Fine-tunings in Nuclear Physics

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supported by CAS, PIFI

by DFG, SFB 1639

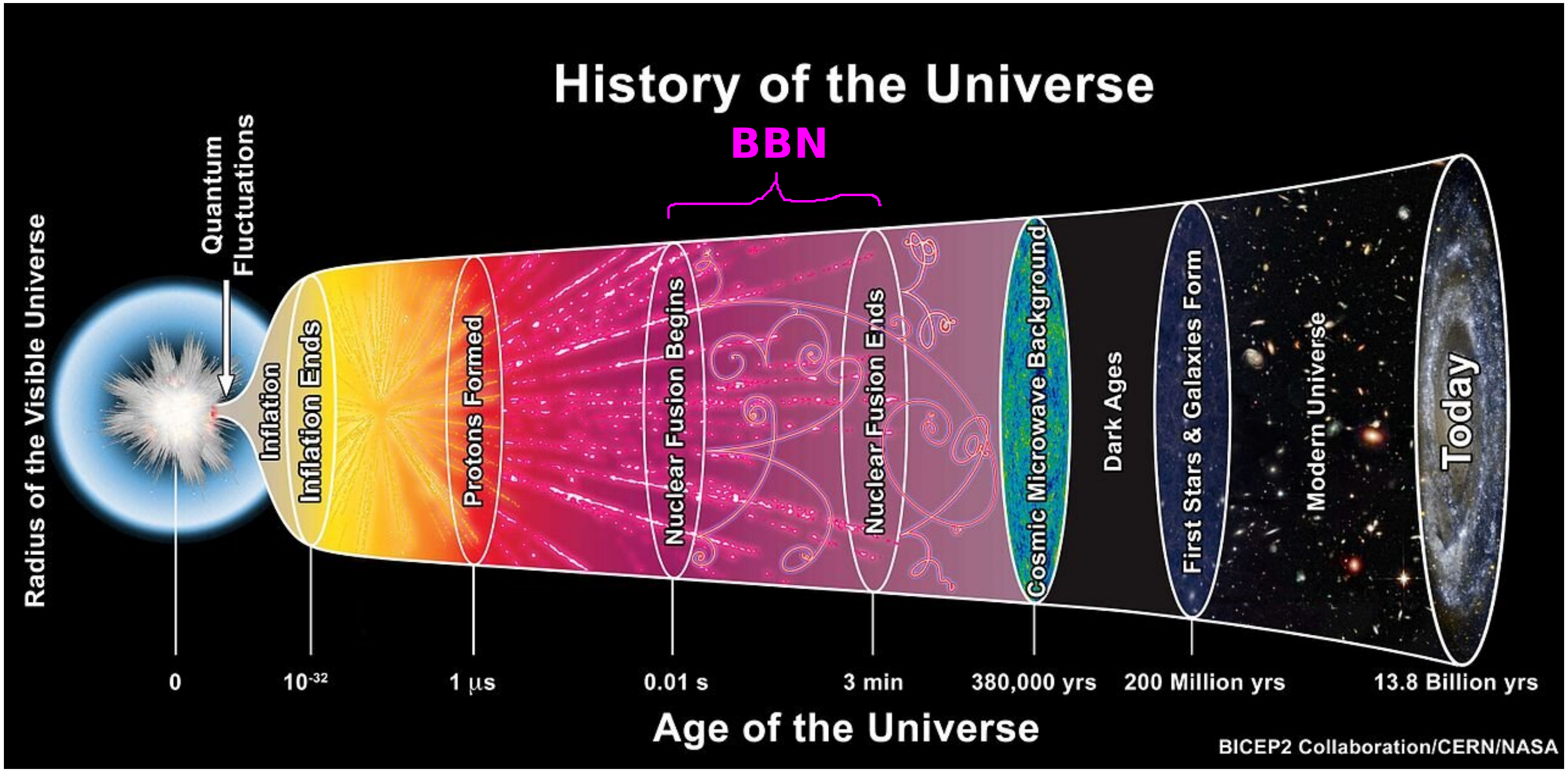
by ERC, EXOTIC

by NRW-FAIR

Contents

Definition of the physics problem

History of the Universe



- BBN is a fine probe of our understanding of fundamental physics

  Olive et al. (2000), Iocco et al. (2009), Cyburt et al. (2016), Pitrou et al (2018), ...

- Are the fundamental constants really constant? Dirac (1973), and many others

Primordial element generation

Fusion of d, $^3\mathrm{H}$, $^3\mathrm{He}$, $^4\mathrm{He}$, $^6\mathrm{Li}$, $^7\mathrm{Li}$ and $^7\mathrm{Be}$

3 min

1 min:
"deuterium bottleneck":
$n + p \to d + \gamma$ possible

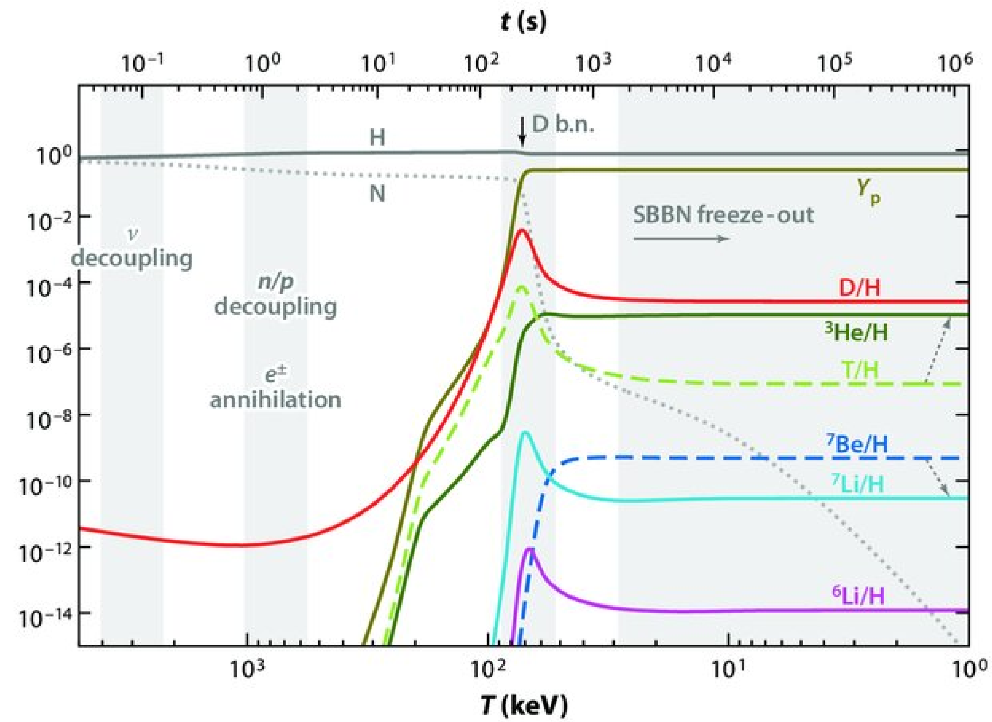
1 s: $n \leftrightarrow p$
freeze-out

- deuteron weakly bound

$\hookrightarrow$ deuterium bottleneck

$\hookrightarrow$ **first nuclear fine-tuning**

- also small $m_n - m_p$ ....

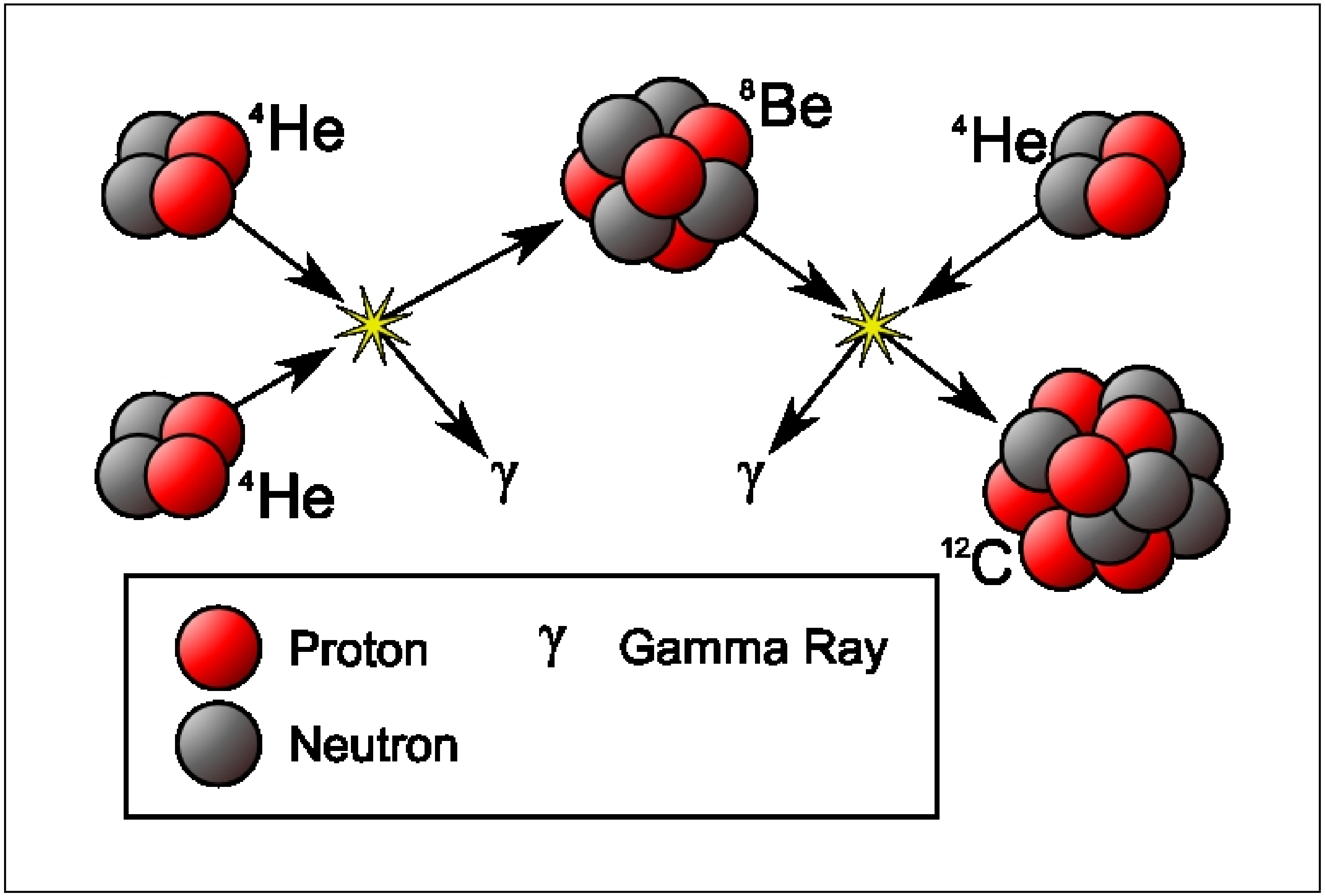


Pospelov, Pradler (2010)

Carbon nucleosynthesis

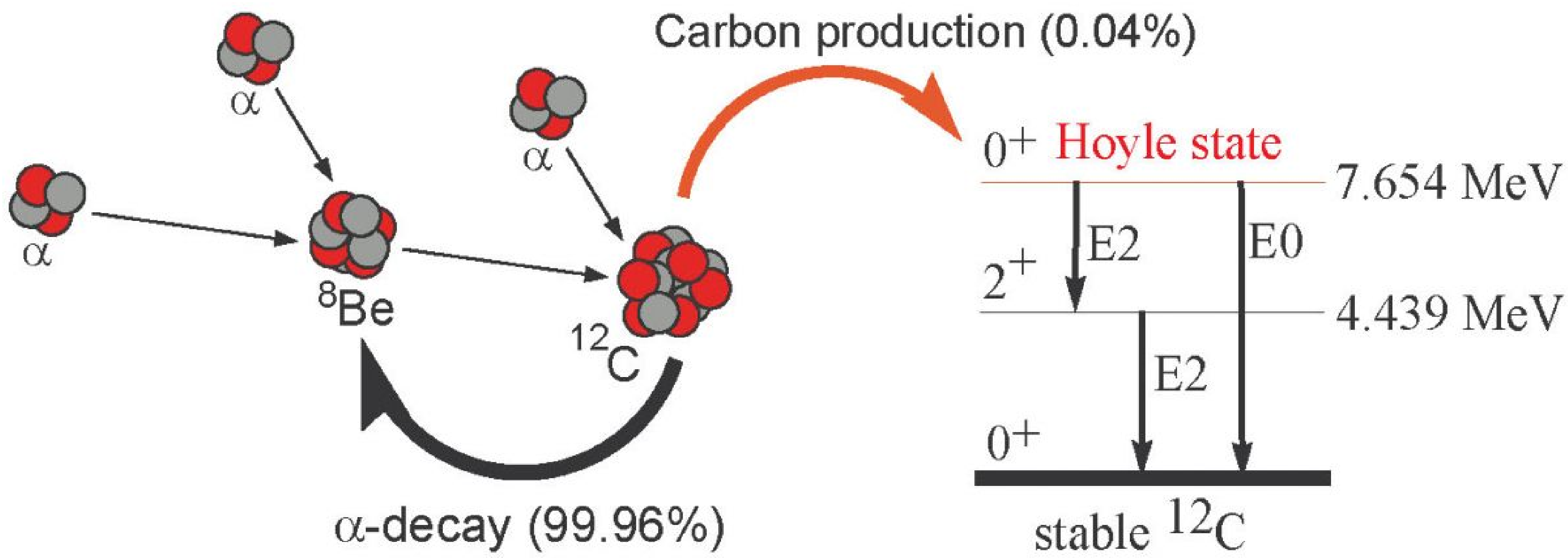
- Carbon is generated through the fusion of 3 helium nuclei (alpha-particles)

$\longrightarrow$ a short movie



@ Wikipedia

The triple-alpha process



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- The ^{8}Be nucleus is instable, long lifetime $(10^{-16}$ s) $\rightarrow$ 3 alpha-particles must meet
- The Hoyle state is located just above the continuum threshold
  $\rightarrow$ the excited carbon nuclei decay in various ways

$\hookrightarrow$ **we encounter 2 fine-tunings!**

BBN: some technicalities

Evolution of the abundances

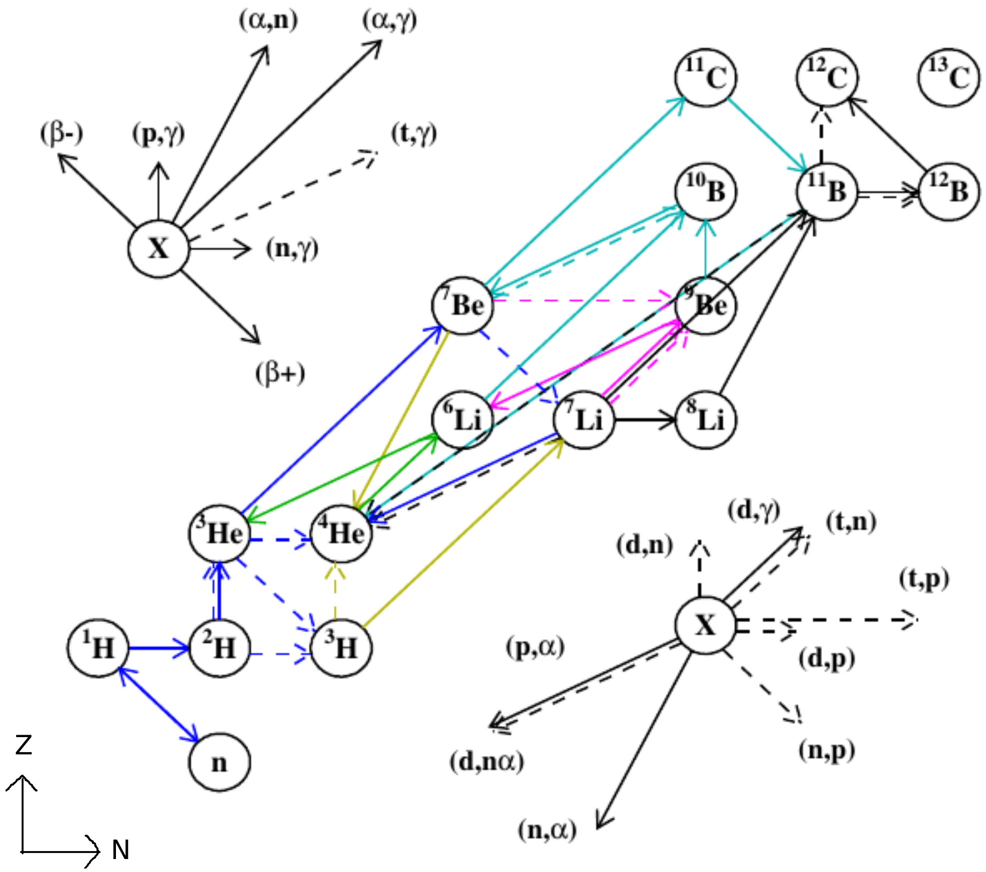
- Abundance defined via $Y_i = \dfrac{n_i}{n_b}$

  n_i = density of species i

  n_b = total baryon density

- Evolution depends on
  - cosmological model: Hubble expansion
  - **particle reactions** ($\Gamma_{ij\to kl} = n_b \langle \sigma v \rangle_{ij\to kl}$) and **decays** ($\Gamma_{i\to\ldots}$)

$\Rightarrow$ need to solve the system of rate equations:



Pitrou et al. (2018)

$$\boxed{\dot{Y}_i \supset -Y_i\Gamma_{i\to\ldots} + Y_j\Gamma_{j\to i+\ldots} + Y_kY_l\Gamma_{lk\to ij} - Y_iY_j\Gamma_{ij\to kl}}$$

Evolution of the abundances - results

• 5 different codes:

NUC123 (Kawano-code) [FORTRAN, 88 rate eqs.]

Kawano, FERMILAB-PUB-92-004-A (1992)

PRIMAT [Mathematica, 423 rate eqs.]

Pitrou et al., Phys. Rept. 754 (2018) 1

AlterBBN [C, 100 rate eqs.]

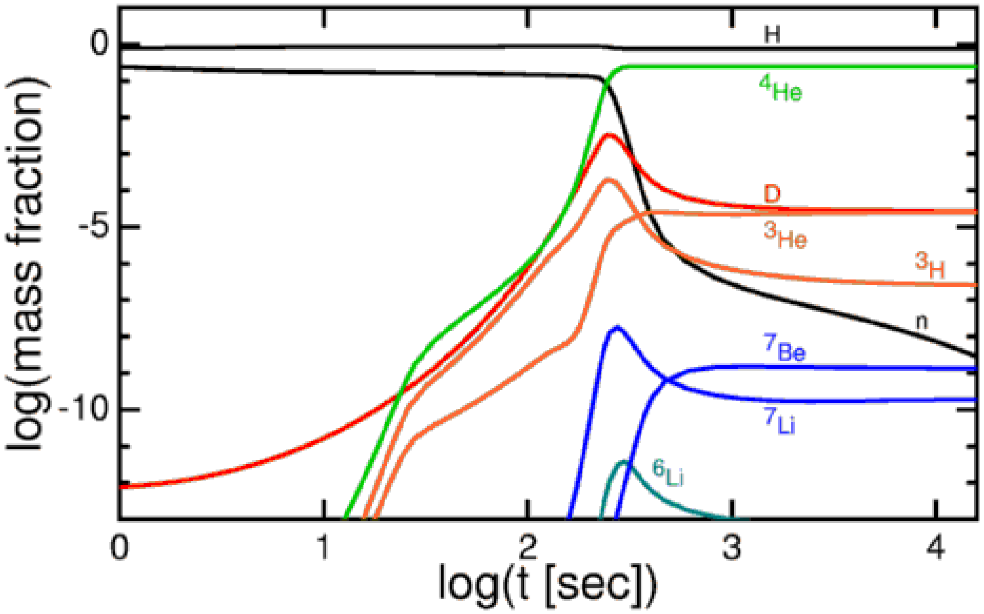
Arbey et al., Comp. Phys. Comm. 248 (2020) 106902

PArthENoPE [FORTRAN, 100 rate eqs.]

Gariazzo et al., Comp. Phys. Comm. 271 (2022) 108205

PRyMordial [Python, 423 rate eqs.]

Burns et al., Eur. Phys. J. C **84** (2024) 86



Burles, Nollett, Turner (2001)

• Altho' the results are by and large consistent in spite of the differences (# of rate eqs, QED, ...)

