Two topics in strong interactions physics with electromagnetic probes

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- Introductory remarks
- The proton radius: A theoreticians view
- The proton radius from J/ψ decays
- The minimal nuclear interaction
- *Ab initio* calculation of the ^{4}He transition form factor
- Summary and outlook

Introductory remarks

Electron scattering off nucleons and nuclei

- Electron scattering is a versatile tool to

  $\Rightarrow$ reveal the structure of the nucleon

  $\Rightarrow$ reveal the structure of atomic nuclei

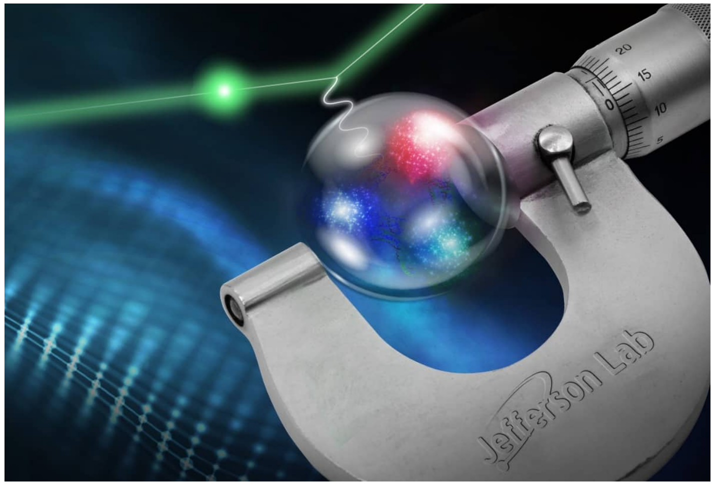
  $\Rightarrow$ information encoded in **form factors**, ...

- Often complimentary information through final-state interactions (FSI) in reactions or decays

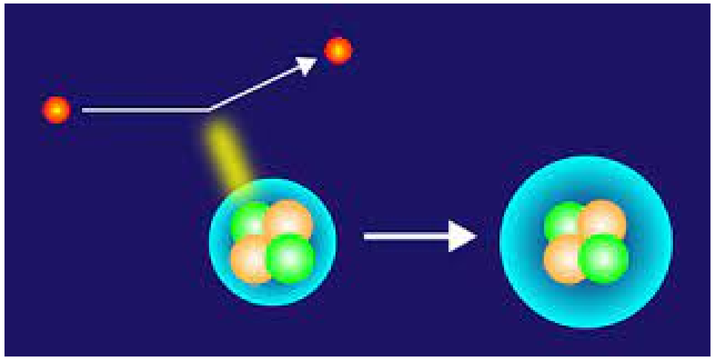
- this talk addresses two topics of high current interest:

  – a new method to measure the proton charge radius

  – an *ab initio* calculation of the ^{4}He transition ff



© Physics World



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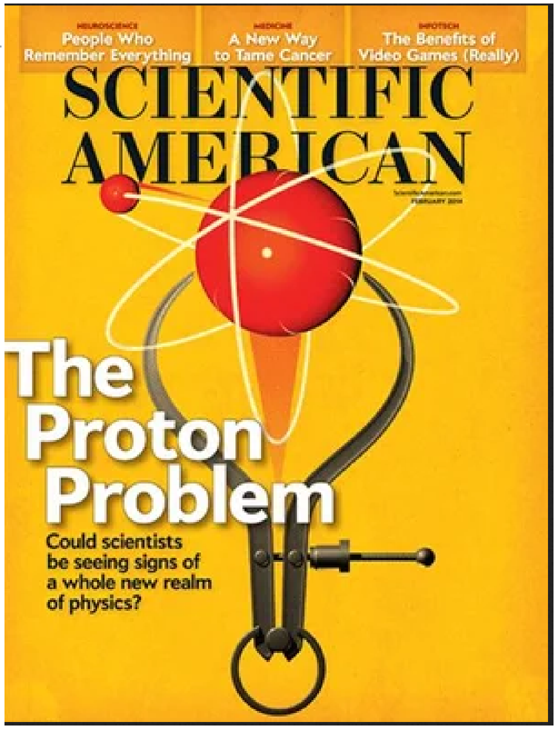
The proton radius "puzzle"

- The so-called proton radius puzzle: Much ado about nothing?

Proton radius puzzle

From Wikipedia, the free encyclopedia

The **proton radius puzzle** is an unanswered problem in physics relating to the size of the proton.[1] Historically the proton charge radius was measured by two independent methods, which converged to a value of about 0.877 femtometres (1 fm = 10^{-15} m). This value was challenged by a 2010 experiment using a third method, which produced a radius about 4% smaller than this, at 0.842 femtometres.[2] New experimental results reported in the autumn of 2019 agree with the smaller measurement, as does a re-analysis of older data published in 2022. While some believe that this difference has been resolved,[3] this opinion is not yet universally held.[4][5]



- Or stated differently: It's all about precision

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Science Bulletin www.scibull.com

News & Views

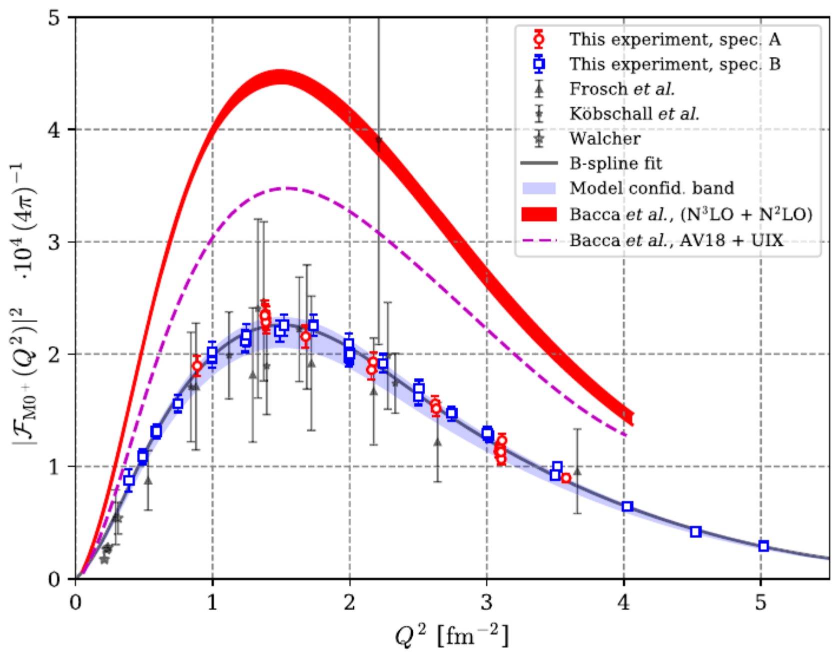
The proton radius: from a puzzle to precision

Hans-Werner Hammer [a,b], Ulf-G. Meißner [c,d,e,*]

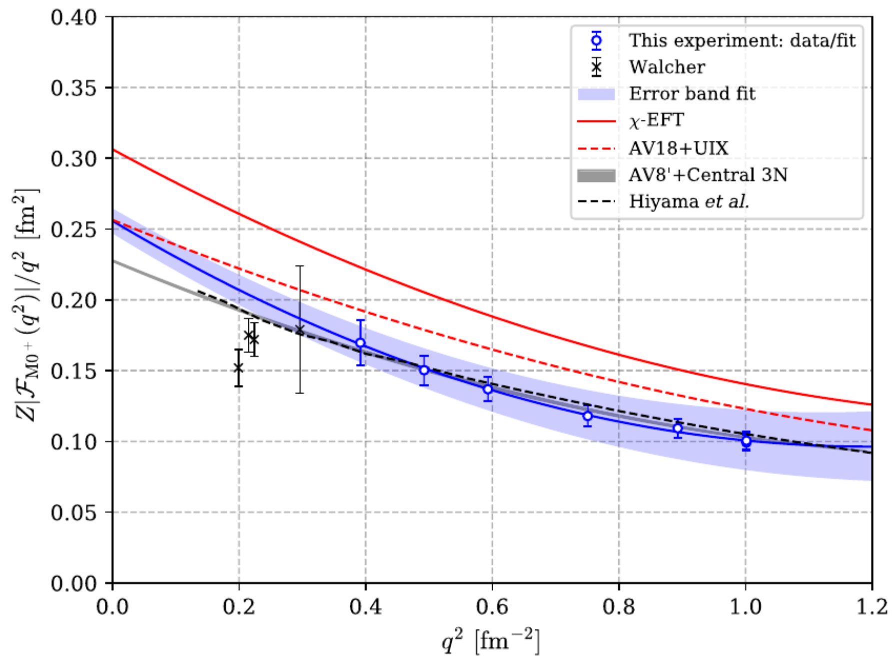
The ^{4}He form factor puzzle

- Recent Mainz measurements of $F_{M0}(0_2^+ \to 0_1^+)$ appear to be in stark disagreement with *ab initio* nuclear theory Kegel et al., Phys. Rev. Lett. **130** (2023) 152502

