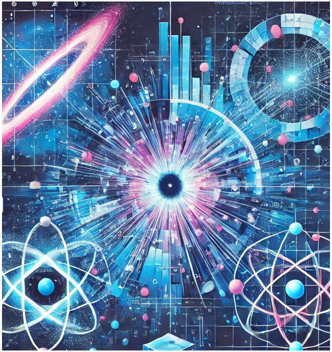
This talk

We have studied BBN under variation of

- the electromagnetic coupling constant α [1]
  - ☞ also using results from Halo EFT calculations[2]
- the strange-quark mass[3]

Goal: find a bound on these variations through comparing calculations with experimental values for light element abundances

$\Rightarrow$ How did we use input from Nuclear Lattice EFT?[4]



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[1] Meißner, Metsch, HM 2023; Bergström, Iguri, Rubenstein, 1999; Nollett, Lopez, 2002; Dent,Stern,Wetterich, 2007; Coc et al., 2007;

[2] Meißner, Metsch , HM 2024; Hammer, Ji, Phillips, 2017; [3] Meißner, Metsch, HM 2025, [4] Lähde, Meißner 2019

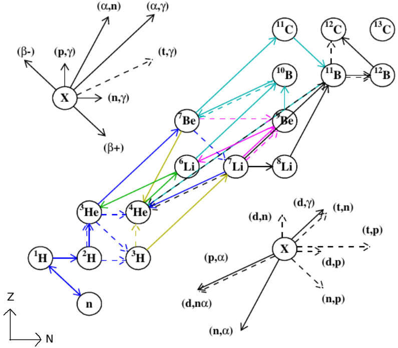
Introducing BBN – Evolution of Abundances

- abundance $Y_i = n_i / n_b$, with n_i density of nucleus i and n_b total baryon density
- Need to solve system of rate equations

$$\dot{Y}_i \supset -Y_i \Gamma_{i \to \dots} + Y_j \Gamma_{j \to i + \dots} + Y_k Y_l \Gamma_{kl \to ij} - Y_i Y_j \Gamma_{ij \to kl}$$

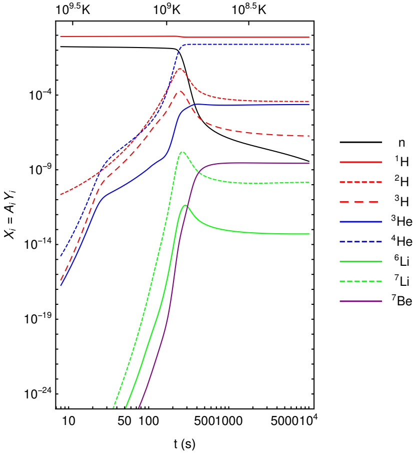
- Used different codes[1] to get an estimate of systematical errors

[1] PRIMAT: Pitrou et al., 2018b, AlterBBN: Arbey et al., 2020, PArthENoPE: Gariazzo et al., 2022, NUC123: Kawano, 1992 and PRyMordial: Burns, Tait, and Valli, 2023



: Taken from Pitrou et al., 2018a

Introducing BBN – The Timescales



: produced by PRIMAT

- $t \leq 1\,\mathrm{s}$

  Weak $n \leftrightarrow p$ reactions
  - number density ratio $\frac{n_n}{n_p} = e^{-Q_n/T}$, Q_n: mass difference
  - at 1 s or $T \approx 1$ MeV: freeze-out and free neutron decay

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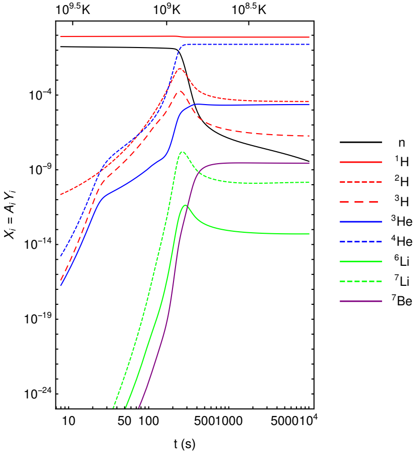

