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JÜLICH Forschungszentrum

The nucleus as a quantum laboratory

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

by CAS, PIFI

中国科学院 CHINESE ACADEMY OF SCIENCES

by DFG, SFB 1639

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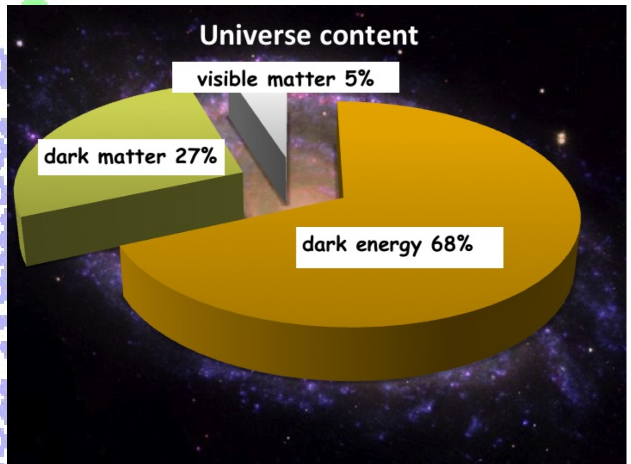
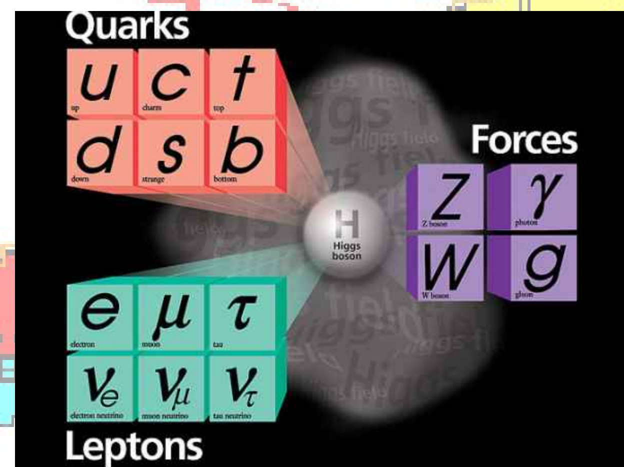
Introductory remarks

Why nuclear physics?

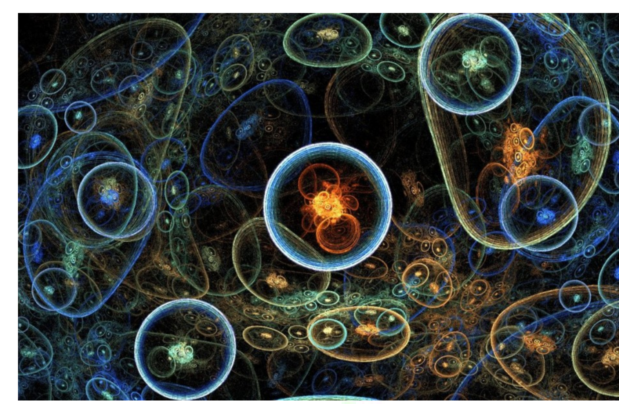
- **The matter we are made off**



- **The last frontier of the SM**



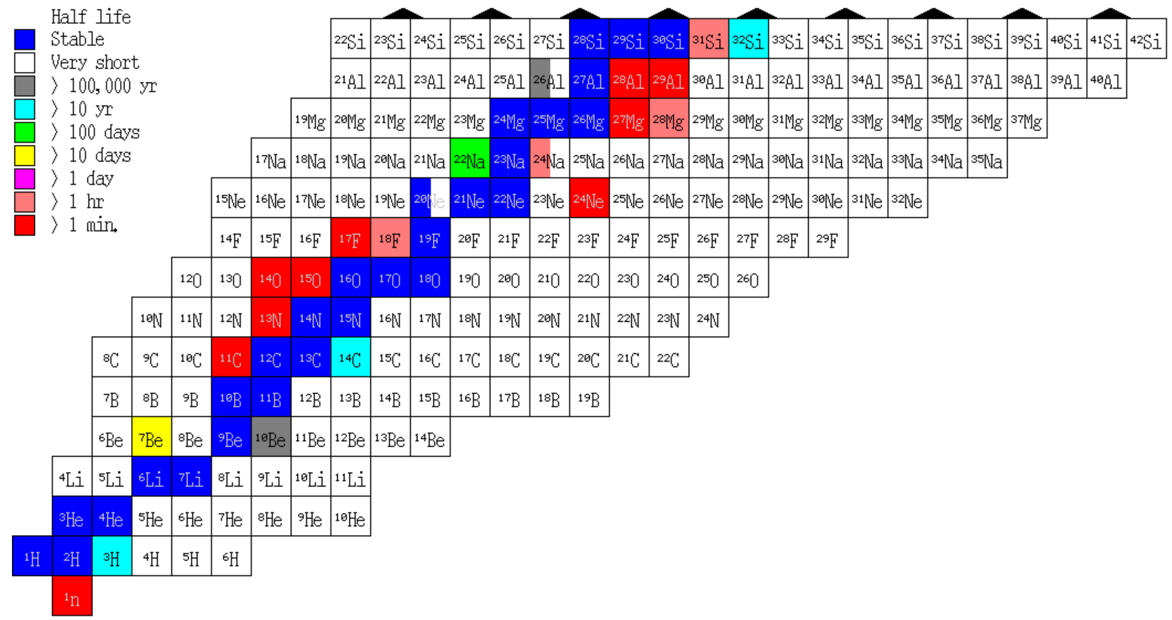
- **Access to the Multiverse**



⇒ Precision mandatory

Ab initio nuclear structure & reactions

- Nuclear structure:



© National Nuclear Data Center

  - ⋆ limits of stability
  - ⋆ 3-nucleon forces
  - ⋆ alpha-clustering

  ⋮

- Nuclear reactions: Scattering processes relevant for nuclear astrophysics
  - ⋆ alpha-particle scattering: ^{4}He + ^{4}He → ^{4}He + ^{4}He
  - ⋆ triple-alpha reaction: ^{4}He + ^{4}He + ^{4}He → ^{12}C + γ
  - ⋆ alpha-capture on carbon: ^{4}He + ^{12}C → ^{16}O + γ

  ⋮

The nucleus as a quantum laboratory

- The nucleus is a challenging and fascinating many-body system

  $\hookrightarrow$ non-perturbative strong interactions balanced by the Coulomb force

  $\hookrightarrow$ many interesting phenonema: drip lines, clustering, reactions, ...

  $\hookrightarrow$ a plethora of few-body/many-body methods already exists

- Macroscopic nuclear matter = neutron stars

  $\hookrightarrow$ gained prominence again in the multi-messenger aera

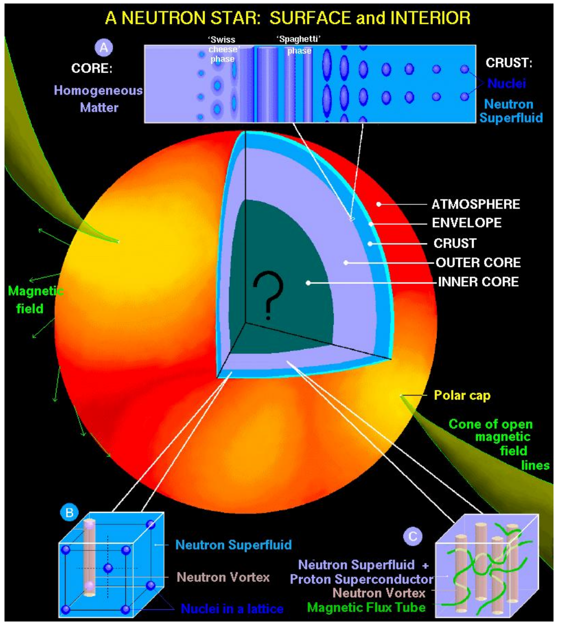
  $\hookrightarrow$ must be able to describe these with the same methods

- I will advocate here a new quantum many-body appraoch

  $\hookrightarrow$ synthezies chiral EFT w/ stochastic methods

  $\hookrightarrow$ allows to tackle nuclear structure *and* reactions

  $\hookrightarrow$ allows to access the multiverse



A brief introduction to nuclear physics

Nuclear physics – a primer

- Nuclei are self-bound system of fermions (protons & neutrons): (Heisenberg (1934)

$$N(\text{ucleon}) = \begin{pmatrix} p(\text{roton}) \\ n(\text{eutron}) \end{pmatrix}$$

- Bound by the **strong** force (now understood as residual color force of QCD)

- Repulsion also from the **Coulomb** force ($Z_p = +e$, $Z_n = 0$)

- Nuclear binding energies

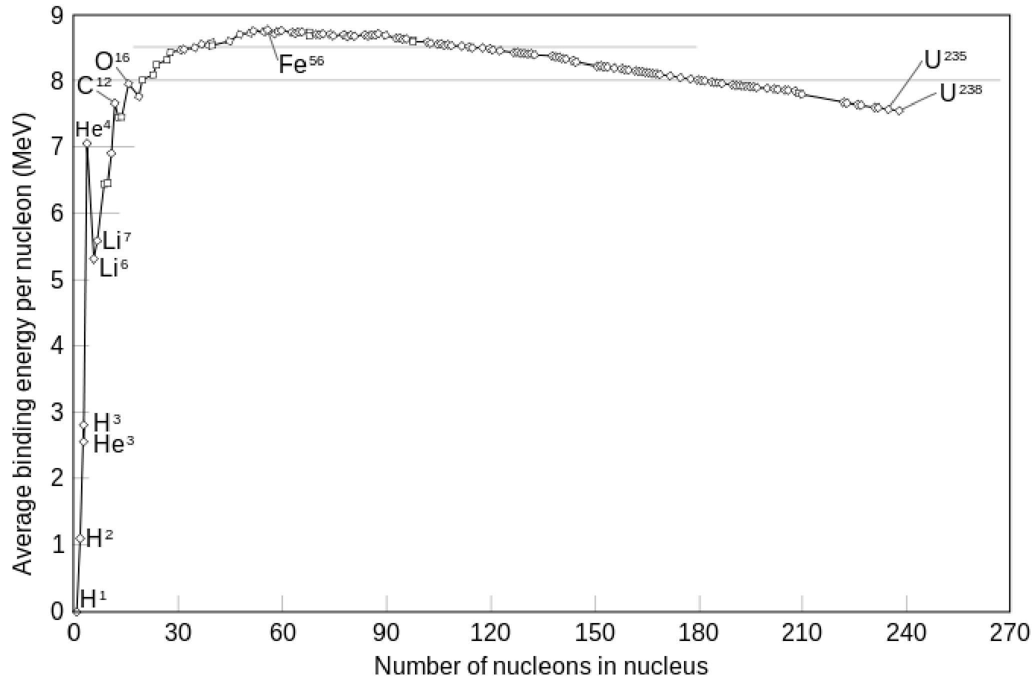
  $\ll$ nuclear masses

  $m_p = 939.57$ MeV

  $m_n = 928.27$ MeV

  [in nucl. phys. $\hbar = c = 1$]

- DOFs are protons, neutrons and mesons, NOT quarks and gluons! [resolution!]



Nuclear physics – a primer

- Non-relativistic system → nuclear Hamiltonian takes the form:

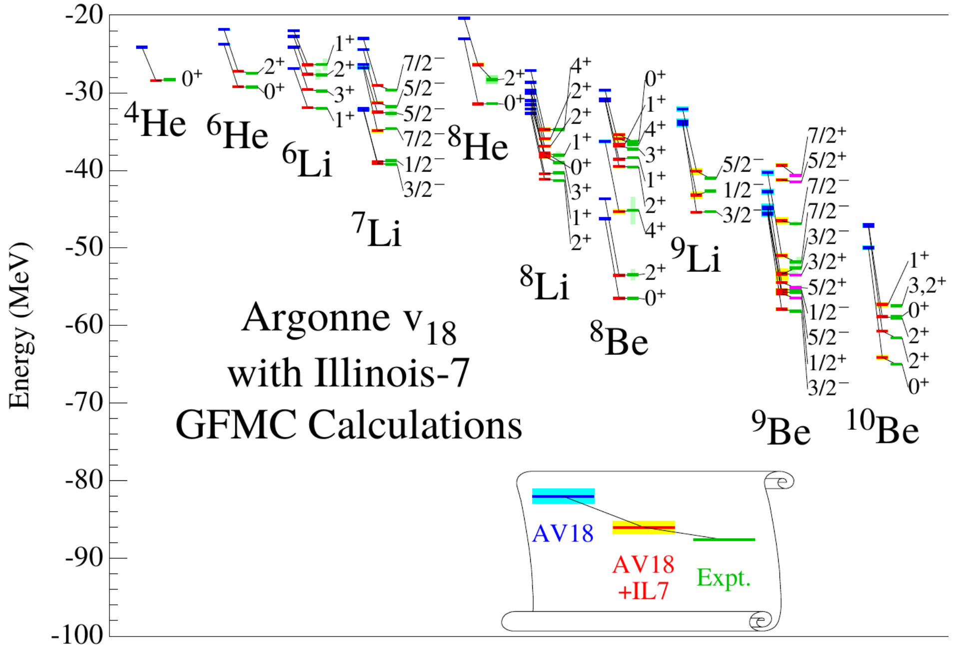
$$H_{\text{nuclear}} = \sum_{i=1}^{A} \frac{p_i^2}{2m_i} + V \ , \quad V = V_{\text{NN}} + V_{\text{NNN}} + \ldots$$

- Dominant two-nucleon potential V_{NN}, but small three-nucleon force V_{NNN} is required (see pheno forces right)

- The nuclear Hamiltonian can be **systematically** analyzed using the **symmetries** of the strong interactions

  Weinberg, Phys. Lett. B **251** (1990) 288

  Weinberg, Nucl. Phys. B **363** (1991) 3

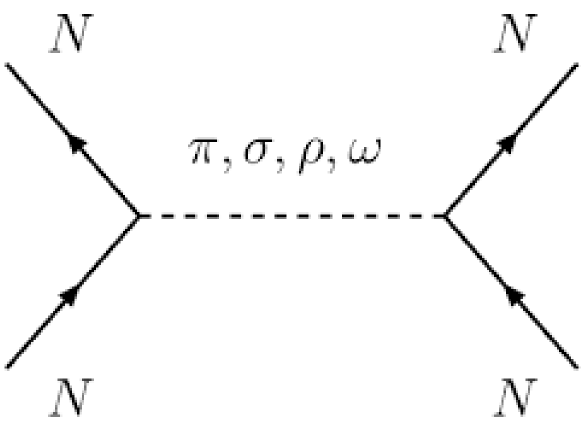


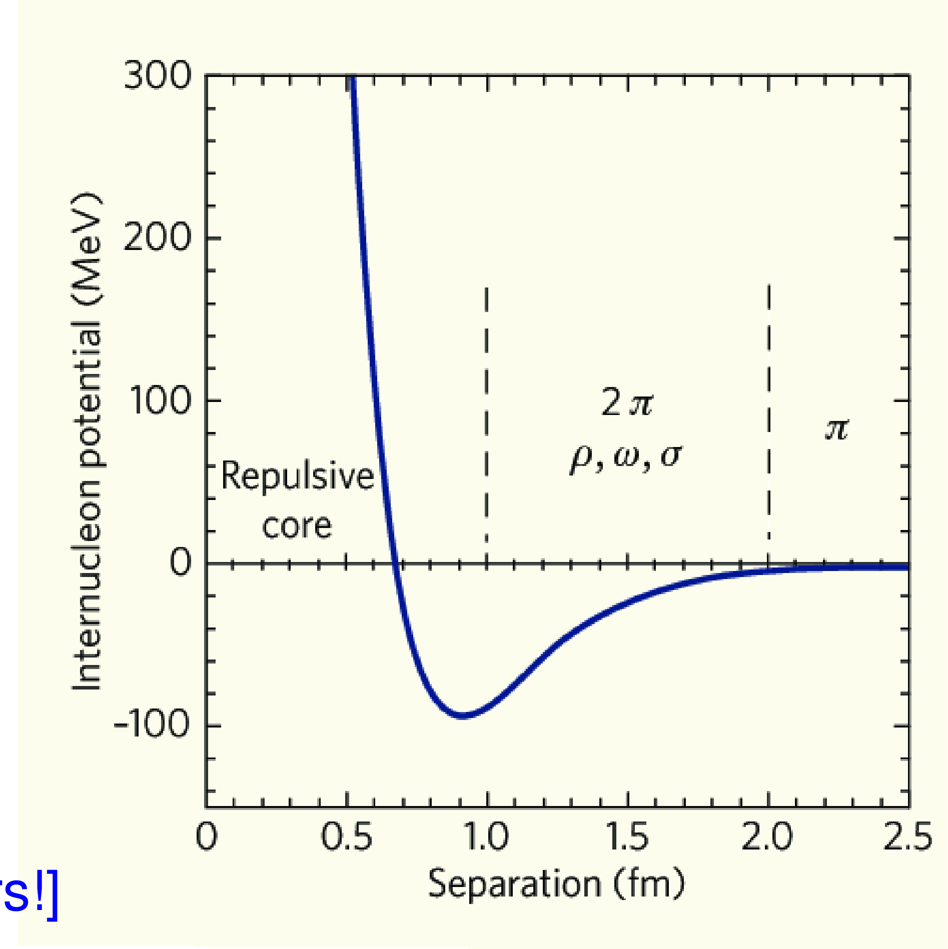
Carlson et al., Rev. Mod. Phys. **87** (2015) 1067

Nuclear interactions – a primer

- Boson-exchange picture of the nucleon-nucleon interaction (V_{NN}):

Bonn, Paris, Nijmegen, Stony Brook, Idaho, ...