- Monopole transition ff



- low-momentum expansion



$\Rightarrow$ **A low-energy puzzle for nuclear forces?**

The proton radius and its relatives

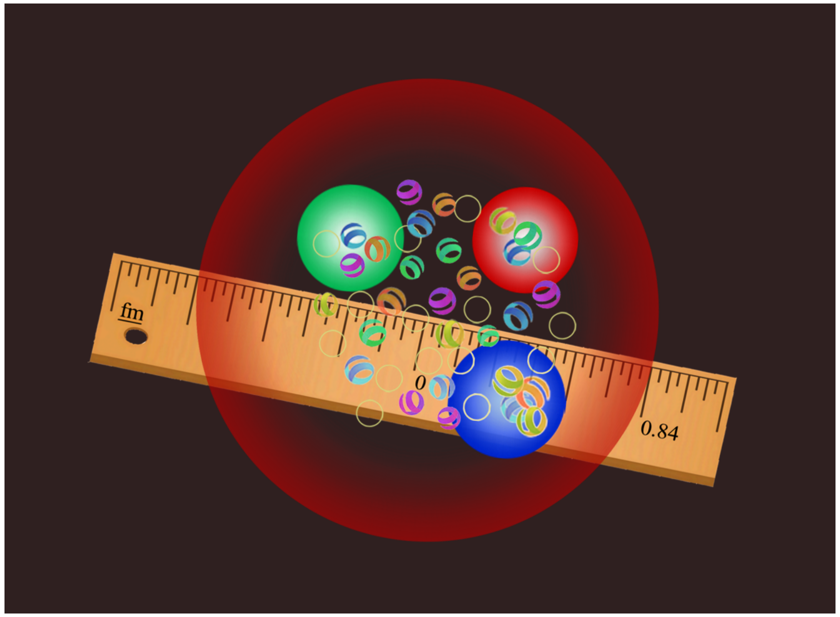


Fig. courtesy Yong-Hui Lin

Proton charge radius

- Definition: $r_p^2 \equiv -6\, G'_E(0)$ [not discussing charge distribution here!]

- Measurements:

  – Leptonic hydrogen Lamb shift (LS) [in principle 2 numbers: r_p & R_∞]

$$\Delta E_{LS} = \Delta E_1 + \Delta E_2 C(r_p^2) + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

$$C(r_p^2) = c_1 + c_2 r_p^2 + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

  – Lepton-proton scattering (Rosenbluth separation)

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\rm Mott}}{d\Omega}\frac{1}{1+\tau}\left(G_E^2 + \frac{\tau}{\varepsilon}G_M^2\right)(1+\delta_{\rm rad.}) + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

- The neglected sibling, the proton magnetic radius:

$$(r_p^M)^2 \equiv -(6/\mu_p)\, G'_M(0)$$

Proton charge & magnetic radius from DR

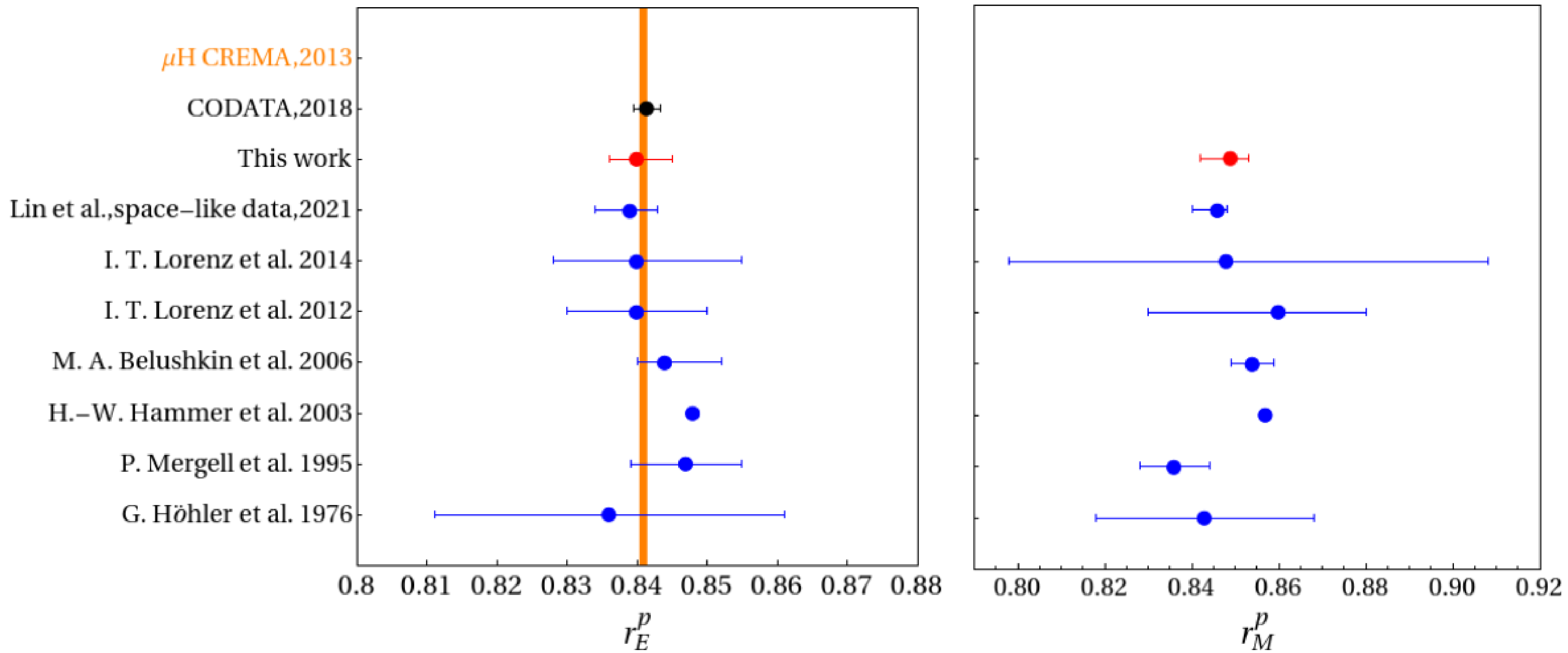
Lin, Hammer, UGM, Phys. Lett. B **816** (2021) 136254 [2102.11642 [hep-ph]]
Phys. Rev. Lett. **128** (2022) 052002 [2109.12961 [hep-ph]]

- Dispersion relations – determination incl. statistical and systematic errors:

$$r_E^p = 0.840^{+0.003+0.002}_{-0.002-0.002}\ \text{fm}\ ,\quad r_M^p = 0.849^{+0.003+0.001}_{-0.003-0.004}\ \text{fm}$$

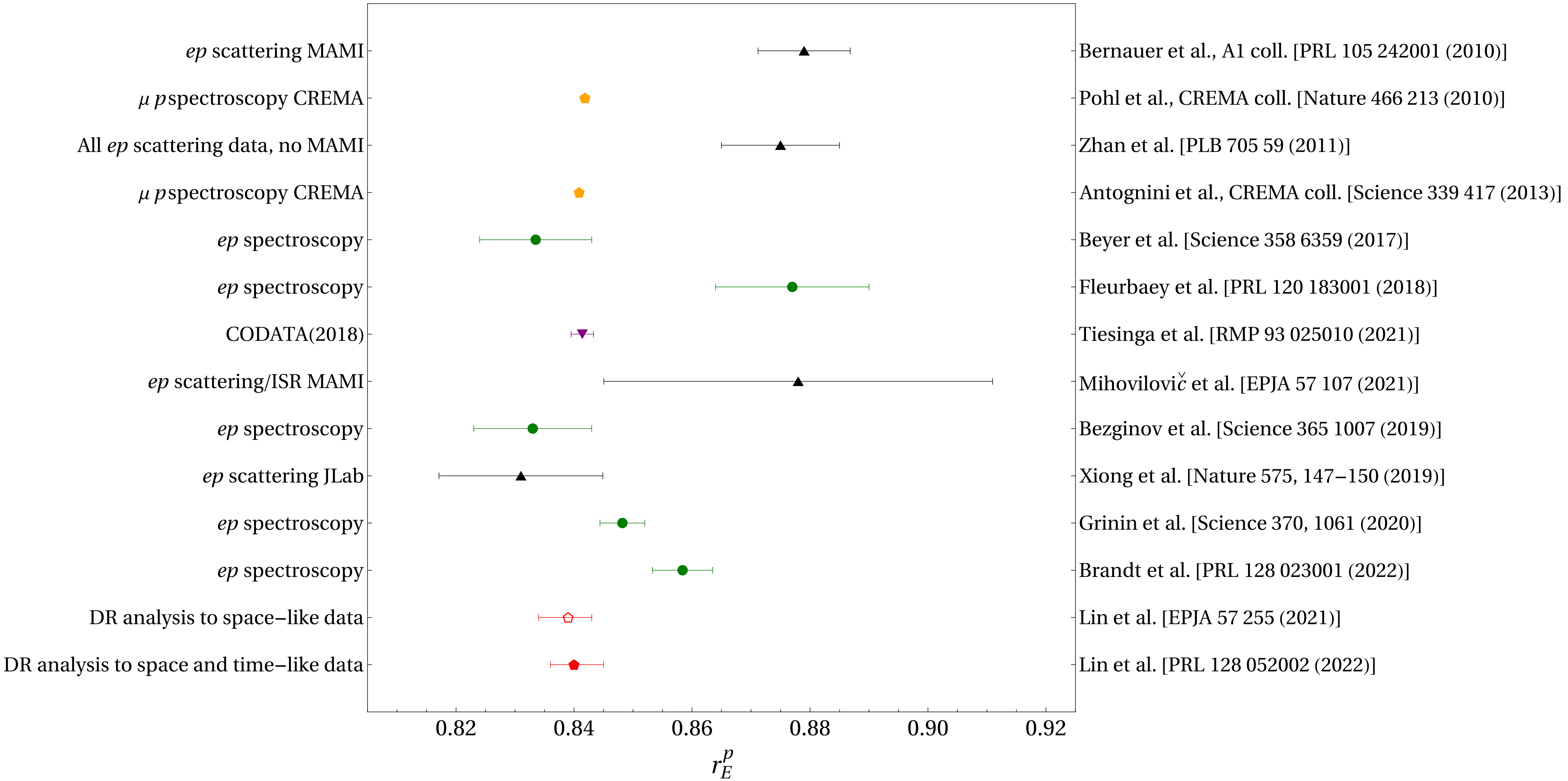
- Comparison to earlier DR determinations (some data)