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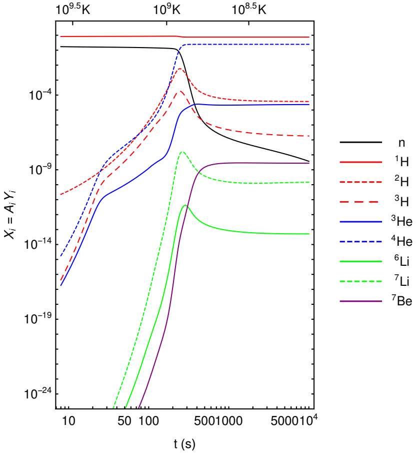
- $t = 1\,\mathrm{min}$

  Deuterium bottleneck: $n + p \rightarrow d + \gamma$ efficient

Introducing BBN – The Timescales



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  ☞ number density ratio $\frac{n_n}{n_p} = e^{-Q_n/T}$, Q_n: mass difference

  ☞ at 1 s or $T \approx 1\,\mathrm{MeV}$: freeze-out and free neutron decay

- $t = 1\,\mathrm{min}$

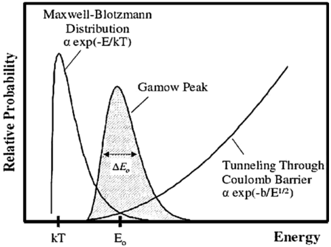
  Deuterium bottleneck: $n + p \rightarrow d + \gamma$ efficient

- $t \lesssim 3\,\mathrm{min}$

  Fusion of light elements (up to $^7\mathrm{Be}$)

Variation of α – What to consider

- Nuclear reaction rates: Coulomb barrier → energy-dependent penetration factor in cross section[1]
- Radiative capture



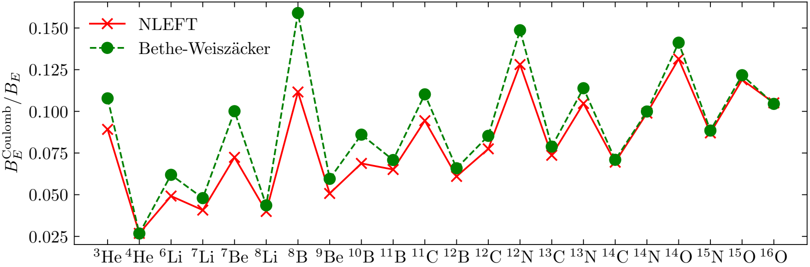
- $n \leftrightarrow p$ and β-decay rates: final (initial) state interactions between charged particles
- Indirect effects: binding energies[2] and Q_n (QED contribution)[3]

$$\Delta Q_n = Q_n^{\text{QED}} \cdot \delta\alpha = -0.58(16)\,\text{MeV} \cdot \delta\alpha$$

[1] Blatt and Weisskopf, 1979; [2] Elhatisari et al., 2024; [3] Gasser, Leutwyler, and Rusetsky, 2021

Coulomb contributions to binding energies

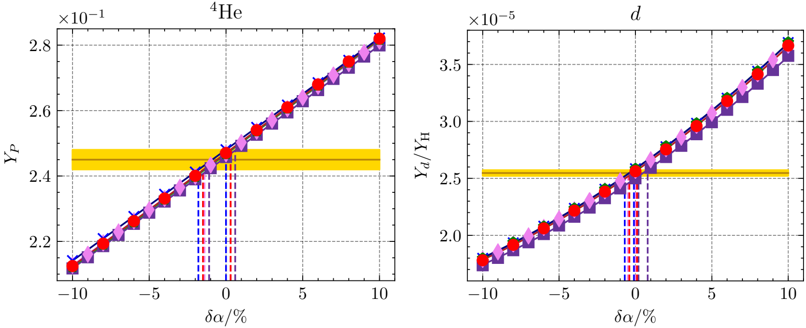
$$\Delta Q = \delta\alpha \left(-\sum_i B_E^{\mathrm{Coulomb},i} + \sum_j B_E^{\mathrm{Coulomb},j} \right)$$