$\hookrightarrow$ when considering the variation of fundamental parameters such as $\alpha_{\rm EM}$,
we must use all these codes to have a grip on the systematics

Fundamental forces and constants I

- Must consider the strong, electromagnetic and weak interactions (v = Higgs VEV)

- **Strong interactions**: $\Lambda_{\rm QCD}$, m_q $(q = u, d, s)$, $m_f = g_f v$ (keep g_f constant)

$$\hookrightarrow \boxed{\Lambda_{\rm QCD} \sim \left(1 + \frac{\delta v}{v}\right)^{1/4}}$$

Agrawal et al. (1998), Burns et al. (2024)

- In almost all nuclear reactions, strong isospin violation $m_d/m_u \simeq 2$

  can be neglected because $\displaystyle\frac{m_u - m_d}{\Lambda_{\rm QCD}} \simeq \frac{1}{100}$

- $m_d \neq m_u$ features prominently in neutron β-decay

- Dominant strange quark effect through the nucleon mass (trace anomaly)

$$\boxed{m_N^{u,d,s} = \langle N(p)|m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s|N(p)\rangle}$$

- Quark mass variations = variations of the $\underbrace{\text{weak scale } v}_{\text{Higgs VEV}}$ (Yukawa's fixed)

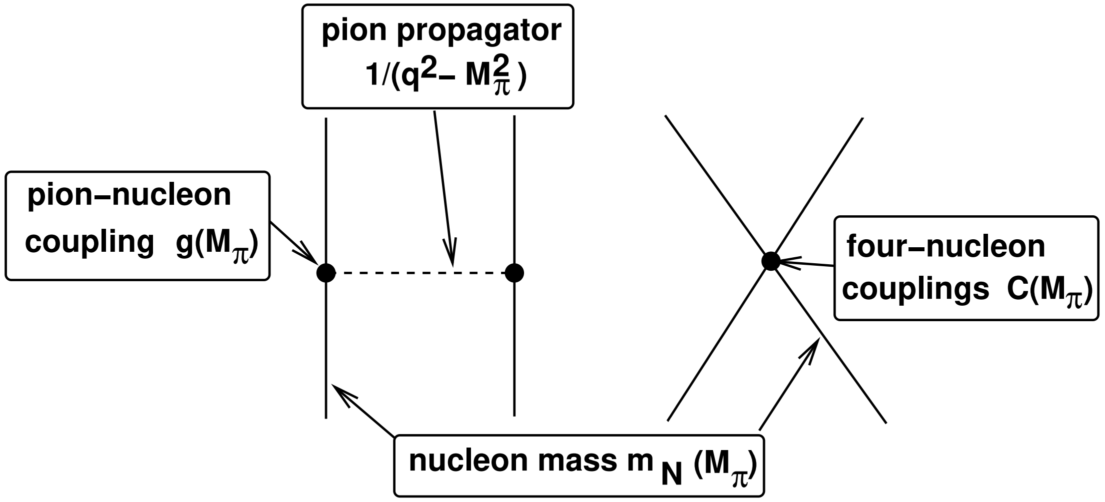
A closer look at the strong interactions

- Nuclear forces are given by chiral EFT based on Weinberg's power counting

Weinberg 1991

$\Rightarrow$ Pion-exchange contributions and short-distance multi-N operators

- graphical representation of the quark mass dependence of the LO potential



- always use the Gell-Mann–Oakes–Renner relation: $\boxed{M^2_{\pi^\pm} \sim (m_u + m_d)}$

- fulfilled to better than 94% in QCD

Colangelo, Gasser, Leutwyler 2001

- use input from lattice QCD and chiral perturbation theory

Fundamental forces and constants II

- **EM interactions**: $\hookrightarrow$ variations of the fine-structure constant $\boxed{\alpha_{\rm EM}}$
  around its present day value $\alpha_0 = 7.2973525693(11) \times 10^{-3}$

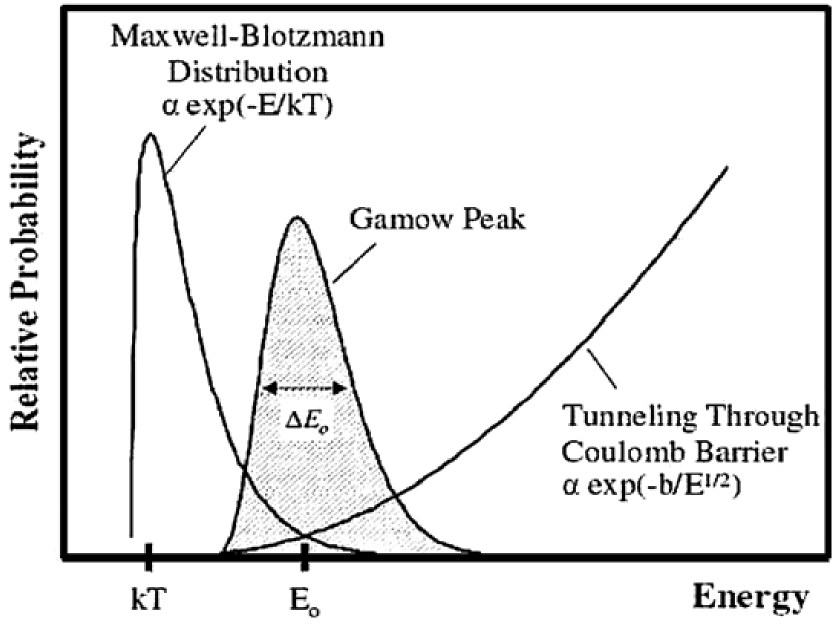
- Where does $\alpha_{\rm EM}$ appear in BBN?

  - Nuclear Rates: Coulomb barrier
    $\longrightarrow$ Gamow factor Gamow (1928)

  - Radiative capture reactions $\sigma \propto \alpha_{\rm EM}$

  - Weak rates: final-state Coulomb interaction
    in $n \leftrightarrow p$ rates and β-decays

  - Indirectly: n-p mass difference $Q_n = m_n - m_p$,
    EM contribution to nuclear binding energies
    $\longrightarrow$ reaction Q-values



from Trache (2010)

- **Weak interactions**: $\hookrightarrow$ variations of the Fermi constant $\boxed{G_F}$, m_f

BBN: results

UGM, Metsch, Eur. Phys. J. A **58** (2022) 212 [2208.12600 [nucl-th]]

UGM, Metsch, Meyer, Eur. Phys. J. A **59** (2023) 223 [2305.15849 [hep-th]

Meyer, UGM, JHEP **06** (2024) 074, JHEP **01** (2025) 033 (err) [2403.09325 [hep-ph]]

UGM, Metsch, Meyer, Eur. Phys. J. A **60** (2024) 200 [2405.09971 [nucl-th]]

UGM, Metsch, Meyer, JHEP **06** (2025) 244 [2502.15409 [hep-ph]]

UGM, Metsch, Meyer, Eur. Phys. J. A **61** (2025) 5, 122 [2503.15162 [nucl-th]]

Results I: Variations of α

- Consider first the variation in the fine-structure constant α

- Parameter fit at fixed $\eta = 6.14 \cdot 10^{-10}$ and $\tau_n = 879.4$ s:

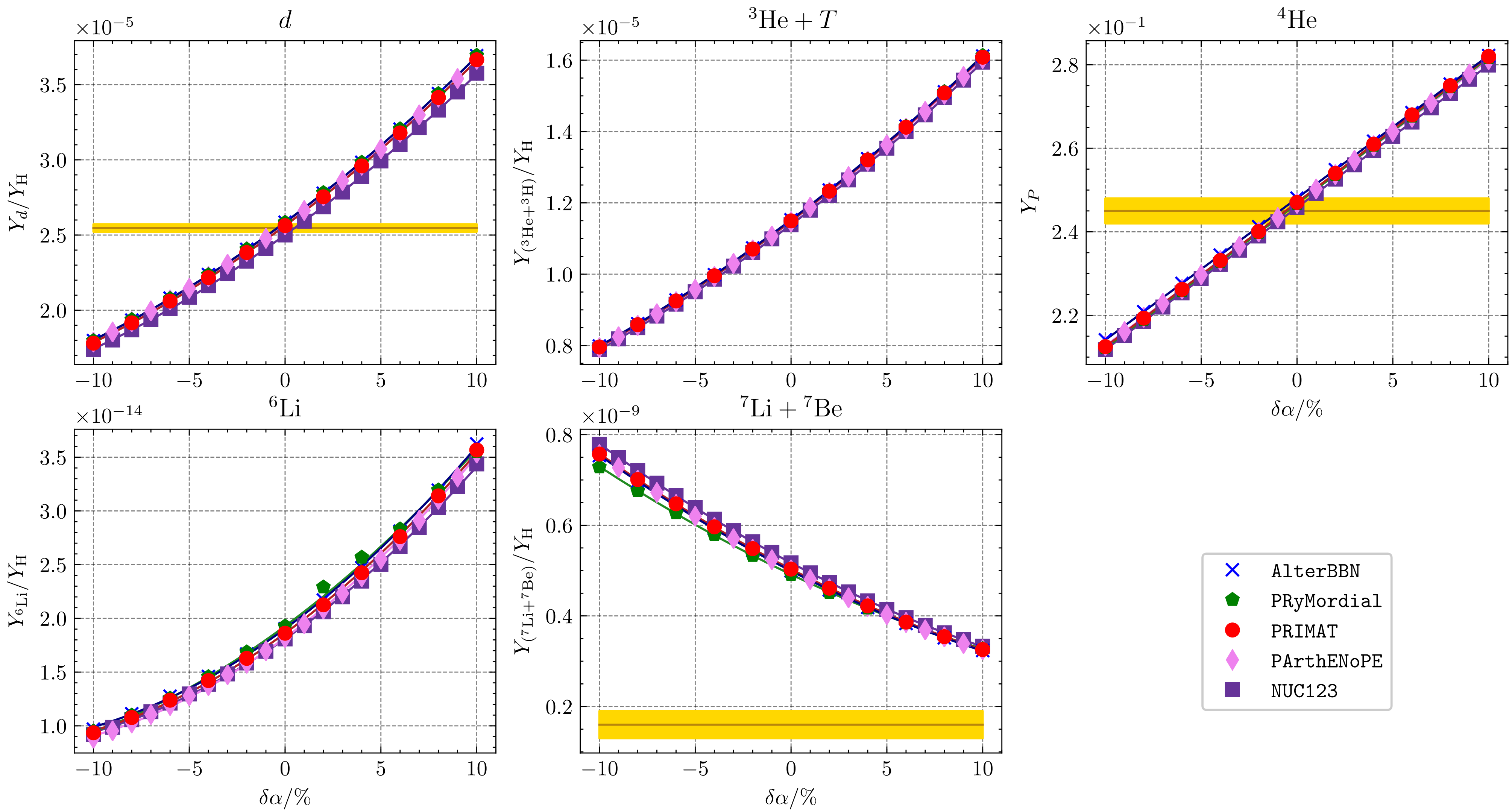
$$\boxed{\frac{Y(\alpha) - Y(\alpha_0)}{Y(\alpha_0)} = a \cdot \frac{\delta\alpha}{\alpha_0} + b \cdot \left(\frac{\delta\alpha}{\alpha_0}\right)^2}$$

- Consider variations in α up to $|\delta\alpha/\alpha_0| \leq 0.1$

- Main results:
  - Temperature-dependence of reaction rates at varying α important
  - For most elements, change in the nuclear reaction rates is the biggest effect
  - ^{4}He abundance indeed very sensitive to ΔQ_n
  - Lithium problem persists

Results II: Variations of α

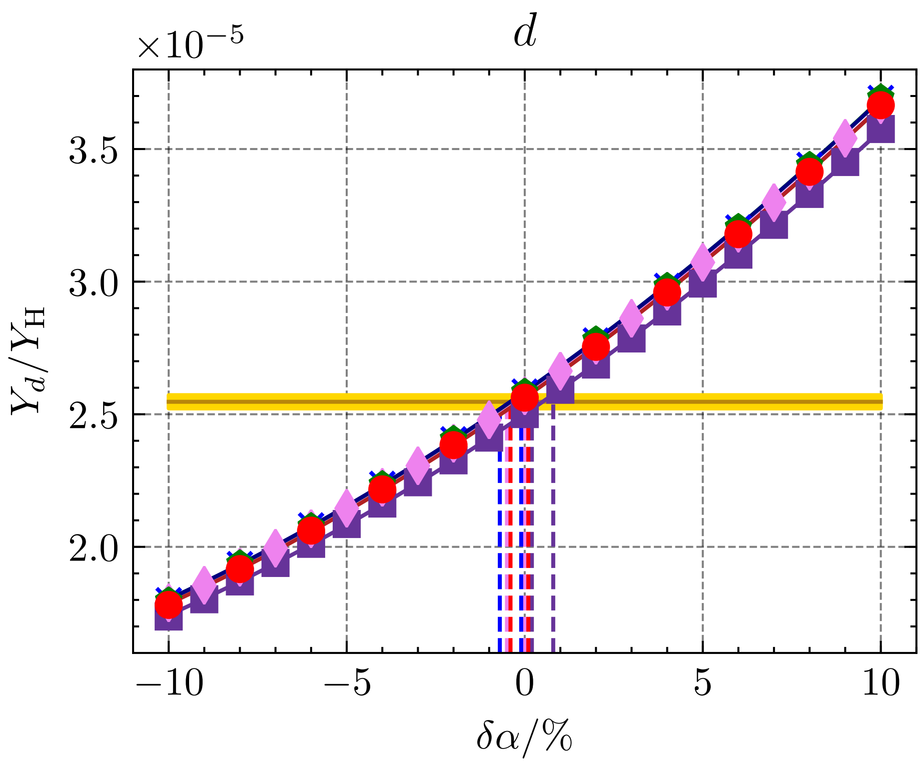
- α-dependence of the abundances:

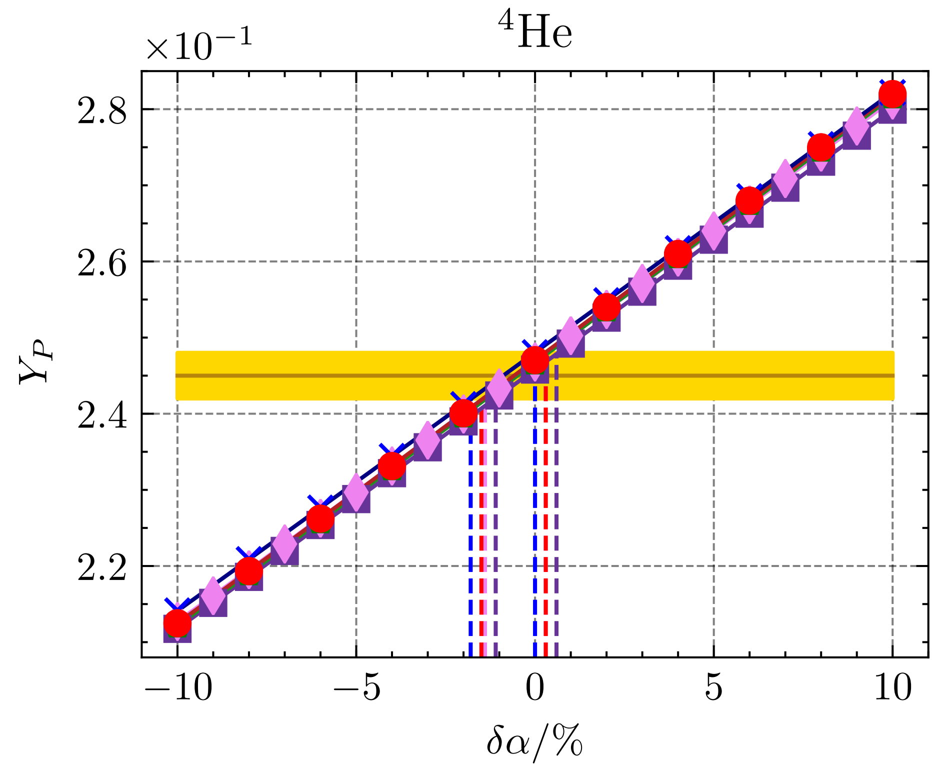


$\hookrightarrow$ largely independent of the codes

Results III: Variations of α

- Extract the allowed α-variation:

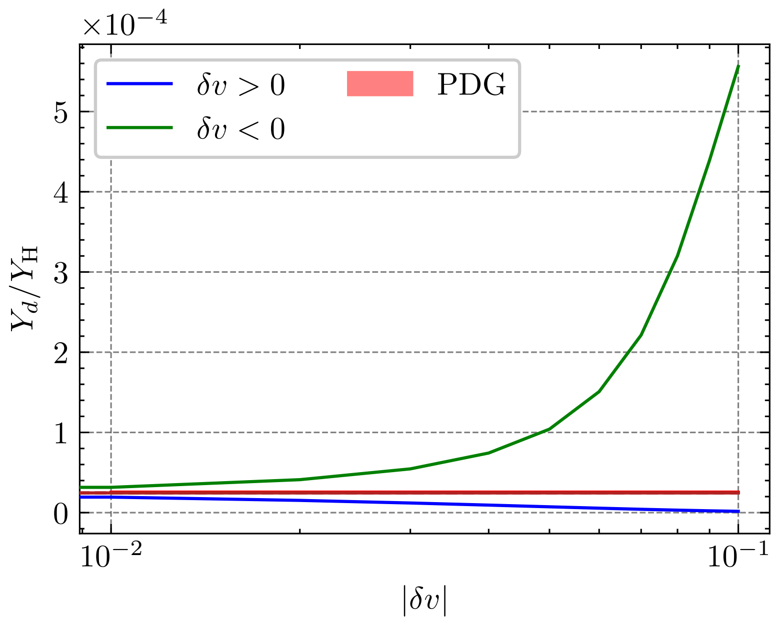


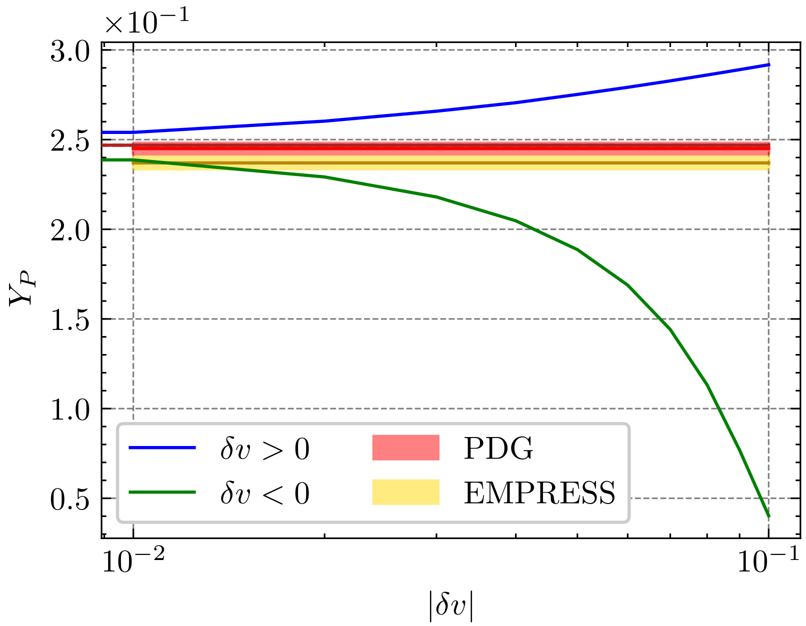


- 1σ-bounds on α-variation $\Rightarrow$ from ^{4}He: $\boxed{|\delta\alpha| < 1.8\%}$
- Stronger bound than found earlier (updated reaction rates, smaller $Q_n^{\rm QED}$, T-dependence of the reactions)

Results IV: Bounds on the Higgs VEV variations

- Consider d and ^{4}He abundances





$$\delta v/v \in [-0.0007, -0.0002]$$

$$\delta v/v \in [-0.0069, 0.0039] \quad \text{[PDG]}$$
$$\delta v/v \in [-0.0138, -0.0030] \quad \text{[Empress]}$$

$\hookrightarrow$ much more stringent than found earlier (B_d changes in $np \leftrightarrow d\gamma$)

$\hookrightarrow$ same strong bounds on the light quark masses m_u, m_d

$\hookrightarrow$ in contrast to earlier works, the bound from Y_d/Y_H is tighter than from Y_p

Results V: Bounds on strange quark mass variations

- Map the m_s dependence onto the nucleon mass of binding energies and other observables

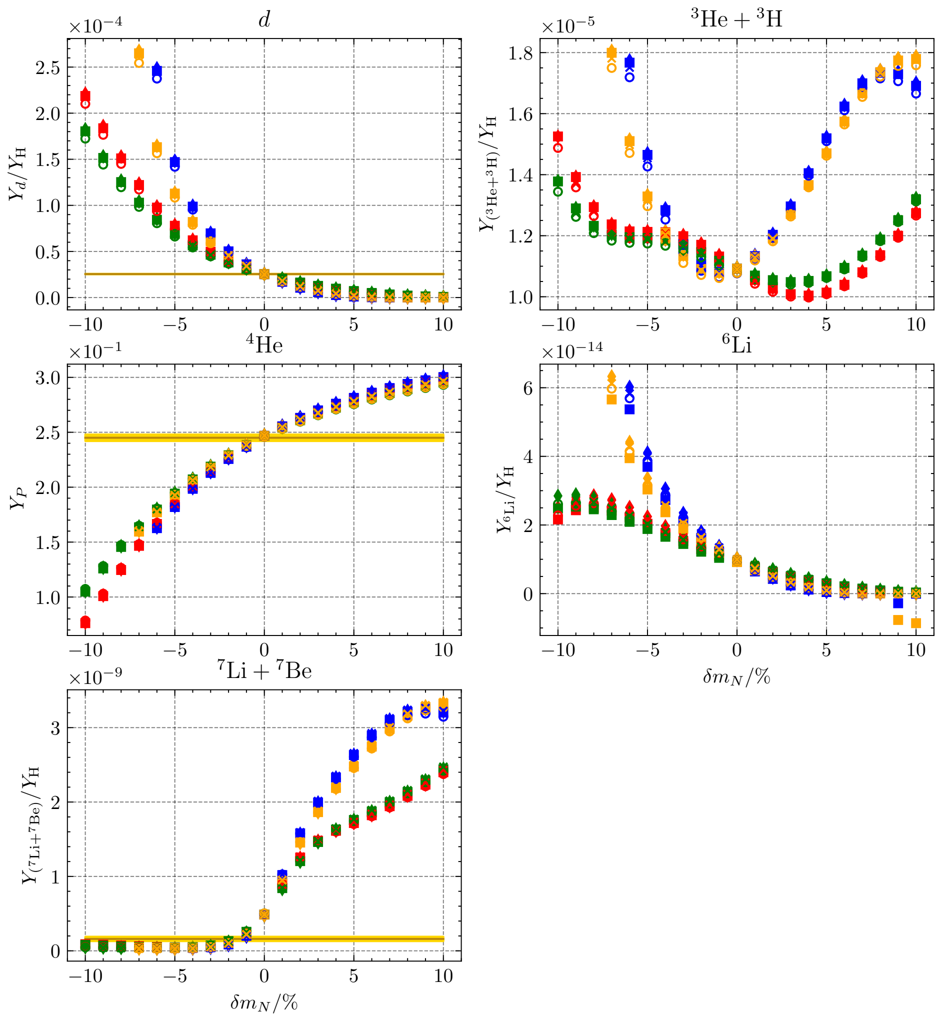
$\hookrightarrow$ model the m_N dependence of nuclear reaction rates

$\hookrightarrow$ four different ways of doing that (NLEFT, pionless EFT, square-well potential)

$\hookrightarrow$ use four different codes

$\hookrightarrow$ get an upper bound on m_s variations:

$$\boxed{\left|\frac{\Delta m_s}{m_s}\right| \leq 5.1\ \%}$$



$\hookrightarrow$ also bound on variations of $\Lambda_{\rm QCD}$: Less than 1.13% between the BB and now