Review on DR: Lin, Hammer, UGM, EPJA **57** (2021) 255



Proton charge radius cont'd

- Comparison to recent measurements:



Neutron radii

- The charge squared neutron $(r_E^n)^2$ radius was mostly input in DR analyses, but not the magnetic one

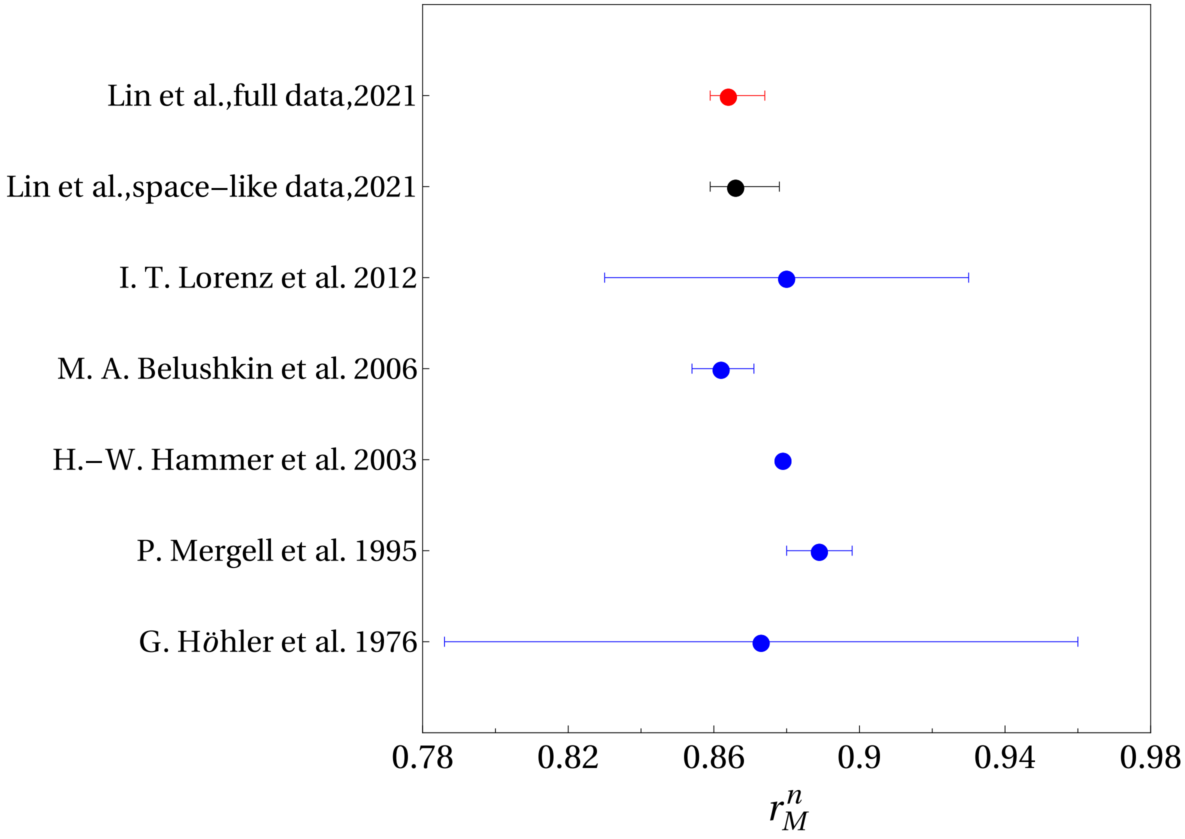
$$r_M^n = 0.864^{+0.004+0.006}_{-0.004-0.001}\ \mathrm{fm}$$

$\hookrightarrow$ rather stable over time

$\hookrightarrow$ but larger variation

$\hookrightarrow$ always the largest em radius!

$\hookrightarrow$ lattice QCD gives rather comparable isovector radii (p& n)



Comparison with lattice QCD

- Compare isovector radii, these are free of disconnect diagrams
- Show only calculations at the physical pion mass

	r_E^V [fm]	r_M^V [fm]
Disp. rel.	0.900(2)(2)	0.854(1)(3)
Lattice/Mainz (new) [0]	0.882(12)(15)	0.814(7)(5)
Lattice/Cyprus [1]	0.920(19)(–)	0.742(27)(–)
Lattice/Mainz [2]	0.894(14)(12)	0.813(18)(7)
Lattice/ETMC [3]	0.827(47)(5)	—-
Lattice/PACS [4]	0.785(17)(21)	0.758(33)(286)
Lattice/MIT [5]	0.787(87)	—-

[0] D. Djukanovic et al., arXiv:2309.07491

[1] C. Alexandrou et al., in preparation (values PRELIMINARY, 09/22)

[2] D. Djukanovic et al., Phys. Rev. D **103** (2021) 094522 [2102.07460 [hep-lat]].

[3] C. Alexandrou et al., Phys. Rev. D **101** (2020) 114504 [2002.06984 [hep-lat]]

[4] E. Shintani et al., Phys. Rev. D **99** (2019) 014510 [E] Phys. Rev. D **102** (2020) 019902

[5] N. Hasan et al., Phys. Rev. D **97** (2018) 034504 [1711.11385 [hep-lat]]

A new magnetic puzzle? let's wait ...

The proton radius from J/ψ decays

Y.-H. Lin, F.-K. Guo, UGM, arXiv:2309.07850

related work: J. Guttmann, M. Vanderhaeghen, Phys. Lett. B **719** (2013) 136

The proton charge radius from J/ψ decays

Lin, Guo, UGM, arXiv:2309.07850

- BESIII has a tremendous sample ($\sim 10^{10}$) of J/ψ decays:

  $\hookrightarrow$ study the sensitivity of $J/\psi \to p\bar{p}e^+e^-$ to the nucleon em form factors

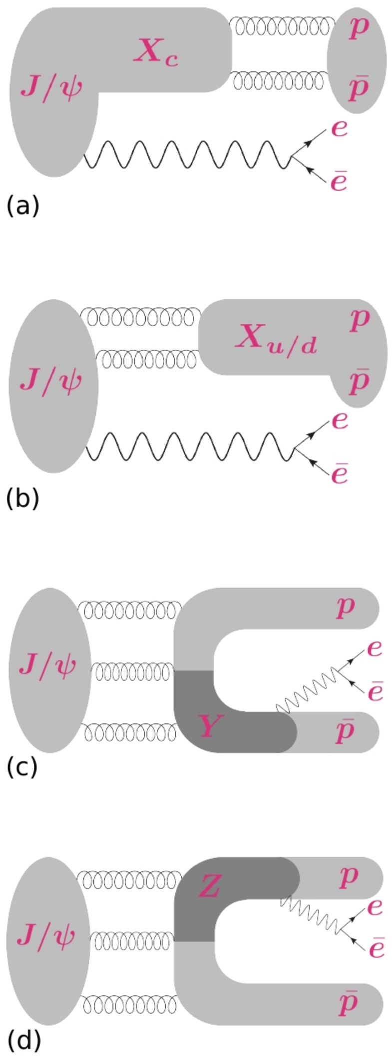
  $\hookrightarrow$ e^+e^- threshold at 1.05×10^{-6} GeV^2 (never reached !)

- X-type: the same for $p\bar{p}$ and $n\bar{n}$

- Y- and Z-type (assume CPT):

  $\hookrightarrow$ proton $\to$ EMFFs

  $\hookrightarrow$ Delta $\to$ model the N^* background



so that

$$|\mathcal{M}|^2 = |\mathcal{M}_{Y+Z}|^2 + \underbrace{\left(\mathcal{M}_{Y+Z}\mathcal{M}_X^* + \mathcal{M}_{Y+Z}^*\mathcal{M}_X\right)}_{\mathcal{M}_{\rm mix}} + |\mathcal{M}_X|^2$$

$\hookrightarrow$ subtracting the $J/\psi \to n\bar{n}e^+e^-$ data

The pertinent kinematic region

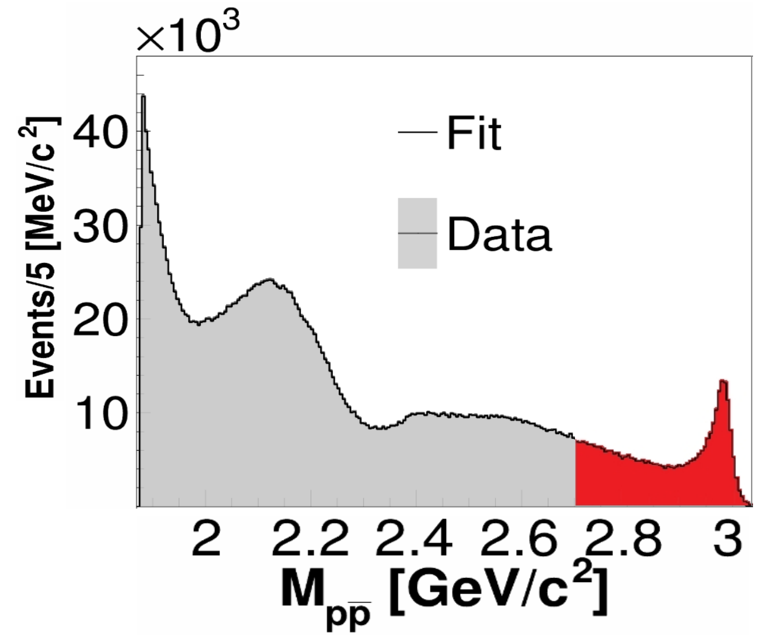
Lin, Guo, UGM, arXiv:2309.07850

- Selection of the $m_{p\bar{p}}$ region: search for best control of the backgrround