$$\sigma(E,\alpha) \propto \underbrace{(E + Q(\alpha))^{p_\gamma}}_{\text{phase space}} \alpha^{q_\gamma} \frac{\sqrt{E_G^{\mathrm{in}}(\alpha)/E}}{\exp\left(\sqrt{E_G^{\mathrm{in}}(\alpha)/E}\right) - 1} \frac{\sqrt{E_G^{\mathrm{out}}(\alpha)/(E + Q(\alpha))}}{\exp\left(\sqrt{E_G^{\mathrm{out}}(\alpha)/(E + Q(\alpha))}\right) - 1}$$

Experimental constraints

- PDG[1]: reliable measurements for $^4\mathrm{He}$, *d* and $^7\mathrm{Li}$ (But: Lithium problem[2])



- 5 codes give similar results
- Only α-variation of $|\delta\alpha| < 1.8\%$ is consistent with experiment

[1] Workman et al., 2022; [2] Fields, 2011

Halo Effective Field Theory (EFT)

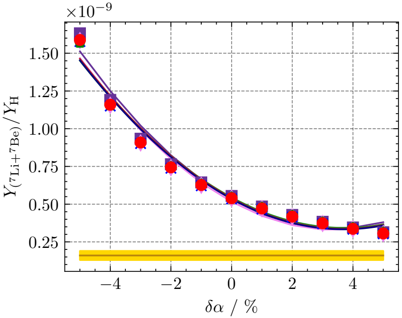
Biggest source of uncertainty: reaction rates and cross sections

$\Rightarrow$ Need theoretical predictions

- So far: only pionless EFT for $n + p \to d + \gamma$ [1]
- Now: include Halo EFT[2] rates for
  - $n + {}^7\mathrm{Li} \to {}^8\mathrm{Li} + \gamma$ [3]
  - $p + {}^7\mathrm{Be} \to {}^8\mathrm{B} + \gamma$ [4]
  - ${}^3\mathrm{H} + {}^4\mathrm{He} \to {}^7\mathrm{Li} + \gamma$ and ${}^3\mathrm{He} + {}^4\mathrm{He} \to {}^7\mathrm{Be} + \gamma$ [5]

[1] Rupak, 2000; [2] review: Hammer, Ji, Phillips, 2017; [3] Fernando, Higa, Rupak 2012; Higa, Premarathna, Rupak, 2021; [4] Higa, Premarathna, Rupak, 2022;

[5] Higa, Rupak, Vaghani, 2018; Premarathna, Rupak, 2020



: Meißner, Metsch, HM 2024

${}^7\mathrm{Li} + {}^7\mathrm{Be}$ abundance diverges?

Where does strangeness appear in BBN?

Main contribution of m_s through strange quark σ-term[1,2,3]

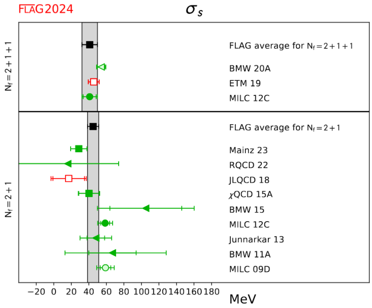
$$\sigma_s = \langle N | m_s \, \bar{s}s | N \rangle = 44.9(64)\,\text{MeV}\ ^{4}$$