The fate of carbon-based life as a function of the quark mass

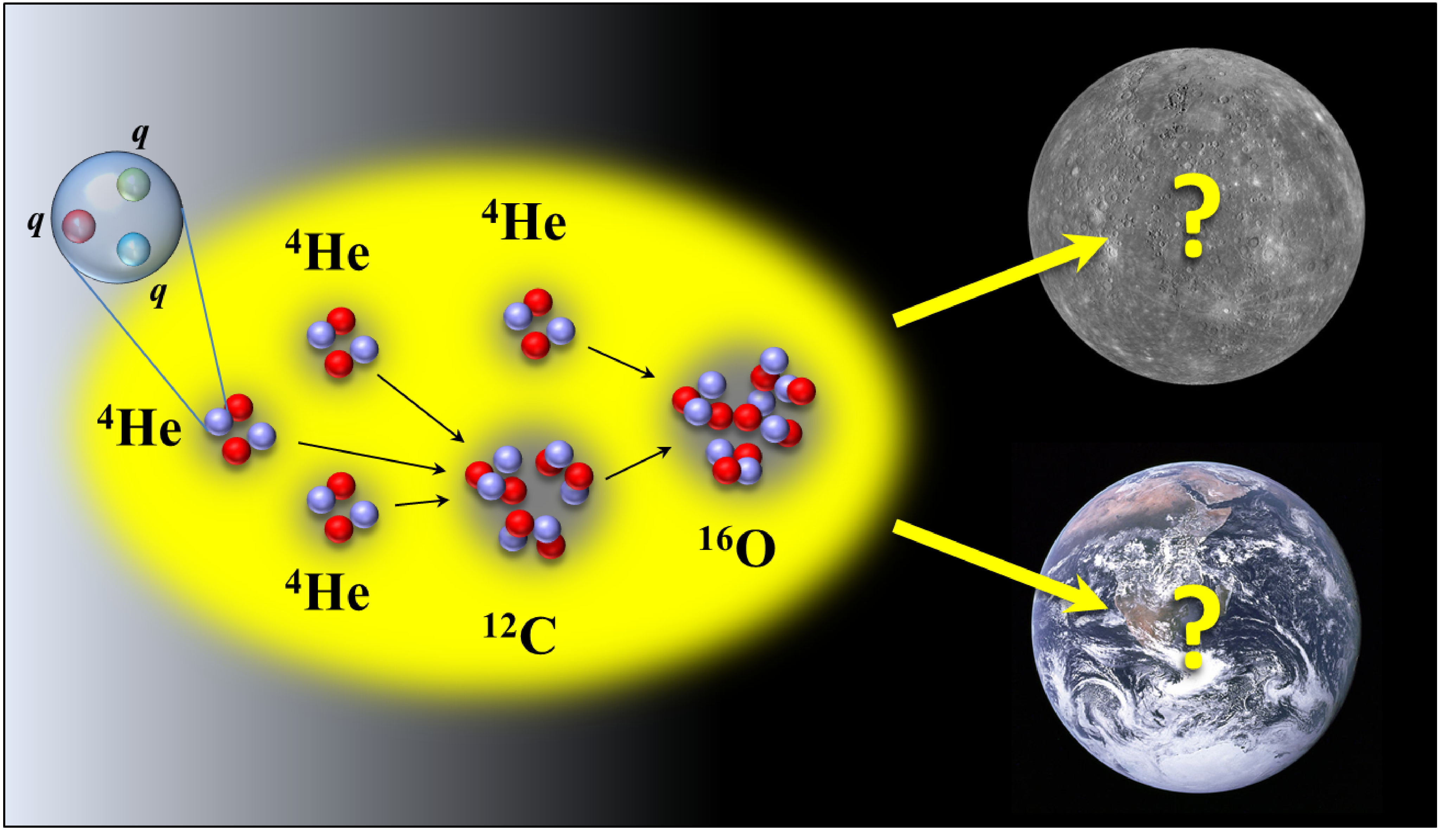
Epelbaum, Krebs, Lähde, Lee, UGM
Phys. Rev. Lett. **110** (2013) 112502; Eur. Phys. J. **A 48** 82 (2013)

update: Lähde, UGM, Epelbaum, Eur. Phys. J. **A 56** (2020) 89

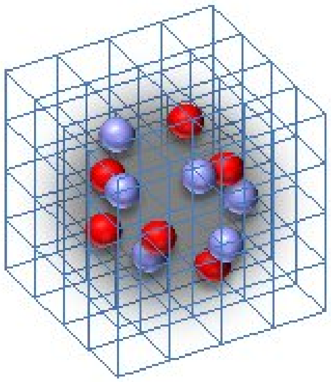
review: UGM, Sci. Bull. **60** (2015) 43

Fine-tuning of the fundamental parameters

Fig. courtesy Dean Lee



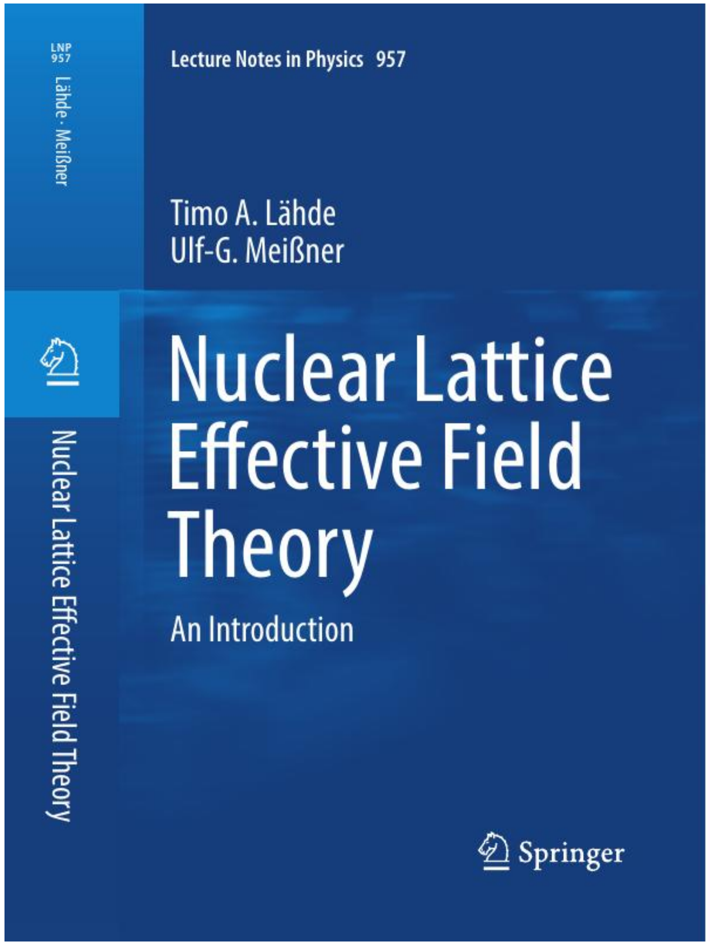
The tool: Nuclear lattice effective field theory



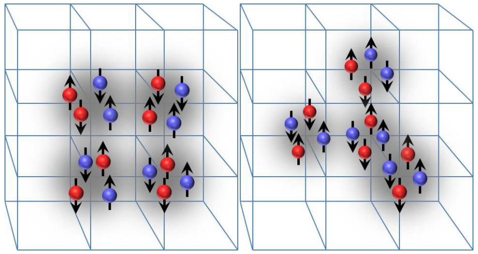
- A new many-body approach: NLEFT
- For all details on chiral EFT on a lattice

T. Lähde & UGM
Nuclear Lattice Effective Field Theory - An Introduction
Springer Lecture Notes in Physics **957** (2019) 1 - 396

- Computational equipment



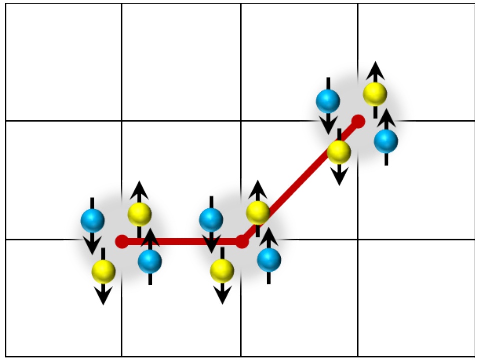
Early results from NLEFT: Validation



☐ Lattice EFT calculations for A=3,4,6,12 nuclei, PRL 104 (2010) 142501

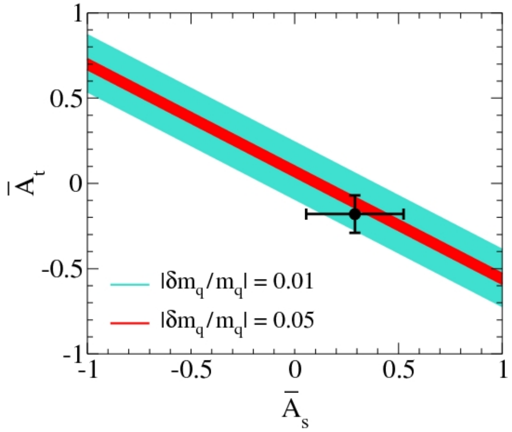
☐ *Ab initio* calculation of the Hoyle state, PRL 106 (2011) 192501

☐ Structure and rotations of the Hoyle state, PRL 109 (2012) 142501



☐ Validity of Carbon-Based Life as a Function of the Light Quark Mass
PRL 110 (2013) 142501

☐ *Ab initio* calculation of the Spectrum and Structure of ^{16}O,
PRL 112 (2014) 142501

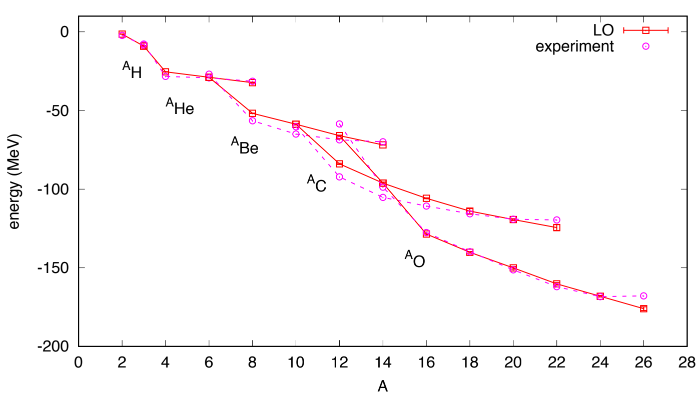


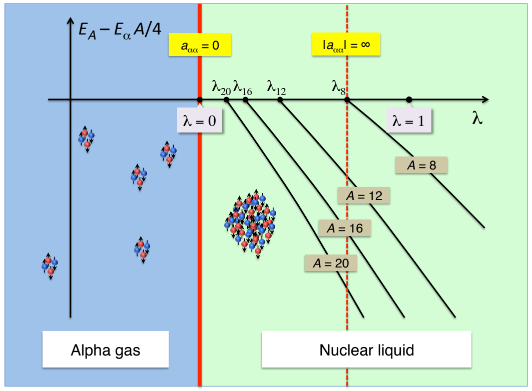
☐ Lattice effective field theory for medium-mass nuclei, PLB 732 (2014) 110

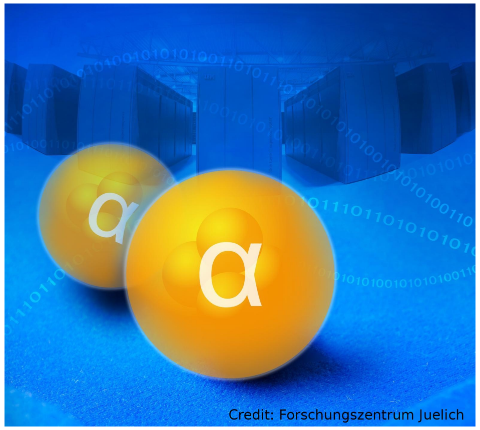
☐ *Ab initio* alpha-alpha scattering, Nature 528 (2015) 111

☐ Nuclear Binding Near a Quantum Phase Transition, PRL 117 (2016) 132501

☐ *Ab initio* calculations of the isotopic dependence of nuclear clustering,
PRL 119 (2017) 222505

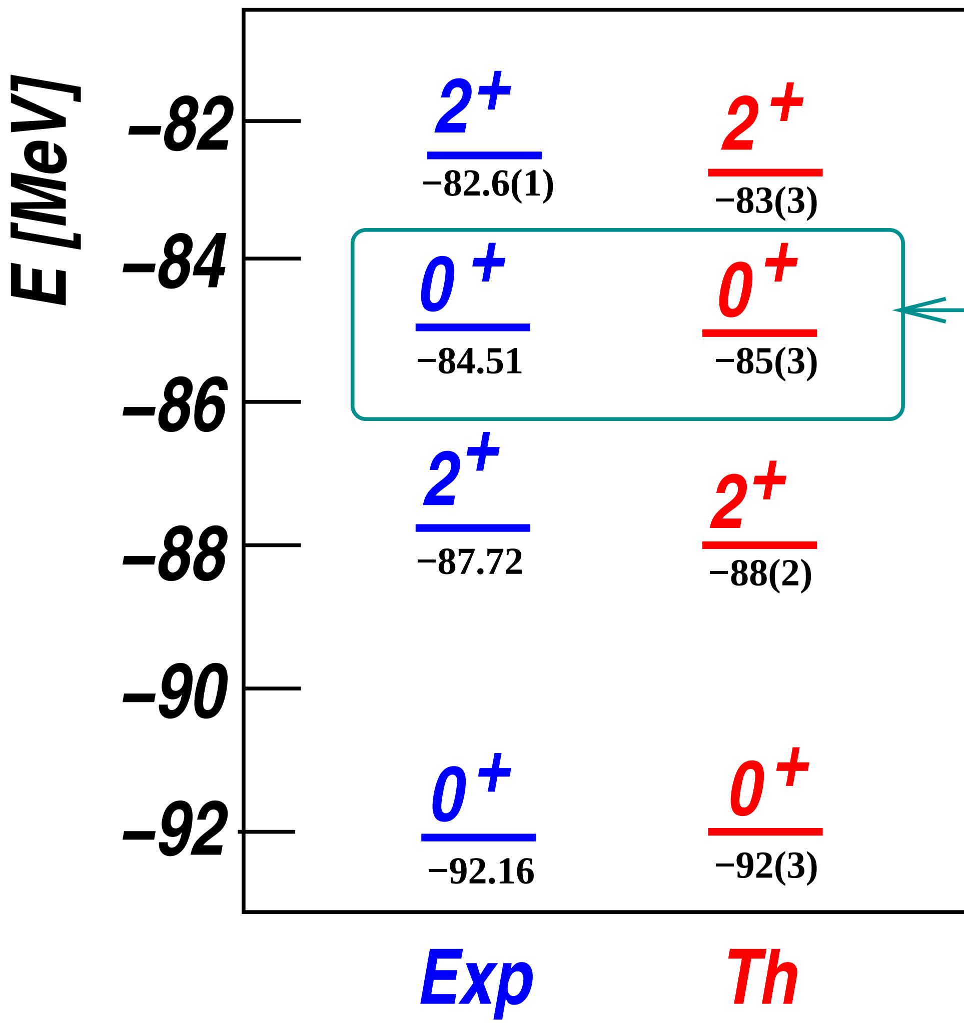






The spectrum of carbon-12 A.D. 2011

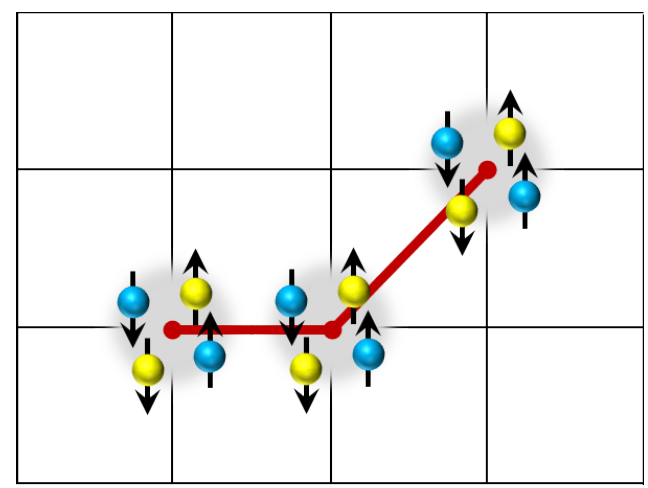
- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and “some” human work)



Hoyle

$\Rightarrow$ First ab initio calculation of the Hoyle state $\surd$

Structure of the Hoyle state:

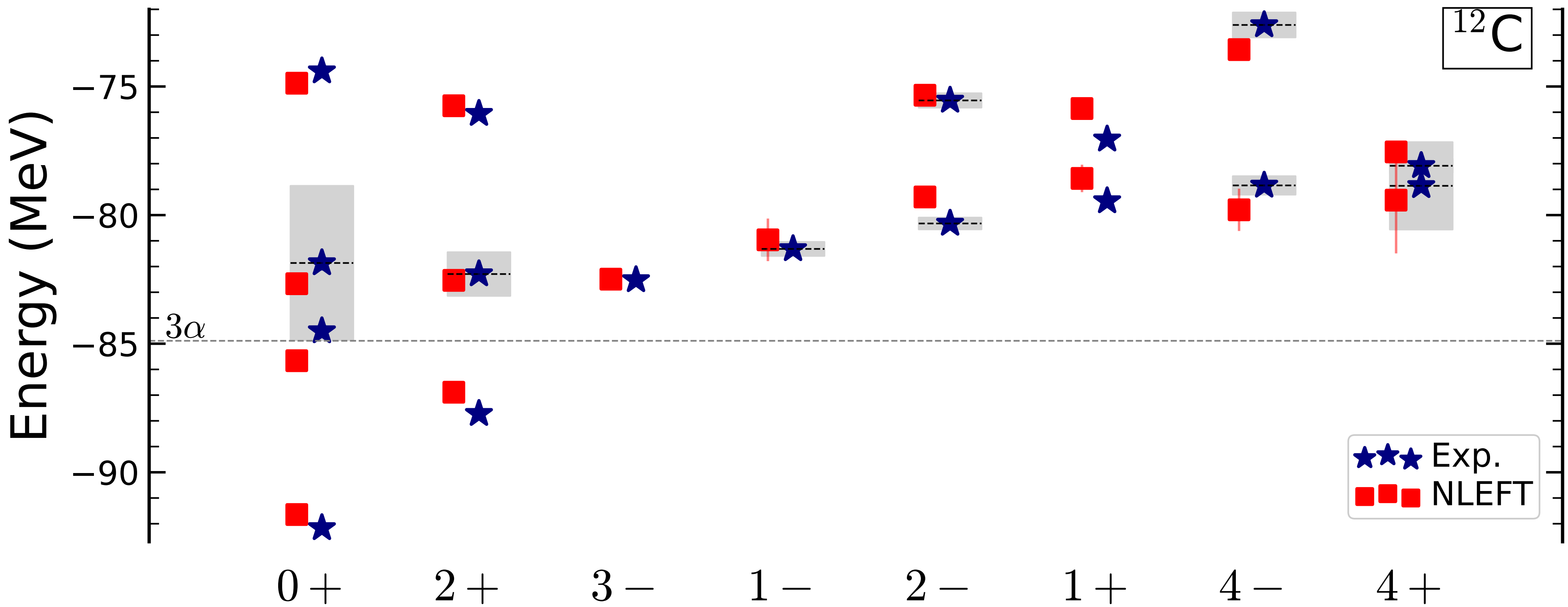


Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501
Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. **109** (2012) 252501

The spectrum of carbon-12 A.D. 2023

- with much improved algorithms and methods:

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777



→ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

Pion mass dependence from MC simulations

- Consider pion mass changes as *small perturbations* for an energy (difference) E_i

$$\left.\frac{\partial E_i}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = \left.\frac{\partial E_i}{\partial M_\pi^{\rm OPE}}\right|_{M_\pi^{\rm phys}} + x_1 \left.\frac{\partial E_i}{\partial m_N}\right|_{m_N^{\rm phys}} + x_2 \left.\frac{\partial E_i}{\partial g_{\pi N}}\right|_{g_{\pi N}^{\rm phys}} + x_3 \left.\frac{\partial E_i}{\partial C_0}\right|_{C_0^{\rm phys}} + x_4 \left.\frac{\partial E_i}{\partial C_I}\right|_{C_I^{\rm phys}}$$

with

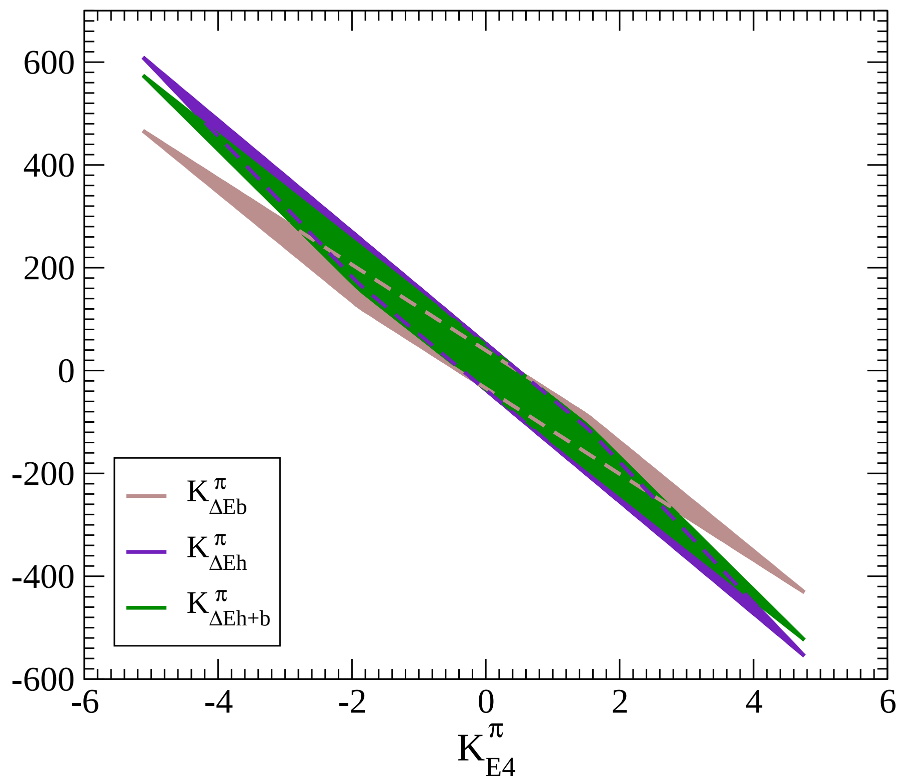
$$x_1 \equiv \left.\frac{\partial m_N}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \; x_2 \equiv \left.\frac{\partial g_{\pi N}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \; x_3 \equiv \left.\frac{\partial C_0}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \; x_4 \equiv \left.\frac{\partial C_I}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

$\Rightarrow$ problem reduces to the calculation of the various derivatives using AFQMC and the determination of the x_i

- x_1 and x_2 can be obtained from LQCD plus CHPT
- x_3 and x_4 can be obtained from NN scattering and its M_π-dependence $\to \bar{A}_{s,t}$