- $p\bar{p}$ invariant mass distribution in $J/\psi \to p\bar{p}\gamma$ from data taken in 2009-2018

Region of $m_{p\bar{p}}$ [GeV]	X resonances
~ 1.95	$X(1385)$
$2.0 \sim 2.2$	$f_0(2020), f_0(2100), f_0(2200)$ $a_1(1930)$ $f_2(2010)$
$2.7 \sim 3.05$	η_c

R. Kappert, PhD thesis, U. Groningen (2022)



- Signal:

$$\frac{d\Gamma_{\rm signal}}{dm_{e^+e^-}} = \int_{2.7\ \rm GeV}^{M_{J/\psi}-m_{e^+e^-}} dm_{p\bar{p}} \int d\cos\theta_p^* d\cos\theta_e' d\phi\, d\Gamma_{\rm signal}$$

$$d\Gamma_{\rm signal} \sim |\mathcal{M}|^2_{\rm signal} = |\mathcal{M}^N_{Y+Z}|^2 + \mathcal{M}^{N+\eta_c}_{\rm mix}$$

The pertinent kinematic region II

- N and Δ vertices in type-Y,Z diagrams:

$$\frac{dR}{dm_{e^+e^-}\,dm_{p\bar{p}}} \sim \int \cdots \frac{d\Gamma^{N}_{Y+Z}}{d\Gamma^{N+\Delta}_{Y+Z}}$$

$$\Gamma^{\mu}_{\gamma NN}(q) = ie\left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m_N} q_\nu F_2(q^2)\right)$$

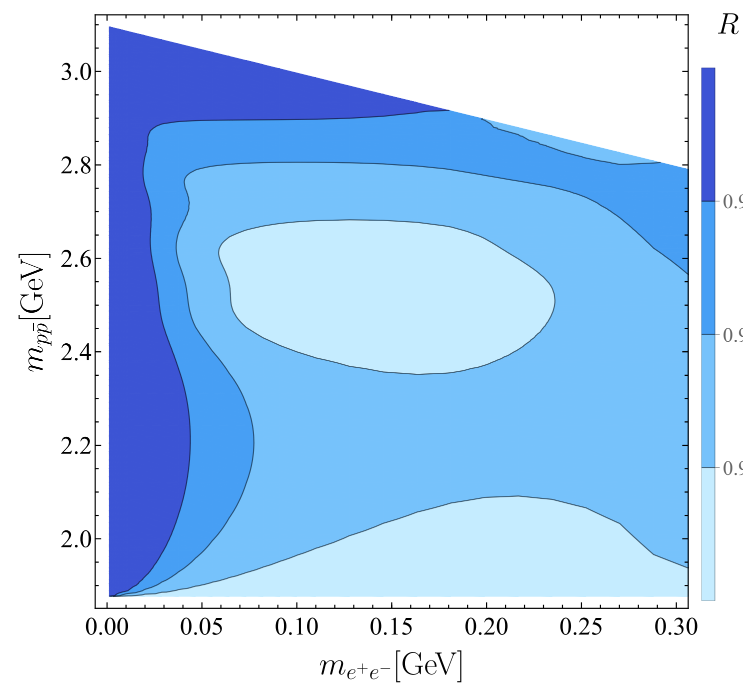
$$\Gamma^{\alpha\mu}_{\gamma\Delta N}(q,p_\Delta) = ie\sqrt{\frac{2}{3}}\frac{3(m_N+m_\Delta)}{2m_N((m_\Delta+m_N)^2-q^2)} \times g_M^{\Delta}(q^2)\epsilon^{\alpha\mu\rho\sigma}p_{\Delta,\rho}q_\sigma$$

Pascalutsa, Vanderhaeghen, Yang, Phys. Rept. **437** (2007) 125

- J/ψ-vertices

$$\Gamma^{\mu}_{J/\psi N\bar{N}}(r,p_0) = g_S\left(\gamma^\mu - \frac{r^\mu}{M_{J/\psi}+2m_N}\right) + g_D e^{i\delta_1}\left(\gamma_\nu - \frac{r_\nu}{M_{J/\psi}+2m_N}\right)t^{\mu\nu}$$

$$\Gamma^{\mu\alpha}_{J/\psi\Delta\bar{N}}(r,p_0) = f_S\gamma_5 g^{\mu\alpha} + f_D e^{i\delta_2}\gamma_5 t^{\mu\alpha}$$

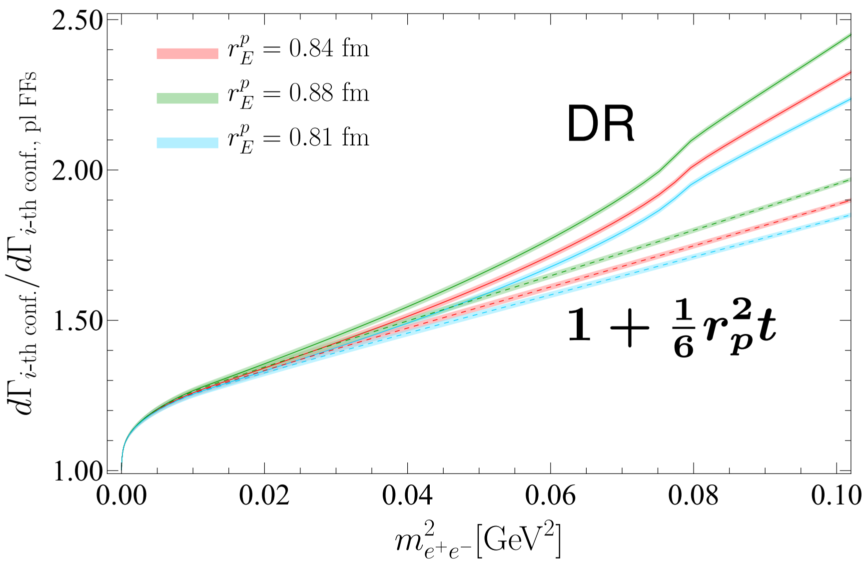


- Parameters determined from $\mathcal{B}(J/\psi \to p\bar{p})$, $\mathcal{B}(J/\psi \to \Delta\bar{p})$ and $\Gamma_{J/\psi}$

$\Rightarrow$ For $m_{p\bar{p}} \gtrsim 2.7\,\mathrm{GeV}$, the proton pole contribution dominates the type-Y,Z diagrams

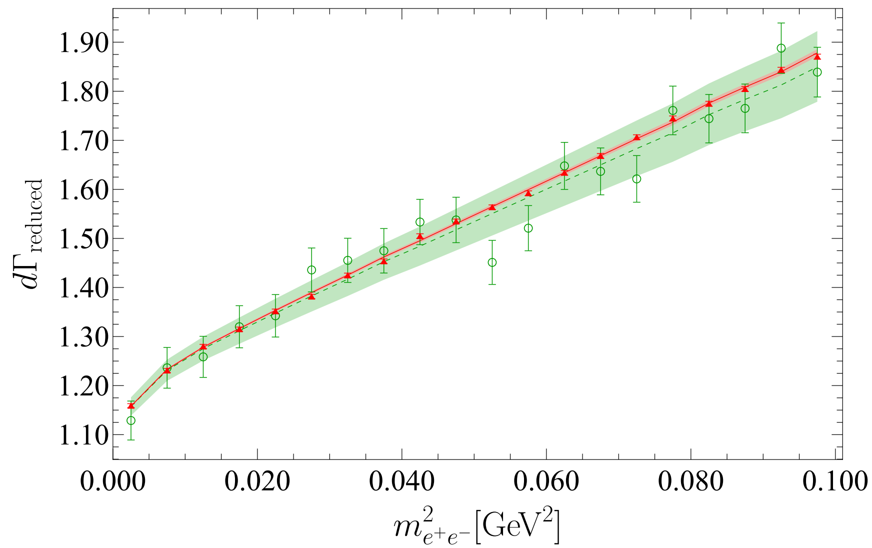
Sensitivity studies

- DR vs polynomial fit



- Normalized to point-like proton
  $\longrightarrow$ better controlf of the systematics
- Parameter variations under control
- Polynomial ansatz too simple!
  $\longrightarrow$ cusp at the opening of the 2π channel!