$\Rightarrow$ changes the nucleon mass m_N:

$$\left|\delta_{m_s}\right| = \frac{\left|\Delta m_N\right|}{\sigma_s}$$

Nucleon mass change in kinetic Hamiltonian affects

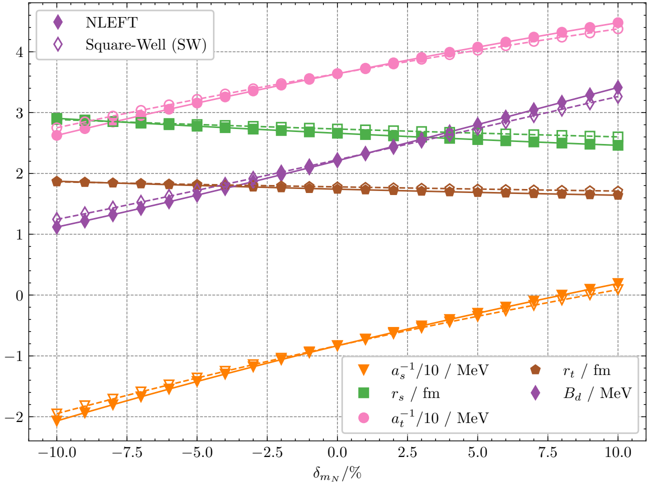
- nucleon-nucleon scattering observables
- nuclear binding energies



: taken from arxiv.org/pdf/2411.04268

[1] Collins, Duncan, Joglekar, 1977; [2] Crewther, 1972; [3] Nielsen, 1977; [4] FLAG collaboration, 2024 ($N_f = 2+1$)

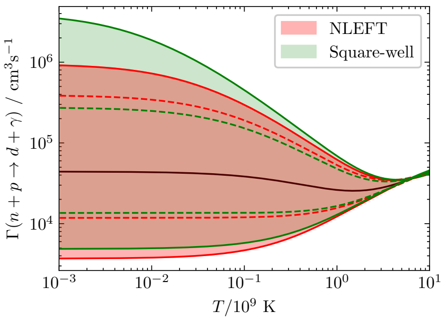
Nucleon-nucleon scattering



$n + p \to d + \gamma$

For $n + p \to d + \gamma$ there exists analytic cross section from pionless EFT[1]

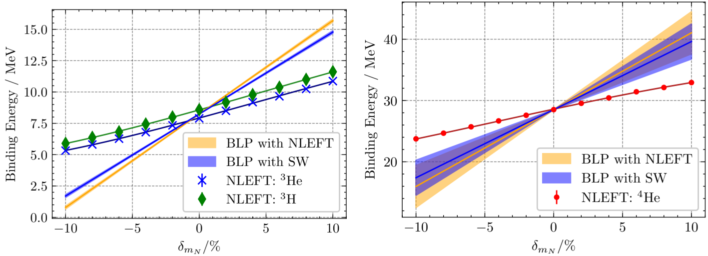
- change in scattering parameters has huge effect
- relevant temperature range: 1.25 to 10×10^9K
- main effect: backwards reaction (deuterium bottleneck)



[1] Rupak, 2000

Binding energies

Again: change in nuclear binding energies due δ_{m_N} in kinetic Hamiltonian



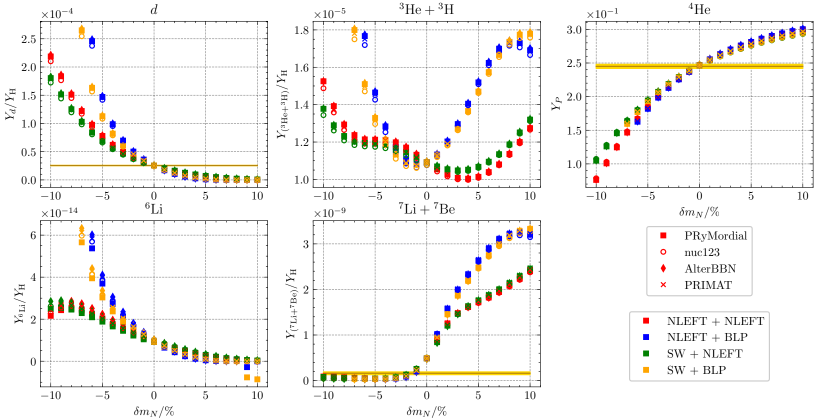
Alternatively, one defines (BLP)[1,2]