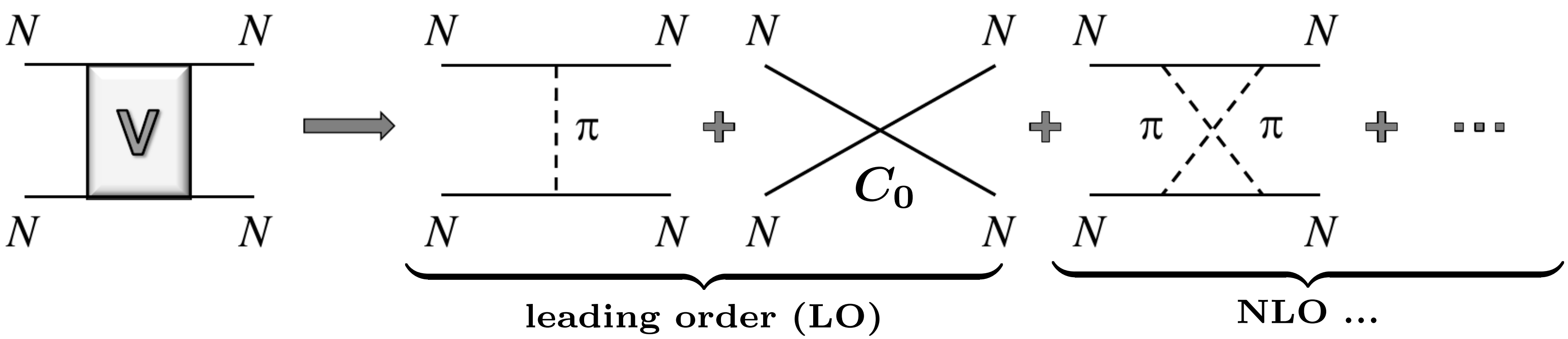




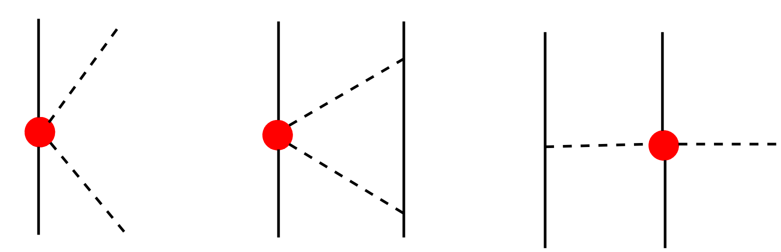
- very successful in describing
  $\sim \mathbf{10.000}$ pp and np scattering data
- hadron extension provides parameters
- time-honored parameterization of V_{NN}
- But: no error estimate ! [th'y must have errors!]
- But: no consistent three-nucleon force exists!

Nuclear interactions – a primer

- Modern picture of the nucleon-nucleon interaction (V_{NN}) [and similarly V_{NNN}] based on chiral Lagrangians (symmetry of QCD !):



- equally successful in describing the $\sim$ **10.000** pp and np scattering data
- short- and medium-range interactions parameterized by contact terms $\rightarrow$ fit LECs C_i
- two- and three-nucleon forces are related! $\surd$
- power counting allows for error estimates $\surd$
- worked out to high orders = high precision! $\rightarrow$ next slide

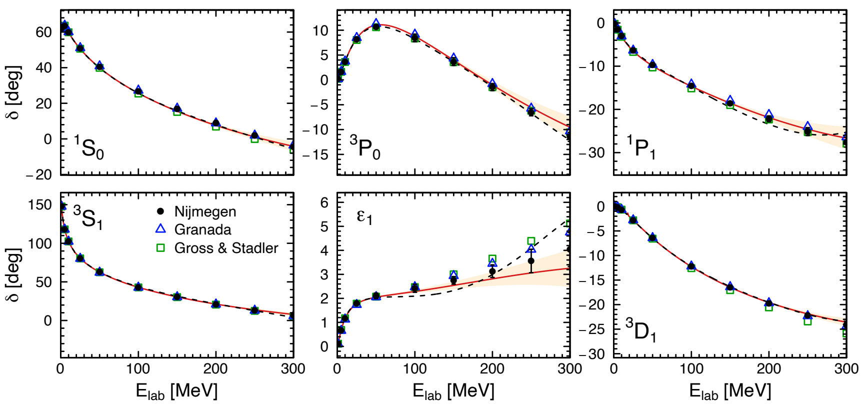


Nuclear interactions – a primer

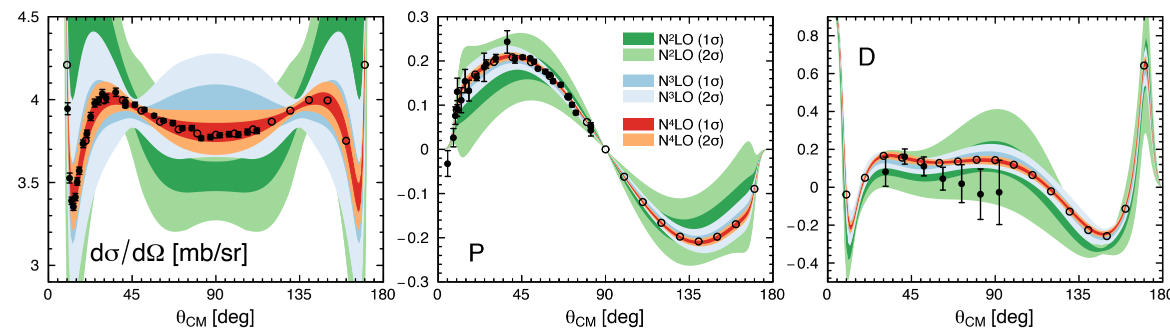
- State-of-the-art chiral EFT results (N4LO+)

Reinert, Krebs, Epelbaum, Eur. Phys. J. A **54** (2018) 86; Phys. Rev. Lett. **126** (2021) 092501

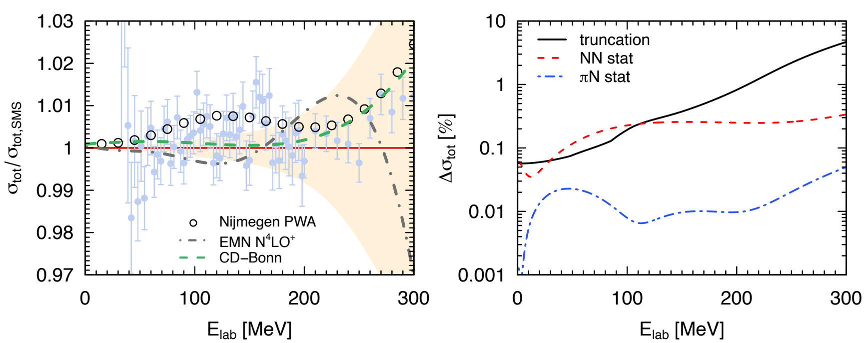
Neutron-proton phase shifts at N⁴LO⁺



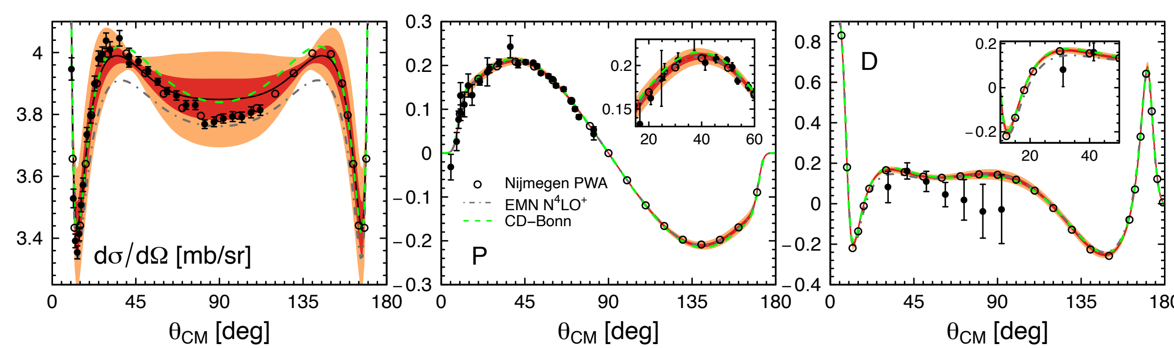
Proton-proton scattering observables around $E_{\rm lab} = 143$ MeV



Neutron-proton total cross section at N⁴LO⁺



Proton-proton scattering observables around $E_{\rm lab} = 143$ MeV



$\hookrightarrow$ often more precise than the data!

Nuclear many-body methods – a primer

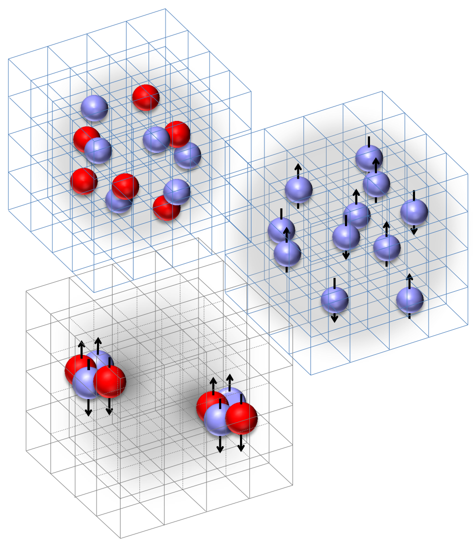
- Solve the Schrödinger equation for A nucleons in a given nucleus:

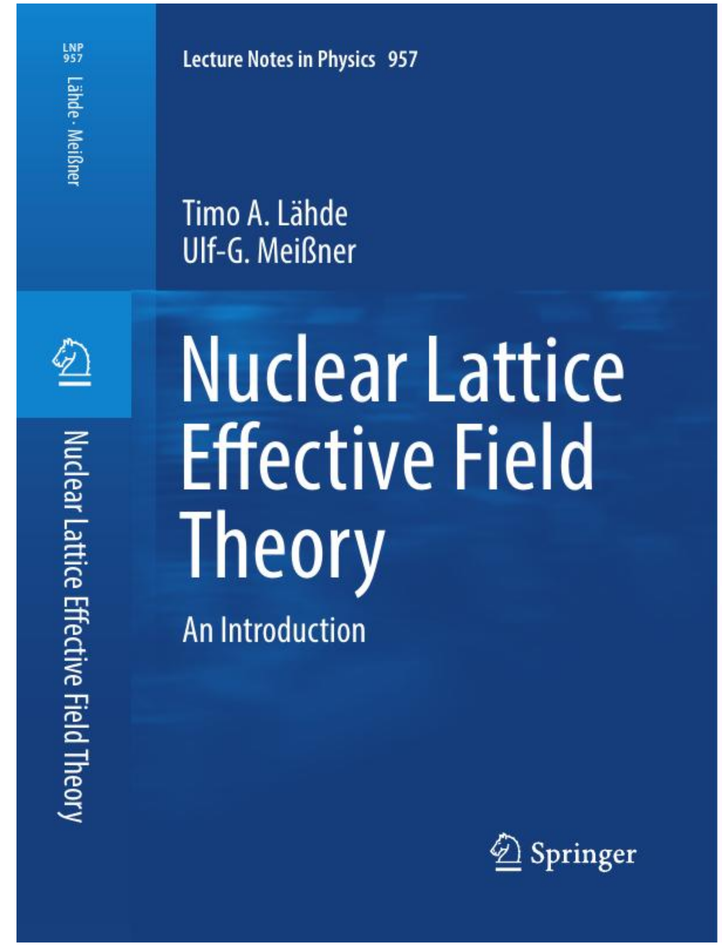
$$\left(-\sum_{i=1}^{A}\frac{\nabla_i^2}{2m_N}+V\right)|\Psi\rangle = E|\Psi\rangle\ , \qquad V = V_{\rm NN}+V_{\rm NNN}+\ldots$$

- a variety of classical many-body approaches:
  - — the shell-model (independent particles) Goeppert Mayer, Jensen (1949)  1963
  - — the deformed shell-model (rotational bands) Nilsson (1957)
  - — collective excitations (deformations, vibrations) Bohr, Mottelson (1958)  1975
  - — coupled-cluster approach (correlations) Koester, Kümmel (1958)
  - — density-functional approach (correlations) Kohn, Sham (1963)  1998
  - — and various others....

$\hookrightarrow$ all have limitations, we want to do better (exact solutions w/ modern forces)

Chiral EFT on a lattice





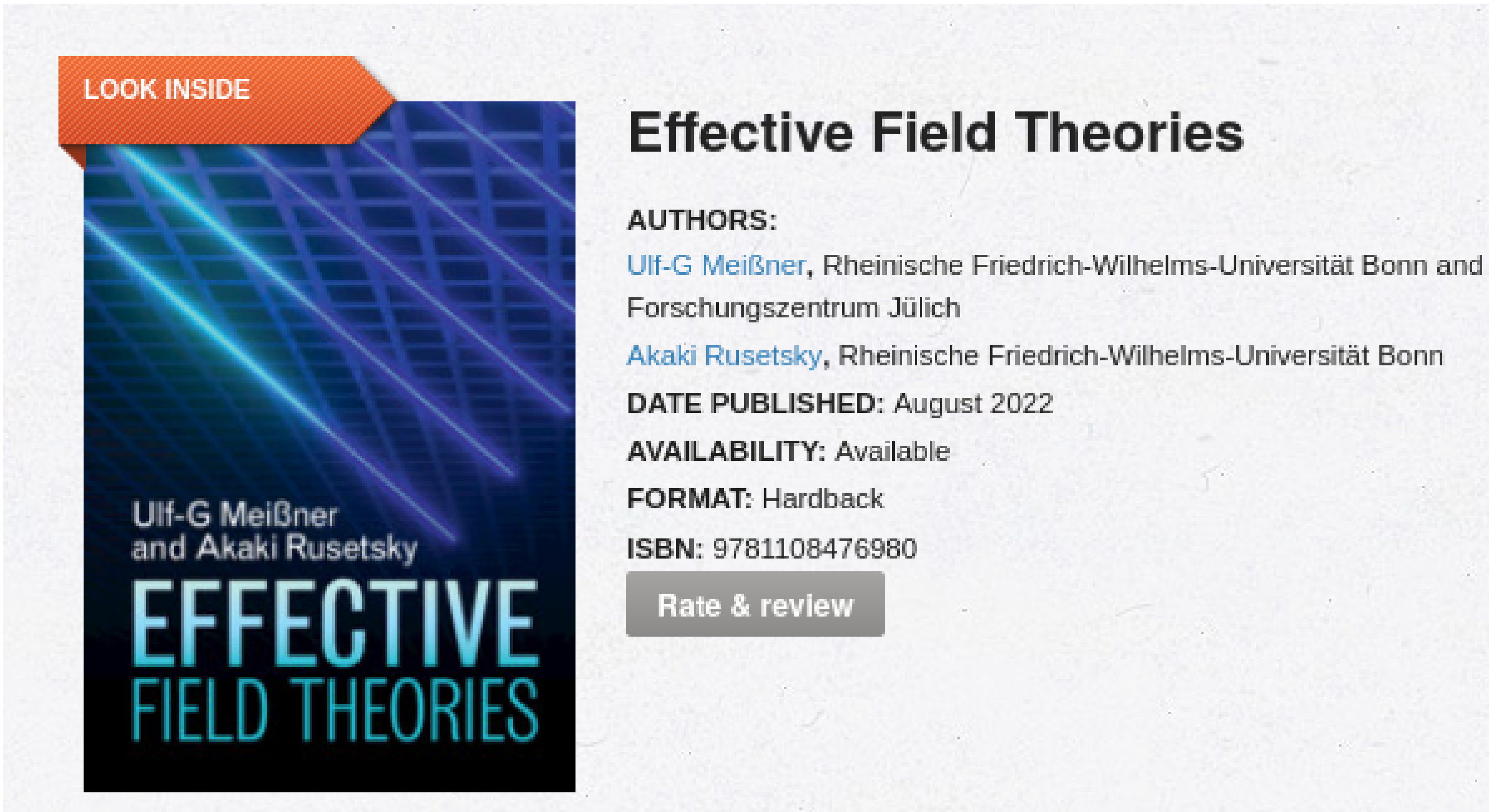
T. Lähde & UGM

Nuclear Lattice Effective Field Theory - An Introduction

Springer Lecture Notes in Physics **957** (2019) 1 - 396

More on EFTs

- Much more details on EFTs in light quark physics:

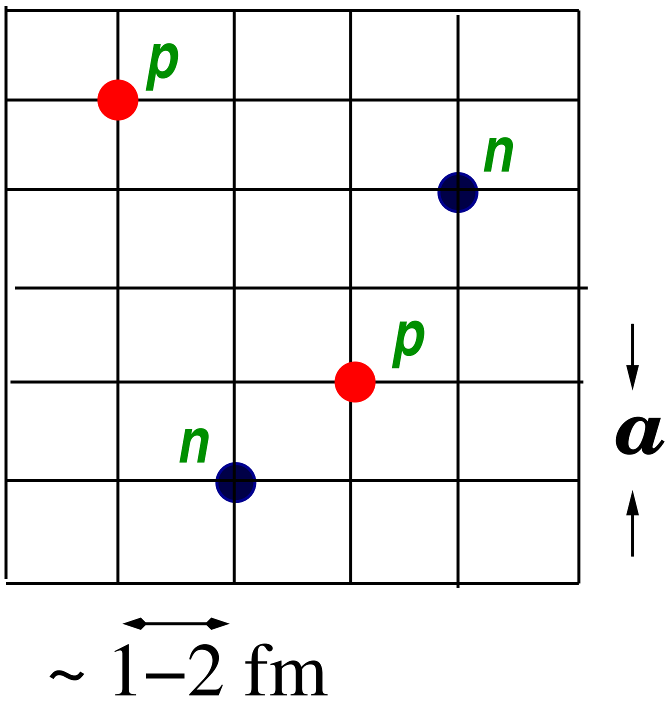


https://www.cambridge.org/de/academic/subjects/physics/theoretical-physics-and-mathematical-physics/effective-field-theories

Nuclear lattice effective field theory (NLEFT)

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem



- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

$\rightarrow$ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters

$$p_{\max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$

- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

Transfer matrix method

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons
  [or a more sophisticated (correlated) initial/final state]

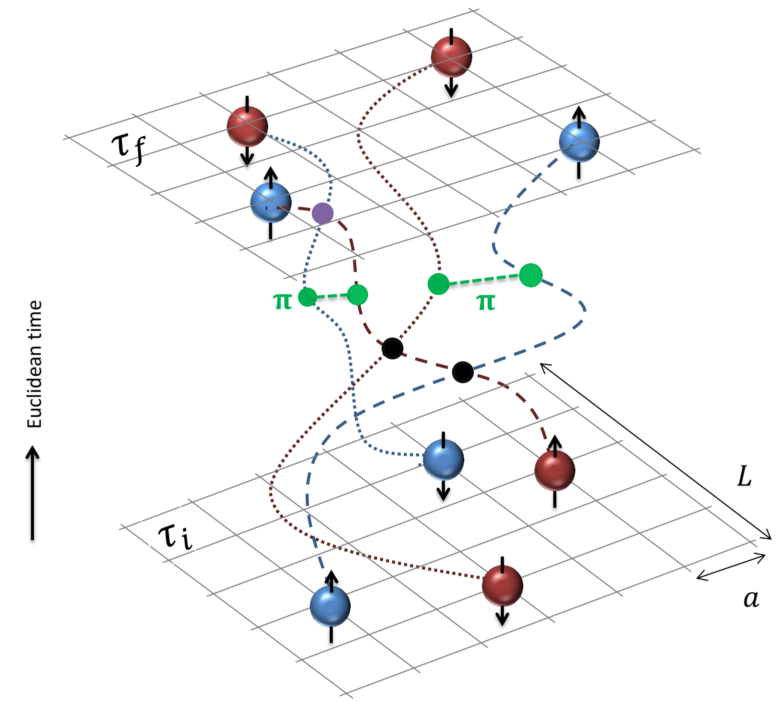
Euclidean time

- Transient energy

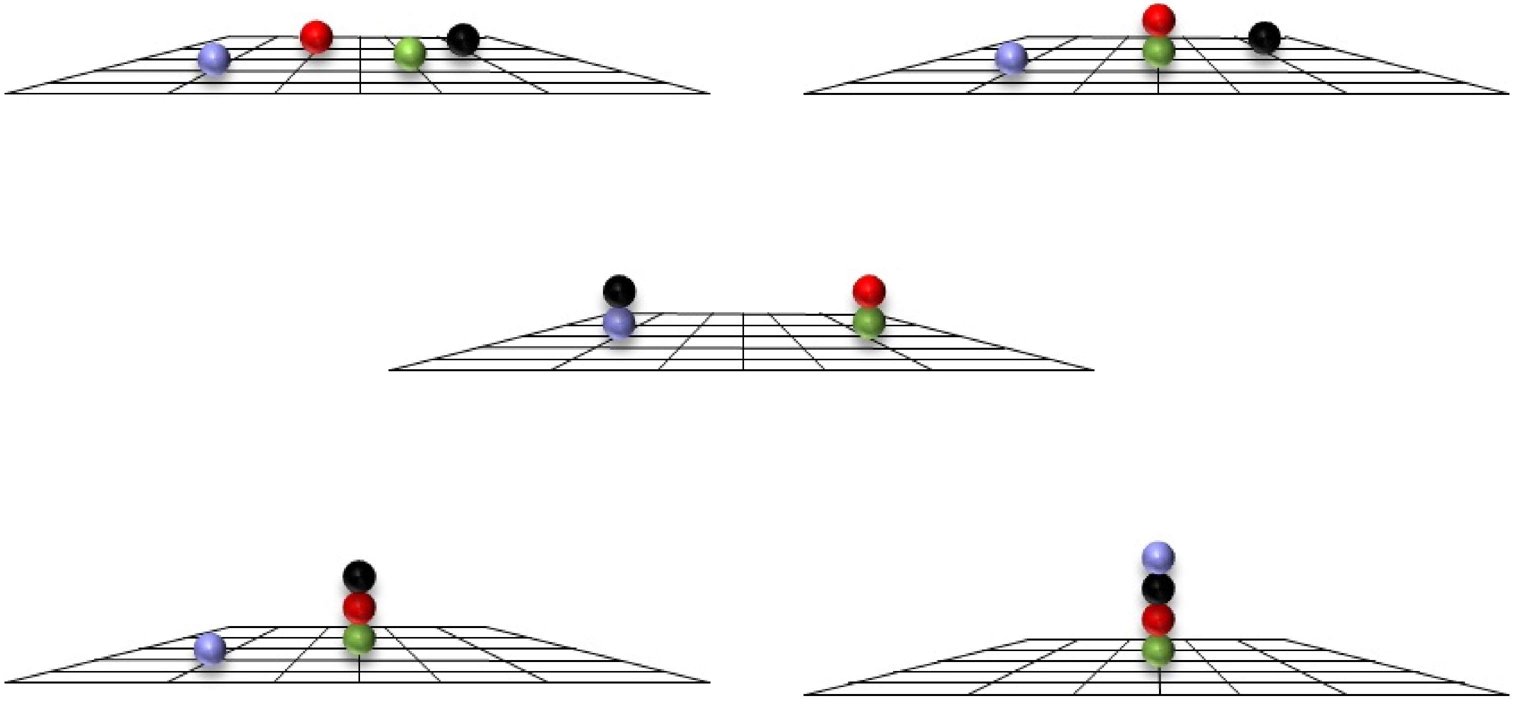
$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

$\to$ ground state: $E_A^0 = \lim_{\tau\to\infty} E_A(\tau)$

- Similarly:
  - insert operators / other observables
  - excited states / transitions



Configurations



⇒ all *possible* configurations are sampled

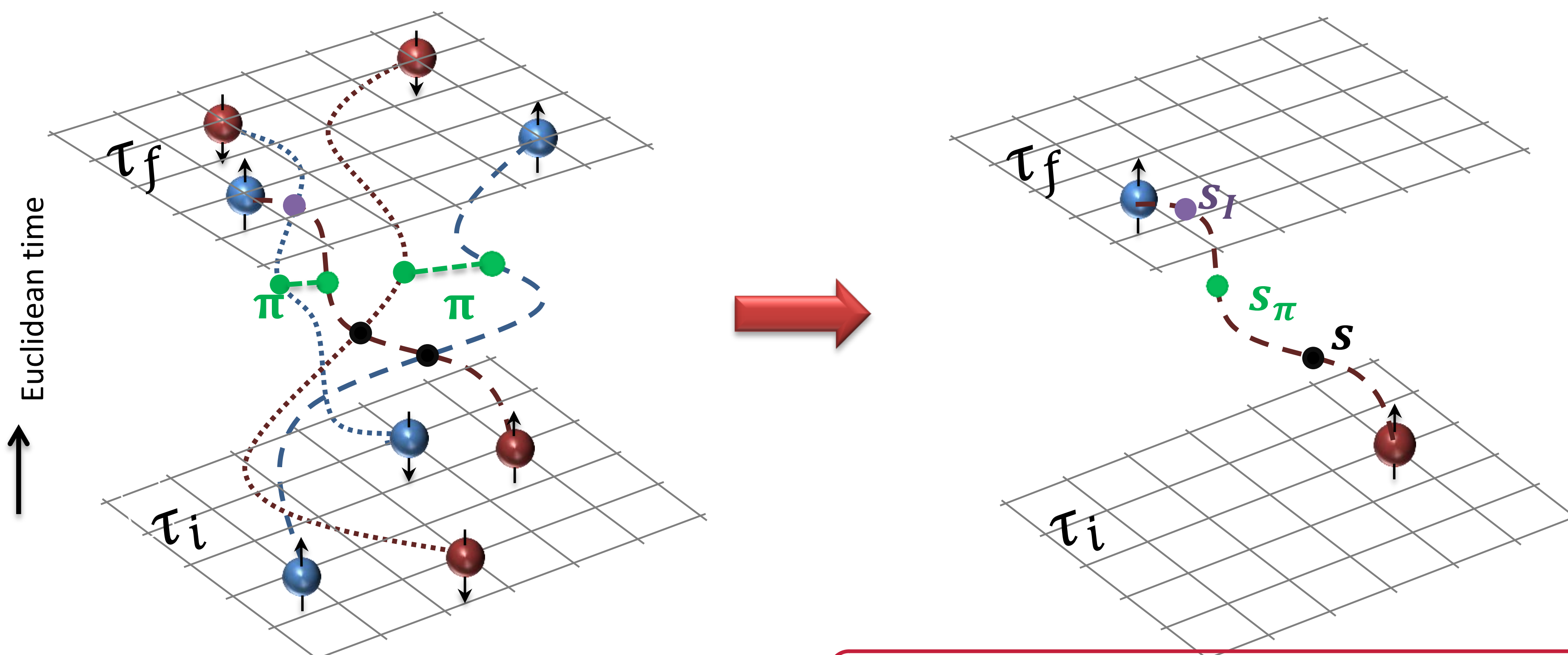
⇒ preparation of *all possible* initial/final states

⇒ *clustering* emerges *naturally*

Auxiliary field method

- Represent interactions by auxiliary fields (Gaussian quadrature):

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds\,\exp\left[-\frac{s^2}{2}+\sqrt{C}\;s\left(N^\dagger N\right)\right]$$



optimally suited for parallel computing!

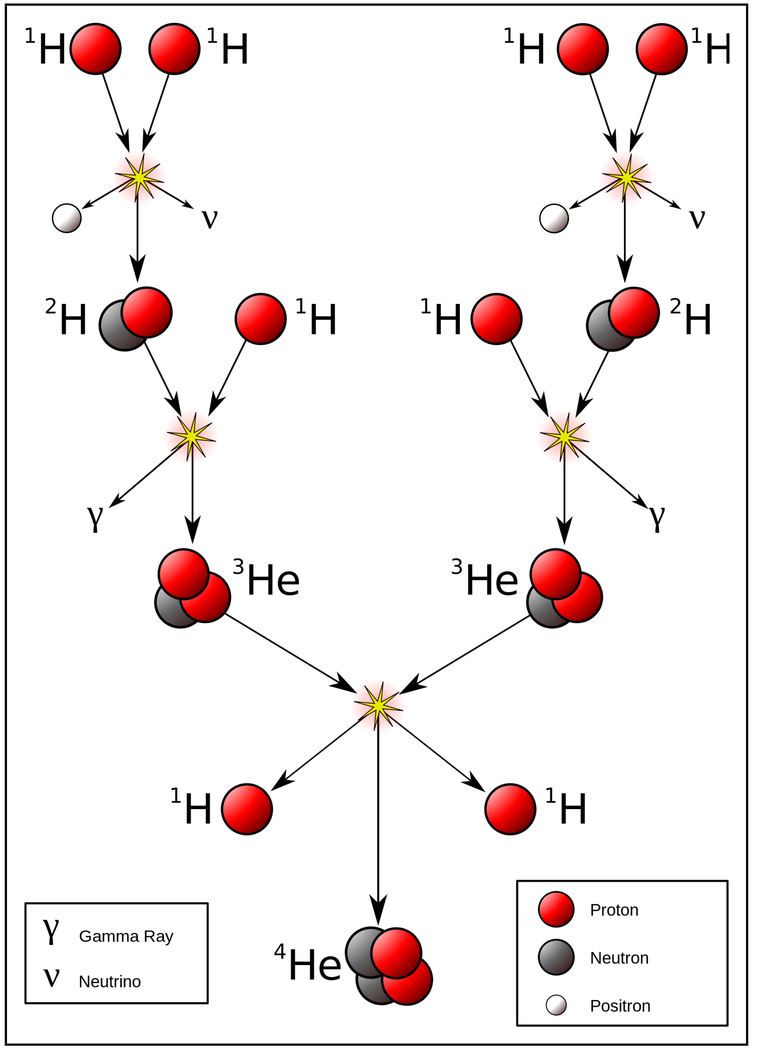
Computational equipment

• Present = JUWELS (modular system) + FRONTIER + ...

The Hoyle state and the generation of carbon in stars

Element generation

- Elements are generated in the Big Bang & in stars through the **fusion** of protons & nuclei [pp chain or CNO-cycle]
- All is simple until ^{4}He
- Only elements up to Be are produced in the Big Bang [BBNucleosynthesis]
- **Life-essential** elements like ^{12}C and ^{16}O are generated in hot, old stars (triple-alpha reaction !)
- Note also that nuclei make up the visible matter in the Universe



[from Wikipedia]

A short history of the Hoyle state

- Heavy element generation in massive stars: triple-α process

Bethe 1938, Öpik 1952, Salpeter 1952, Hoyle 1954, ...