- Fits to synthetic data



- 10^4 events at BESIII
  $\hookrightarrow r_p = (0.828 \pm 0.040)$ fm
- 10^6 events at STCF   Achasov et al., 2303.15790 [hep-ex]
  $\hookrightarrow r_p = (0.846 \pm 0.004)$ fm

Essentials of Nuclear Binding

B. N. Lu, N. Li, S. Elhatisari, D. Lee, E. Epelbaum, UGM,
Phys. Lett. **B 797** (2019) 134863

A minimal nuclear interaction

- Basic idea:

  ↪ explore the approximate SU(4) spin-isospin symmetry of the nuclear forces

  Wigner (1936)

  ↪ particular friendly for MC simulations (suppression of sign oscillations)

  Chen, Lee, Schäfer, Phys. Rev. Lett. **93** (2004) 242302

  ↪ the ^{4}He nucleus is a prime candidate ($I = S = 0$)

- Ingredients:

  ↪ 2N & 3N forces (contact interactions)

  ↪ local & non-local smearing (generates range of these forces)

  ↪ use later as the LO action free of sign problem (simple Hamiltonian)

Essential elements for nuclear binding I

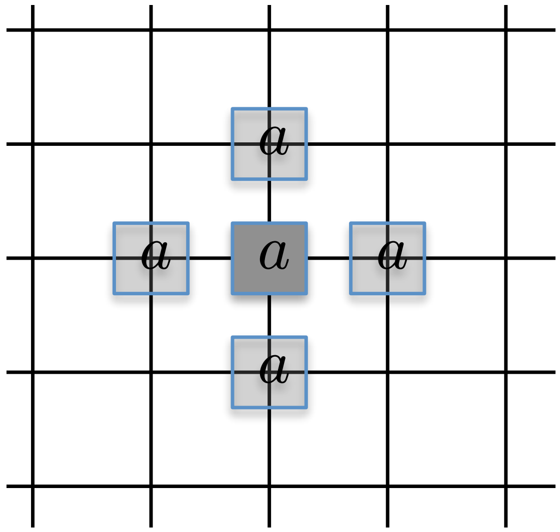
Lu, Li, Elhatisari, Epelbaum, Lee, UGM. Phys. Lett. B **797** (2019) 134863 [arXiv:1812.10928]

- Highly SU(4) symmetric LO action without pions, local and non-local smearing:

$$H_{\rm SU(4)} = H_{\rm free} + \frac{1}{2!} C_2 \sum_n \tilde\rho(n)^2 + \frac{1}{3!} C_3 \sum_n \tilde\rho(n)^3$$

$$\tilde\rho(n) = \sum_i \tilde a_i^\dagger(n) \tilde a_i(n) + s_L \sum_{|n'-n|=1} \sum_i \tilde a_i^\dagger(n') \tilde a_i(n')$$

$$\tilde a_i(n) = a_i(n) + s_{NL} \sum_{|n'-n|=1} a_i(n')$$



- Only **four** parameters!

  C_2 and C_3 = strength of the leading two- and three-body interactions

  s_L and s_{NL} = strength of the local and the non-local interaction

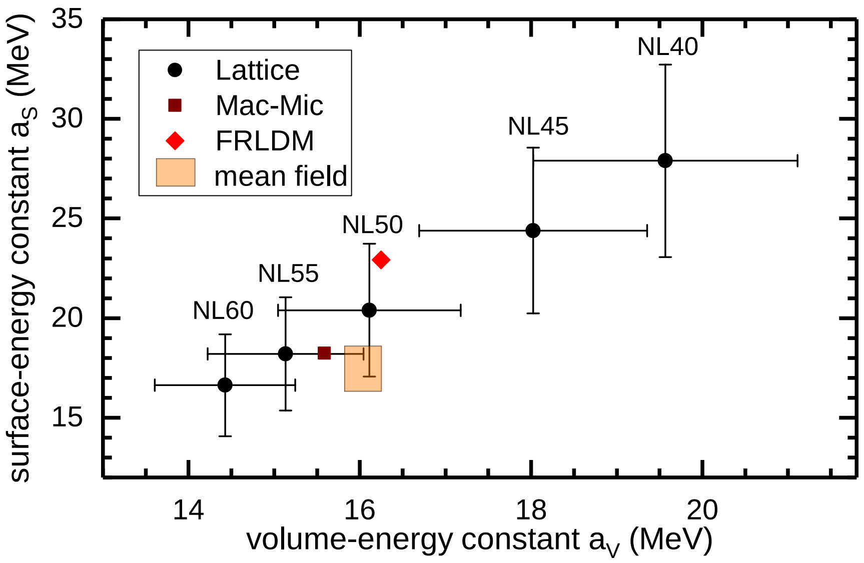
Essential elements for nuclear binding II

- Fixing the parameters:
  - ⋆ interaction strength C_2 and range s_L from the average S-wave scattering lengths and effective ranges (requires SU(4) breaking later)
  - ⋆ interaction strength C_3 from the ^{3}H binding energy
  - ⋆ interaction range s_{NL} can not be determined in light nuclei

  ↪ calculate the volume- and surface energy of mid-mass nuclei $16 \leq A \leq 40$

- compare w/ existing calculations:

↪ $\boxed{s_{NL} = 0.5}$



Mac-Mic: Wang et al., Phys. Lett. B **734** (2014) 215

FRLDM: Möller et al., Atom Data Nucl. Data Tabl. **59** (1995) 184

mean field: Bender et al., Rev. Mod. Phys. **75** (2003) 121

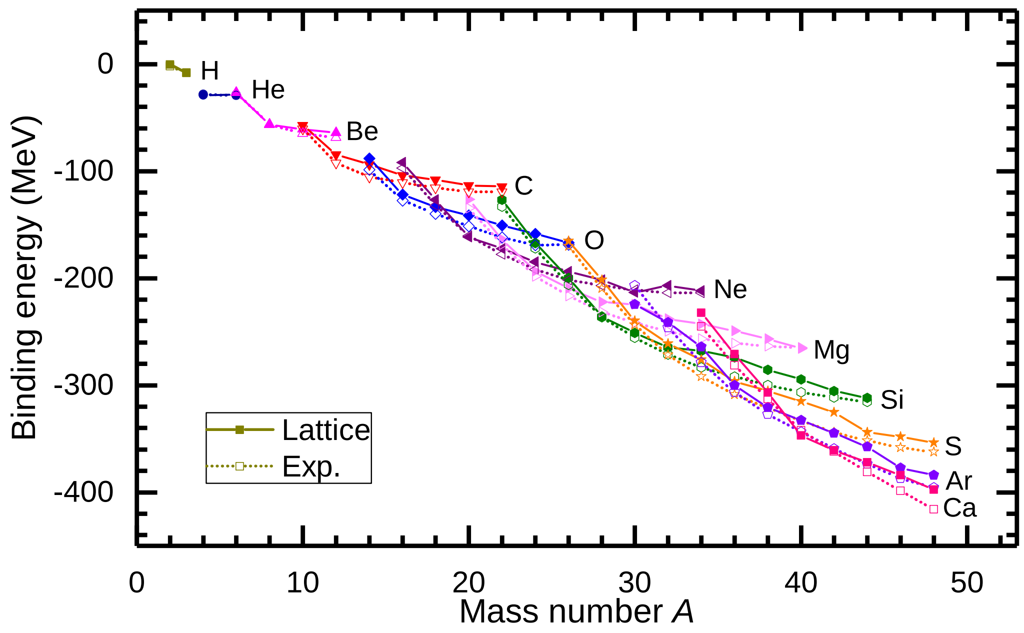
Energies for selected nuclei

- Calculated binding energies for 3N & alpha-type nuclei:

	B [MeV]	Coul. [MeV]	B/Exp.
^{3}H	8.48(2)*	0.0	1.00
^{3}He	7.75(2)	0.73(1)	1.00
^{4}He	28.89(1)	0.80(1)	1.02
^{16}O	121.9(3)	13.9(1)	0.96
^{20}Ne	161.6(1)	20.2(1)	1.01
^{24}Mg	193.5(17)	28.0(2)	0.98
^{28}Si	235.8(17)	37.1(4)	1.00
^{40}Ca	346.8(8)	71.7(6)	1.01

[* = input]

- Binding energies for 86 even-even nuclei



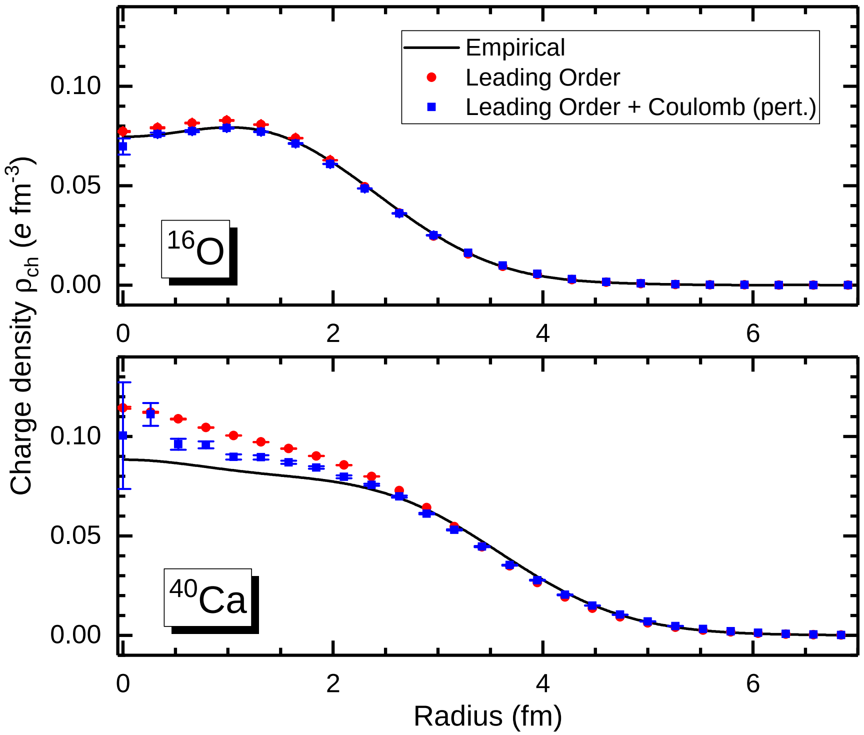
- selected nuclei: amazingly precise, all deviations $\leq 4\%$ (except ^{12}C)
- even-even isotopic chains come out amazingly precise, general trends reproduced
  $\hookrightarrow$ on the proton-rich side better than on the neutron-rich one $\rightarrow$ spin-dep. effects
- but remember: this is only leading order!