$$K^{m_N}_{B_{^3\mathrm{He}}} = K^{m_N}_{a_s} K^{a_s}_{B_{^3\mathrm{He}}} + K^{m_N}_{B_d} K^{B_d}_{B_{^3\mathrm{He}}}$$
$$K^{m_N}_{B_{^4\mathrm{He}}} = K^{m_N}_{a_s} K^{a_s}_{B_{^4\mathrm{He}}} + K^{m_N}_{B_d} K^{B_d}_{B_{^4\mathrm{He}}}$$

$$K^{a_s}_{B_{^3\mathrm{He}}} = 0.12(1)\,, \quad K^{B_d}_{B_{^3\mathrm{He}}} = 1.41(1)\,;$$
$$K^{a_s}_{B_{^4\mathrm{He}}} = 0.037(11)\,, \quad K^{B_d}_{B_{^4\mathrm{He}}} = 0.74(22)\,.$$

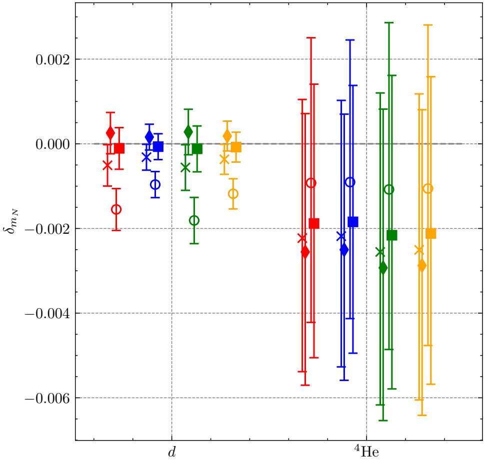
[1] Berengut et al., 2013; [2] Bedaque, Luu, Platter, 2011

Results



d
$\times 10^{-4}$
Y_d/Y_H
$^3He + ^3H$
$\times 10^{-5}$
$Y_{(^3He+^3H)}/Y_H$
^4He
$\times 10^{-1}$
Y_P
^6Li
$\times 10^{-14}$
$Y_{^6Li}/Y_H$
$^7Li + ^7Be$
$\times 10^{-9}$
$Y_{(^7Li+^7Be)}/Y_H$
$\delta m_N/\%$
PRyMordial
nuc123
AlterBBN
PRIMAT
NLEFT + NLEFT
NLEFT + BLP
SW + NLEFT
SW + BLP

Constraints



- Constraints very narrow
- Now: deuterium constraints δ_{m_N} much more than ^{4}He
- $\Rightarrow$ upper bound for strange quark mass variation:

$$|\delta_{m_s}| = \frac{|\Delta m_N|}{\sigma_s} < 5.1\%$$

To summarize...

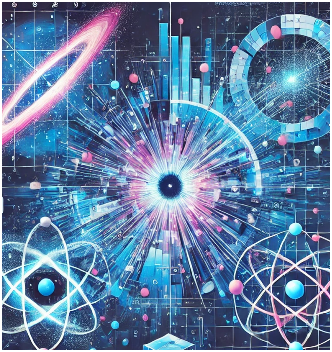
- simulated Big Bang Nucleosynthesis with 5 different codes as laboratory
- considered variation of fundamental constants and found
  - for the fine-structure constant

$$|\delta\alpha| < 1.8\%$$

  - for the strange quark mass

$$|\delta_{m_s}| < 5.1\%$$

  to be consistent with measurements, using NLEFT as input
- Now: How fine-tuned is our universe?