Correlations

- vary the quark mass derivatives of $\bar{A}_{s,t} = \partial a_{s,t}^{-1}/\partial M_\pi|_{M_\pi^{\rm phys}}$ within $-1,\ldots,+1$:



$$\Delta E_b = E(^8\mathrm{Be}) - 2E(^4\mathrm{He})$$

$$\Delta E_h = E(^{12}\mathrm{C}^*) - E(^8\mathrm{Be}) - E(^4\mathrm{He})$$

$$\Delta E_{h+b} = E(^{12}\mathrm{C}^*) - 3E(^4\mathrm{He})$$

$$\boxed{\frac{\partial O_H}{\partial M_\pi} = K_H^\pi \frac{O_H}{M_\pi}}$$

- clear correlations: the two fine-tunings are not independent

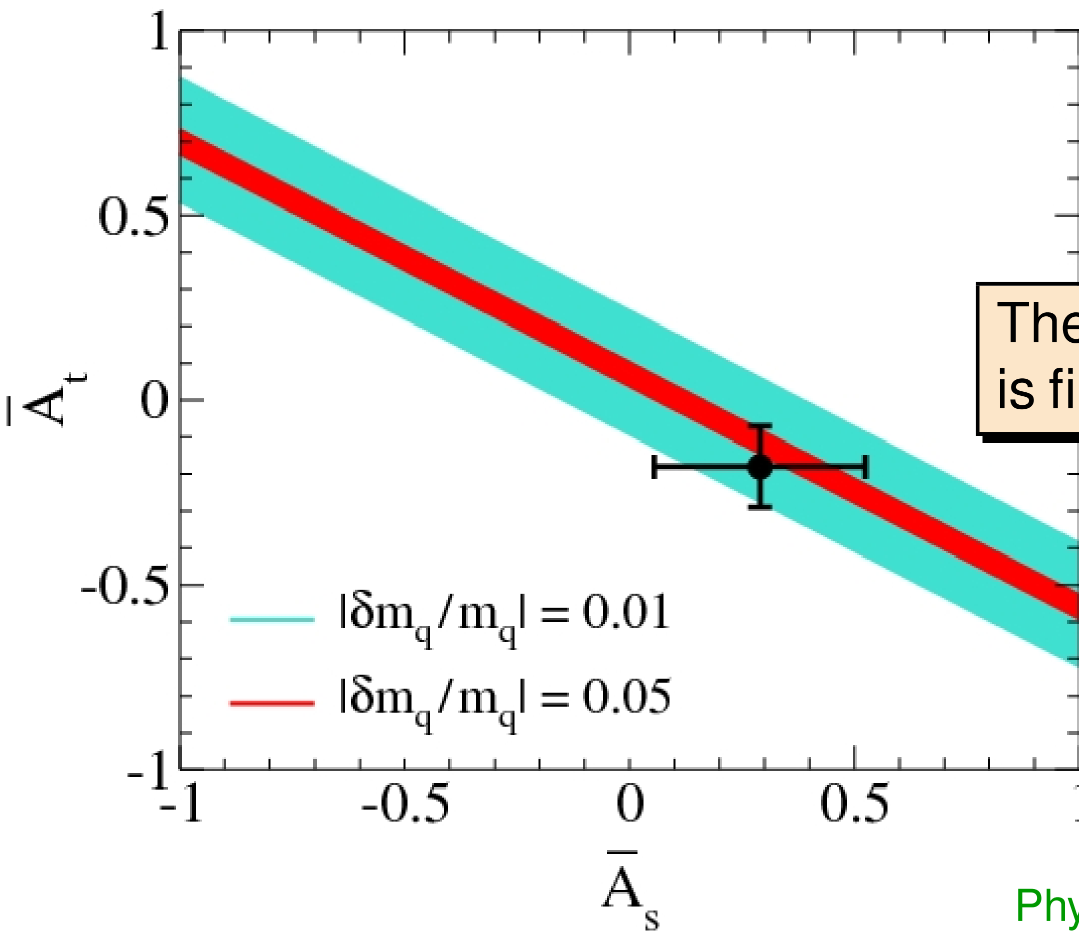
$\Rightarrow$ has been speculated before but could not be calculated Weinberg (2001)

The end-of-the-world plot I

- $|\delta(\Delta E_{h+b})| < 100$ keV

Oberhummer et al., Science (2000)

$$\rightarrow \boxed{\left|\left(0.571(14)\bar{A}_s + 0.934(11)\bar{A}_t - 0.069(6)\right)\frac{\delta m_q}{m_q}\right| < 0.0015}$$



$$\bar{A}_{s,t} \equiv \left.\frac{\partial a_{s,t}^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

The light quark mass is fine-tuned to $\simeq 2-3\,\%$

Similarly: $\alpha_{\rm EM}$ is fine-tuned to $\simeq 2.5\%$

Berengut et al., Phys. Rev. **D 87** (2013) 085018

An update on fine-tunings in the triple-alpha process

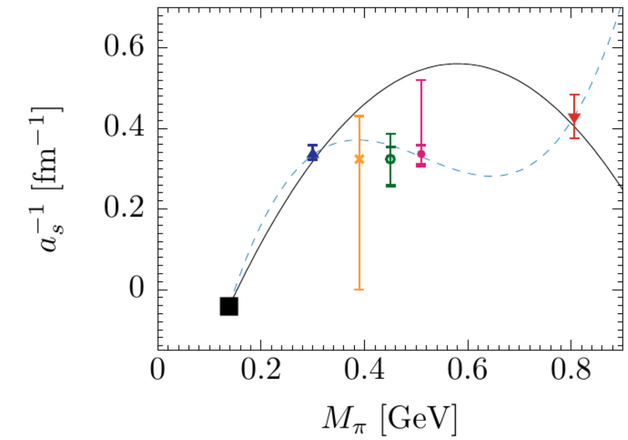
Lähde, UGM, Epelbaum, Eur. Phys. J **A 56** (2020) 89

- Use lattice data to determine $\bar{A}_s$ and $\bar{A}_t$:

$$\bar{A}_s = 0.54(24)\,, \quad \bar{A}_t = 0.33(16)$$

$\hookrightarrow$ $\bar{A}_s$ is consistent w/ earlier determination

$\hookrightarrow$ $\bar{A}_t$ changes sign compared to earlier determination

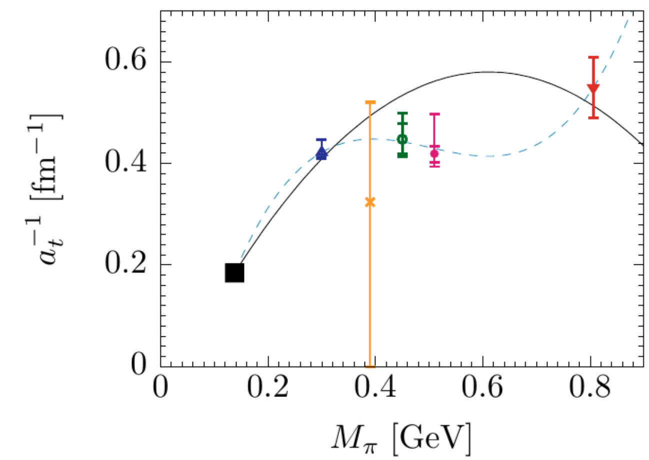


- update x_1 and x_2 using better LQCD data:

$$x_1 = 0.84(7)\,, \quad x_2 = -0.053(16)$$

$\hookrightarrow$ x_1 and x_2 more precise

$\hookrightarrow$ x_2 now has a definite sign



$\Rightarrow$ update end-of-the-world plot

Beane et al. (2012)
Yamazaki et al. (2015)
Orginos et al. (2015)
Beane et al. (2013)
Yamazaki et al. (2012)

New end-of-the-world plots

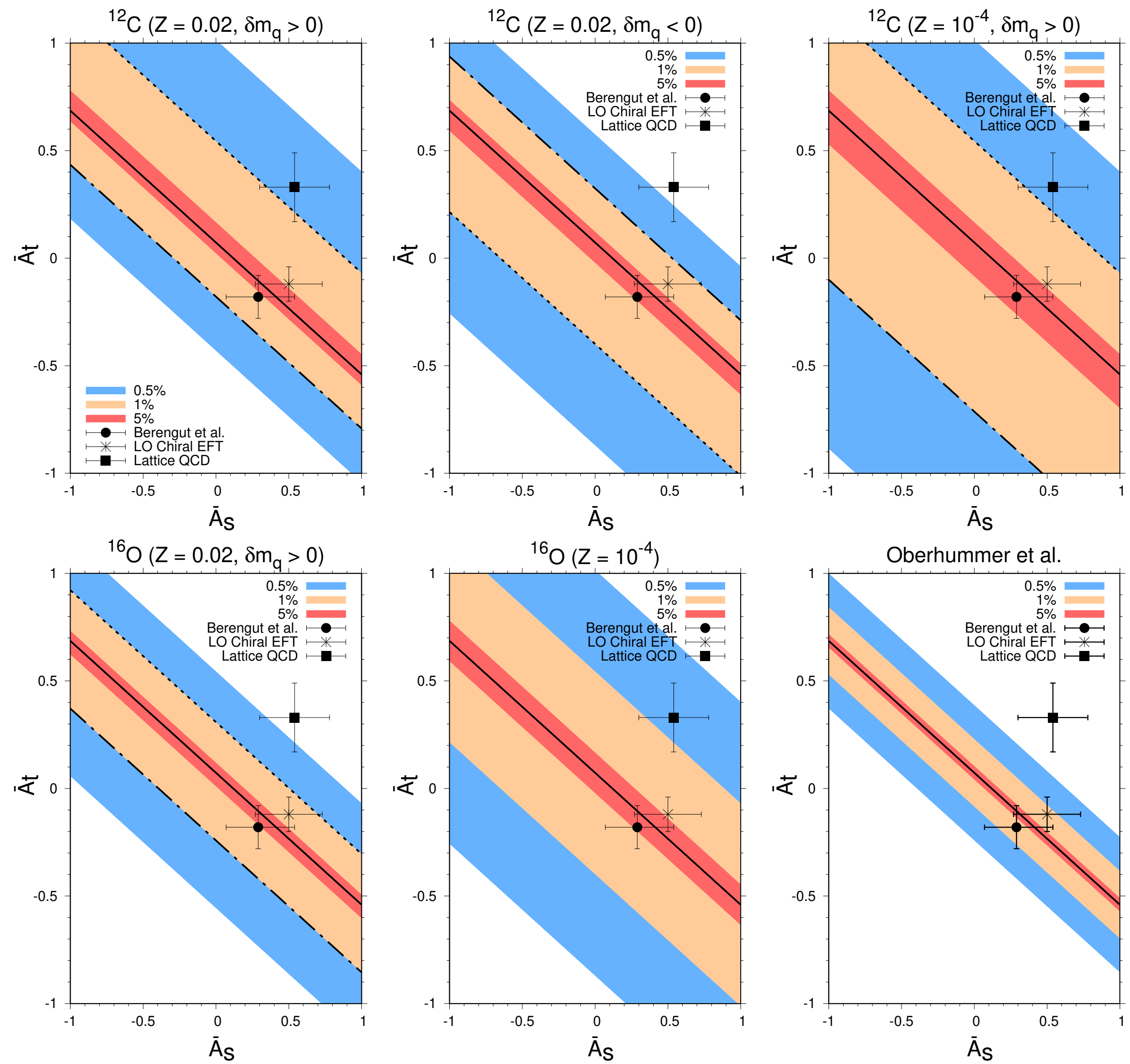
Lähde, UGM, Epelbaum, Eur. Phys. J **A 56** (2020) 89

- Improved simulations of carbon and oxygen generation in massive stars

$\rightarrow$ constraints now depend on Z, the nucleus and the sign of δm_q

Huang, Adams, Grohs, Astropart. Phys. **105** (2019) 13

- lattice values for $\bar{A}_{s,t}$:



The light quark mass is fine-tuned to $\simeq 0.5\,\%$

- Bound on $\alpha_{\rm EM}$ softened ($\sim 7.5\,\%$)

$\Rightarrow$ need better determinations of $\bar{A}_{s,t}$ from lattice QCD with pion masses closer to the physical point!

A dose of philosophy: The anthropic principle

Anthropic principle

- The anthropic principle:

"The observed values of all physical and cosmological quantities are not equally probable but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirements that the universe be old enough for it to have already done so."

Carter 1974, Barrow & Tippler 1988, ...

$\Rightarrow$ does this lead to physical/testable consequences?

Some examples of anthropic reasoning

VOLUME 59, NUMBER 22 PHYSICAL REVIEW LETTERS 30 NOVEMBER 1

Anthropic Bound on the Cosmological Constant

Steven Weinberg
Theory Group, Department of Physics, University of Texas, Austin, Texas 78712
(Received 5 August 1987)

In recent cosmological models, there is an "anthropic" upper bound on the cosmological constant Λ. It is argued here that in universes that do not recollapse, the only such bound on Λ is that it should not be so large as to prevent the formation of gravitationally bound states. It turns out that the bound is quite large. A cosmological constant that is within 1 or 2 orders of magnitude of its upper bound would help with the missing-mass and age problems, but may be ruled out by galaxy number counts. If so, we may conclude that anthropic considerations do not explain the smallness of the cosmological constant.

1099 citations

Nature Vol. 278 12 April 1979 605

review article

The anthropic principle and the structure of the physical world

B. J. Carr* & M. J. Rees
Institute of Astronomy, Madingley Road, Cambridge, UK

The basic features of galaxies, stars, planets and the everyday world are essentially determined by a few microphysical constants and by the effects of gravitation. Many interrelations between different scales that at first sight seem surprising are straightforward consequences of simple physical arguments. But several aspects of our Universe—some of which seem to be prerequisites for the evolution of any form of life—depend rather delicately on apparent 'coincidences' among the physical constants.

The Anthropic Landscape of String Theory

L. Susskind
Department of Physics
Stanford University
Stanford, CA 94305-4060

Abstract

In this lecture I make some educated guesses, about the landscape of string theory vacua. Based on the recent work of a number of authors, it seems plausible that the lanscape is unimaginably large and diverse. Whether we like it or not, this is the kind of behavior that gives credence to the Anthropic Principle. I discuss the theoretical and conceptual issues that arise in developing a cosmology based on the diversity of environments implicit in string theory.

1167 citations

PHYSICAL REVIEW D VOLUME 57, NUMBER 9 1 MAY 1998

Viable range of the mass scale of the standard model

V. Agrawal,[1] S. M. Barr,[1] John F. Donoghue,[2] and D. Seckel[1]
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(Received 30 July 1997; published 1 April 1998)

In theories in which different regions of the universe can have different values of certain physical parameters, we would naturally find ourselves in a region where they take values favorable for life. We explore the range of such viable values of the mass parameter in the Higgs potential, μ^2. For $\mu^2<0$, the requirement that complex elements be formed suggests that the Higgs vacuum expectation value v must have a magnitude less than 5 times its observed value. For $\mu^2>0$, baryon stability requires that $|\mu| \ll M_P$, the Planck mass. Smaller values of $|\mu^2|$ may or may not be allowed depending on issues of element synthesis and stellar evolution. We conclude that the observed value of μ^2 appears reasonably typical of the viable range, and a multiple-domain scenario may provide a plausible explanation for the closeness of the QCD scale and the weak scale.
[S0556-2821(98)05509-X]

A prime exampe of the AP

- Hoyle (1953):

Prediction of an excited state in the carbon spectrum neccessary to generate a sufficient amount of heavy elements (^{12}C, ^{16}O,...) in stars

- was later heralded as the prime example for the AP:

"As far as we know, this is the only genuine anthropic principle prediction"
Carr & Rees 1989

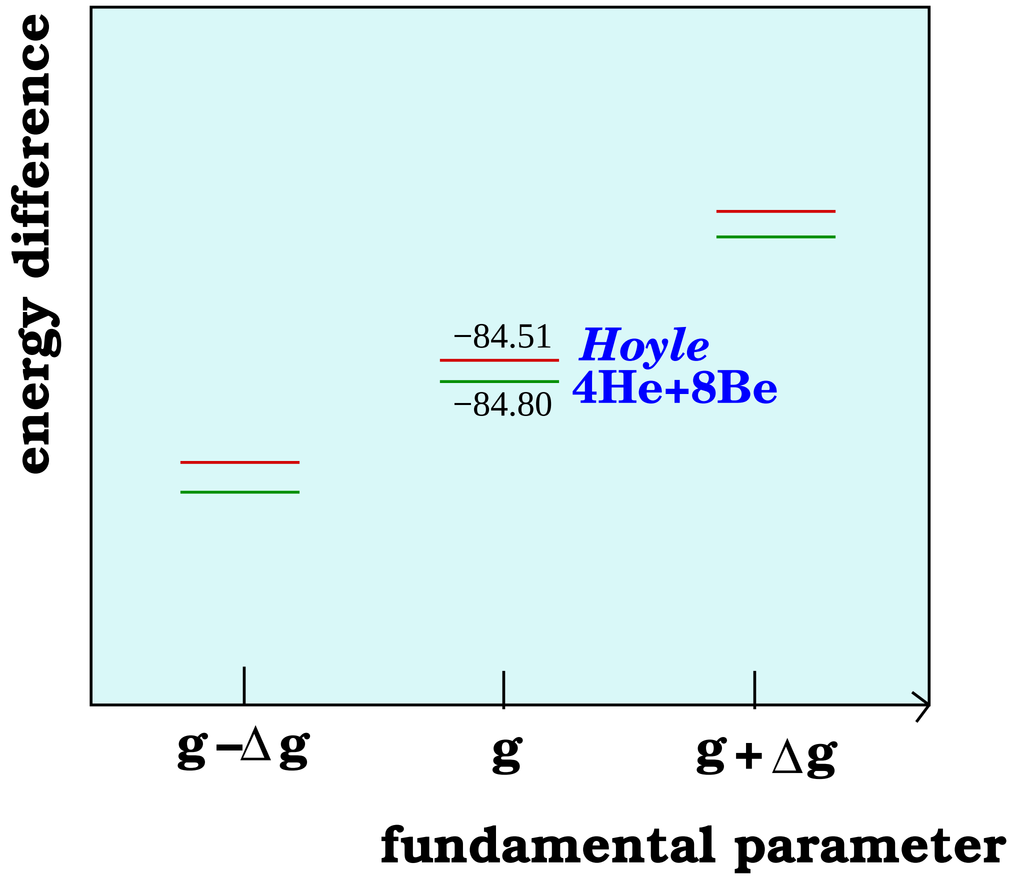
"In 1953 Hoyle made an anthropic prediction on an excited state – 'level of life' – for carbon production in stars"
Linde 2007

"A prototype example of this kind of anthropic reasoning was provided by Fred Hoyle's observation of the triple alpha process..."
Carter 2006

$\Rightarrow$ can we find out / test whether this is true?

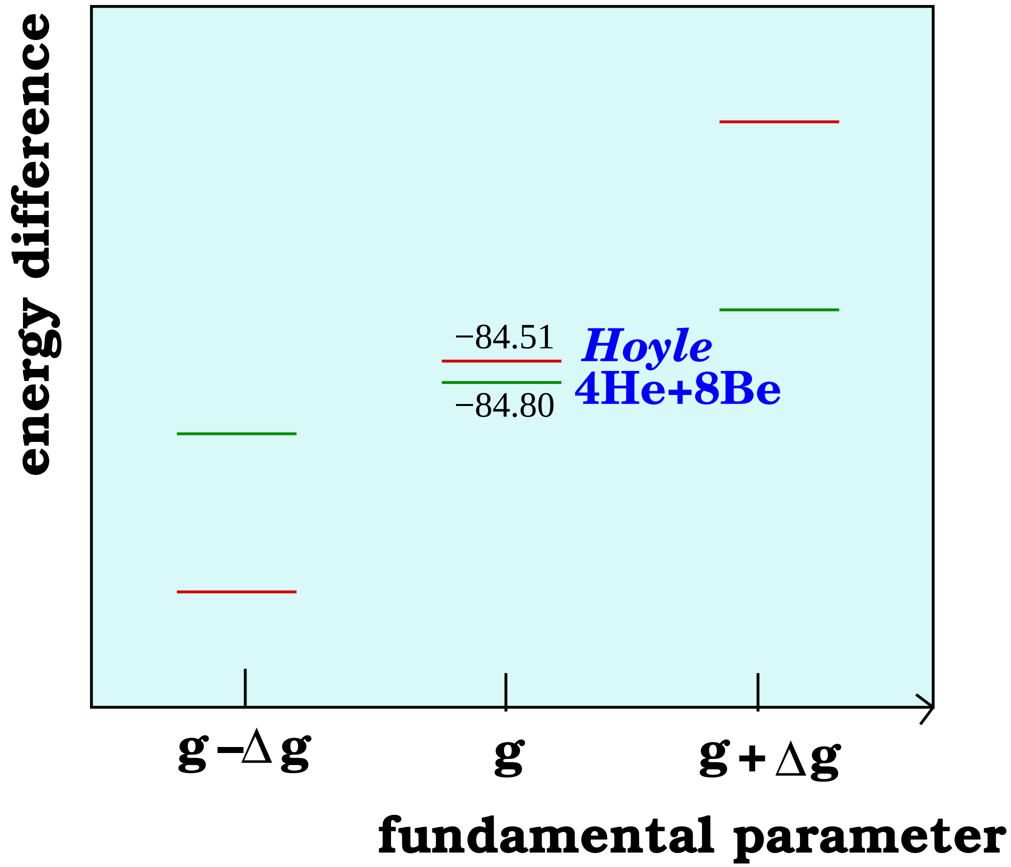
The non-anthropic scenario

- Weinberg's assumption: The Hoyle state stays close to the 4He+8Be threshold



The anthropic scenario

- The AP strikes back: The Hoyle state quickly moves away from the 4He+8Be threshold



AP at work

- Our calculations support the anthropic scenario:

$$E_R = E(^{12}C^*) - 3E(^4\mathrm{He}) = 379.47\,\mathrm{keV} + \boxed{\mathrm{C}}\,\frac{dm_q}{m_q}[\%]\,\mathrm{keV}$$

$$\Delta E_h = E(^{12}\mathrm{C}^*) - E(^8\mathrm{Be}) - E(^4\mathrm{He}) = 289\,\mathrm{keV} + \boxed{0.8\,\mathrm{C}}\,\frac{dm_q}{m_q}[\%]\,\mathrm{keV}$$

- C depends on the precise values of $\bar{A}_{s,t}$, here $C = -372$ (from LQCD+LETs)
- there is one combination of $\bar{A}_s$ and $\bar{A}_t$, where $C = 0$

  $\hookrightarrow$ no more quark mass dependence

  $\hookrightarrow$ does not seem to be realized in Nature (not yet entirely excluded)

Summary & outlook

- Instead of a summary and outlook

Eur. Phys. J. A (2025) 61:122
https://doi.org/10.1140/epja/s10050-025-01587-5

THE EUROPEAN PHYSICAL JOURNAL A

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Invited Viewpoint and Perspective

Fine-tunings in nucleosynthesis and the emergence of life: status and perspectives

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Received: 21 March 2025 / Accepted: 12 May 2025

Communicated by Thomas Duguet

Abstract We discuss the fine-tunings of nuclear reactions in the Big Bang and in stars and draw some conclusions on the emergence of the light elements and the life-relevant elements carbon and oxygen. We also stress how to improve these calculations in the future. This requires a concerted effort of different communities, especially in nuclear reaction theory, lattice QCD for few-nucleon systems, stellar evolution calculations, particle physics and philosophy.