Radii for selected nuclei

- Calculated charge radii
  for 3N & alpha-type nuclei:

	R_{ch}	Exp.	R_{ch}/Exp.
^{3}H	1.90(1)	1.76	1.08
^{3}He	1.99(1)	1.97	1.01
^{4}He	1.72(3)	1.68	1.02
^{16}O	2.74(1)	2.70	1.01
^{20}Ne	2.95(1)	3.01	0.98
^{24}Mg	3.13(2)	3.06	1.02
^{28}Si	3.26(1)	3.12	1.04
^{40}Ca	3.42(3)	3.48	0.98

- Charge distributions
  for ^{16}O and ^{40}Ca



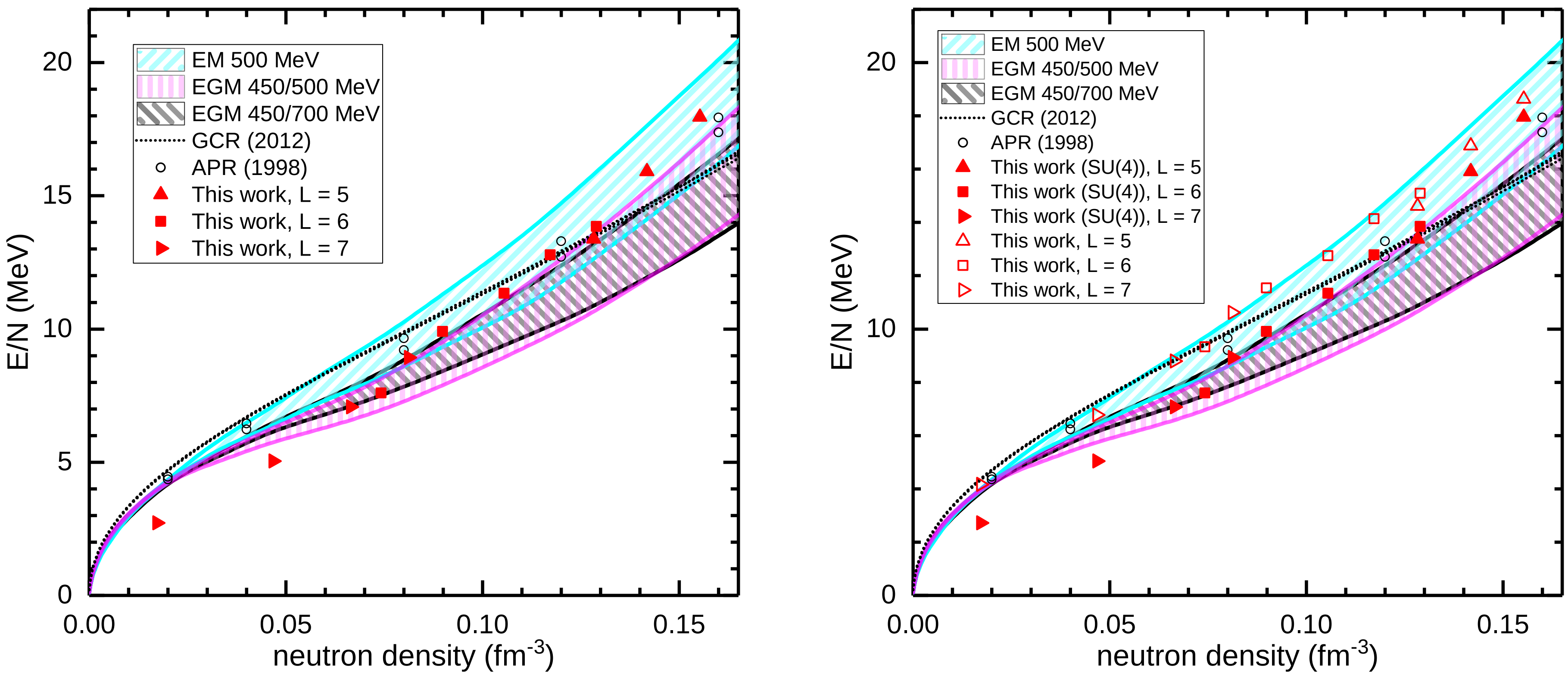
- Radii quite well described (except ^{12}C)

↪ overcomes earlier problems (see PRL 109 (2012) 252501, 112 (2014) 102501)

- Also a fair description of the charge distributions at LO!

Neutron matter

- 14 to 66 neutrons in $L = 5, 6, 7 \rightarrow \rho = 0.02 - 0.15\,\text{fm}^{-3}$



- exact SU(4)

  $\hookrightarrow$ deviations at low densities

- SU(4) breaking term $\rightarrow a_{nn}$ $\checkmark$

  $\hookrightarrow$ good overall description

APR = Akmal, Pandharipande, Ravenhall, Phys. Rev. C **58** (1998) 1804; GCR = Gandolfi, Carlson, Reddy, Phys. Rev. C **85** (2012) 032801; all others in: Tews et al., Phys. Rev. Lett. **110** (2013) 032504.

Ab initio calculation of the ^{4}He transition form factor

UGM, S. Shen, S. Elhatisari, D. Lee, 2309.01558 [nucl-th]

Basic considerations

- Use the essential elements action, **all parameters fixed!**

- Calculate the transition ff and its low-energy expansion form the transition density

$$\rho_{\rm tr}(r) = \langle 0_1^+ | \hat\rho(\vec r) | 0_2^+ \rangle$$

$$F(q) = \frac{4\pi}{Z}\int_0^\infty \rho_{\rm tr}(r) j_0(qr) r^2 dr = \frac{1}{Z}\sum_{\lambda=1}^{\infty}\frac{(-1)^\lambda}{(2\lambda+1)!} q^{2\lambda}\langle r^{2\lambda}\rangle_{\rm tr}$$

$$\frac{Z|F(q^2)|}{q^2} = \frac{1}{6}\langle r^2\rangle_{\rm tr}\left[1 - \frac{q^2}{20}\mathcal{R}_{\rm tr}^2 + \mathcal{O}(q^4)\right]$$

$$\mathcal{R}_{\rm tr}^2 = \langle r^4\rangle_{\rm tr}/\langle r^2\rangle_{\rm tr}$$

- The first excited state sits in the continuum & close to the ^3H-p threshold

  $\hookrightarrow$ use large volumes $L = 10, 11, 12$ or $L = 13.2$ fm, 14.5 fm, 15.7 fm

  $\hookrightarrow$ the lattice spacing is fixed to $a = 1.32$ fm, corresponding $\Lambda = \pi/a = 465$ MeV

The first excited state

- 3 coupled channels with $\mathbf{0^+}$ q.n's $\to$ accelerates convergence as $L_t \to \infty$
- Shell-model wave functions (4 nucleons in $1s_{1/2}$, twice 3 in $1s_{1/2}$ and 1 in $2s_{1/2}$)

L [fm]	$E(0_1^+)$ [MeV]	$E(0_2^+)$ [MeV]	ΔE [MeV]
13.2	−28.32(3)	−8.37(14)	0.28(14)
14.5	−28.30(3)	−8.02(14)	0.42(14)
15.7	−28.30(3)	−7.96(9)	0.39(9)

$\hookrightarrow$ statistical and large-L_t errors

$\hookrightarrow$ agreement w/ experiment: $E(0_1^+) = 28.3$ MeV, $\Delta E = 0.4$ MeV

$\hookrightarrow$ ΔE consistent w/ no-core Gamov shell model

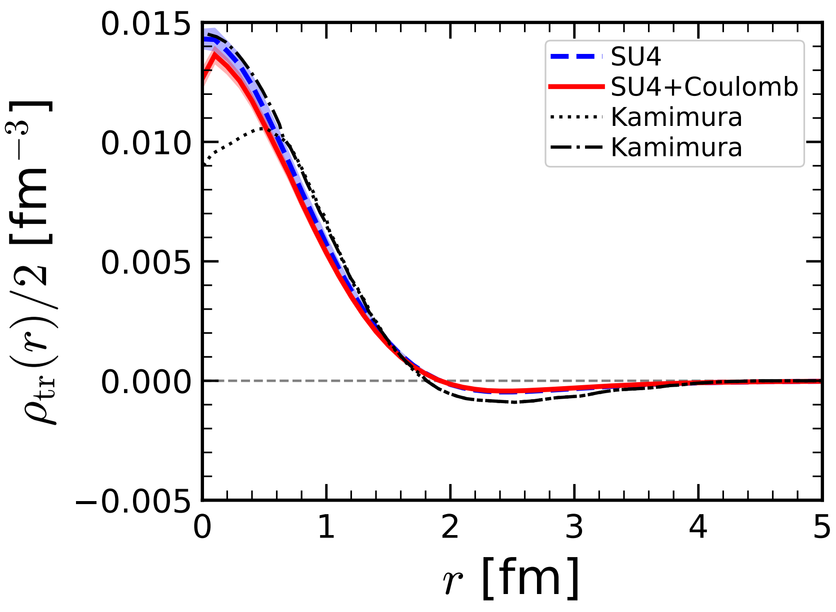
2306.05192 [nucl-th]