^{4}He + ^{4}He $\rightleftharpoons$ ^{8}Be
^{8}Be + ^{4}He $\rightleftharpoons$ ^{12}C+γ
^{12}C + ^{4}He $\rightleftharpoons$ ^{16}O+γ [further step to get oxygen]

$\hookrightarrow$ this generates **order of magnitude** too few carbon and oxygen

- Hoyle's great idea:

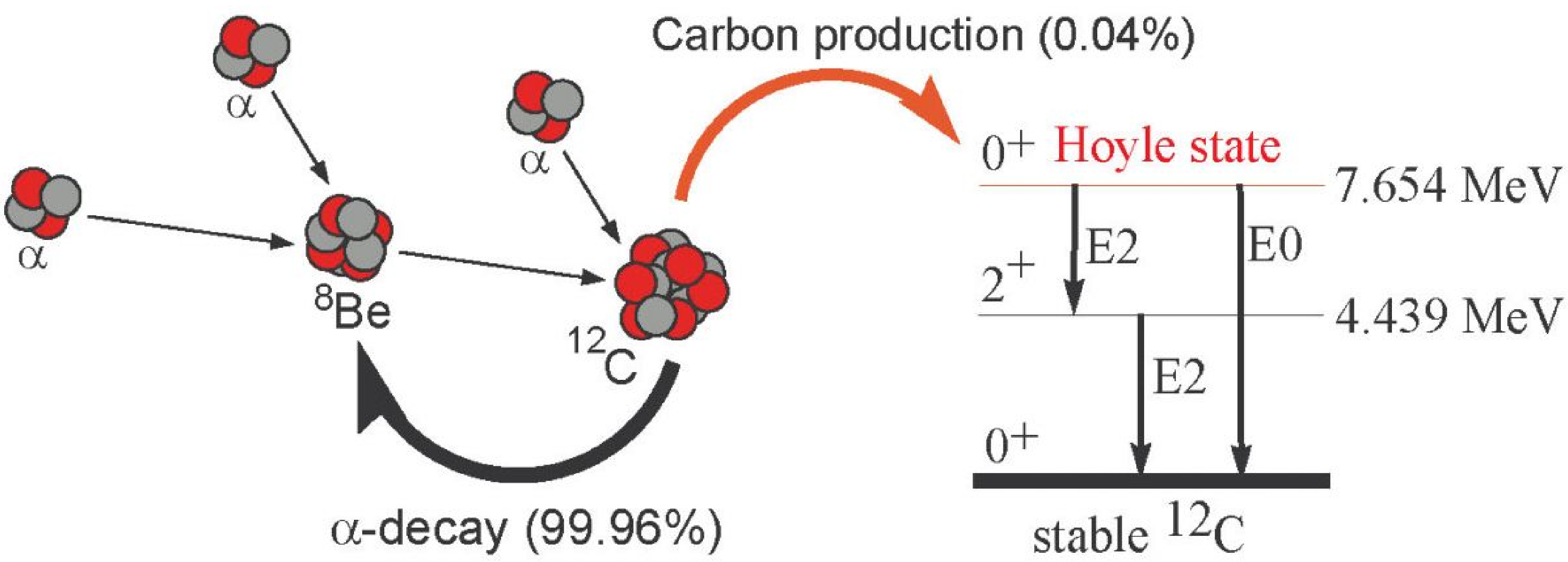
$\Rightarrow$ need a resonance close to the ^{8}Be + ^{4}He threshold at $E_R = 0.35$ MeV
$\Rightarrow$ ^{8}Be + ^{4}He $\rightleftharpoons$ ^{12}C* $\rightarrow$^{12}C+γ
$\Rightarrow$ this corresponds to a 0^+ excited state 7.7 MeV above the g.s.

- a corresponding state was experimentally confirmed at Caltech at $E - E(\text{g.s.}) = 7.653 \pm 0.008$ MeV

Dunbar et al. 1953, Cook et al. 1957

- this state was an enigma for *ab initio* calculations until 2011 (just hold your breath)

The triple-alpha process



©ANU

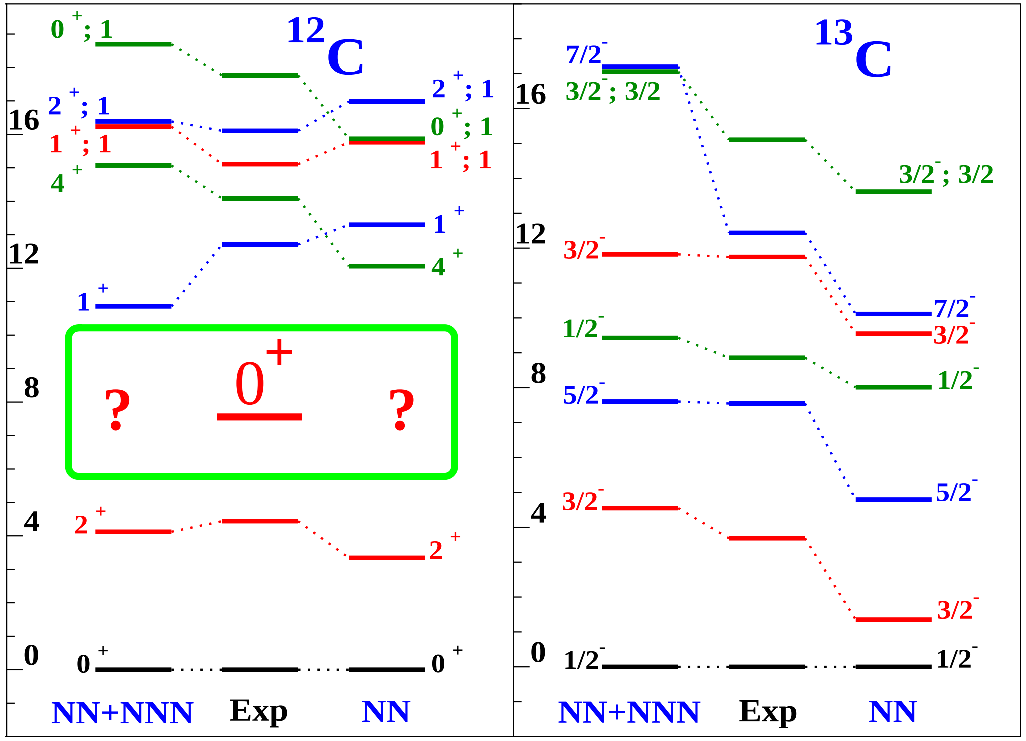
- this reaction involves two **fine-tunings** [are they related?]
  1) the ^{8}Be nucleus is instable, long lifetime $\to$ 3 alphas must meet
  2) the Hoyle state sits just above the continuum threshold
     $\hookrightarrow$ about 4 out of 10000 decays produce stable carbon

- the reaction rate: $$\tau_{3\alpha} \sim \exp\left\{-\left(E_{\rm Hoyle} - E_{(8{\rm Be}+4{\rm He})}\right)/kT\right\}$$

An enigma for nuclear theory

- Ab initio calculation in the no-core shell model: $\approx 10^7$ CPU hrs on JAGUAR

P. Navratil et al., Phys. Rev. Lett. **99** (2007) 042501; R. Roth et al., Phys. Rev. Lett. **107** (2011) 072501



$\Rightarrow$ excellent description, but no trace of the Hoyle state

A breakthrough based on NLEFT

- Take home message:

  If you have a new method, you must solve a problem others could not!

PRL 106, 192501 (2011) — Selected for a Viewpoint in *Physics* — PHYSICAL REVIEW LETTERS — week ending 13 MAY 2011

Ab Initio Calculation of the Hoyle State

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[4]*Institut für Kernphysik, Institute for Advanced Simulation and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*
(Received 24 February 2011; published 9 May 2011)

The Hoyle state plays a crucial role in the helium burning of stars heavier than our Sun and in the production of carbon and other elements necessary for life. This excited state of the carbon-12 nucleus was postulated by Hoyle as a necessary ingredient for the fusion of three alpha particles to produce carbon at stellar temperatures. Although the Hoyle state was seen experimentally more than a half century ago nuclear theorists have not yet uncovered the nature of this state from first principles. In this Letter we report the first *ab initio* calculation of the low-lying states of carbon-12 using supercomputer lattice simulations and a theoretical framework known as effective field theory. In addition to the ground state and excited spin-2 state, we find a resonance at −85(3) MeV with all of the properties of the Hoyle state and in agreement with the experimentally observed energy.

DOI: 10.1103/PhysRevLett.106.192501 — PACS numbers: 21.10.Dr, 21.45.−v, 21.60.De, 26.20.Fj

PRL 109, 252501 (2012) — PHYSICAL REVIEW LETTERS — week ending 21 DECEMBER 2012

Structure and Rotations of the Hoyle State

Evgeny Epelbaum,[1] Hermann Krebs,[1] Timo A. Lähde,[2] Dean Lee,[4] and Ulf-G. Meißner[5,2,3]
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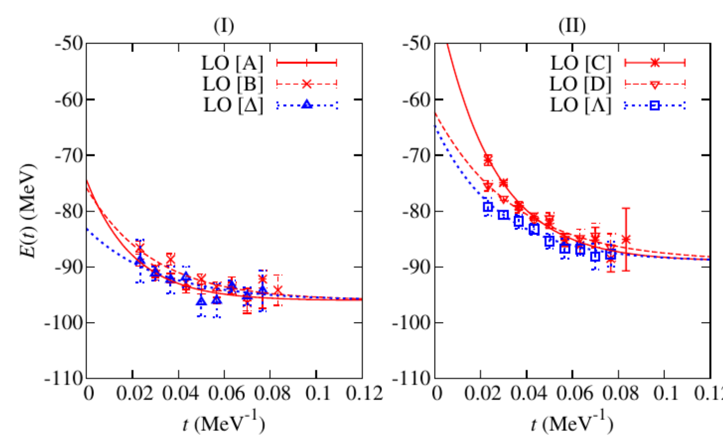
[2]*Institut für Kernphysik, Institute for Advanced Simulation, Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[3]*JARA—High Performance Computing, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[4]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA*
[5]*Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*
(Received 14 August 2012; published 17 December 2012)

The excited state of the ^{12}C nucleus known as the "Hoyle state" constitutes one of the most interesting, difficult, and timely challenges in nuclear physics, as it plays a key role in the production of carbon via fusion of three alpha particles in red giant stars. In this Letter, we present *ab initio* lattice calculations which unravel the structure of the Hoyle state, along with evidence for a low-lying spin-2 rotational excitation. For the ^{12}C ground state and the first excited spin-2 state, we find a compact triangular configuration of alpha clusters. For the Hoyle state and the second excited spin-2 state, we find a "bent-arm" or obtuse triangular configuration of alpha clusters. We also calculate the electromagnetic transition rates between the low-lying states of ^{12}C.

DOI: 10.1103/PhysRevLett.109.252501 — PACS numbers: 21.10.Dr, 21.60.De, 23.20.−g, 27.20.+n

- N2LO potential (2NFs+3NFs)
  - ↪ all LECs determined before
- Independent particle and cluster initial states
  - ↪ results do not depend on this!
  - ↪ clustering emerges (see later)

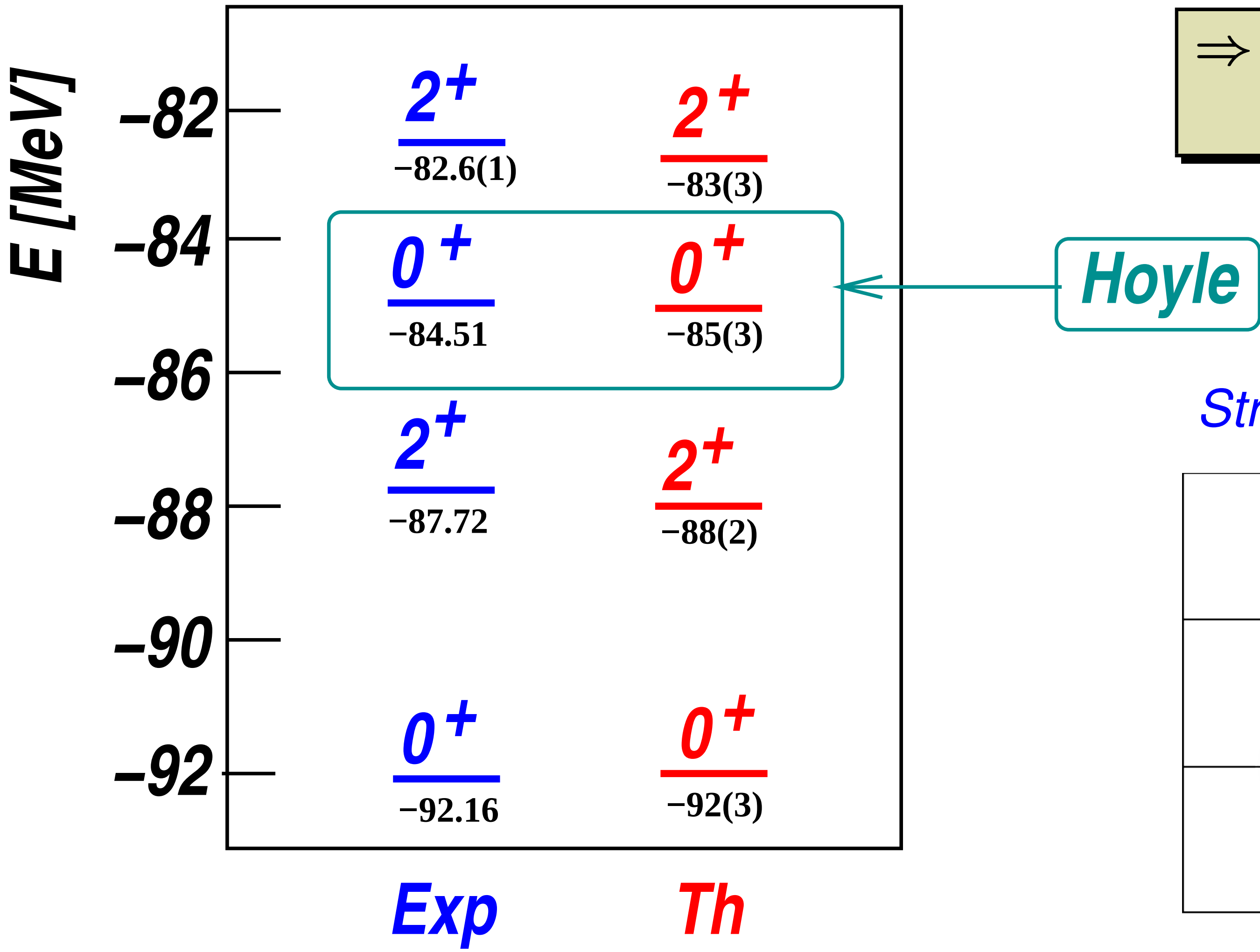


A breakthrough - spectrum of carbon-12

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501

Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. **109** (2012) 252501

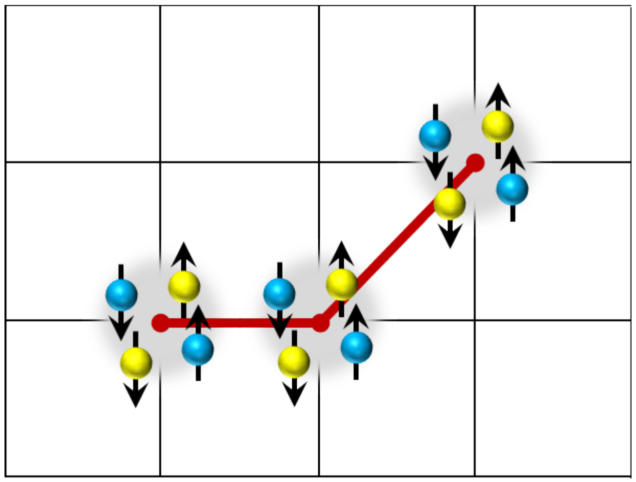
- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and "some" human work)



$\Rightarrow$ First ab initio calculation of the Hoyle state $\surd$

[see also Feldmeier & Neff, FMD]

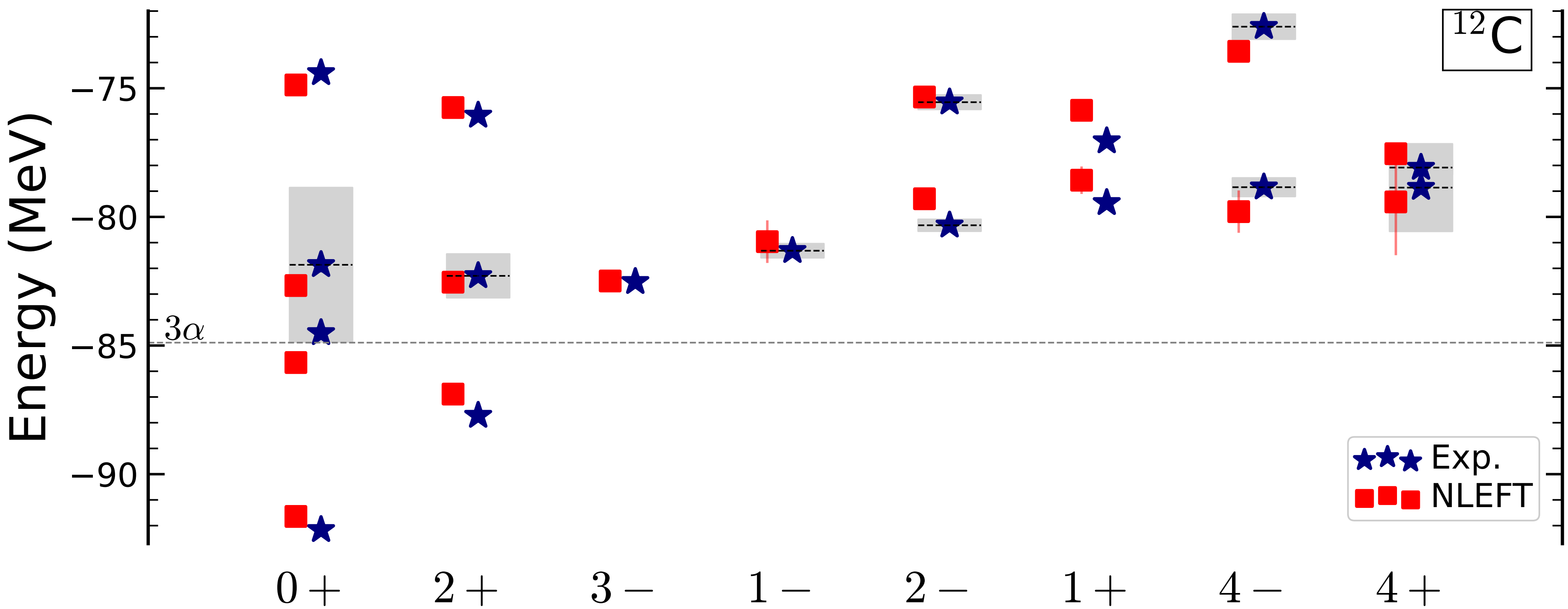
Structure of the Hoyle state:



Spectrum of ^{12}C reloaded

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777

- New algorithms, new methods, finer & larger lattices $\hookrightarrow$ improved description of ^{12}C

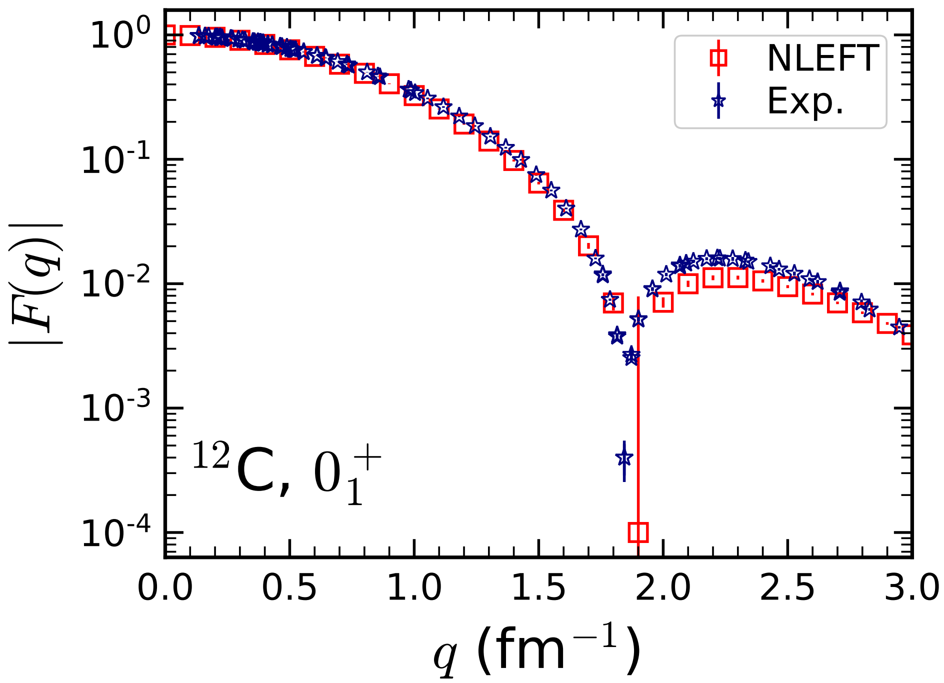


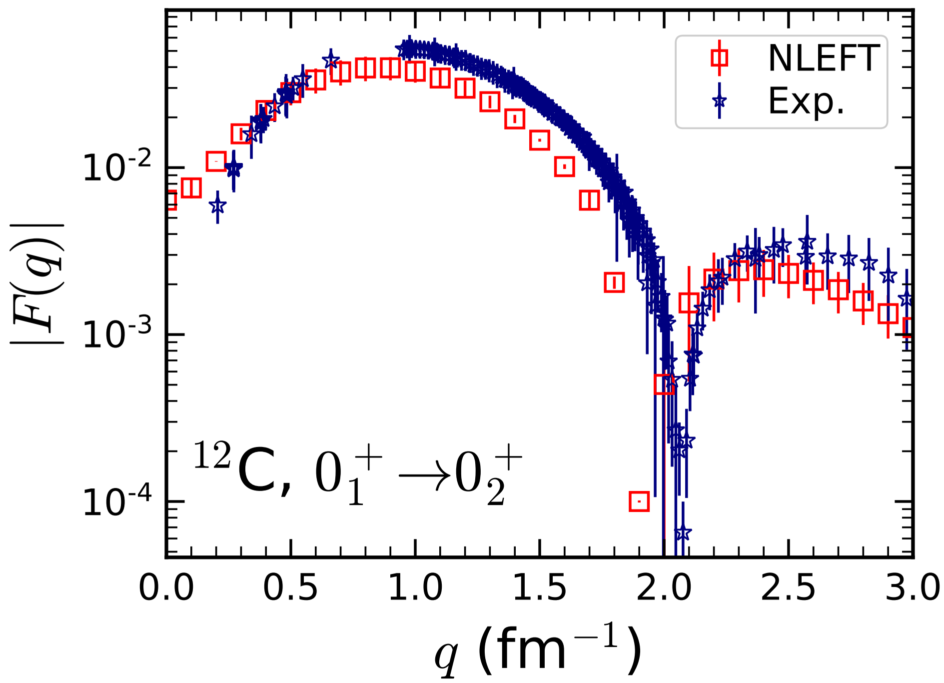
$\rightarrow$ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

Electromagnetic properties

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777

- Form factors and transition ffs [parameter-free]:



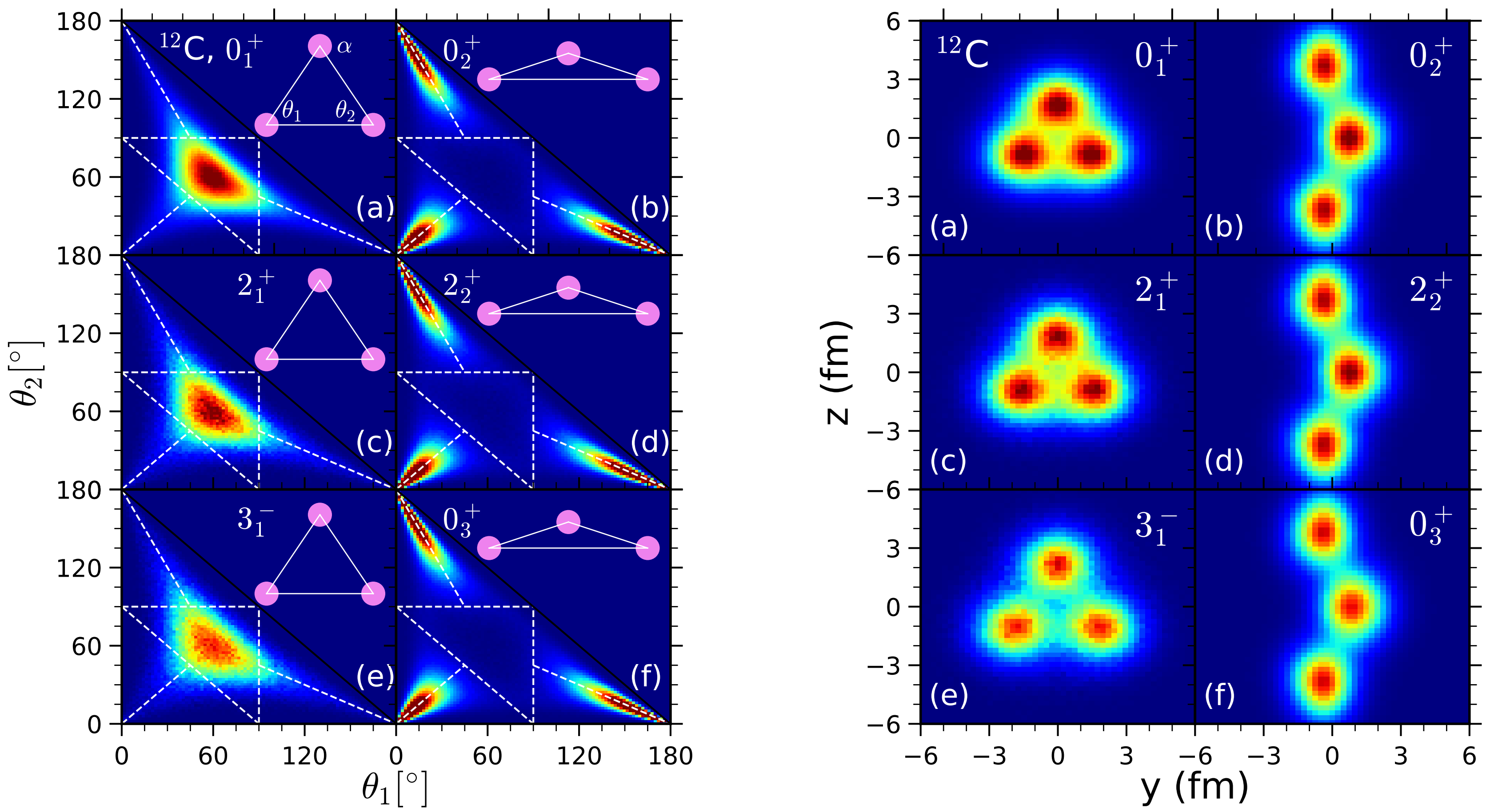


Sick, McCarthy, Nucl. Phys. A **150** (1970) 631
Strehl, Z. Phys. **234** (1970) 416
Crannell et al., Nucl. Phys. A **758** (2005) 399

Chernykh et al., Phys. Rev. Lett. **105** (2010) 022501

Emergence of geometry

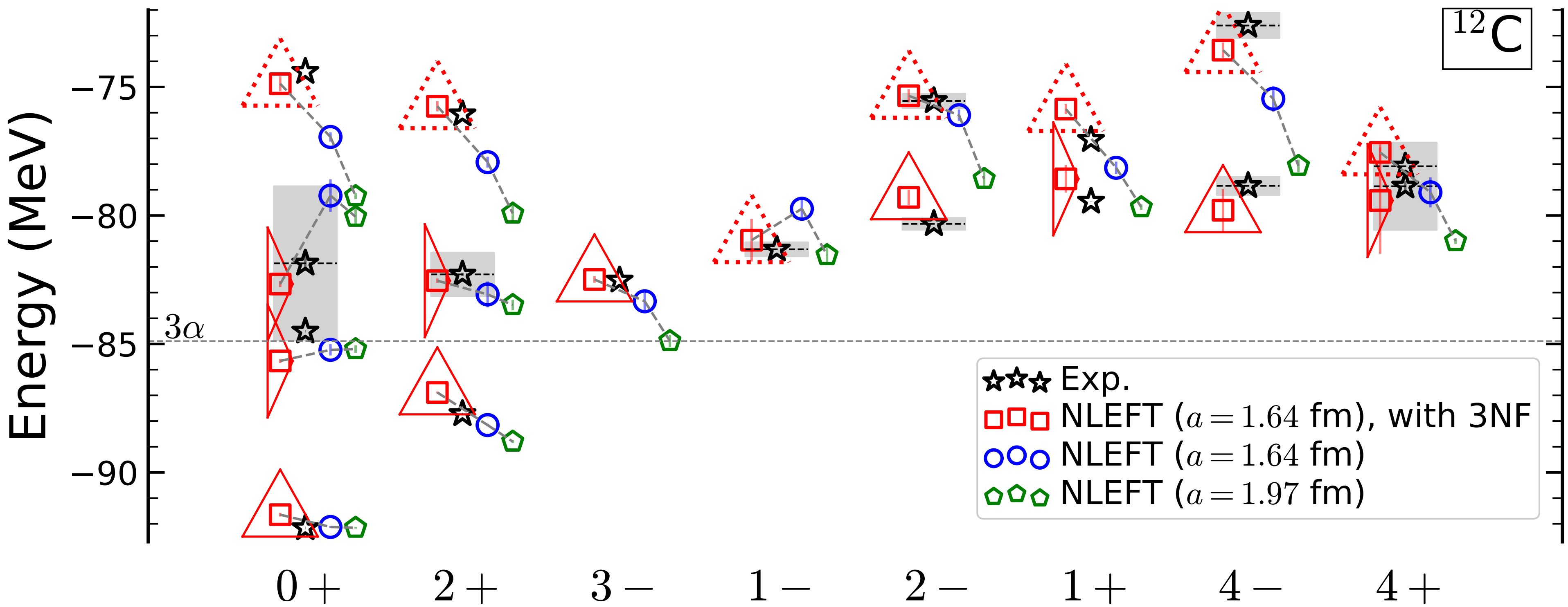
- Use the pinhole algorithm to measure the distribution of α-clusters/matter:



$\hookrightarrow$ clear signal of alpha (^{4}He) particle clustering in these states!

Emergence of duality

- ^{12}C spectrum shows a cluster/shell-model duality

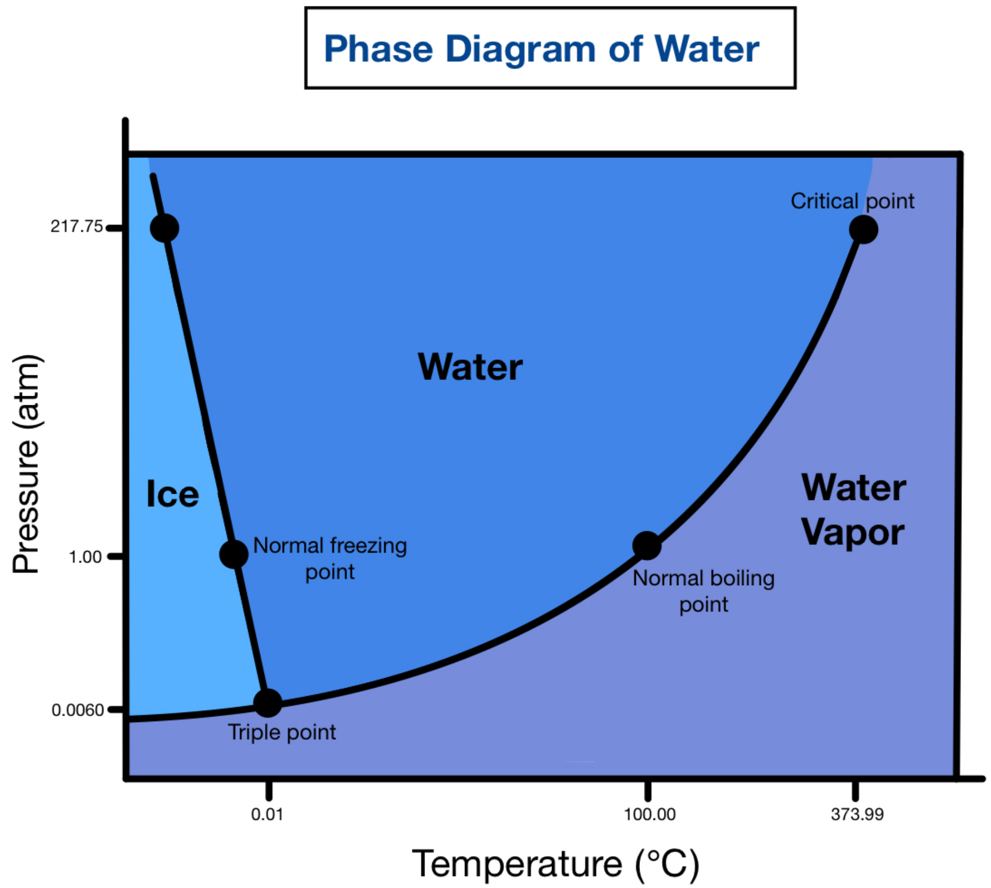


- dashed triangles: strong 1p-1h admixture in the wave function

Nuclear binding: A quantum phase transition

Phase transitions

- The classical example



ⓒCaroline Monahan

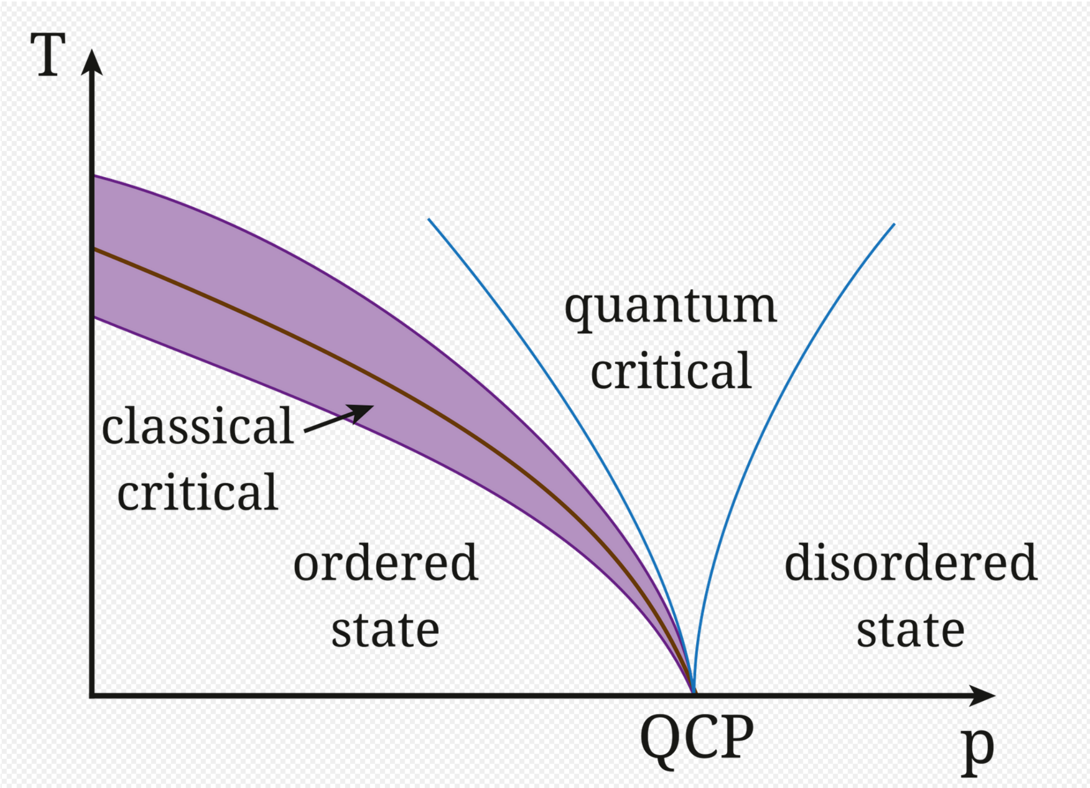
- External conditions (like temperature, pressure) change the state of matter

  e.g. ice $\xrightarrow{T}$ water

- and many other examples (pick your favorite)

What is a quantum phase transition?