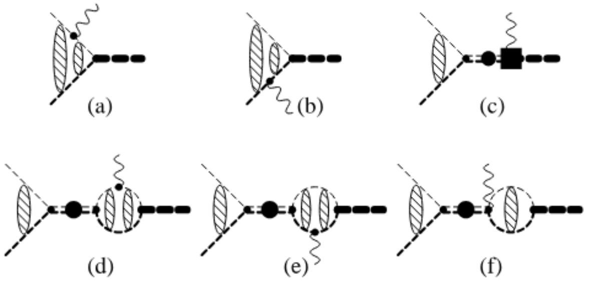
Bang and in stars – that is why one often says that we are made of stardust. Of particular relevance are the ^{12}C and ^{16}O nuclei, which form the basis of the life on Earth as we know it. These elements are generated in hot, old stars through the triple-alpha process and the radiative capture process $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$, respectively. Both ^{12}C and ^{16}O are alpha-type nuclei, that is to a good approximation they can be described as bound states of three, respectively four, ^{4}He particles. ^{4}He is already generated abundantly shortly after the Big Bang,

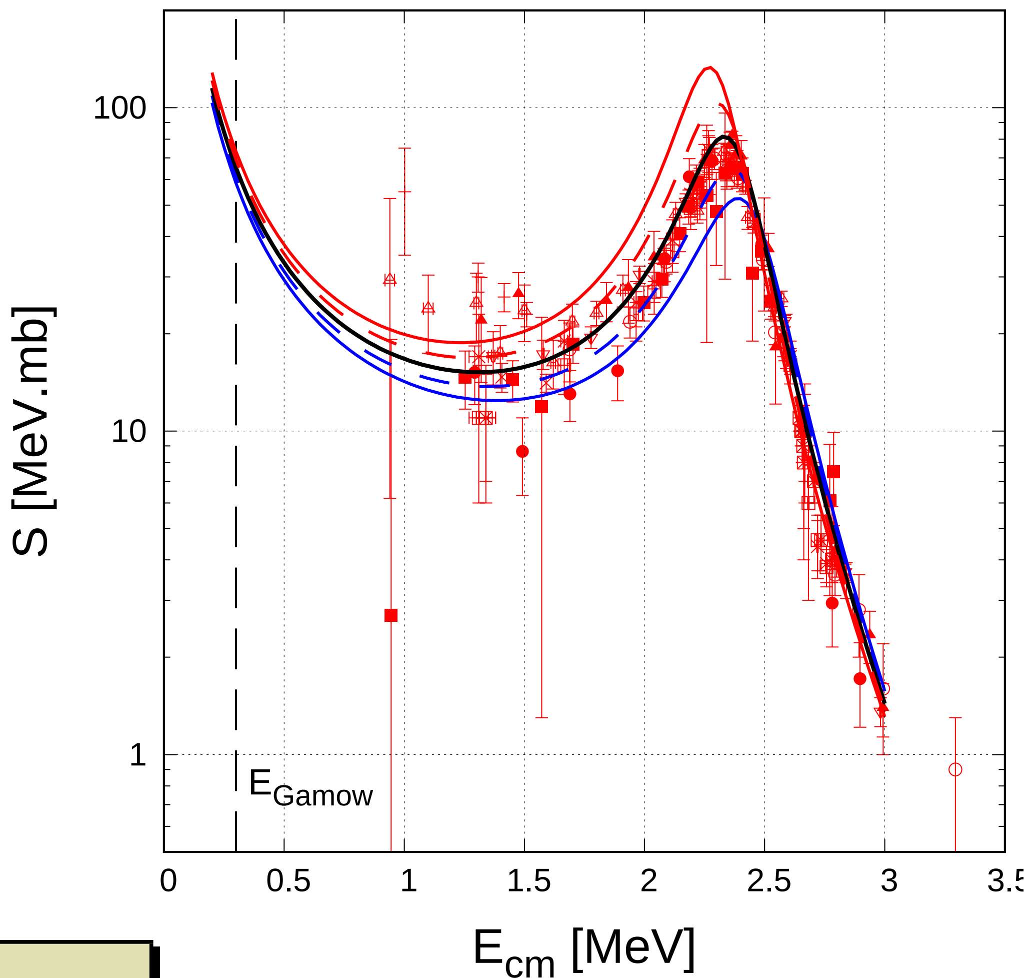
$\hookrightarrow$ and a “Schmankerl” to finish up

How fine-tuned is the holy grail of nuclear astrophysics?

- Use cluster EFT for $\alpha + {}^{12}\mathrm{C} \to {}^{16}\mathrm{O} + \gamma$

Ando, PRC 100 (2019) 015807; PRC 197 (2023) 945808
Ando, PRC 109 (2024) 015801; 2502.08190 [nucl-th]]





$\hookrightarrow$ study the α dependence of the S-factor

(α appears in many places...)

$$S(E) = \sigma(E) E \exp(2\pi k_C/p)$$
$$k_C = Z_1 Z_2 \alpha \mu,\ E = p^2/2\mu$$
$$\mu = m_1 m_2/(m_1 + m_2)$$

$\Rightarrow$ extremely fine-tuned: $\left|\frac{\delta\alpha}{\alpha}\right| < 0.2$ per mille

SPARES

The electromagnetic fine-structure constant in primordial nucleosynthesis revisited

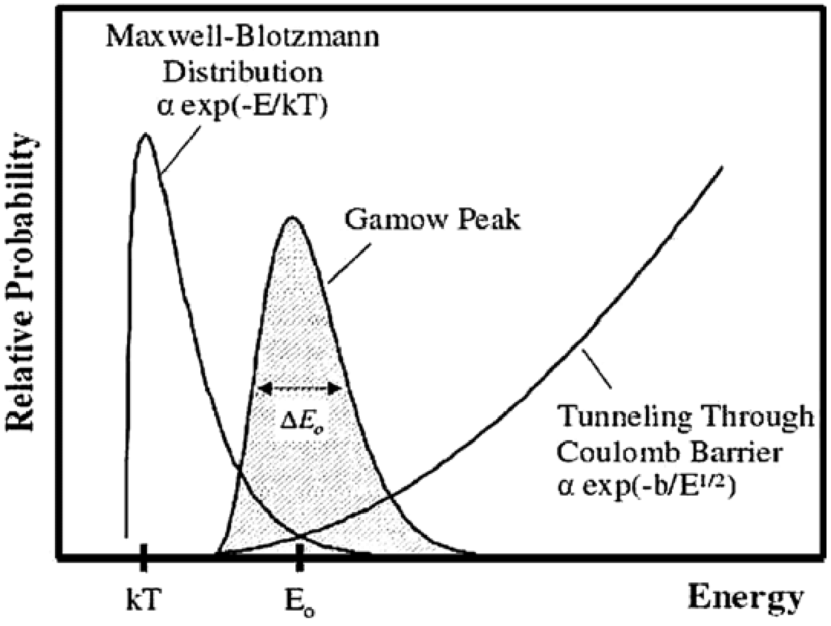
UGM, Metsch, Meyer, Eur. Phys. J. A **59** (2023) 223 [2305.15849 [hep-th]]

The electromagnetic fine-structure constant

- Want to study variations of $\alpha_{\rm EM}$ about its present day value:

$$\alpha_0 = 7.2973525693(11) \times 10^{-3}$$

$\hookrightarrow$ find a bound on $\delta\alpha/\alpha_0$ by comparison with the measured Y_n



from Trache (2010)

- Where does α appear in BBN?

  - Nuclear Rates: Coulomb barrier $\to$ Gamow factor — Gamow (1928)

  - Weak rates: final-state Coulomb interaction in $n \leftrightarrow p$ rates and β-decays

  - Indirectly: n-p mass difference $Q_n = m_n - m_p$, EM contribution to nuclear binding energies $\to$ reaction Q-values

Nuclear reaction rates: Coulomb barrier

- Reminder:

$$\boxed{\gamma_{ab\to cd}(T) = \langle\sigma v\rangle \propto \int_0^\infty dE\,\sigma_{ab\to cd}(E)\cdot E\cdot \mathrm{e}^{-\frac{E}{k_B T}}} \qquad E = \frac{1}{2}\mu_{ab}v^2$$

$$\mu_{ab} = \frac{m_a m_b}{m_a + m_b}$$

- Coulomb barrier

$\hookrightarrow$ The cross section is proportional to the penetration factor (in/out)

$$\sigma \propto v_0 = \frac{2\pi\eta}{\mathrm{e}^{2\pi\eta} - 1}$$

with the Sommerfeld parameter

$$\eta = \frac{Z_a Z_b \alpha_{\rm EM} c}{\hbar v} = \frac{1}{2\pi}\sqrt{E_G/E}$$

and the Gamow energy

$$E_G = 2\pi^2\, Z_a^2\, Z_b^2\, \alpha_{\rm EM}^2\, \mu_{ab}\, c^2$$

Nuclear reaction rates: Radiative capture

- Coupling $\propto e \to$ cross section $\sigma \propto e^2 \propto \alpha_{\rm EM}$

- Capture processes are peripheral $\to$ parameterized in $f(\delta\alpha_{\rm EM}) \simeq 1$

Nollett, Lopez (2002)

- Assume dipole dominance

- For the leading $np \to d\gamma$ reactions use again EFT formalism   Rupak (2000)

$\Rightarrow$ α-dependence of cross sections ($q_\gamma = 1$ for radiative capture, 0 else)

$$\boxed{\sigma(\alpha_{\rm EM}, E) \propto \left(\frac{\sqrt{E_G^{\rm in}/E}}{{\rm e}^{\sqrt{E_G^{\rm in}/E}} - 1}\right) \cdot \left(\frac{\sqrt{E_G^{\rm in}/(E+Q)}}{{\rm e}^{\sqrt{E_G^{\rm in}/(E+Q)}} - 1}\right) \cdot \left(\alpha_{\rm EM} f(\delta\alpha_{\rm EM})\right)^{q_\gamma}}$$

with $Q = m_a + m_b - m_c - m_d$

$\hookrightarrow$ penetration factors must be modified in there is a neutron in the initial and/or final state

$\hookrightarrow$ for details, see UGM, Metsch, Meyer (2023)

Weak decay rates

- β-decay rate in terms of the Fermi function

Segrè (1964)

$$\lambda = \frac{G_F^2\,|\mathcal{M}_{fi}|^2}{2\pi^3 c^3 \hbar^7} \underbrace{\int_0^{p_{e,\max}} \left(W - \sqrt{m_e^2 c^4 + p_e^2 c^2}\right)^2 F(Z,\alpha,p_e)\, p_e^2\, dp_e}_{=f(\alpha,Q)}$$

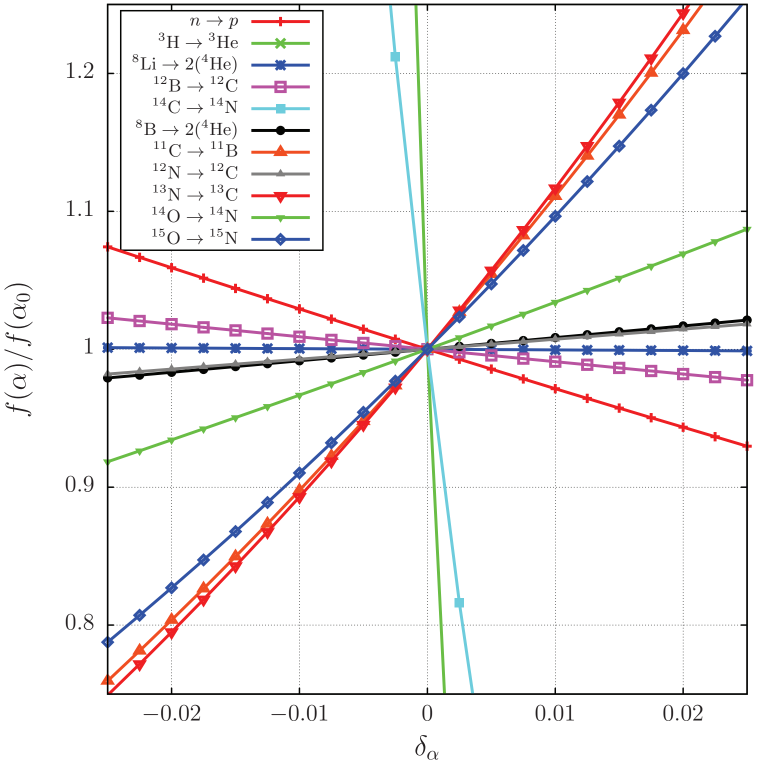
$$p_{e,\max} = \frac{1}{c}\sqrt{W^2 - m_e^2 c^4}\,, \quad W \simeq m_a - m_b = Q$$

- Fermi function (for $Z\alpha \ll 1$):

$$F(\pm Z,\alpha,\epsilon_e) \simeq \frac{\pm 2\pi\nu}{1-\exp(\mp 2\pi\nu)}\,, \quad \nu = \frac{Z\alpha\epsilon_e}{\sqrt{\epsilon_e^2 - 1}}$$

$\hookrightarrow$ α-dependent rates:

$$\boxed{\lambda(\alpha) = \lambda(\alpha_0)\frac{f(\alpha,Q)}{f(\alpha_0,Q)}}$$



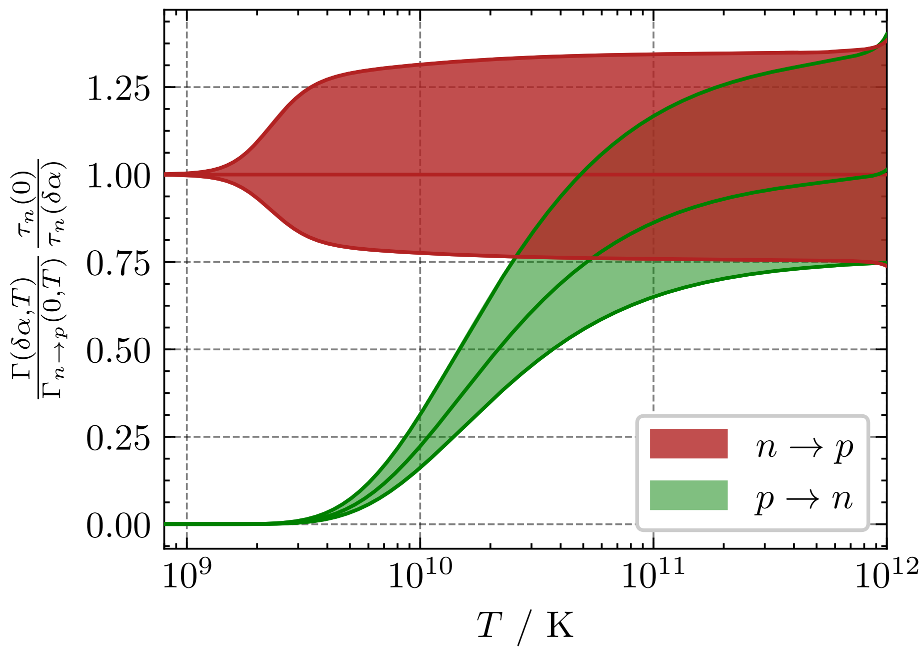
Neutron decay and $n \leftrightarrow p$ rates

- Consider free neutron decay:

$$\tau_n(\alpha) = \tau_n(\alpha_0)\,\frac{f(\alpha, Q)}{f(\alpha_0, Q)}$$

- But this ignores the Fermi-Dirac distribution of the neutrino and the electron

$\Rightarrow$ strong T-dependence in the α-variation for high T



- $Q_n = m_n - m_p$ has a QED contribution, use the new value:

$$\boxed{\Delta Q_n = Q_n^{\rm QED} \cdot \delta\alpha = -0.58(16)\ {\rm MeV} \cdot \delta\alpha}$$

Gasser et al. (2021)

$\hookrightarrow$ affects the weak $n \leftrightarrow p$ rates $\rightarrow$ ^{4}He abundance very sensitive

$\hookrightarrow$ affects the Q-values of the β-decays

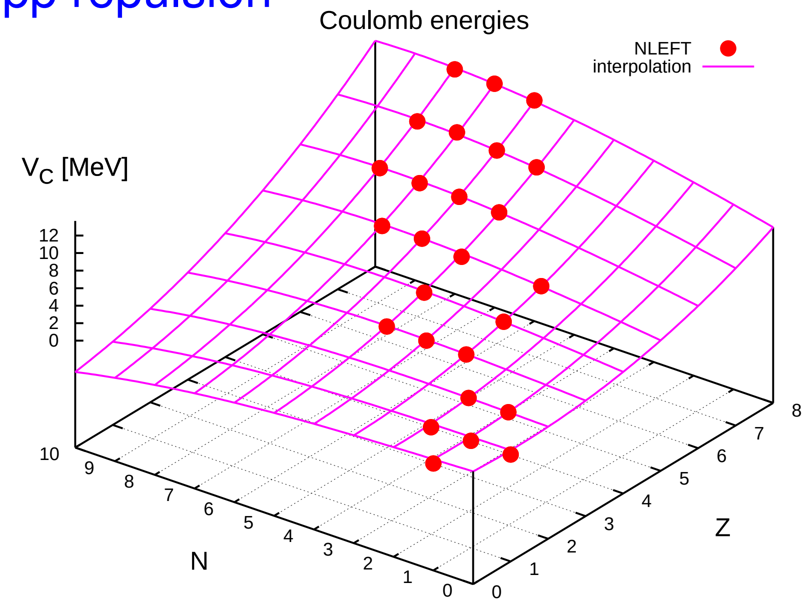
$\hookrightarrow$ α-dependence of $m_N = (m_p + m_n)/2$ in $np \rightarrow d\gamma$ can be neglected

Indirect effects – binding energies

- EM (Coulomb) contributions to nuclear BEs from pp repulsion
- calculated in NLEFT Elhatisari et al., Nature **630** (2024) 59

$\hookrightarrow$ change in Q-values

$$\Delta Q = \delta\alpha \left(-\sum_i B_C^i + \sum_j B_C^j \right)$$



$\Rightarrow$ Nuclear reaction cross sections:

$$\boxed{\sigma(E,\alpha) \propto \underbrace{(E+Q(\alpha))^{p_\gamma}}_{\text{phase space}} \alpha^{q_\gamma} \frac{\sqrt{E_G^{\rm in}/E}}{\exp\left(\sqrt{E_G^{\rm in}/E}\right)-1} \frac{\sqrt{E_G^{\rm out}/(E+Q(\alpha))}}{\exp\left(\sqrt{E_G^{\rm out}/(E+Q(\alpha))}\right)-1}}$$

$p_\gamma = 3\,,\quad q_\gamma = 1$ for radiative capture

$p_\gamma = 1/2\,,\quad q_\gamma = 0$ for other reactions

Comparison to earlier works

- Compare the coefficient a for various codes and earlier works

Code/work	d	^{3}H+^{3}He	^{4}He	^{6}Li	^{7}Li+^{7}Be
PRIMAT	3.658	3.534	1.408	6.953	-4.302
AlterBBN	3.644	3.526	1.373	6.856	-4.322
Dent et al. (2007)	3.612	0.948	1.898	6.681	-11.307
Nollett et al. (2002)	3.993	1.033	–	–	-9.296
Bergstoem et al. (2002)	5.129	0.778	1.956	–	-13.619

- largest differences in ^{3}H+^{3}He (unmeasured) and ^{7}Li+^{7}Be (Li problem)

- Our bounds are stronger due to:

  $\hookrightarrow$ updated experimental values for masses, constants, ..., smaller $Q_n^{\rm QED}$

  $\hookrightarrow$ different reaction rates due to cross section parametrizations

  $\hookrightarrow$ calculating the corrections exactly or using T-dependent approximations

A SHORT HISTORY of the HOYLE STATE

- Heavy element generation in massive stars: triple-α process

Bethe 1938, Öpik 1952, Salpeter 1952, Hoyle 1954, $\ldots$

$$
\begin{aligned}
&{}^4\mathrm{He} + {}^4\mathrm{He} \rightleftharpoons {}^8\mathrm{Be} \\
&{}^8\mathrm{Be} + {}^4\mathrm{He} \rightleftharpoons {}^{12}\mathrm{C}^* \rightarrow {}^{12}\mathrm{C} + \gamma \\
&{}^{12}\mathrm{C} + {}^4\mathrm{He} \rightleftharpoons {}^{16}\mathrm{O} + \gamma
\end{aligned}
$$

- Hoyle's contribution: calculation of relative abundances of ^{4}He, ^{12}C and ^{16}0
  $\Rightarrow$ need a resonance close to the ^{8}Be + ^{4}He threshold at $E_R = 0.35$ MeV
  $\Rightarrow$ this corresponds to a $J^P = 0^+$ excited state 7.7 MeV above the g.s.

- a corresponding state was experimentally confirmed at Caltech at
  $E - E(\mathrm{g.s.}) = 7.653 \pm 0.008$ MeV Dunbar et al. 1953, Cook et al. 1957

- still on-going experimental activity, e.g. EM transitions at SDALINAC

M. Chernykh et al., Phys. Rev. Lett. 98 (2007) 032501

- and how about theory ? $\rightarrow$ this talk

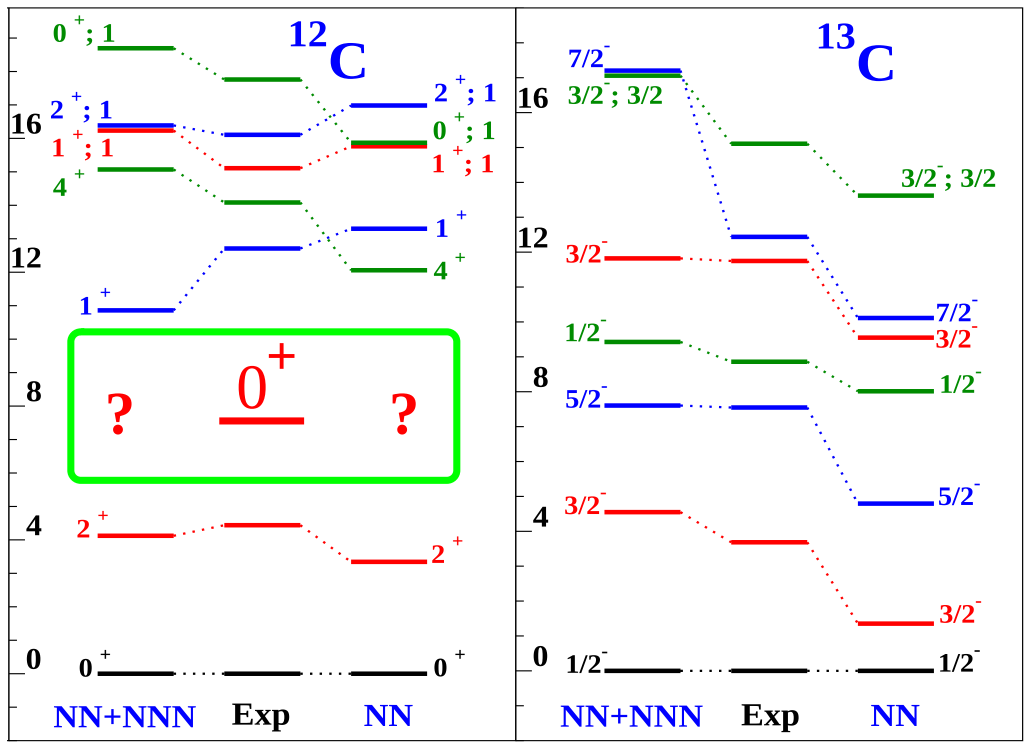
- side remark: NOT driven by anthropic considerations

H. Kragh, Arch. Hist. Exact Sci. 64 (2010) 721

AN ENIGMA for NUCLEAR THEORY

- Ab initio calculation in the no-core shell model: $\approx 10^7$ CPU hrs on JAGUAR

P. Navratil et al., Phys. Rev. Lett. **99** (2007) 042501; R. Roth et al., Phys. Rev. Lett. **107** (2011) 072501



$\Rightarrow$ excellent description, but no trace of the Hoyle state

RESULTS for HEAVIER NUCLEI

- calculate BBN response matrix of primordial abundances Y_a at fixed baryon/photon ratio :

$$\frac{\delta \ln Y_a}{\delta \ln m_q} = \sum_{X_i} \frac{\partial \ln Y_a}{\partial \ln X_i} K^q_{X_i}$$

$\Rightarrow$

X	d	^{3}He	^{4}He	^{6}Li	^{7}Li
a_s	−0.39	0.17	0.01	−0.38	2.64
$B_{\rm deut}$	−2.91	−2.08	0.67	−6.57	9.44
$B_{\rm trit}$	−0.27	−2.36	0.01	−0.26	−3.84
$B_{^3\rm He}$	−2.38	3.85	0.01	−5.72	−8.27
$B_{^4\rm He}$	−0.03	−0.84	0.00	−69.8	−57.4
$B_{^6\rm Li}$	0.00	0.00	0.00	78.9	0.00
$B_{^7\rm Li}$	0.03	0.01	0.00	0.02	−25.1
$B_{^7\rm Be}$	0.00	0.00	0.00	0.00	99.1
τ	0.41	0.14	0.72	1.36	0.43

updated Kawano code

Kawano, FERMILAB-Pub-92/04-A

RESULTS

- putting pieces together:

$$\boxed{\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.455(35)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.744(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.056(10)}$$

$$\boxed{\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.117(34)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.189(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.012(9)}$$

  - x_1 and x_2 only affect the small constant terms
  - also calculated the shifts of the individual energies (not shown here)

INTERPRETATION

- $(\partial\Delta E_h/\partial M_\pi)/(\partial\Delta E_b/\partial M_\pi) \simeq 4$
  $\Rightarrow$ ΔE_h and ΔE_b cannot be independently fine-tuned

- Within error bars, $\partial\Delta E_h/\partial M_\pi$ & $\partial\Delta E_b/\partial M_\pi$ appear unaffected
  by the choice of x_1 and x_2 $\rightarrow$ indication for α-clustering

- the triple alpha process is controlled by :

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^\star - 3E_4$$

$$\boxed{\left.\frac{\partial\Delta E_{h+b}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.571(14)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.934(11)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.069(6)}$$

$\Rightarrow$ quark mass dependence of the scattering lengths discussed earlier

Probing nuclear observables via primordial nucleosynthesis

UGM, Metsch, Eur. Phys. J. A **58** (2022) 212 [arXiv:2208.12600 [nucl-th]]

Variations of nuclear observables

- Study the nuclear abundances Y_i as a function of nuclear observables:

  $\to$ binding energies, scattering lengths, neutron lifetime, ...