$\hookrightarrow$ consistent w/ the Efimov tetramer analysis $\Delta E = 0.38(2)$ MeV

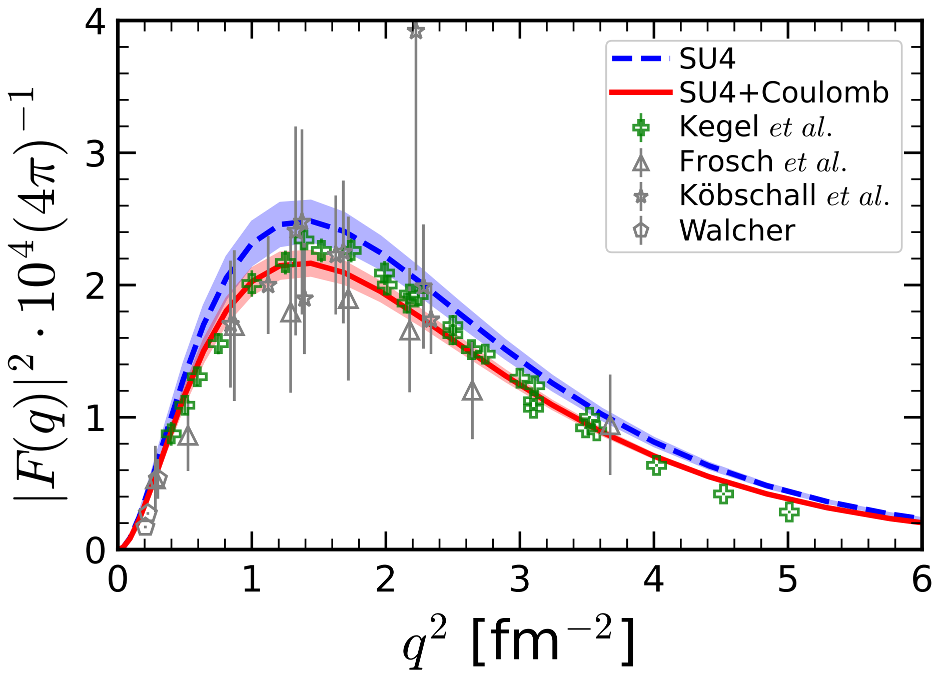
von Stecher, D'Incao, Greene, Nat. Phys. **5** (2009) 417; Hammer, Platter, EPJA **32** (2007) 113

The transition form factor

- Transition charge density



- Transition form factor



↪ agrees with the reconstructed one from Kamimura PTEP **2023** (2023) 071D01

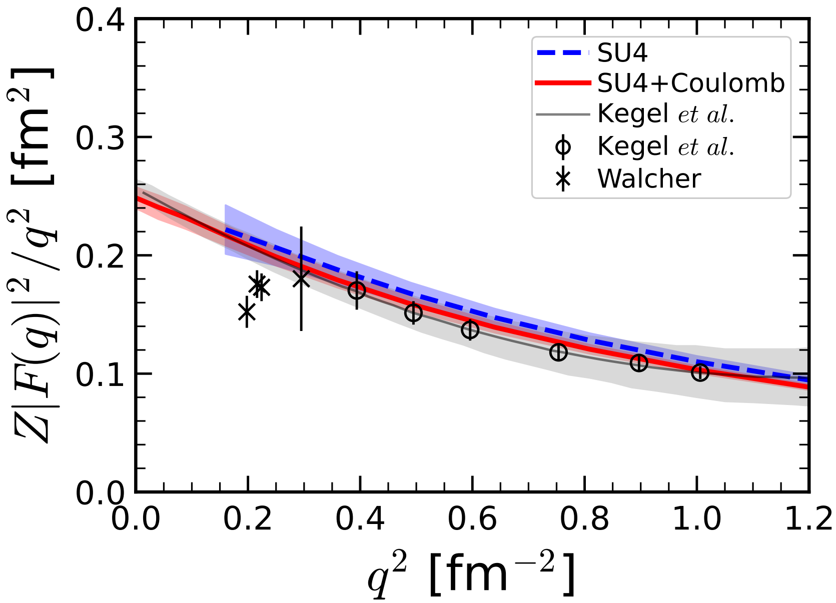
↪ very small central depletion (no zero)

↪ excellent description of the data

↪ Coulomb required plus smaller uncertainty (improved signal)

The transition form factor II

- Small momentum expansion



	$\langle r^2 \rangle_{tr}$ [fm^2]	$\mathcal{R}_{tr}$ [fm]
Experiment	1.53 ± 0.05	4.56 ± 0.15
Th (AV8'+ centr. 3N)*	1.36 ± 0.01	4.01 ± 0.05
Th (AV18 + UIX)	1.54 ± 0.01	3.77 ± 0.08
Th (NLEFT)	1.49 ± 0.01	4.00 ± 0.04

*Hiyama, Gibson, Kamimura, PRC **70** (2004) 031001

$\hookrightarrow$ Also consistent description of the low-energy data

$\hookrightarrow$ **No puzzle** to the nuclear forces!

$\hookrightarrow$ Can be improved using N3LO action + wave function matching

Elhatisari et al., 2210.17488 [nucl-th]

Summary & outlook

- Presented a new method to determine the proton charge radius from $J/\psi \to p\bar{p}e^+e^-$ and $J/\psi \to n\bar{n}e^+e^-$ decays

  $\hookrightarrow$ studied sensitivity for BESIII and the future STCF $\to$ promising

  $\hookrightarrow$ method can be applied to other charged hadrons such as $\Sigma^\pm, \Xi^-$

- Investigated the ^{4}He transition form factor using NLEFT w/o tuning any parameter

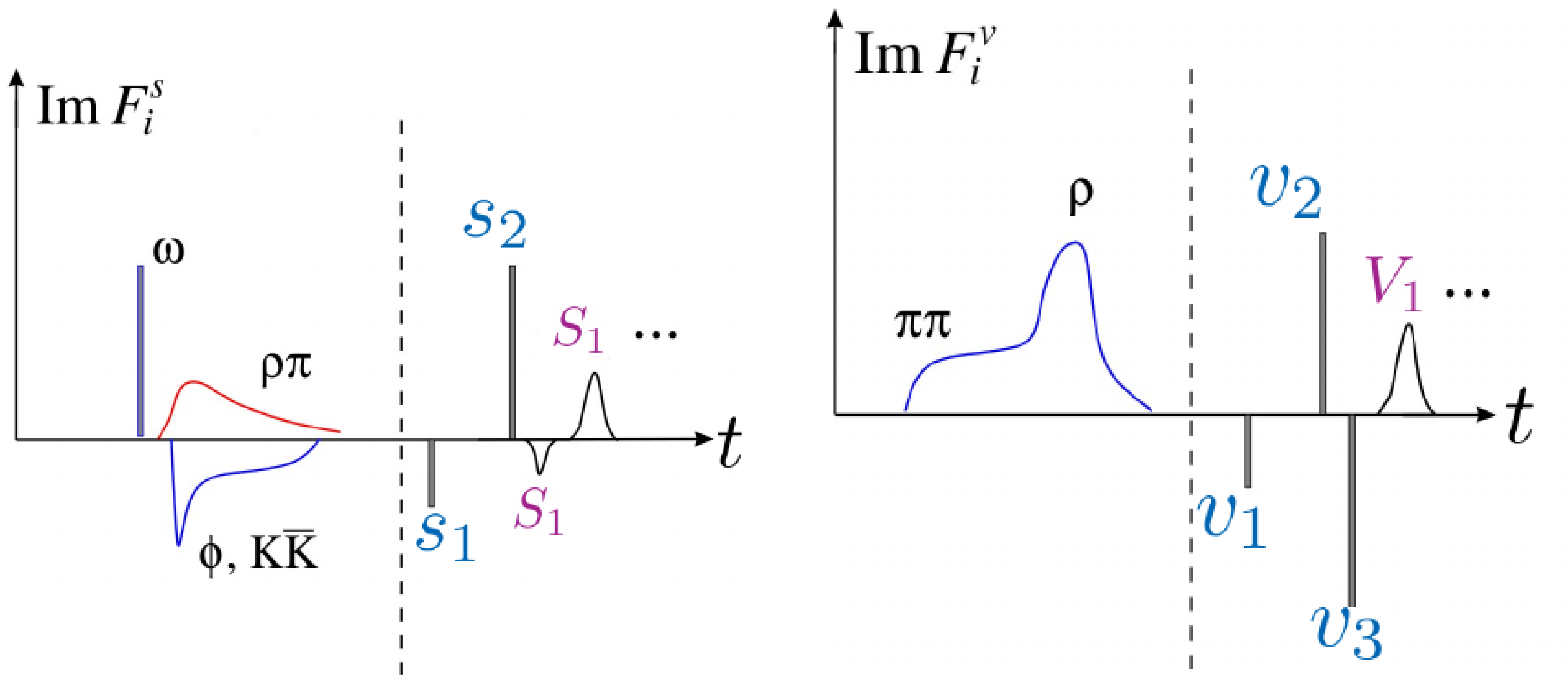
  $\hookrightarrow$ minimal nuclear interaction gives a good description of the data