A quantum phase transition (QPT) is a phase transition between different quantum phases (phases of matter at zero temperature). Contrary to classical phase transitions, quantum phase transitions can only be accessed by varying a physical parameter – such as magnetic field or pressure – at absolute zero temperature.



Vectorized version by AG Caesar, original by DG85 from Wikipedia

- at $T = 0$ any system is in its lowest energy state

$\hookrightarrow$ any QPT is driven by quantum fluctuations (Heisenberg's uncertainty principle)

- much thought after in cond. mat./ UC atom systems (spin chains, spin ice, ...)

Vojta, Rept. Prog. Phys. **66** (2003) 2069

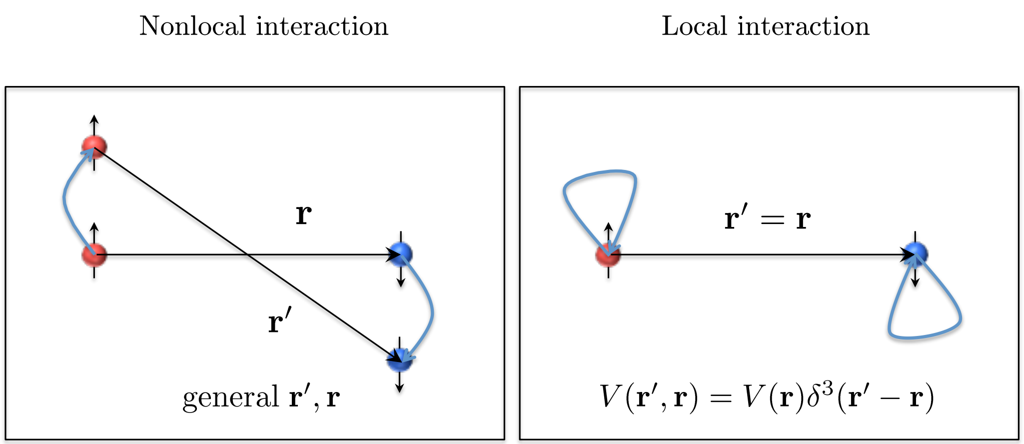
- but nuclear physics provides a real-life example!

Prelude: Local and non-local interactions

- General potential: $V(\vec{r},\vec{r}\,')$
- Two types of interactions:

  local: $\vec{r} = \vec{r}\,'$

  non-local: $\vec{r} \neq \vec{r}\,'$



- Nuclear physics: both types appear: Pion exchange is non-local, often used as local

$\hookrightarrow$ covariant form

$$V(\vec{q},\vec{q}\,') = -\frac{g_{\pi N}^2}{4m_N^2}\frac{\omega_N'\omega_N}{(\vec{q}\,'-\vec{q})^2+M_\pi^2}\left(\frac{\vec{\sigma}_1'\cdot\vec{q}}{\omega_N'}-\frac{\vec{\sigma}_1\cdot\vec{q}}{\omega_N}\right)\left(\frac{\vec{\sigma}_2'\cdot\vec{q}}{\omega_N'}-\frac{\vec{\sigma}_2\cdot\vec{q}}{\omega_N}\right)\xrightarrow{\rm FT} V(r',r)$$

$\hookrightarrow$ static pion-exchange ($\omega_N' \simeq \omega_N \simeq 2m_N, \vec{k} = \vec{q}\,' - \vec{q}$)

$$V(\vec{q})^{\rm loc} = -\frac{g_{\pi N}^2}{4m_N^2}\frac{(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{k})}{\vec{k}^2+M_\pi^2}\xrightarrow{\rm FT} V(r)$$

- let us explore this freedom on the lattice (optimal tool!)

Local and non-local interactions on the lattice

Elhatisari, et al., Phys. Rev. Lett. **117** (2016) 132501 [arXiv:1602.04539]

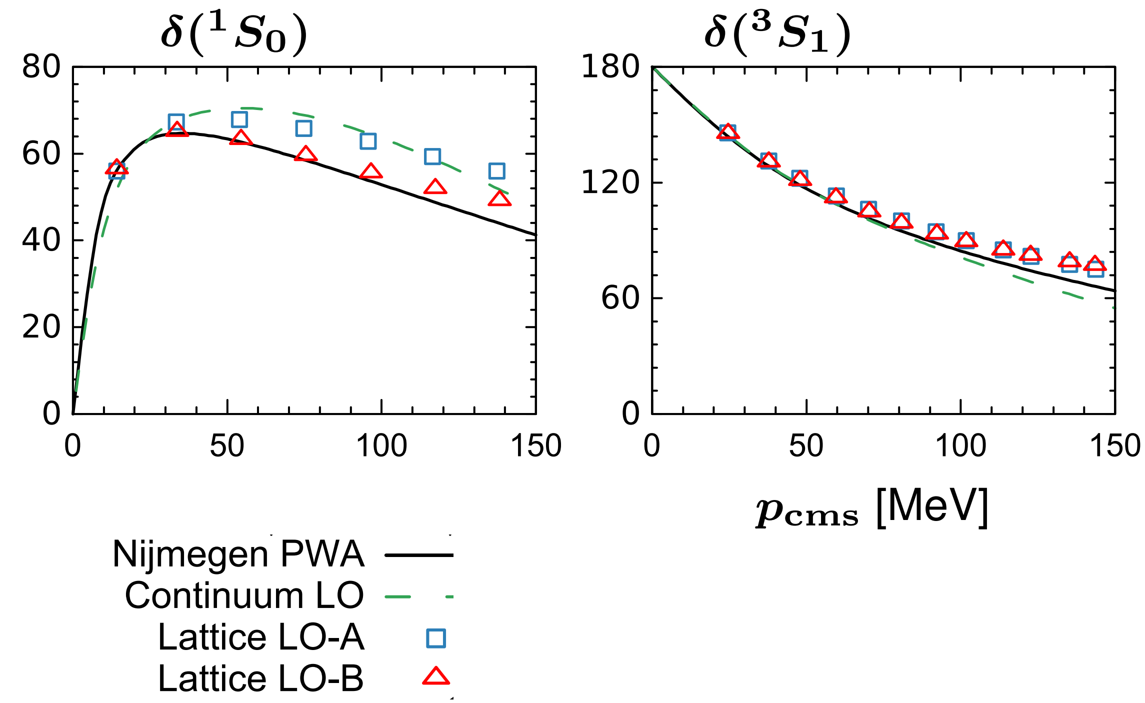
- Use the lattice and a simplified interactions [not high precsion]
- Taylor two very different interactions:

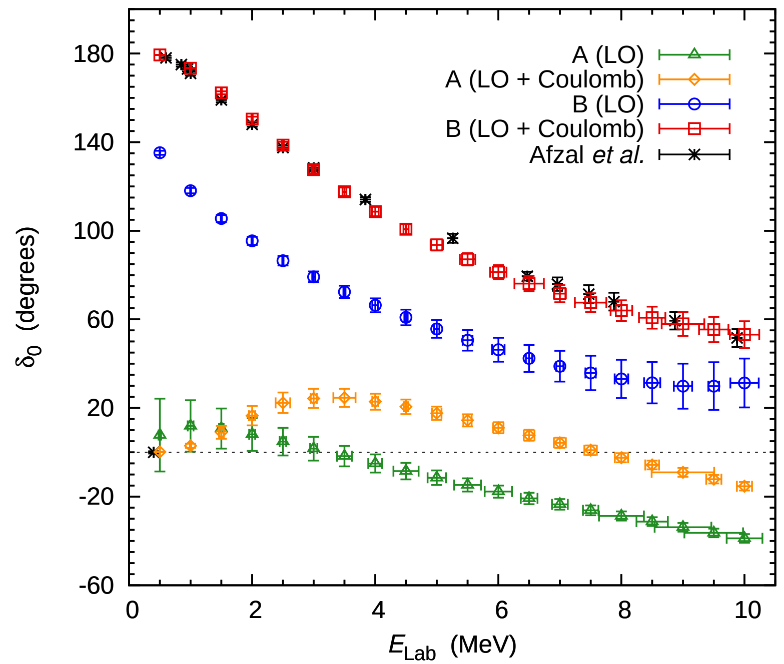
Interaction A at LO (+ Coulomb)	Interaction B at LO (+ Coulomb)
Non-local short-range interactions + One-pion exchange interaction (+ Coulomb interaction)	Non-local short-range interactions + Local short-range interactions + One-pion exchange interaction (+ Coulomb interaction)
$\to$ tuned to NN phase shifts	$\to$ tuned to NN + α-α phase shifts

Phase shifts for interactions A and B

Elhatisari, et al., Phys. Rev. Lett. **117** (2016) 132501 [arXiv:1602.04539]

- NN and α-α phase shifts:





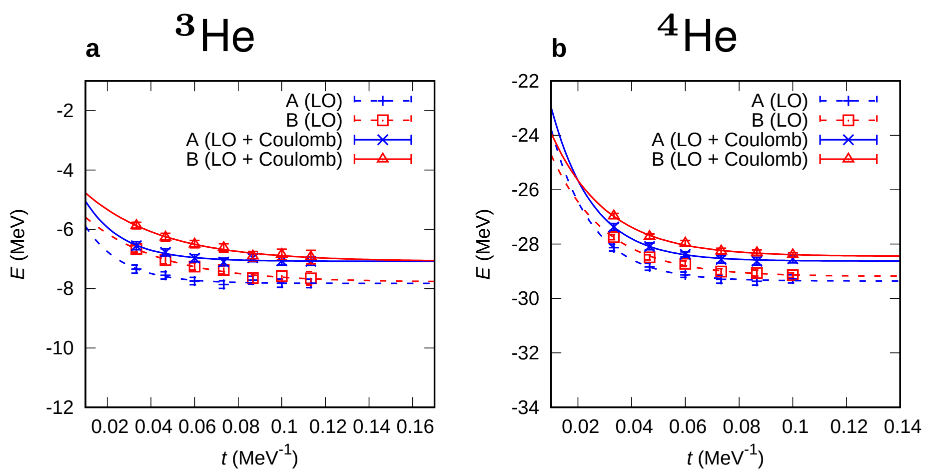
$\rightarrow$ both interactions do well for NN, but differ for α-α scattering

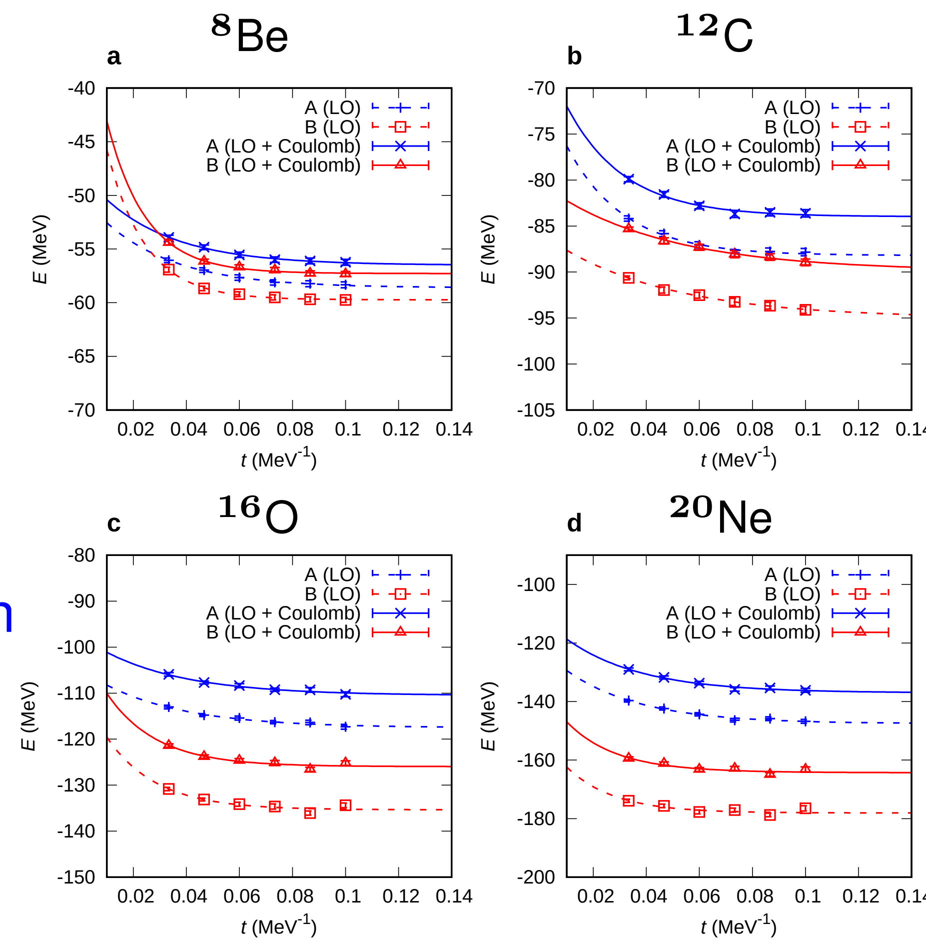
$\rightarrow$ α-α interaction is sensitive to the degree of locality of the NN interaction

$\hookrightarrow$ consequences for nuclei?