- Use four different BBN codes to study the systematic error

  $\hookrightarrow$ only if this is small / controlled, it makes sense to look at variations of $\alpha_{\rm EM}$ etc.

- Nuclear reactions rates: $\Gamma_{ab\to cd} = n_B\,\gamma_{ab\to cd}$

$$\gamma_{ab\to cd} = N_A\sqrt{\frac{8}{\pi\mu_{ab}(kT)^3}}\int_0^\infty {\rm d}E\,E\,\sigma_{ab\to cd}(E)\,{\rm e}^{-\frac{E}{kT}}$$

$$\mu_{ab} = m_a m_b/(m_a+m_b)$$

- Inverse reaction (with spin multiplicity g_i):

$$\gamma_{cd\to ab}(T) = \left(\frac{\mu_{ab}}{\mu_{cd}}\right)^{\frac{3}{2}}\frac{g_a\,g_b}{g_c\,g_d}\,{\rm e}^{-\frac{Q}{kT}}\,\gamma_{ab\to cd}(T)$$

- Consider now changes in the binding energies by $\pm$ 1 permille $\to$ Q-factors change

Variations of the binding energies / Q-values

- Direct reactions $a+b\to c+d$

$$\gamma_{ab\to cd}(\widetilde{Q};T) \approx \gamma_{ab\to cd}(Q_0;T) + \sqrt{\frac{8}{\pi\,\mu_{ab}\,(kT)^3}}\int_0^\infty \mathrm{d}E\,E\,\sigma(Q_0;E) \times \left(\frac{\Delta Q}{2\,(Q_0+E)} + \frac{\Delta Q\,\sqrt{E_G}}{2\,(Q_0+E)^{\frac{3}{2}}}\right)\mathrm{e}^{-\frac{E}{kT}}$$

$$= \gamma_{ab\to cd}(Q_0;T) + \Delta\gamma(T)_{ab\to cd}$$

$$\widetilde{Q} = Q_0 + \Delta Q$$ change of the Q-value

$$E_G = 2\pi^2\,Z_c^2\,Z_d^2\,\alpha_{\mathrm{EM}}^2\,\mu_{cd}\,c^2$$ Gamov energy in the exit channel

- In all earlier investigations, the T-dependence of $\Delta\gamma(T)_{ab\to cd}$ was neglected

- Similar for radiative capture reactions $a+b\to c+\gamma$
  and weak decay rates $a\to b+e^{\pm}+\overset{(-)}{\nu}$

Reactions considered

- Radiative capture reations:

$$n+p\to d+\gamma\,,\;p+d\to {}^3\text{He}+\gamma\,,\;p+{}^3\text{H}\to {}^4\text{He}+\gamma\,,\;{}^3\text{H}+{}^4\text{He}\to {}^7\text{Li}+\gamma\,,$$
$${}^3\text{He}+{}^4\text{He}\to {}^7\text{Be}+\gamma\,,\;\;d+{}^4\text{He}\to {}^6\text{Li}+\gamma\,,\;\;p+{}^7\text{Li}\to {}^4\text{He}+{}^4\text{He}+\gamma\,,$$
$$p+{}^6\mathbf{Li}\to {}^7\mathbf{Be}+\gamma$$

- Charge exchange reactions:

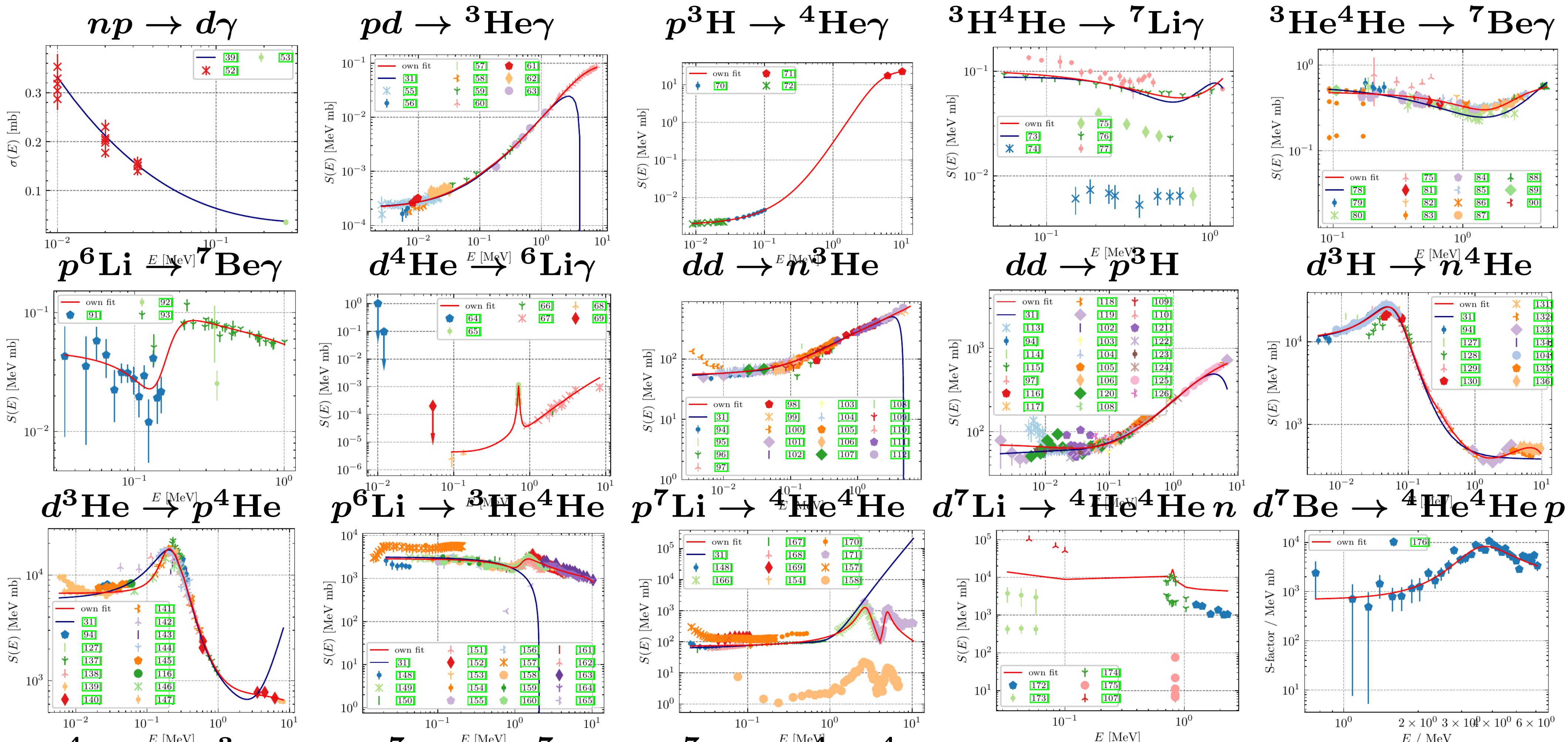
$$n+{}^3\text{He}\to {}^3\text{H}+p\,,\;n+{}^7\text{Be}\to {}^7\text{Li}+p$$

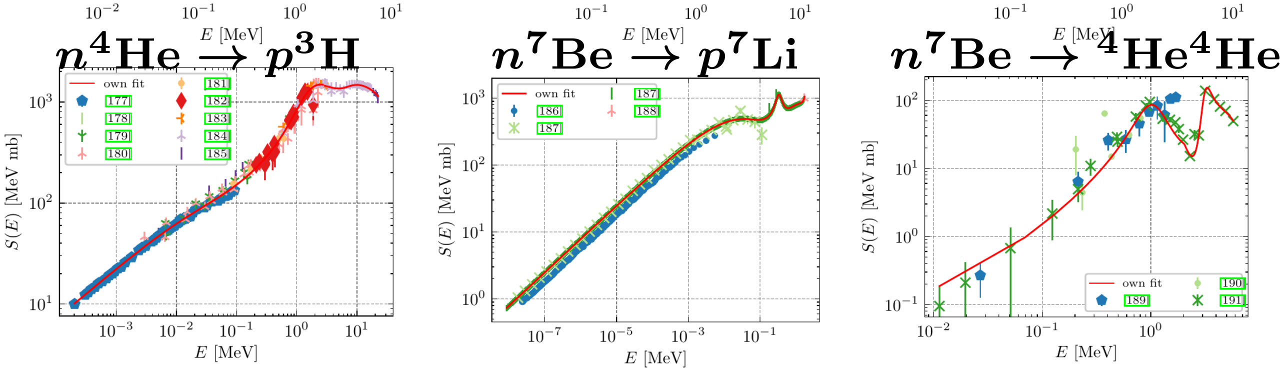
- Direct reactions:

$$d+d\to {}^3\text{H}+p\,,\;d+d\to {}^3\text{He}+n,\;\;d+{}^3\text{H}\to {}^4\text{He}+n\,,\;d+{}^3\text{He}\to {}^4\text{He}+p\,,$$
$$p+{}^7\text{Li}\to {}^4\text{He}+{}^4\text{He}\,,\;\;p+{}^6\text{Li}\to {}^3\text{He}+{}^4\text{He}\,,\;\;d+{}^7\text{Be}\to {}^4\text{He}+{}^4\text{He}+p\,,$$
$$n+{}^7\text{Be}\to {}^4\text{He}+{}^4\text{He}\,,\;\;d+{}^6\text{Li}\to {}^3\text{He}+{}^4\text{He}+n$$

- plus weak decays ....

Reaction parameterizations





$$S(E) = E\sigma(E)\exp(\sqrt{E_G/E})$$
$$= \text{rat. polynom} + \text{BW(s)}$$
$$E_G = 1 \text{ for neutron capture}$$

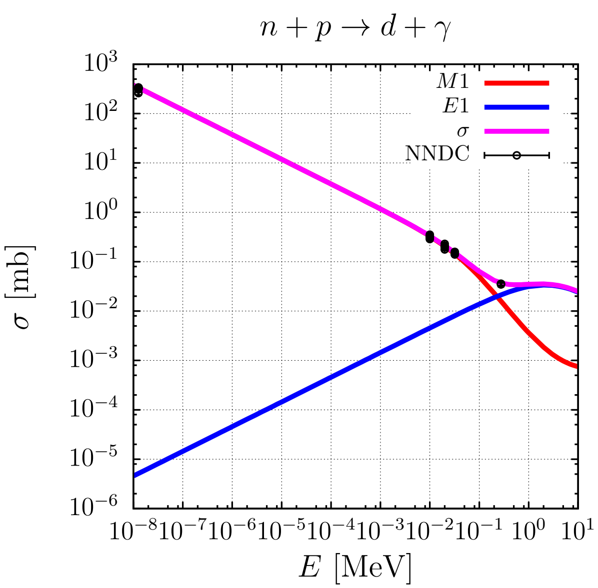
[31] Serpico et al., JCAP **12**, 010 (2004)

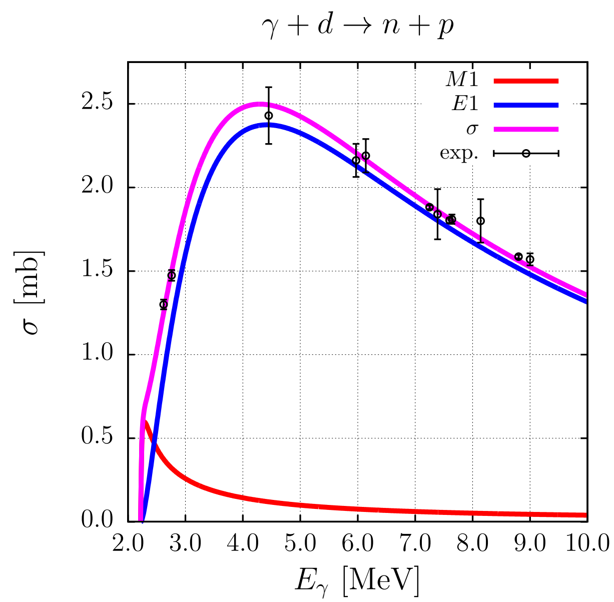
The leading reaction $n + p \to d + \gamma$

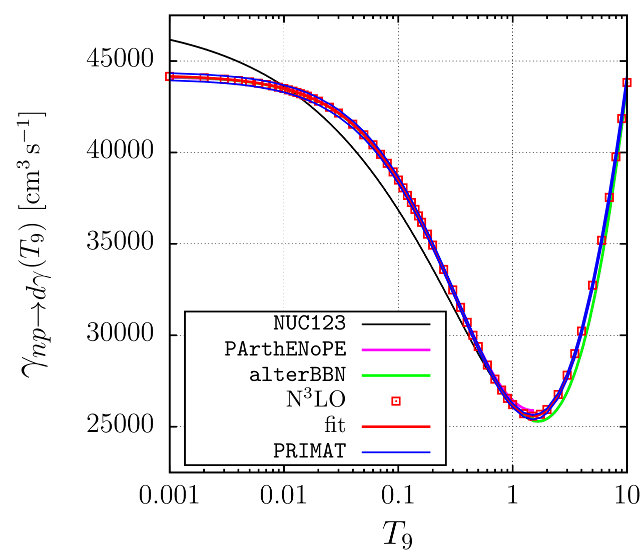
- Use the pionless EFT description up to N^{3}LO

Chen, Savage (1999), Rupak (2000)

$$\sigma_{np\to d\gamma}(E) = \frac{4\pi\,\alpha_{\rm EM}\,(\gamma^2+p^2)^3}{\gamma^3\,m_n^4\,p}\left[|\chi_{M1}|^2 + |\chi_{E1}|^2\right]\,,\quad \chi_{E1,M1} = f(a_s, B_d)$$





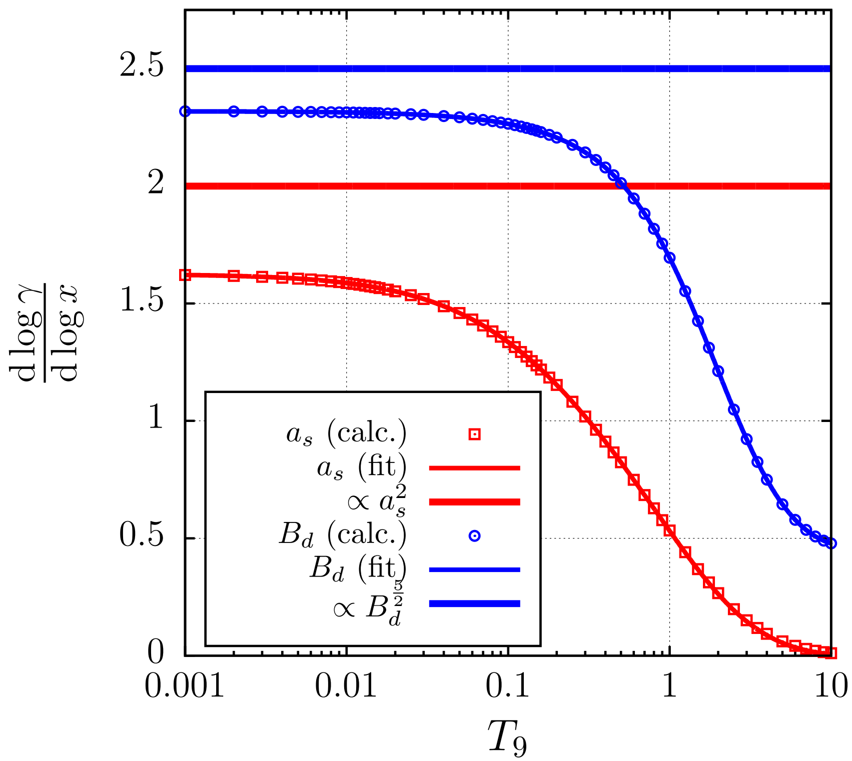


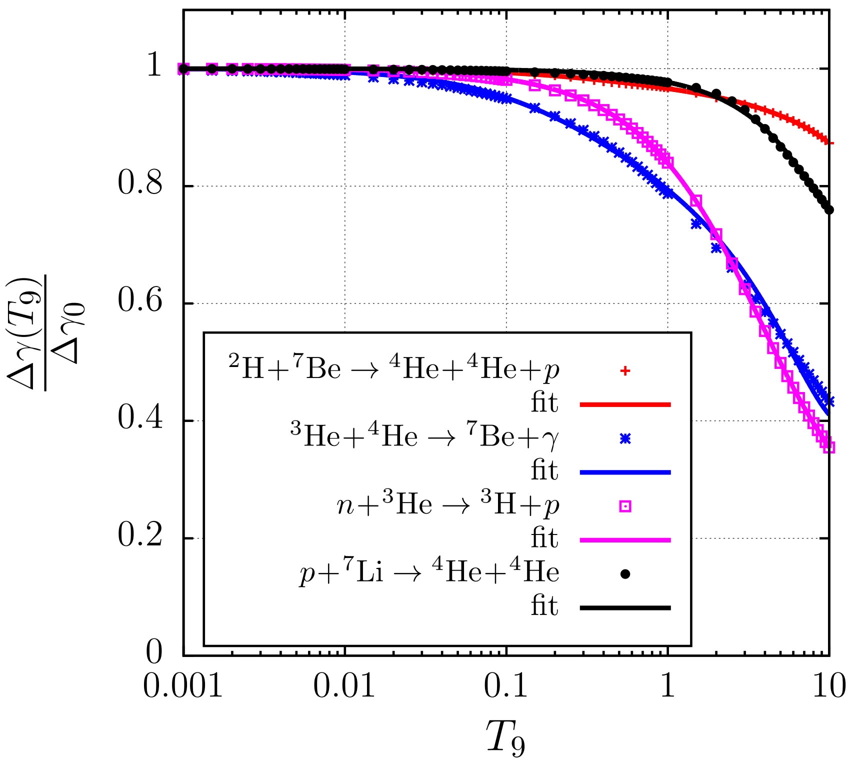
- $M1$ dominance at low energies, all codes agree on the T-dependence

- LO: $\gamma_{M1;np\to d\gamma} \propto B_d^{\frac{5}{2}}\,a_s^2\,,\quad \frac{\partial \log\gamma}{\partial \log a_s} = 2\,,\quad \frac{\partial \log\gamma}{\partial \log B_d} = \frac{5}{2}$ appear T-indep.