  $\hookrightarrow$ no problem to the nuclear forces

Spares

Summary: spectral functions

- Cartoons of the isoscalar/isovector spectral functions:

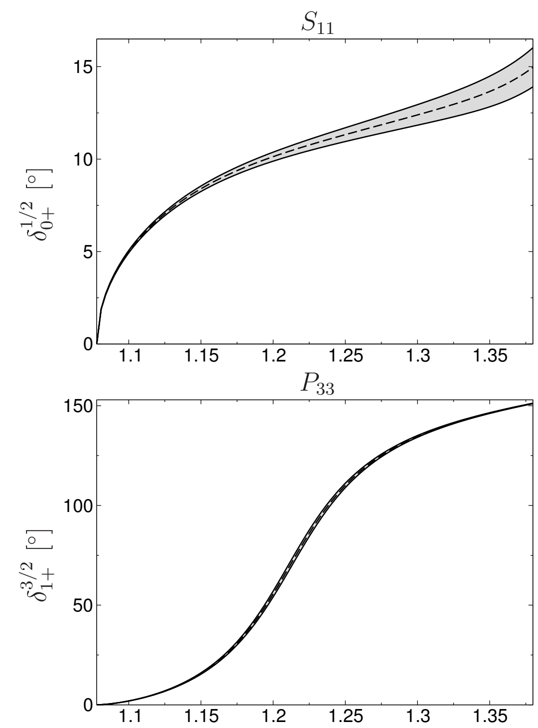


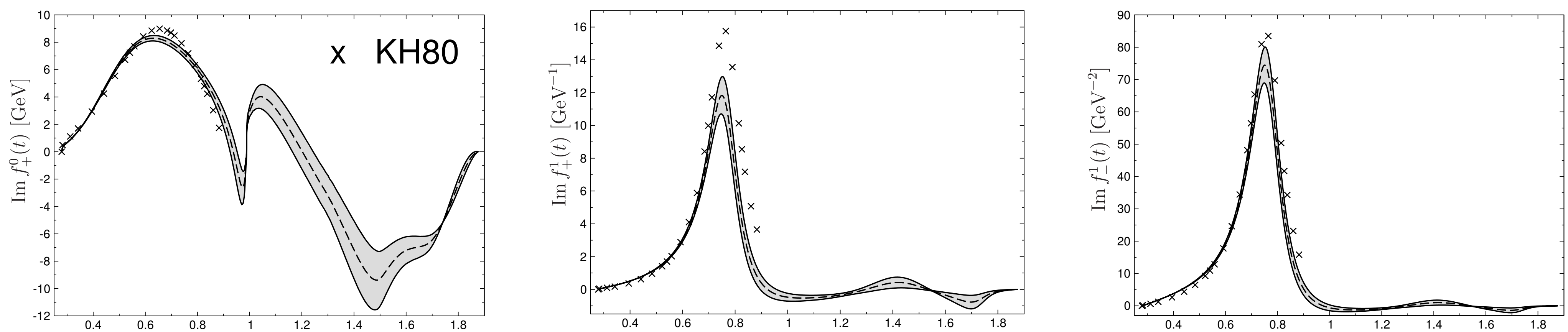
Once more on the isovector spectral functions

Hoferichter, Kubis, Ruiz de Elvira, Hammer, UGM, Eur. Phys. J. A **52** (2016)331 [arXiv:1609.06722 [nucl-th]]

Roy-Steiner equation analysis

- improve the isovector spectral functions by
  - $\hookrightarrow$ updated πN amplitudes from Roy-Steiner equations
  - $\hookrightarrow$ include modern data (esp. pionic hydrogen & deuterium)
  - $\hookrightarrow$ better treatment of isospin-violating effects
  - $\hookrightarrow$ construct the pion FF from precise knowledge of $\delta_1^1(s)$
  - $\hookrightarrow$ perform systematic error analysis

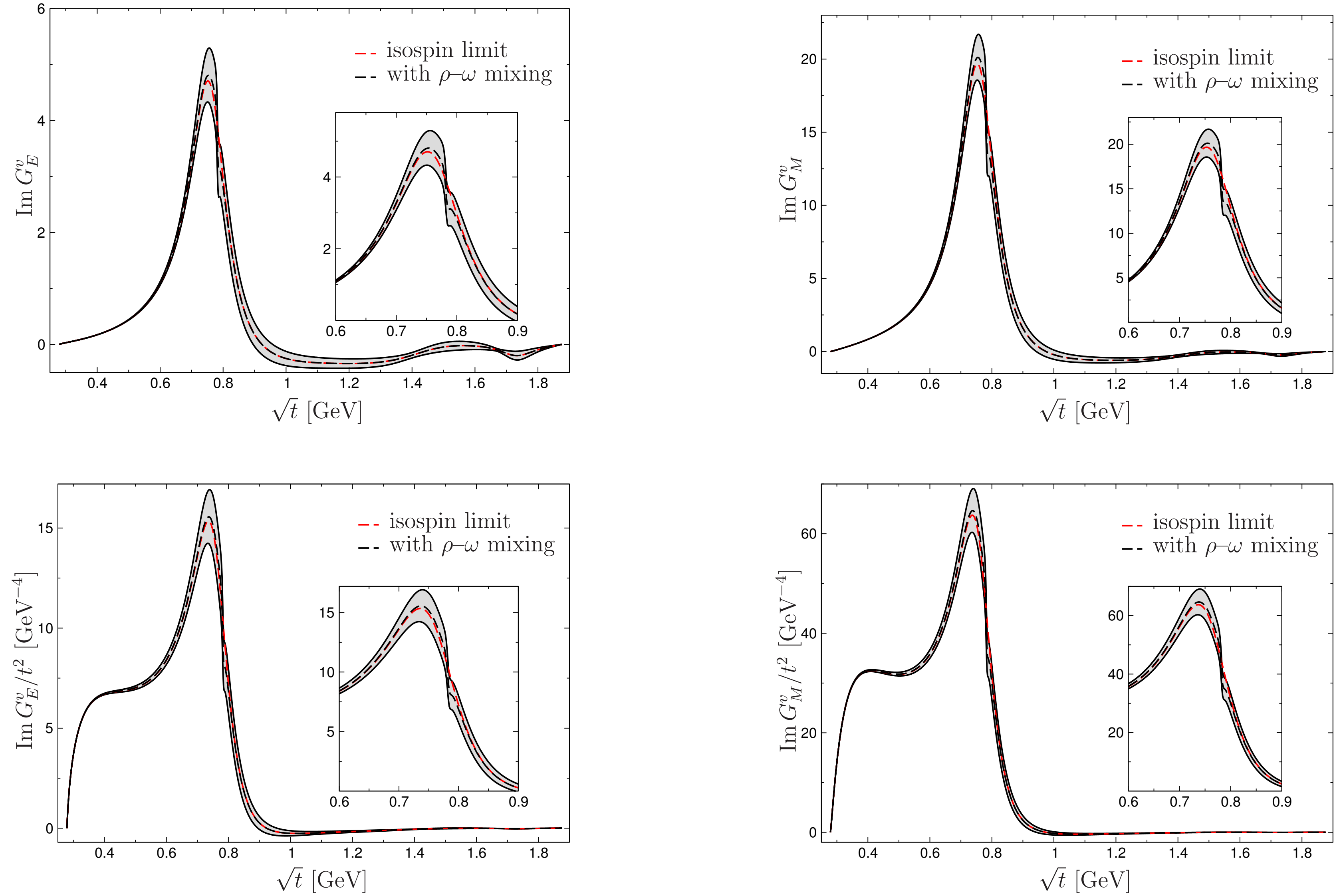




Hoferichter, Ruiz de Elvira, Kubis, UGM, Phys. Rev. Lett. **115** (2015) 092301; Phys. Rev. Lett. **115** (2015) 192301; Phys. Rept. **625** (2016) 1; J.Phys. G**45** (2018) 024001

New isovector spectral functions

- Precise determinations of the isovector spectral functions



Proton charge radius - leptonic Lamb shift

- Energy levels in hydrogen:

$$\boxed{E_{n\ell j} = R_\infty \left(-\frac{1}{n^2} + f_{n\ell j}\left(\alpha, \frac{m_e}{m_p}, \ldots\right) + \delta_{\ell 0} \frac{C_{\rm NS}}{n^3} r_p^2 \right)}$$

$$f_{n\ell j}\left(\alpha, \frac{m_e}{m_p}, \ldots\right) = X_{20}\alpha^2 + X_{30}\alpha^3 + X_{31}\alpha^3 \ln\alpha + X_{40}\alpha^4 + \ldots$$

– n, ℓ, j - principal, orbital, total ang. momentum quantum numbers

– $f_{n\ell j}$ - relativistic corr's, vacuum effects, other QED corrections

– m_e/m_p enters through the coefficients X_{20}, X_{30}, ... (recoil)

– $C_{\rm NS}$ calculable leading order correction due to the finite r_p

– higher order charge distributions are included in $f_{n\ell j}$

$\Rightarrow$ must measure at least 2 transitions to pin down the two unknowns

$\Rightarrow$ this is done in recent measurements, but not before! [inconsistency]