Ground state energies I

- Ground state energies for alpha-type nuclei plus ^{3}He:





- small errors, excellent thermalization

⇒ Both interactions significantly reduce the Monte Carlo sign oscillations (original motivation)

Ground state energies II

- Ground state energies for alpha-type nuclei (in MeV):

	A (LO)	A (LO+C.)	B (LO)	B (LO+C.)	Exp.
^{4}He	$-29.4(4)$	$-28.6(4)$	$-29.2(1)$	$-28.5(1)$	-28.3
^{8}Be	$-58.6(1)$	$-56.5(1)$	$-59.7(6)$	$-57.3(7)$	-56.6
^{12}C	$-88.2(3)$	$-84.0(3)$	$-95.0(5)$	$-89.9(5)$	-92.2
^{16}O	$-117.5(6)$	$-110.5(6)$	$-135.4(7)$	$-126.0(7)$	-127.6
^{20}Ne	$-148(1)$	$-137(1)$	$-178(1)$	$-164(1)$	-160.6

$\hookrightarrow$ B (LO+Coulomb) quite close to experiment (within 2% or better)

$\hookrightarrow$ A (LO+Coulomb) also fine for lighter nuclei, deviations for $A \geq 12$

$\hookrightarrow$ A (LO) describes a Bose condensate of particles:

$$E(^8\mathrm{Be})/E(^4\mathrm{He}) = 1.997(6) \quad E(^{12}\mathrm{C})/E(^4\mathrm{He}) = 3.00(1)$$

$$E(^{16}\mathrm{O})/E(^4\mathrm{He}) = 4.00(2) \quad E(^{20}\mathrm{Ne})/E(^4\mathrm{He}) = 5.03(3)$$

Consequences for nuclei & nuclear matter

- Define a one-parameter family of interactions that interpolates between the interactions A and B:

$$V_\lambda = (1-\lambda)\, V_A + \lambda\, V_B$$

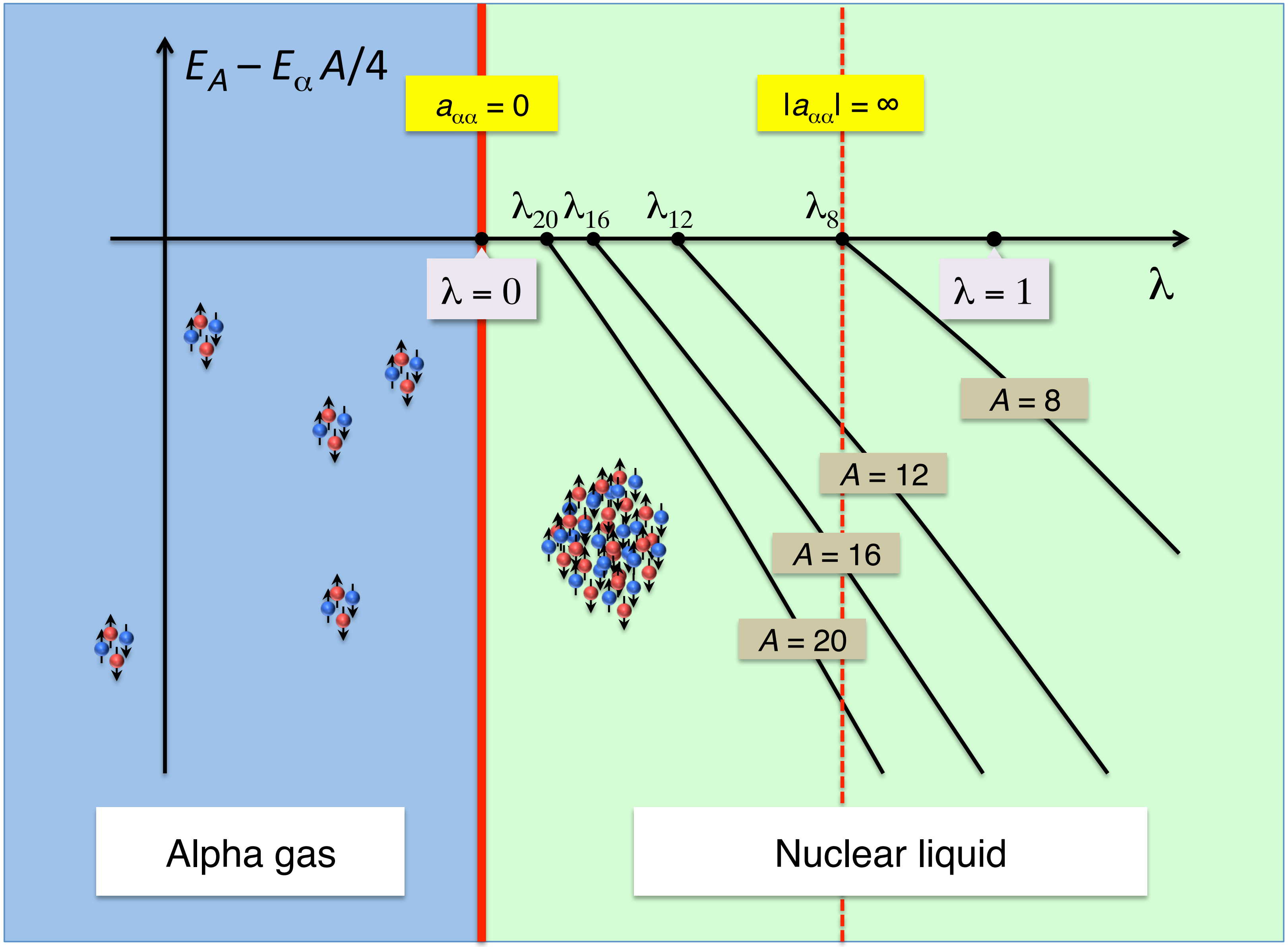
- To discuss the many-body limit, we turn off the Coulomb interaction and explore the zero-temperature phase diagram

- As a function of λ, there is a **quantum phase transition** at the point where the alpha-alpha scattering length vanishes

Stoff, Phys. Rev. A **49** (1994) 3824

- The transition is a first-order transition from a Bose-condensed gas of alpha particles to a nuclear liquid

Zero-temperature phase diagram



$$\lambda_8 = 0.7(1)$$
$$\lambda_{12} = 0.3(1)$$
$$\lambda_{16} = 0.2(1)$$
$$\lambda_{20} = 0.2(1)$$
$$\lambda_\infty = 0.0(1)$$

Further consequences

- By adjusting the parameter λ in *ab initio* calculations, one can move the of any α-cluster state up and down to alpha separation thresholds.

$\rightarrow$ This can be used as a new window to view the structure of these exotic nuclear states

- In particular, one can tune the α-α scattering length to infinity!

$\rightarrow$ In the absence of Coulomb interactions, one can thus make contact to **universal Efimov physics**:

for a review, see Braaten, Hammer, Phys. Rept. **428** (2006) 259

Hoyle state of ^{12}C	$\xrightarrow{\lambda}$	Universal Efimov trimer
Second 0^+ of ^{16}O	$\xrightarrow{\lambda}$	Universal Efimov tetramer

The anthropic principle: A glimpse into the multiverse

The Anthropic Principle (AP)

- so **many** parameters in the Standard Model, the landscape of string theory, . . .

⇒ The anthropic principle:

“The observed values of all physical and cosmological quantities are not equally probable but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirements that the Universe be old enough for it to have already done so.”

Carter 1974, Barrow & Tippler 1988, . . .

⇒ can this be tested? / have physical consequences?

- Ex. 1: “Anthropic bound on the cosmological constant” Weinberg (1987) [1044 cites]
- Ex. 2: “The anthropic string theory landscape” Susskind (2003) [1136 cites]

A prime example of the AP

- Hoyle (1953):

Prediction of an excited level in carbon-12 to allow for a sufficient production of heavy elements (^{12}C, ^{16}O,...) in stars

- was later heralded as a prime example for the AP:

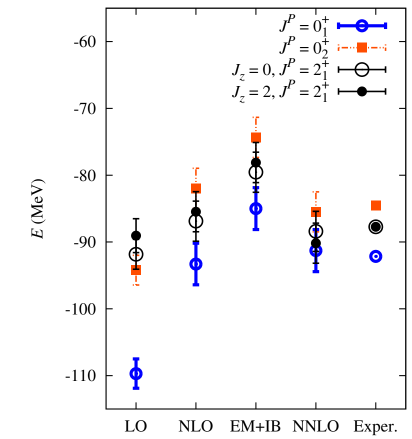
"As far as we know, this is the only genuine anthropic principle prediction"
Carr & Rees 1989

"In 1953 Hoyle made an anthropic prediction on an excited state – 'level of life' – for carbon production in stars"
Linde 2007

"A prototype example of this kind of anthropic reasoning was provided by Fred Hoyle's observation of the triple alpha process..."
Carter 2006

The relevant question

Date: Sat, 25 Dec 2010 20:03:42 -0600
From: Steven Weinberg ⟨weinberg@zippy.ph.utexas.edu⟩
To: Ulf-G. Meissner ⟨meissner@hiskp.uni-bonn.de⟩
Subject: Re: Hoyle state in 12C



Dear Professor Meissner,

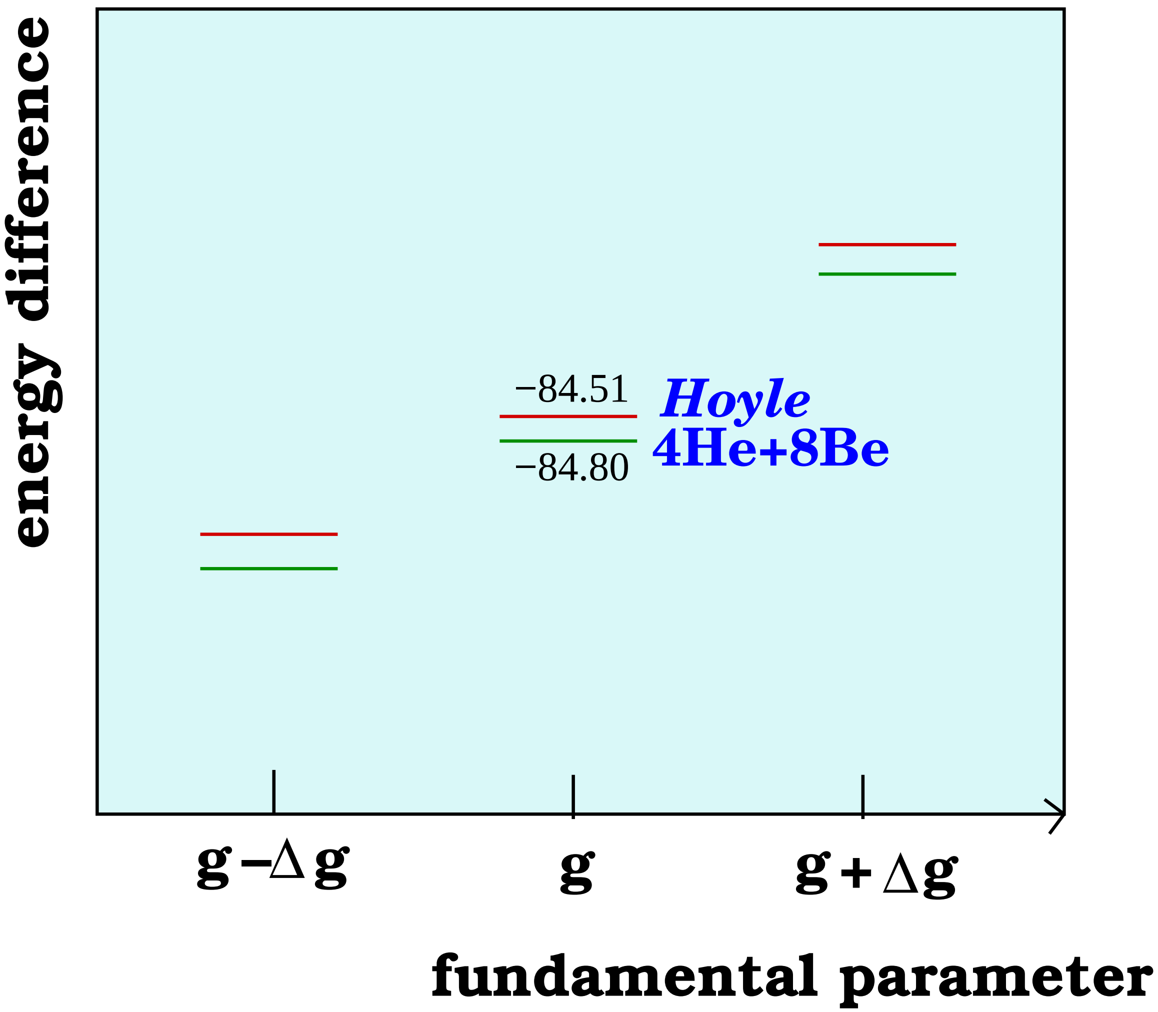
Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in C12, but also of the ground states of He4 and Be8. How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of He4 and Be8 to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of Be8 and He4.

All best,
Steve Weinberg

- How does the Hoyle state move relative to the 4He+8Be threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*

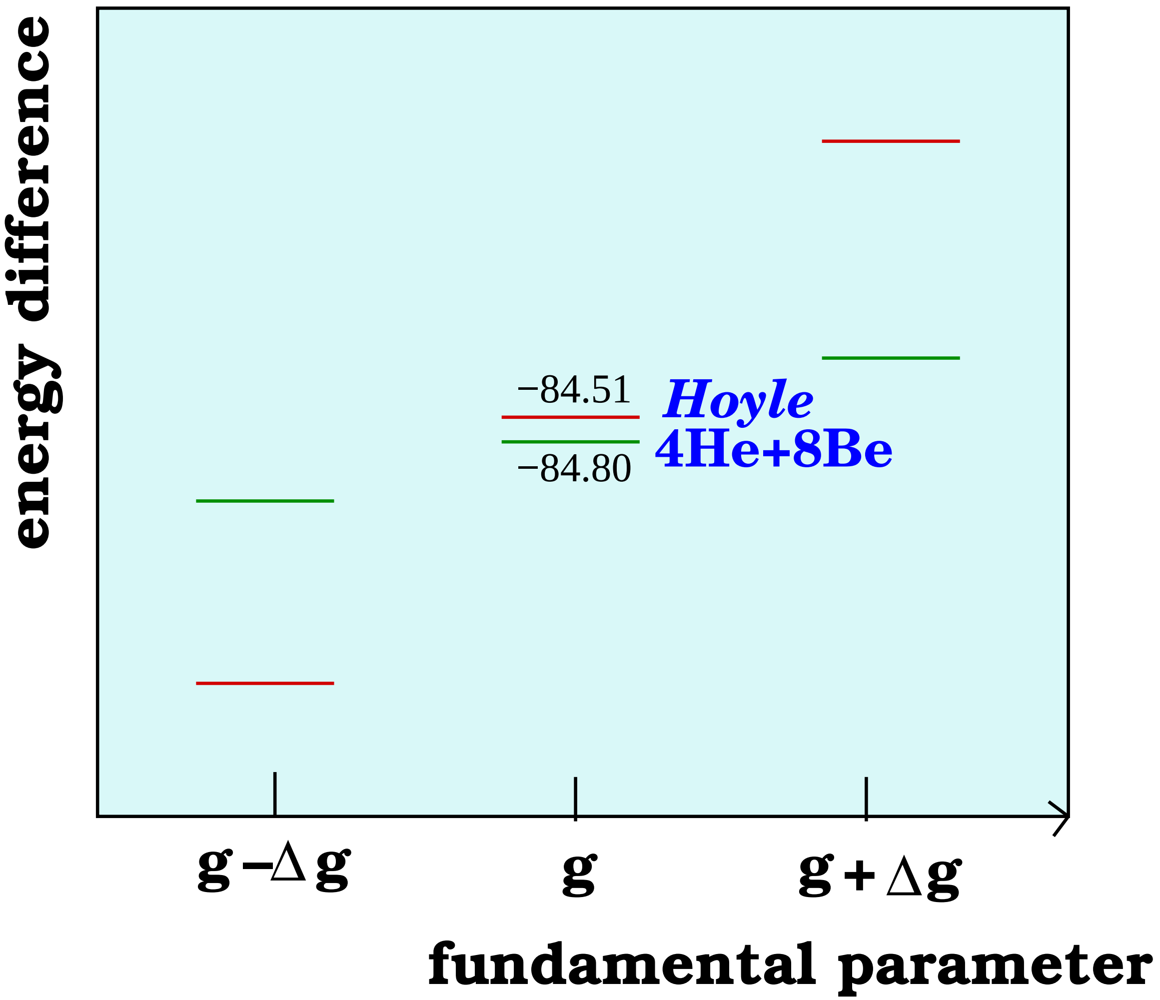
The non-anthropic scenario

- Weinberg's assumption: The Hoyle state stays close to the 4He+8Be threshold



The anthropic scenario

- The AP strikes back: The Hoyle state moves away from the 4He+8Be threshold



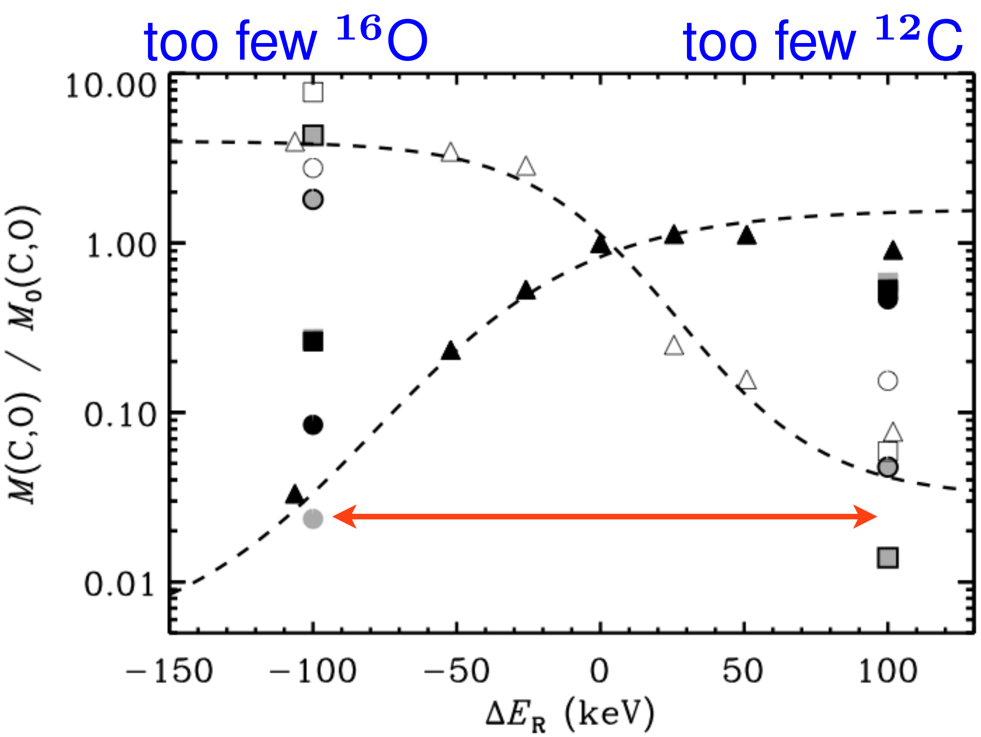
Earlier studies of the AP

- rate of the 3α-process: $r_{3\alpha} \sim \Gamma_\gamma \exp\left(-\frac{\Delta E_{h+b}}{kT}\right)$

$$\Delta E_{h+b} = E^\star_{12} - 3E_\alpha = 379.47(18) \text{ keV}$$

- how much can ΔE_{h+b} be changed so that there is still enough ^{12}C and ^{16}O?