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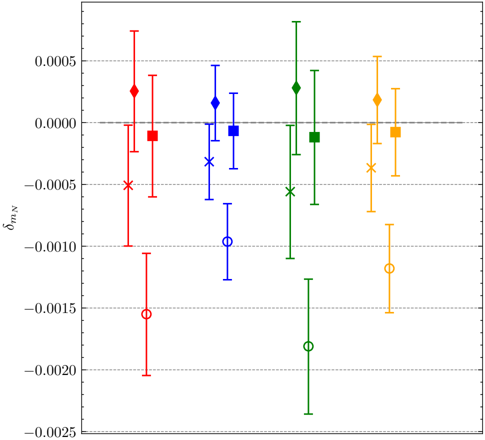
Why deuteron-deuteron reactions?



Constraints from d-abundance for strange quark mass: differences in the code bigger than range of possible variations!

Deuteron abundance is sensitive to choice of rates[1]

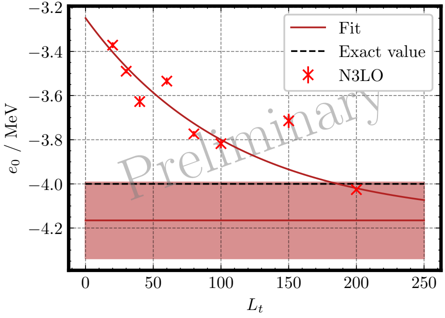
- $d(d,n)^3\mathrm{He}$
- $d(d,p)^3\mathrm{H}$

$\Rightarrow$ Goal: calculating these rates using NLEFT

[1] Pitrou et al., 2021

Challenges and on-going work

First step: $d - d$ elastic scattering



So far for one-cluster APM found

- ideal wave function,
- best working bin size,
- minimal lattice size

Challenges for two-cluster APM

- Deuteron is weakly bound $\Rightarrow$ converges slowly
- Need to collect a lot of statistics

This is now on-going!

Challenges and on-going work

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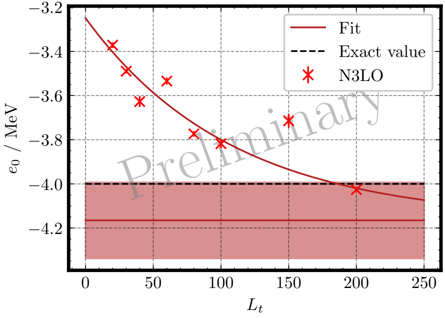

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Thank you for your attention!

Nuclear Reaction Rates – Coulomb Barrier

$$\boxed{\Gamma_{ab\to cd}(T) = N_A \langle \sigma v \rangle \propto \int_0^\infty \mathrm{d}E\, \sigma_{ab\to cd}(E) \cdot E \cdot e^{-\frac{E}{k_B T}}}, \quad E = \frac{1}{2}\mu_{ab} v^2$$

(1) Coulomb Barrier

Cross section is proportional to penetration factor [Blatt and Weisskopf, 1979]

$$\boxed{\sigma \propto v_0 = \frac{2\pi\eta}{e^{2\pi\eta} - 1}},$$

with Sommerfeld parameter

$$\eta = \frac{Z_a Z_b \alpha c}{\hbar v} = \frac{1}{2\pi}\sqrt{E_G/E},$$

and Gamow-energy

$$E_G = 2\mu_{ab} c^2 \pi^2 Z_a^2 Z_b^2 \alpha^2, \quad \mu_{ab} = \frac{m_a m_b}{m_a + m_b}$$

Nuclear Reaction Rates – Radiative Capture

(2) Radiative capture reactions

- Coupling $\propto e \Rightarrow$ Cross section $\sigma \propto \alpha \propto e^2$
- External capture processes [Christy and Duck, 1961]: parameterized in $f(\delta\alpha)$ [Nollett and Lopez, 2002]
- Assume dipole dominance
- For some reactions: Halo EFT cross sections $\Rightarrow$

α-dependence of cross section ($q_\gamma = 1$ for radiative capture, zero else)