Temperature-dependence of nuclear reactions

- Consider the leading and the next 17 reactions in the BBN network
  $\rightarrow$ integrate reaction formulae ($T_9 = T/[10^9\mathrm{K}]$):



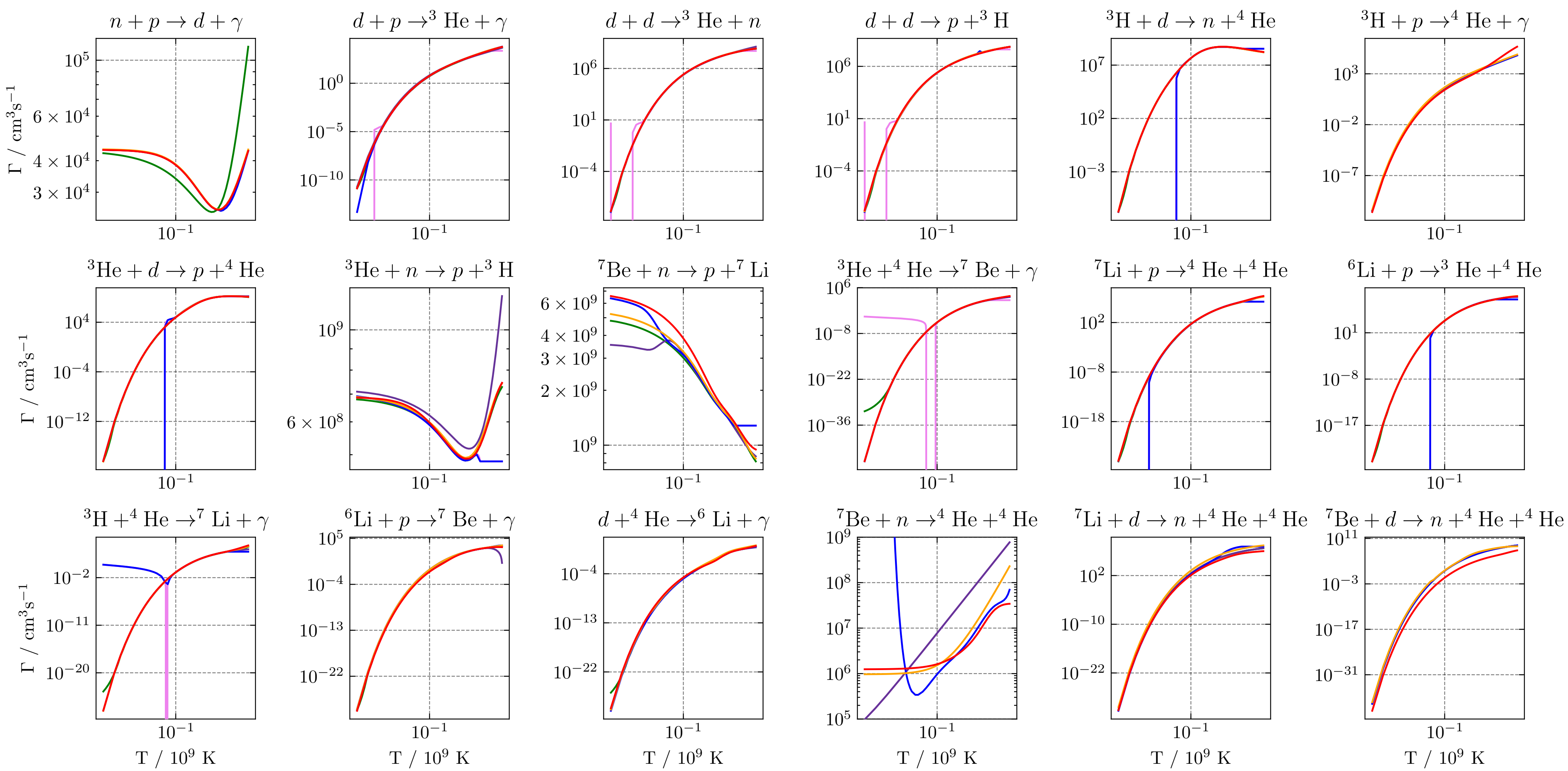


- Strong suppression of the a_s and B_d dependence in the leading reactions due to T

- T-effect appreciable for most reactions for $T_9 \gtrsim 0.1$

Temperature-dependence of nuclear reactions II

- For all codes, this work and NACRE II [Xu et al., Nucl. Phys. A **918** (2013) 61]



BBN response matrix

- Calculate the linear dependence of the Y_n for small changes in a_S, B_d, τ_n:

$$\partial \log Y_n / \partial \log X_k \;, \qquad n \in \{{}^2\text{H}\,, {}^3\text{H} + {}^3\text{He}\,, {}^4\text{He}\,, {}^6\text{Li}\,, {}^7\text{Li} + {}^7\text{Be}\}$$

- Updating all natural constants, atomic & nuclear masses, and the leading reactions from EFT in all codes ($\eta = 6.14 \cdot 10^{-10}$, $\tau_n = 879.4$ s)

code	^{2}H $\times 10^5$	^{3}H $+{}^3$He $\times 10^5$	Y_p	^{6}Li $\times 10^{14}$	^{7}Li $+{}^7$Be $\times 10^{10}$
NUC123	2.550	1.040	0.247	1.101	4.577
PArthENoPE	2.511	1.032	0.247	1.091	4.672
AlterBBN	2.445	1.031	0.247	1.078	5.425
PRIMAT	2.471	1.044	0.247	1.198	5.413
PDG (2022)	2.547		0.245		1.6
$\pm$	0.025		0.003		0.3

- Four codes are largely consistent
- Lithium problem prevails

Fields, Ann. Rev. Nucl. Part. Sci. **61** (2011) 47

Further results

- T-dependence of the reaction rates on changes in B_A for the first ttime considered

$\hookrightarrow \dfrac{\partial \log Y_n}{\partial \log a_s}$ reduced by a factor of three

$\hookrightarrow \dfrac{\partial \log Y_n}{\partial \log B_i}$ reduced by about 10 percent

- η-dependence linear and a minor effect for small changes (for $\eta \cdot 10^{10} = 5.94 - 6.34$)

- Code dependence of $\dfrac{\partial \log Y_n}{\partial X_k}$ is very small

$\Rightarrow$ Now we are in the position to study the dependence of the Y_n on the fundamental parameters, especially on $\alpha_{\rm EM}$

The tool: Nuclear lattice effective field theory

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

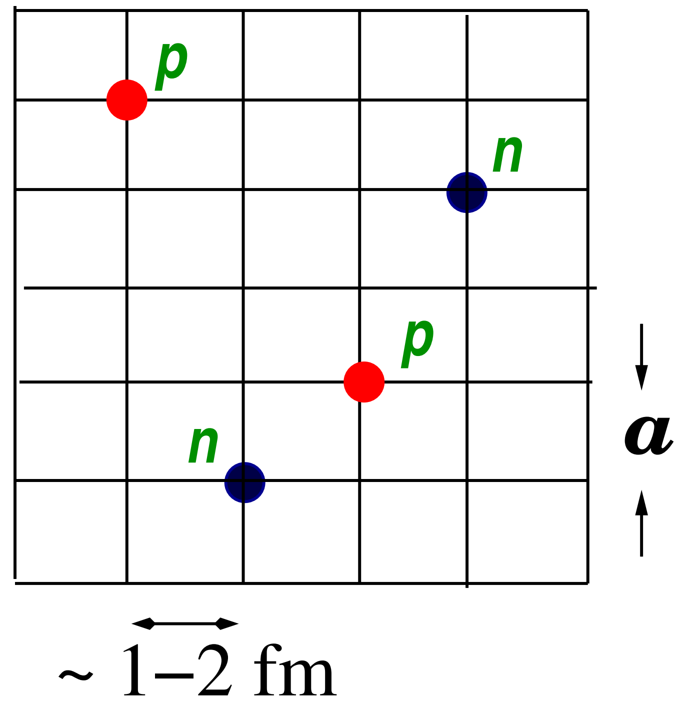
- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

  → see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters



$$p_{\rm max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$

- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

  E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

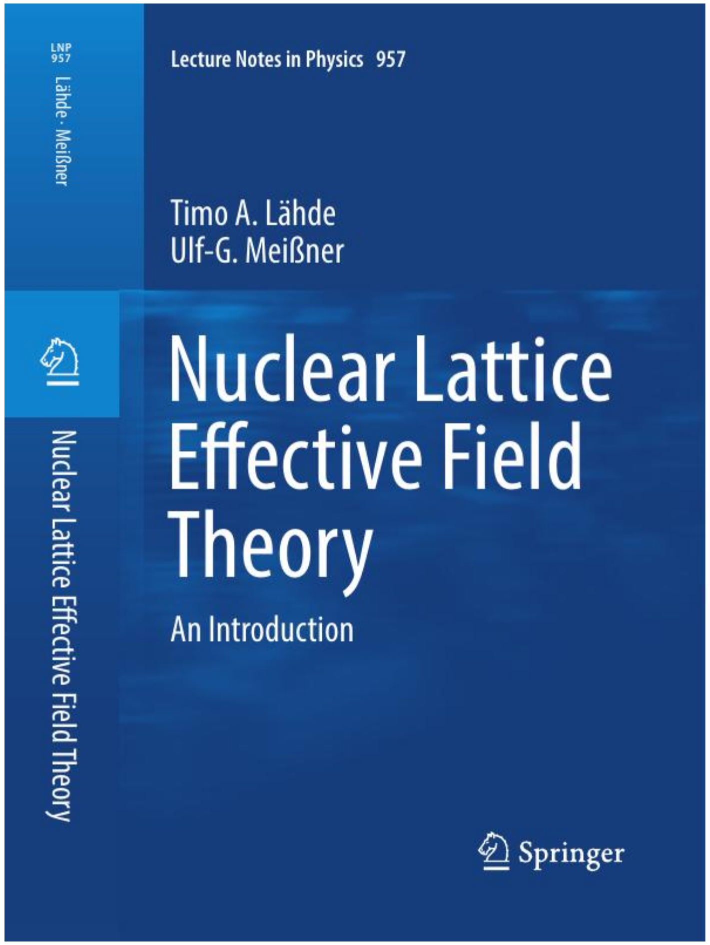
- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

  Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

The tool: Nuclear lattice effective field theory II

- For all details on chiral EFT on a lattice

T. Lähde & UGM
Nuclear Lattice Effective Field Theory - An Introduction
Springer Lecture Notes in Physics **957** (2019) 1 - 396

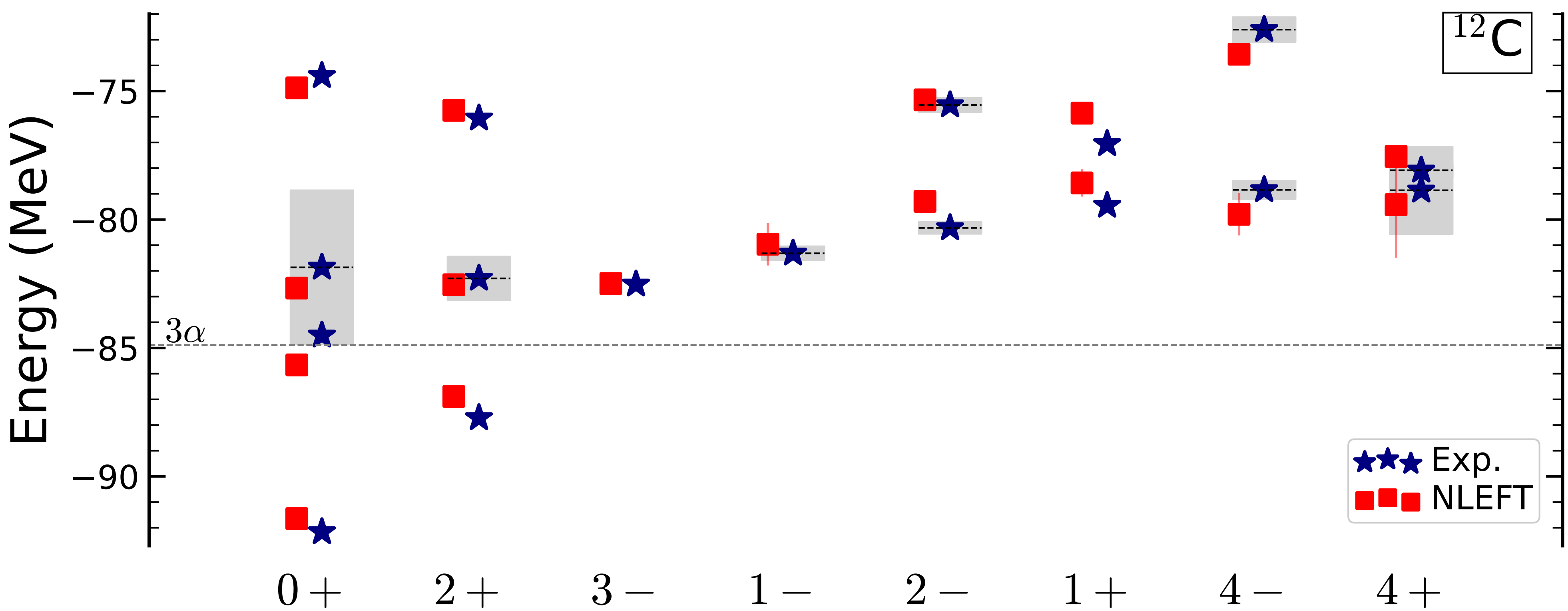


- Computational equipment

NLEFT @ work: The spectrum of carbon-12 A.D. 2023

- with much improved algorithms and methods:

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777



$\rightarrow$ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

Effects of the neutron lifetime

- The neutron width $\Gamma_n \sim 1/\tau_n$ is given by:

$$\Gamma_n = \frac{(G_F \cos\theta_C)^2}{2\pi^3} m_e^5 (1+3g_A^2)\, f\left(\frac{(m_n - m_p)^{\rm QED}}{m_e}\right)$$

- BLP assumed that m_u/m_d stays constant when m_q changes

  $\hookrightarrow$ induces a large dependence of the function f on variation in m_q

  $\hookrightarrow$ is model-dependent, as all other parameters are supposed to be unaffected

- A more natural scenario for $m_u/m_d = \text{constant}$ is that the Higgs VEV changes, while all Yukawa and gauge coupling stay constant

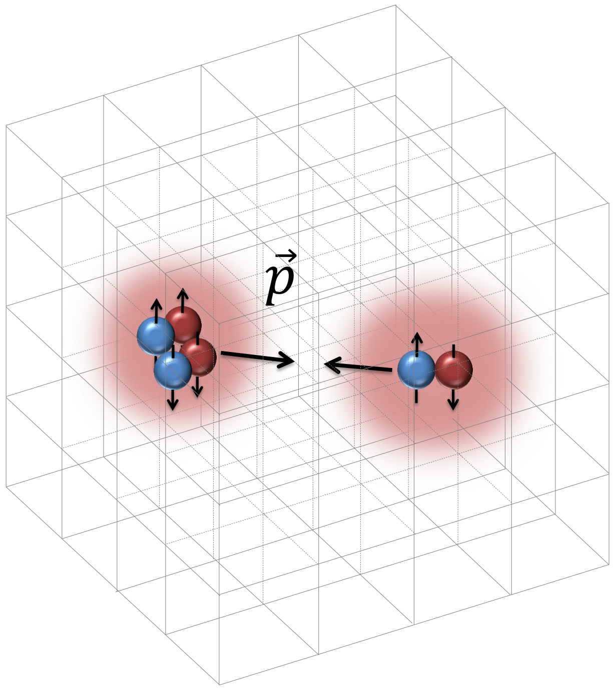
  $\hookrightarrow$ this reduces the dependence of Γ_n on variations of m_q by a factor of 2

  $\hookrightarrow$ the sensitivity to τ_n entirely denotes the ^{4}He sensitivity

Quark mass dependence of alpha-alpha scattering

Nucleus-nucleus scattering on the lattice

- Processes involving α-particles and α-type nuclei comprise a major part of stellar nucleosynthesis, and control the production of certain elements in stars



- Ab initio calculations of scattering and reactions using continuum methods suffer from very unfavorable computational scaling with the number of nucleons A in the clusters (either factorial or exponential in A)

- This is very different in NLEFT:

$$\boxed{\text{Lattice EFT computational scaling} \Rightarrow (A_1 + A_2)^2}$$

Rupak, Lee, Phys. Rev. Lett. **111** (2013) 032502
Pine, Lee, Rupak, Eur. Phys. J. A **49** (2013) 151
Elhatisari, Lee, Phys. Rev. C **90** (2014) 064001
Elhatisari et al., Phys.Rev. C **92** (2015) 054612
Elhatisari, Lee, UGM, Rupak, Eur. Phys. J. A **52** (2016) 174

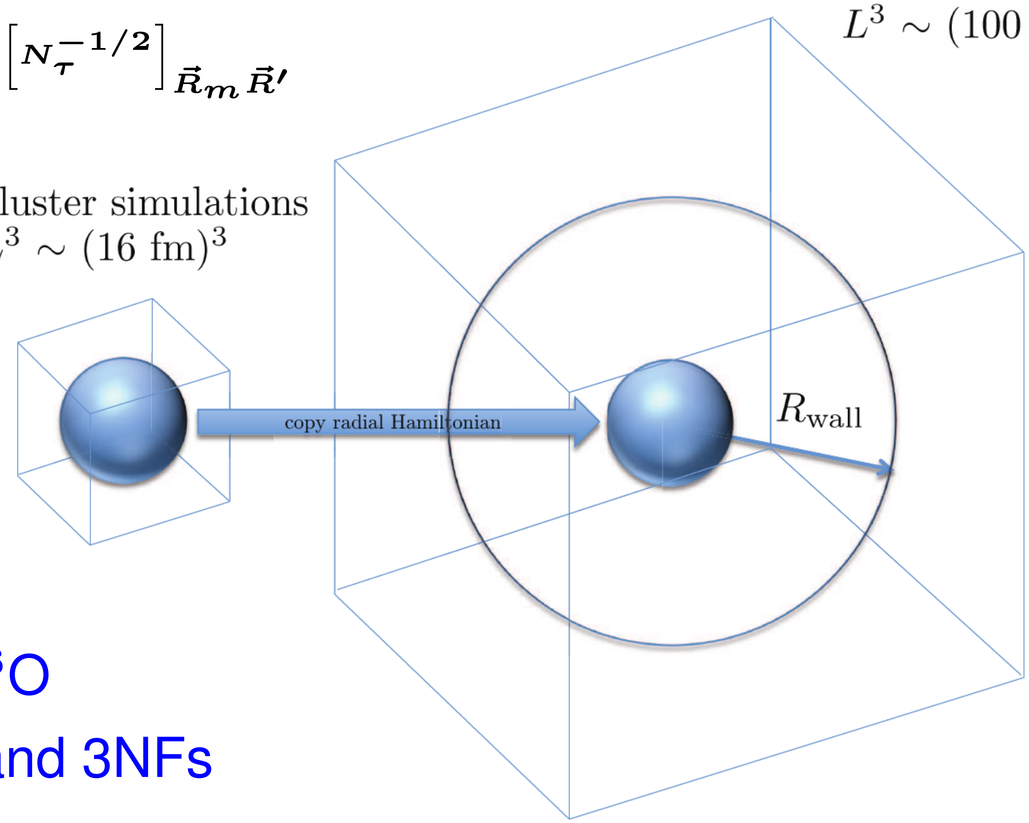
Ab initio alpha-alpha scattering

Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, UGM, Nature **528** (2015) 111

- Construct the so-called adiabatic Hamiltonian

$$[H^a_\tau]_{\vec{R}\vec{R}'} = \sum_{\vec{R}_n \vec{R}_m} \left[N_\tau^{-1/2}\right]_{\vec{R}\vec{R}_n} \left[H_\tau\right]_{\vec{R}_n \vec{R}_m} \left[N_\tau^{-1/2}\right]_{\vec{R}_m \vec{R}'}$$

$\hookrightarrow$ two-cluster simulations



- Long-range Coulomb via spherical wall method (huge box)

Lu, Lähde, Lee, UGM, Phys. Lett. B **760** (2016) 309

$\hookrightarrow$ single cluster simulations

- Same action as used for ^{12}C and ^{16}O chiral N2LO Lagrangian w/ 2NFs and 3NFs

$\hookrightarrow$ all LECs determined before in NN and NNN systems

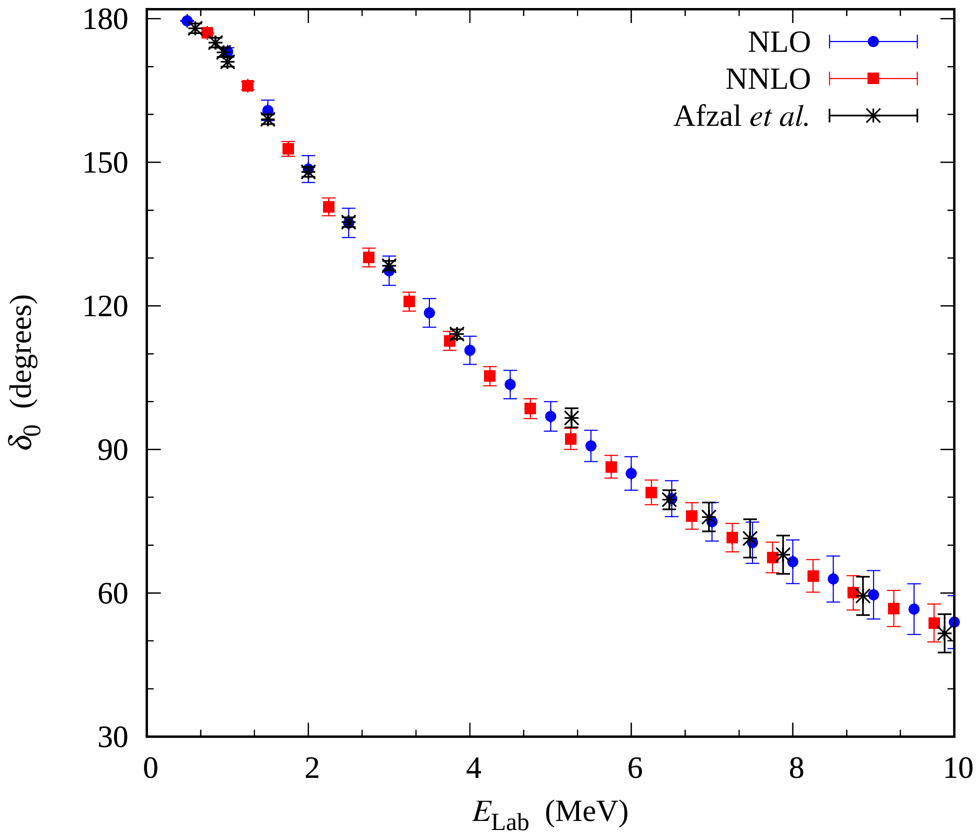
$\hookrightarrow$ parameter-free predictions

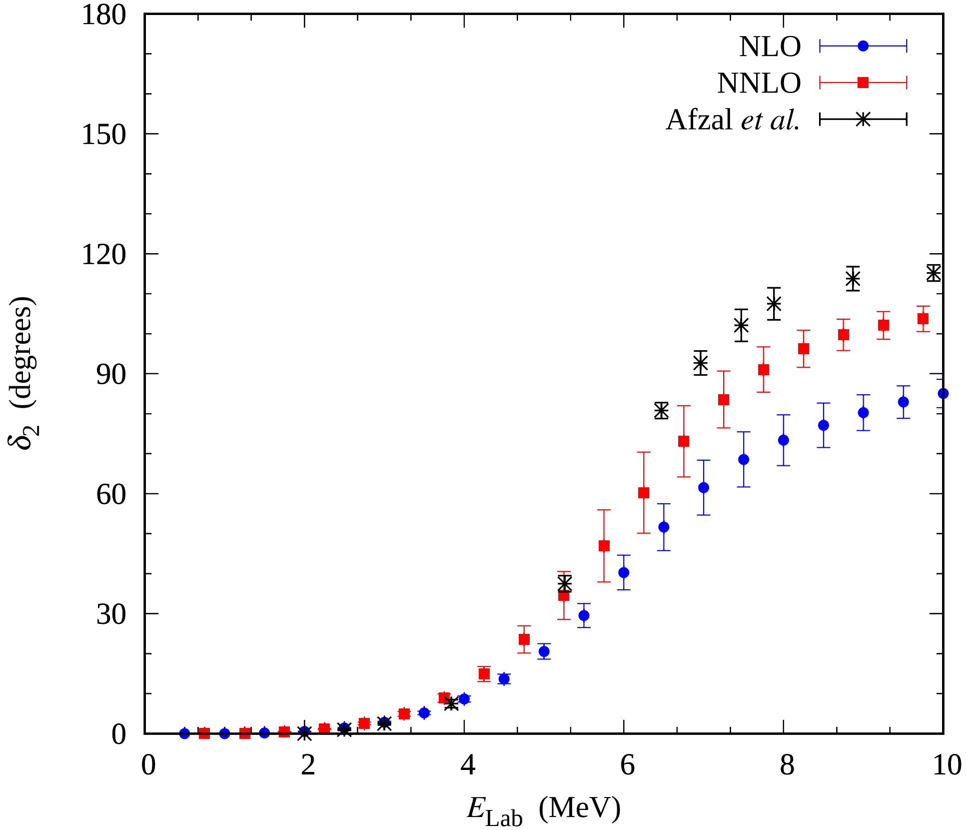
$\hookrightarrow$ first ever *ab initio* calculation of α-α scattering