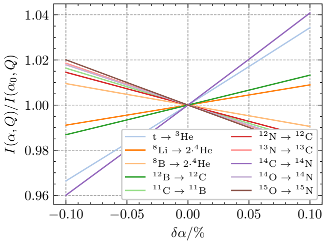
$$\sigma(\alpha, E) \propto \left(\frac{\sqrt{E_G^{\text{in}}/E}}{e^{\sqrt{E_G^{\text{in}}/E}} - 1} \right) \cdot \left(\frac{\sqrt{E_G^{\text{out}}/(E+Q)}}{e^{\sqrt{E_G^{\text{out}}/(E+Q)}} - 1} \right) \cdot (\alpha f(\delta\alpha))^{q_\gamma}$$

$$Q = m_a + m_b - m_c - m_d$$

Weak Rates – Fermi Function

β-decay rate (assume $|M_{fi}|^2$ to be p-independent) [Segrè, 1964]:

$$\lambda = \frac{g^2 |M_{fi}|^2}{2\pi^3 c^3 \hbar^7} \underbrace{\int_0^{p_{e,\max}} \left(W - \sqrt{m_e^2 c^4 + p_e^2 c^2}\right)^2 F(Z, \alpha, p_e) p_e^2 \, \mathrm{d}p_e}_{= I(\alpha, Q)},$$



$$p_{e,\max} = \frac{1}{c}\sqrt{W^2 - m_e^2 c^4}, \; W \approx M_a - M_b = Q$$

Fermi function (for $Z\alpha \ll 1$):

$$F(\pm Z, \alpha, \epsilon_e) \approx \frac{\pm 2\pi\nu}{1 - \exp(\mp 2\pi\nu)}, \quad \nu \equiv \frac{Z\alpha\epsilon_e}{\sqrt{\epsilon_e^2 - 1}}$$

Then:

$$\lambda(\alpha) = \lambda(\alpha_0)\frac{I(\alpha, Q)}{I(\alpha_0, Q)}$$

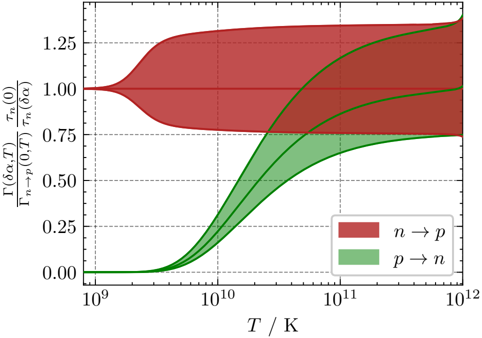
$n \leftrightarrow p$ Rates

Free neutron decay: lifetime

$$\tau_n(\alpha) = \tau_n(\alpha_0)\frac{I(\alpha_0, Q)}{I(\alpha, Q)}$$

But: Ignored Fermi-Dirac distribution of neutrino and electron

$\Rightarrow$ temperature dependence in α-variation for high temperatures



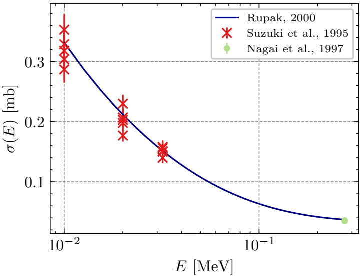
Nuclear Reaction Rates – $n + p \to d + \gamma$

Some corrections due to α variation are energy-dependent
$\Rightarrow$ need reaction cross section!

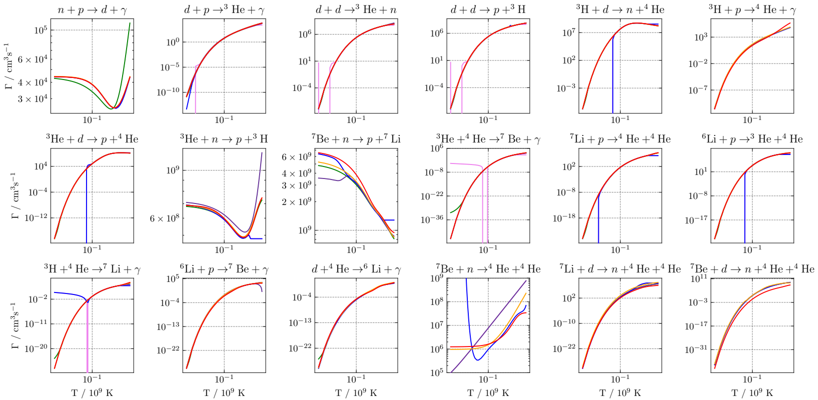
For $n + p \to d + \gamma$:

- Pionless EFT (N^4LO) approach by Rupak, 2000
- $\sigma(n + p \to d + \gamma)$ depends linearly on α

Other reaction cross section need to be parameterized by fitting to data EXFOR database



Nuclear Reaction Rates – Leading Reactions



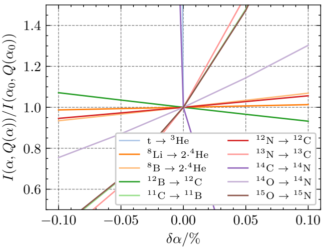
This work ; PRIMAT ; AlterBBN ; PArthENoPE ; NUC123 ; NACRE II ;

(PRyMordial uses the PRIMAT rates)

Indirect Effects – Binding energies [Meißner and Metsch, 2022]

Coulomb interaction between protons in nucleus
$\Rightarrow$ Electromagnetic contribution to binding energy [Elhatisari et al., 2024]
Change in Q-value:

$$\Delta Q = \delta\alpha \left(-\sum_i B_C^i + \sum_j B_C^j \right)$$



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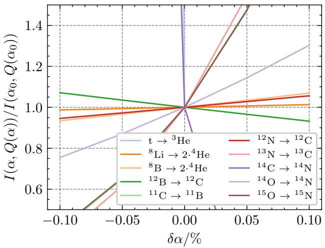

Nuclear reaction cross sections ($p_\gamma = 3, q_\gamma = 1$ for radiative capture, $p_\gamma = 1/2, q_\gamma = 0$ else)

$$\sigma(E,\alpha) \propto \underbrace{(E + Q(\alpha))^{p_\gamma}}_{\text{phase space}} \alpha^{q_\gamma} \frac{\sqrt{E_G^{\text{in}}(\alpha)/E}}{\exp\left(\sqrt{E_G^{\text{in}}(\alpha)/E}\right) - 1} \frac{\sqrt{E_G^{\text{out}}(\alpha)/(E + Q(\alpha))}}{\exp\left(\sqrt{E_G^{\text{out}}(\alpha)/(E + Q(\alpha))}\right) - 1}$$

Indirect Effects – Neutron-proton mass difference

$Q_n = m_n - m_p$ has QED contribution [Gasser, Leutwyler, and Rusetsky, 2021]:

$$\Rightarrow \Delta Q_n = Q_n^{\mathrm{QED}} \cdot \delta\alpha = -0.58(16)\,\text{MeV} \cdot \delta\alpha$$

Affects

- weak $n \leftrightarrow p$ rates
- Q-values of β-decays
- $m_N = (m_n + m_p)/2$ appearing in $n + p \to d + \gamma$ cross section? $\to$ neglect α-dependence!

Measurement of Primordial Abundances

Deuterium *d*:

- Almost completely destroyed in stars
- Observe high red-shift, low-metallicity systems

Helium-4 $^4\mathrm{He}$:

- Recombination lines of He and H in metal-poor extra-galactic HII regions
- Metal Production in stars positively correlated to stellar $^4\mathrm{He}$ contribution
  $\rightarrow$ Primordial abundance found by extrapolation to zero metallicity

Lithium-7 $^7\mathrm{Li}$:

- Observe stars in the galactic halo with very low metallicities
- $^7\mathrm{Li}$ dominant over $^6\mathrm{Li}$
- Lithium problem[1]: theoretical prediction three times higher

[1] LithiumProblem