Phase shifts of alpha-alpha scattering

- S-wave and D-wave phase shifts, updated in 2022

Elhatisari, Lähde, Lee, UGM, Vonk, JHEP **02** (2022) 001





$$E_R^{\rm NNLO} = -0.11(1)\ \text{MeV} \quad [+0.09\ \text{MeV}]$$

$$E_R^{\rm NNLO} = 2.93(5)\ \text{MeV} \quad [2.92(18)\ \text{MeV}]$$

$$\Gamma_R^{\rm NNLO} = 2.00(16)\ \text{MeV} \quad [1.35(50)\ \text{MeV}]$$

Afzal et al., Rev. Mod. Phys. **41** (1969) 247 [data]

Alpha-alpha scattering in the multiverse

Elhatisari, Lähde, Lee, UGM, Vonk, JHEP **02** (2022) 001

- Now vary the light quark mass m_q and the fine-structure constant $\alpha_{\rm EM}$

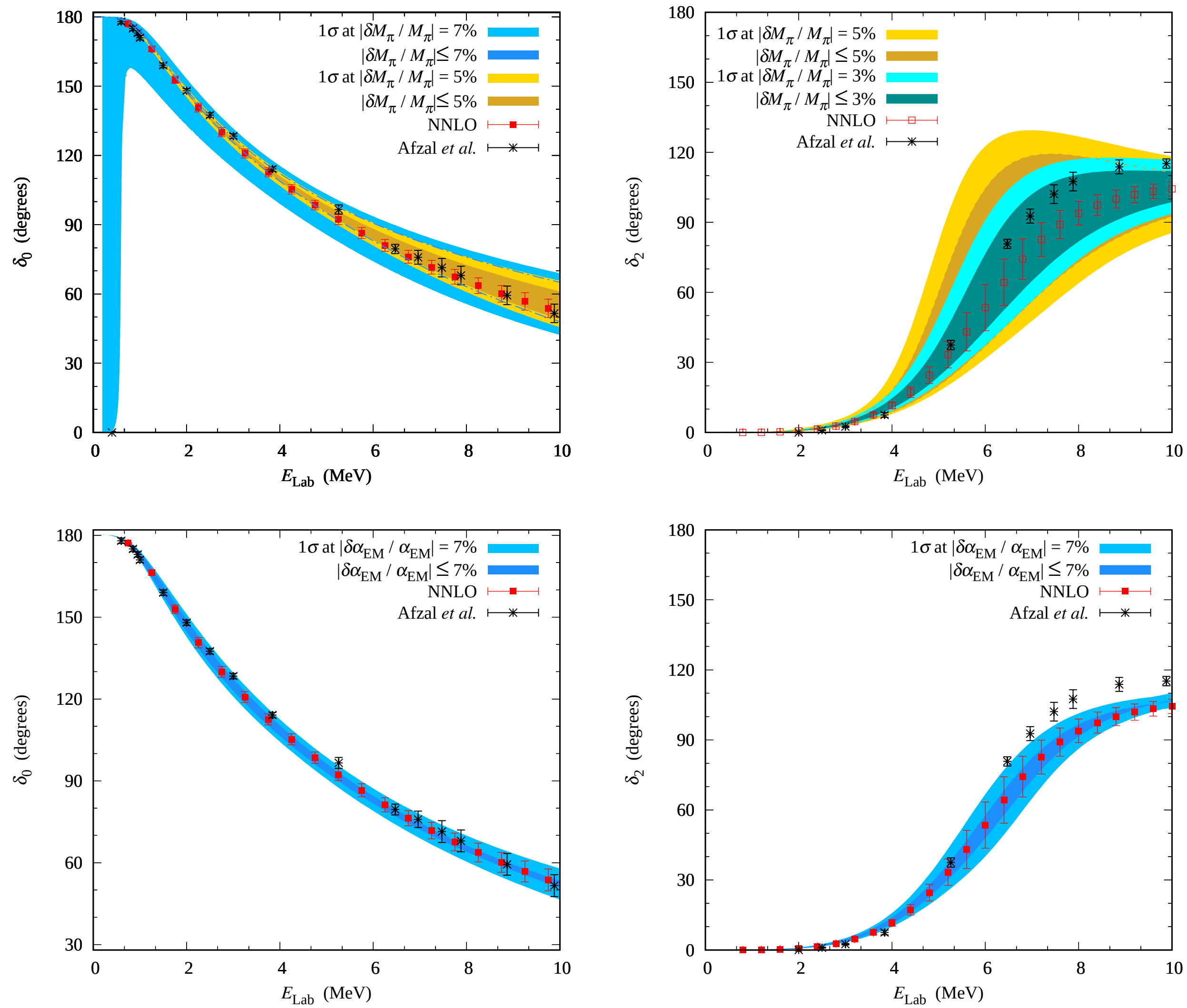
$\Rightarrow$ Dramatic effect in the S-wave
for $\delta M_\pi/M_\pi \simeq 7\%$

$\Rightarrow$ D-wave resonance requires
$\delta M_\pi/M_\pi \lesssim 3\%$

$\Rightarrow$ S- and D-wave phase shifts
tolerate $\delta\alpha_{\rm EM}/\alpha_{\rm EM} \lesssim 7\%$

$\Rightarrow$ weaker bounds as given by the
position of the Hoyle state
but independet of stellar modelling!

- in a next step, consider $\alpha+{}^8$Be$\rightarrow{}^{12}$C
as function of m_q and $\alpha_{\rm EM}$



Primordial nucleosynthesis at varying quark masses

General remarks

- Fundamental parameters of the strong interactions:

  the light quark masses m_u, m_d

  also the strange quark mass m_s

- In almost all nuclear reactions, strong isospin violation $m_d/m_u \simeq 2$ can be neglected because:

$$\boxed{\frac{m_u - m_d}{\Lambda_{\rm QCD}} \simeq \frac{1}{100}}$$

- $m_d \neq m_u$ features prominently in neutron β-decay

- Dominant strange quark effect through the nucleon mass (trace anomaly)

$$\boxed{m_N^{u,d,s} = \langle N(p)|m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s|N(p)\rangle}$$

- Quark mass variations = variations of the $\underbrace{\textbf{weak scale } v}_{\textbf{Higgs VEV}}$ (Yukawa's fixed)

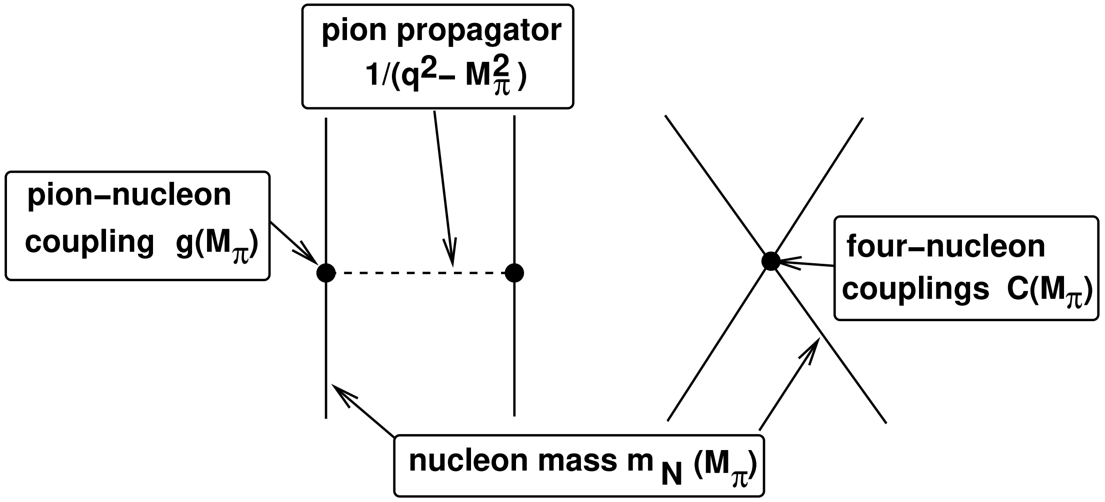
Ingredients

- Nuclear forces are given by chiral EFT based on Weinberg's power counting

Weinberg 1991

$\Rightarrow$ Pion-exchange contributions and short-distance multi-N operators

- graphical representation of the quark mass dependence of the LO potential



- always use the Gell-Mann–Oakes–Renner relation: $\boxed{M_{\pi^\pm}^2 \sim (m_u + m_d)}$

- fulfilled to better than 94% in QCD

Colangelo, Gasser, Leutwyler 2001

- Strong isospin violation and electromagnetic effects can also be included

Higgs VEV variations

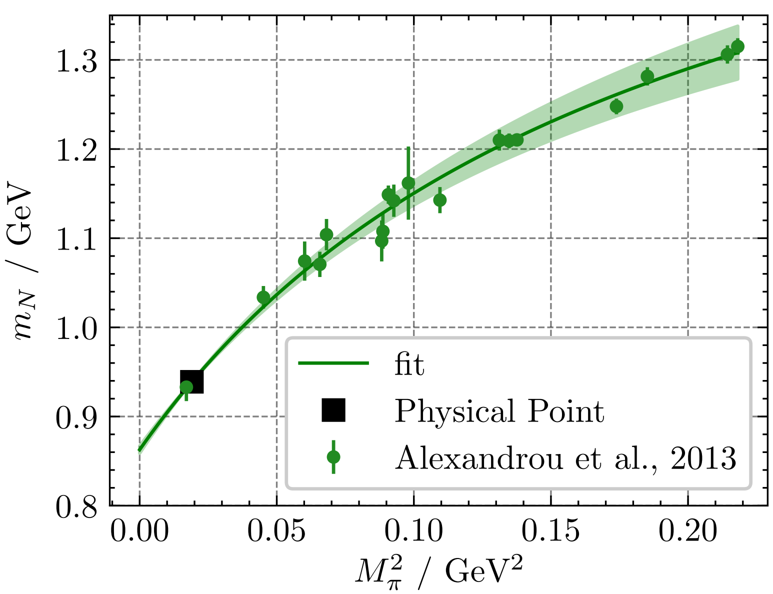
- Where do variations in v appear? $\hookrightarrow$ single nucleon (particle) properties

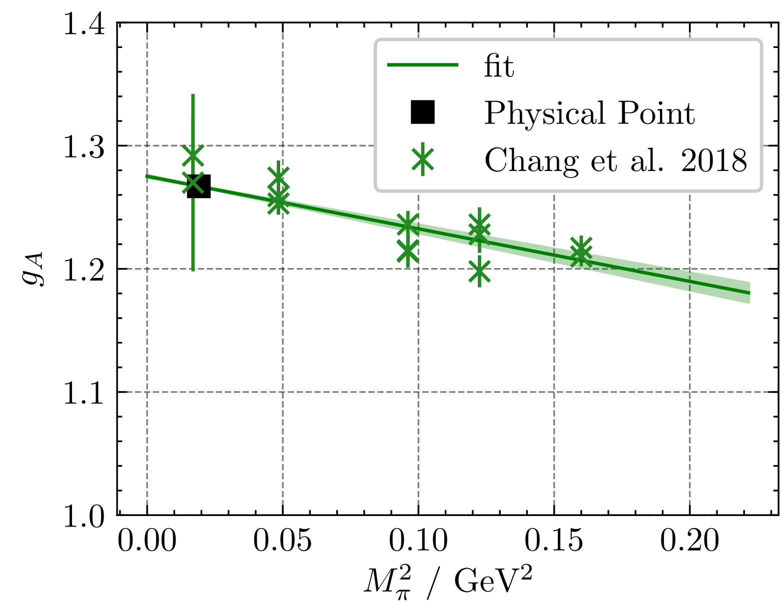
- Fermi constant $G_F \sim (1+\delta v/v)^{-2}$, electron mass, ...

- Neutron-proton mass difference Gasser, Leutwyer, Rusetsky (2021)

$$\frac{Q_N}{\text{MeV}} = \frac{m_n - m_p}{\text{MeV}} = (1.87 \mp 0.16)\left(1 + \frac{\delta v}{v}\right) - (0.58 \pm 0.16)$$

- Nucleon mass m_N and axial-vector coupling constant g_A from lattice QCD





Higgs VEV variations

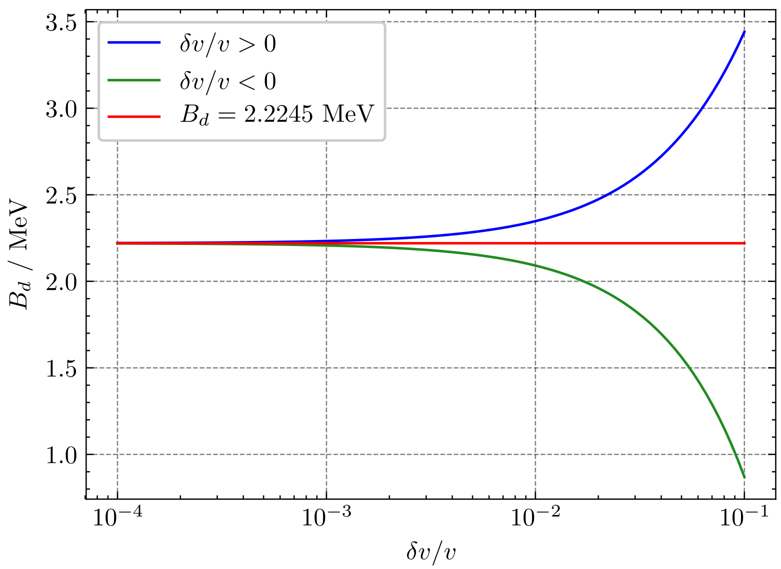
- Where do variations in v appear?

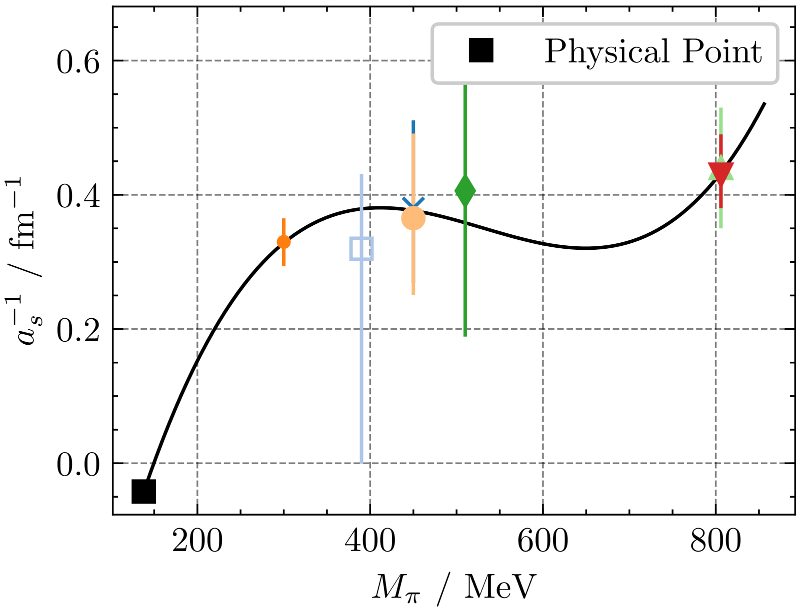
$\hookrightarrow$ two-nucleon observables

- Deuteron binding energy and singlet np scattering length

$\hookrightarrow$ combine LQCD results with NN ERE low-energy theorems

Baru, Epelbaum, Filin, Gegelia (2015,2016)





● PhysRevD.92.014501 □ PhysRevD.85.054511 ▲ PhysRevD.96.114510

× PhysRevD.103.054508 ◆ PhysRevD.86.074514 ▼ PhysRevC.88.024003

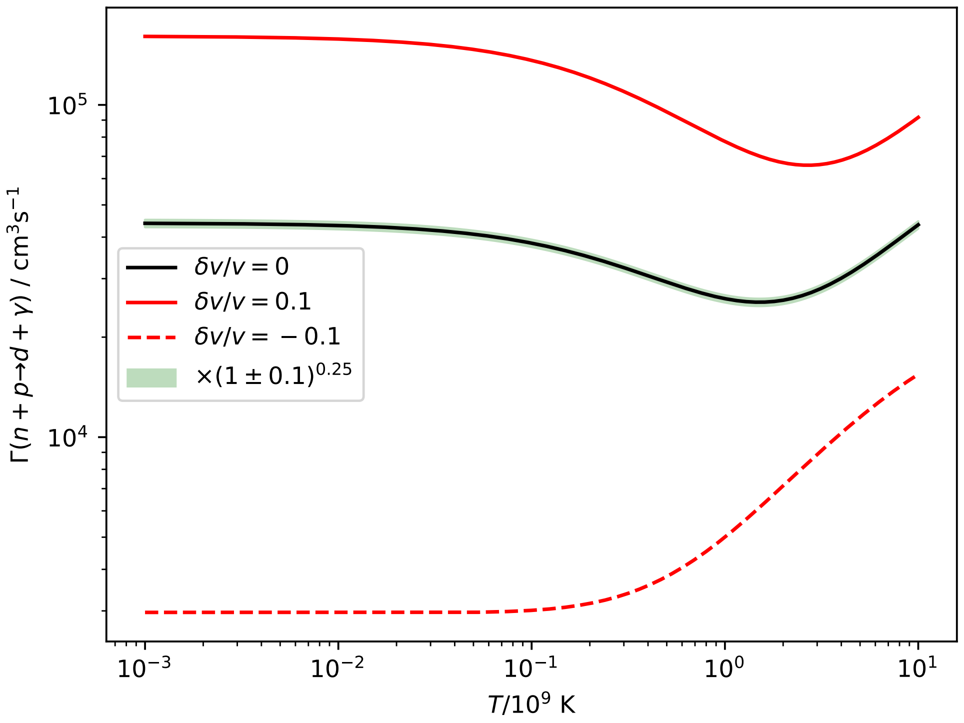
Higgs VEV variations

- Where do variations in v appear?

$\hookrightarrow$ nuclear reactions

- most relevant ist $n + p \to d + \gamma$,
  its rate depends heavily on
  - deuteron binding energy
  - nucleon mass
  - NN scattering parameters

- v dependence much stronger than expected from naive scaling
  Burns et al. (2024)



- **most sensitive** to B_d is the back-reaction at the d bottleneck

$$\boxed{\langle\sigma(d\gamma \to np)v\rangle = \alpha\, T_9^\beta \exp(\kappa/T_9)\langle\sigma(np \to d\gamma)v\rangle\,, \quad \alpha \propto \left(\frac{m_n m_p}{m_d}\right)^{3/2}\,, \quad \kappa \propto B_d}$$

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Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem
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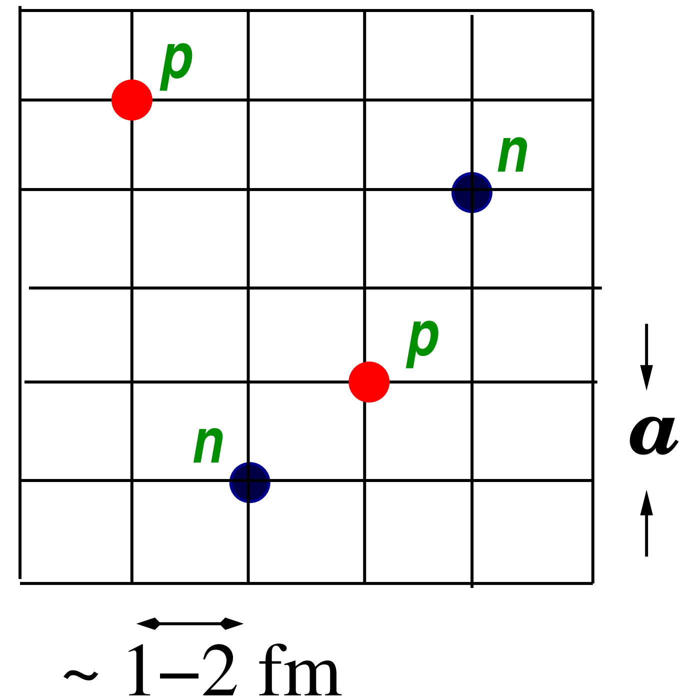

- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

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  Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

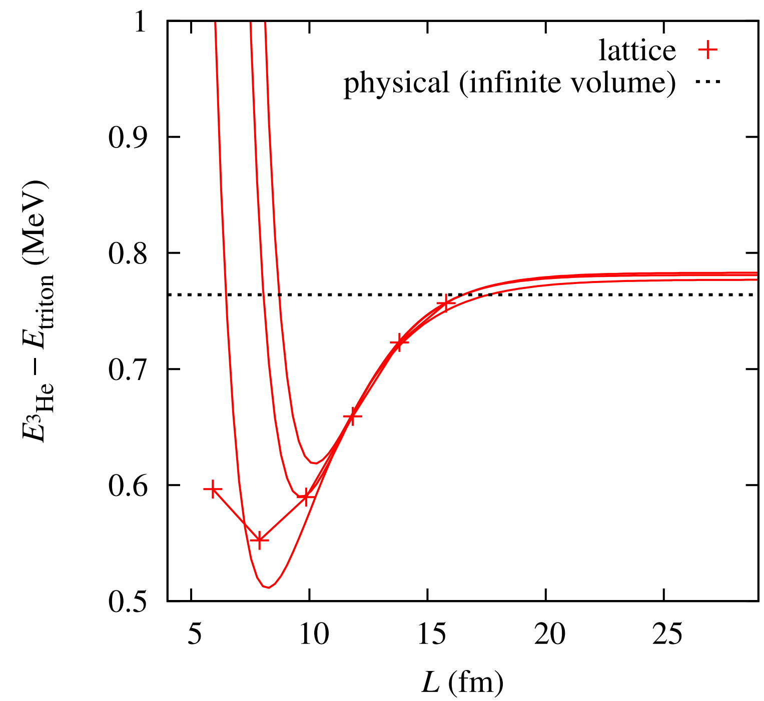
Some early results: Validation of the method

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. A **45** (2010) 335

Lähde, Epelbaum, Krebs, Lee, UGM, Rupak, Phys. Lett. B **732** (2014) 110; Phys. Rev. Lett. **112** (2014) 102501

- Some groundstate energies and differences

E [MeV]	NLEFT	Exp.
^{3}He - ^{3}H	0.78(5)	0.76
^{4}He	$-28.3(6)$	-28.3
^{8}Be	$-55(2)$	-56.5
^{12}C	$-92(3)$	-92.2
^{16}O	$-131(1)$	-127.6
^{20}Ne	$-166(1)$	-160.6
^{24}Mg	$-198(2)$	-198.3
^{28}Si	$-234(3)$	-236.5



- promising results [much improved by now]
- excited states more difficult, but also doable