$\Rightarrow$ $|\Delta E_{h+b}| \lesssim 100$ keV



Oberhummer et al., Science **289** (2000) 88

Csoto et al., Nucl. Phys. A **688** (2001) 560

Schlattl et al., Astrophys. Space Sci. **291** (2004) 27

[Livio et al., Nature **340** (1989) 281]

More recent stellar simulations

- Consider a larger range of masses $M_\star = (15 - 40)\, M_\odot$

- Consider low $Z = 10^{-4}$ and high $Z = Z_\odot \simeq 0.02$ metallicity

- changes depend on Z now

low Z: -300 keV $< \Delta E_R <$ 500 keV (C)

-300 keV $< \Delta E_R <$ 300 keV (O)

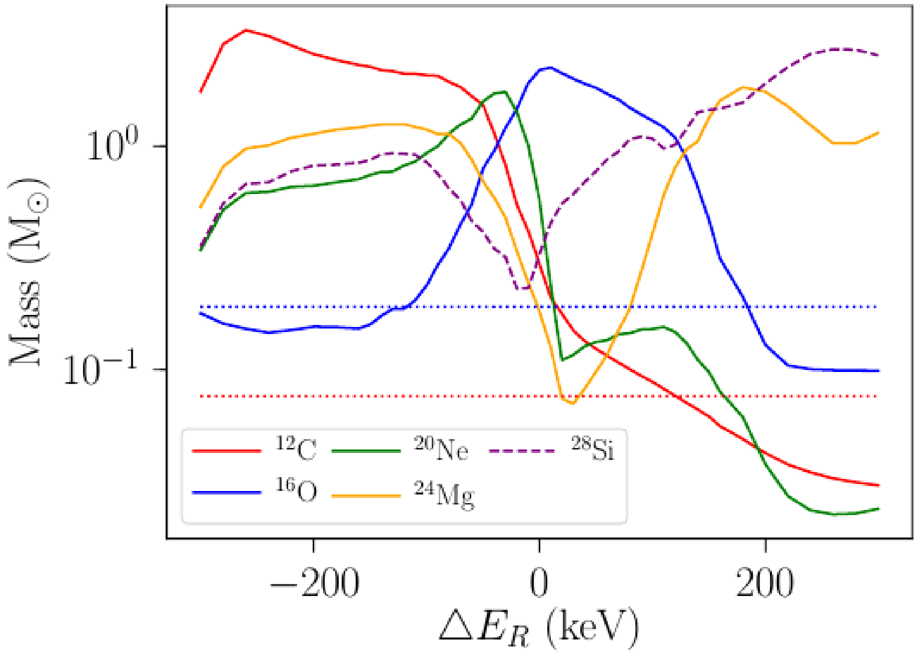
$Z_\odot$: -500 keV $< \Delta E_R <$ 160 keV (C)

-150 keV $< \Delta E_R <$ 200 keV (O)

$\Rightarrow$ carbon constraints somewhat weakened

$\Rightarrow$ stronger constraints from oxygen production

$Z = 10^{-4}$



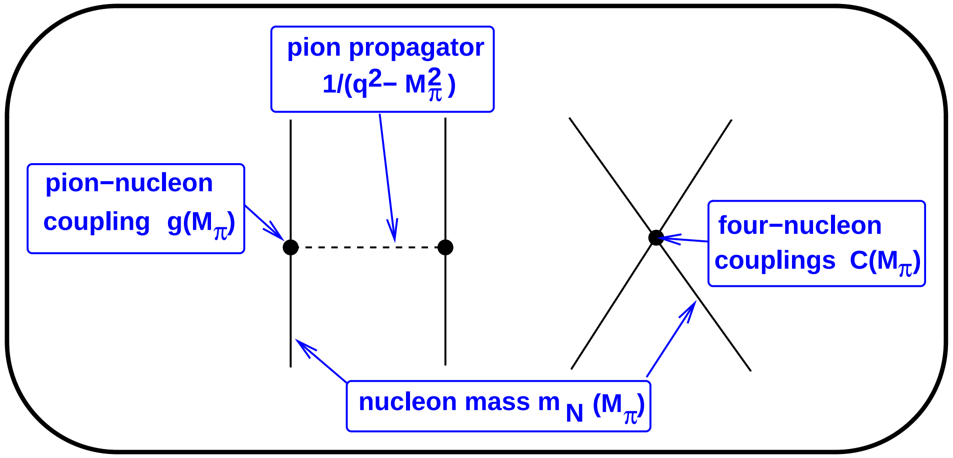
Huang, Adams, Grohs, Astropart. Phys. **105** (2019) 13

Pion mass dependence from MC simulations

- Consider pion mass changes as *small perturbations* for an energy (difference) E_i

$$\left.\frac{\partial E_i}{\partial M_\pi}\right|_{M_\pi^{\rm ph}} = \left.\frac{\partial E_i}{\partial M_\pi^{\rm OPE}}\right|_{M_\pi^{\rm ph}} + x_1 \left.\frac{\partial E_i}{\partial m_N}\right|_{m_N^{\rm ph}}$$

$$+x_2 \left.\frac{\partial E_i}{\partial g_{\pi N}}\right|_{g_{\pi N}^{\rm ph}} + x_3 \left.\frac{\partial E_i}{\partial C_0}\right|_{C_0^{\rm ph}} + x_4 \left.\frac{\partial E_i}{\partial C_I}\right|_{C_I^{\rm ph}}$$



with

$$x_1 \equiv \left.\frac{\partial m_N}{\partial M_\pi}\right|_{M_\pi^{\rm ph}}, \quad x_2 \equiv \left.\frac{\partial g_{\pi N}}{\partial M_\pi}\right|_{M_\pi^{\rm ph}}$$

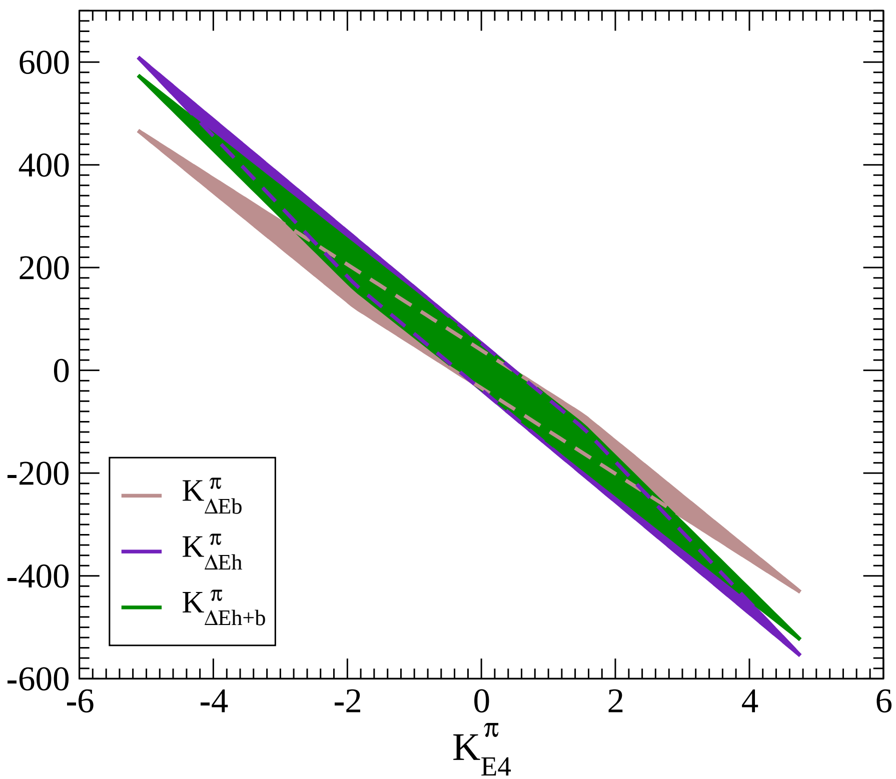
$$x_3 \equiv \left.\frac{\partial C_0}{\partial M_\pi}\right|_{M_\pi^{\rm ph}}, \quad x_4 \equiv \left.\frac{\partial C_I}{\partial M_\pi}\right|_{M_\pi^{\rm ph}}$$

$\Rightarrow$ problem reduces to the calculation of the various derivatives using AFQMC and the determination of the x_i

- x_1 and x_2 can be obtained from LQCD plus CHPT
- x_3 and x_4 can be obtained from NN scattering and its M_π-dependence $\rightarrow \bar{A}_{s,t}$

Correlations

- vary the quark mass derivatives of $\bar{A}_{s,t} = \partial a_{s,t}^{-1}/\partial M_\pi|_{M_\pi^{\rm ph}}$ within $-1,\ldots,+1$:



$$\Delta E_b = E(^8\mathrm{Be}) - 2E(^4\mathrm{He})$$

$$\Delta E_h = E(^{12}\mathrm{C}^*) - E(^8\mathrm{Be}) - E(^4\mathrm{He})$$

$$\Delta E_{h+b} = E(^{12}\mathrm{C}^*) - 3E(^4\mathrm{He})$$

$$\boxed{\frac{\partial O_H}{\partial M_\pi} = K_H^\pi \frac{O_H}{M_\pi}}$$

- clear correlations: the two fine-tunings are not independent

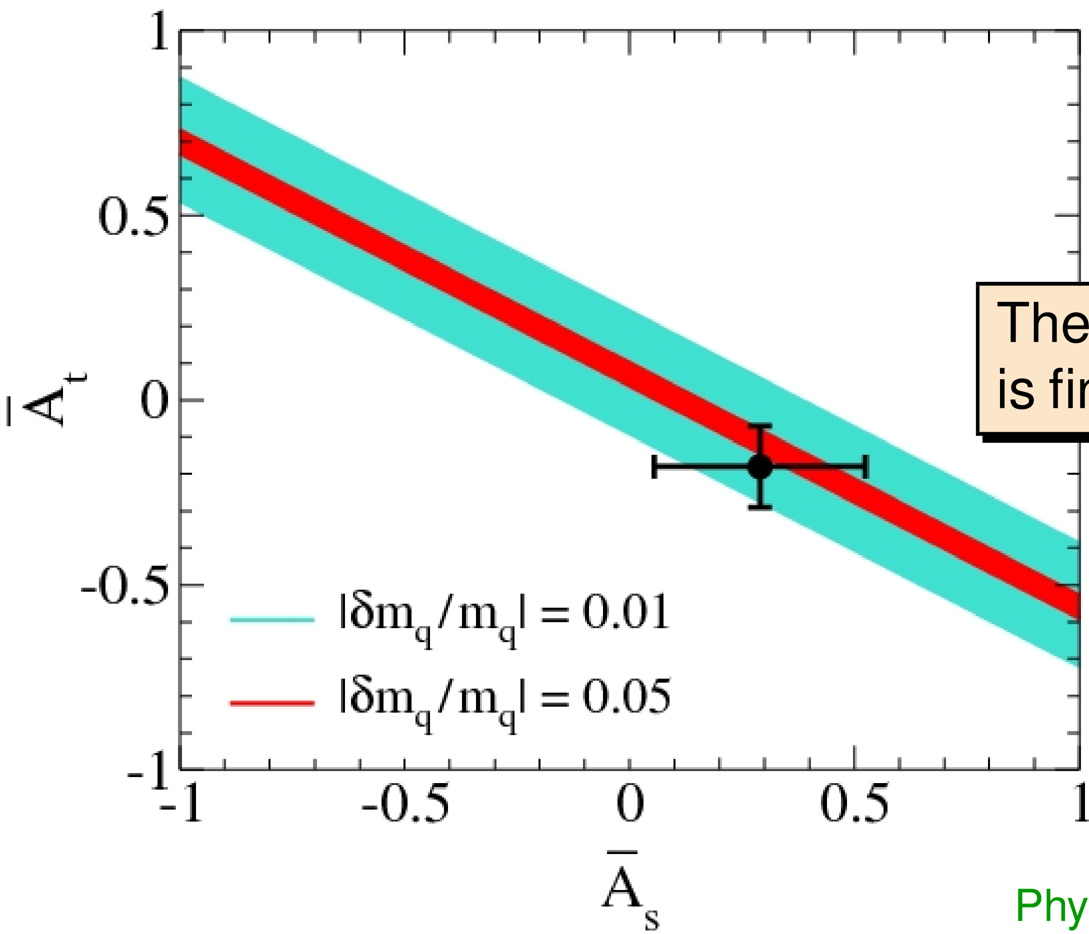
$\Rightarrow$ has been speculated before but could not be calculated Weinberg (2001)

The end-of-the-world plot I

- $|\delta(\Delta E_{h+b})| < 100$ keV

Oberhummer et al., Science (2000)

$$\to \left| \left(0.571(14)\bar{A}_s + 0.934(11)\bar{A}_t - 0.069(6) \right) \frac{\delta m_q}{m_q} \right| < 0.0015$$



$$\bar{A}_{s,t} \equiv \left. \frac{\partial a_{s,t}^{-1}}{\partial M_\pi} \right|_{M_\pi^{\rm phys}}$$

The light quark mass is fine-tuned to $\simeq 2-3\,\%$

Similarly: $\alpha_{\rm EM}$ is fine-tuned to $\simeq 2.5\%$

Berengut et al., Phys. Rev. **D 87** (2013) 085018

An update on fine-tunings in the triple-alpha process

Lähde, UGM, Epelbaum, Eur. Phys. J **A 56** (2020) 89

- Use lattice data to determine $\bar{A}_s$ and $\bar{A}_t$:

$$\bar{A}_s = 0.54(24)\,, \quad \bar{A}_t = 0.33(16)$$

$\hookrightarrow$ $\bar{A}_s$ is consistent w/ earlier determination

$\hookrightarrow$ $\bar{A}_t$ changes sign compared to earlier determination

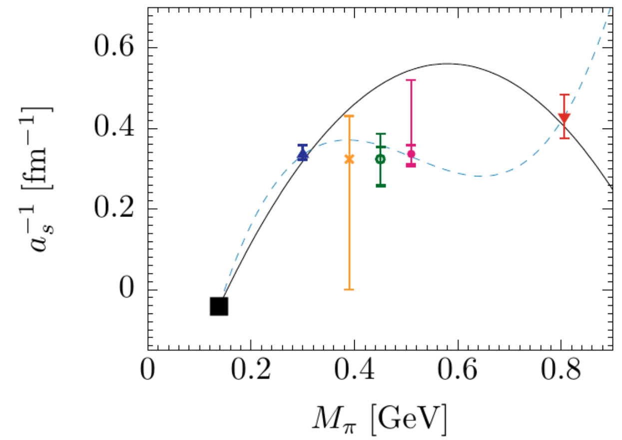
- update x_1 and x_2 using better LQCD data:

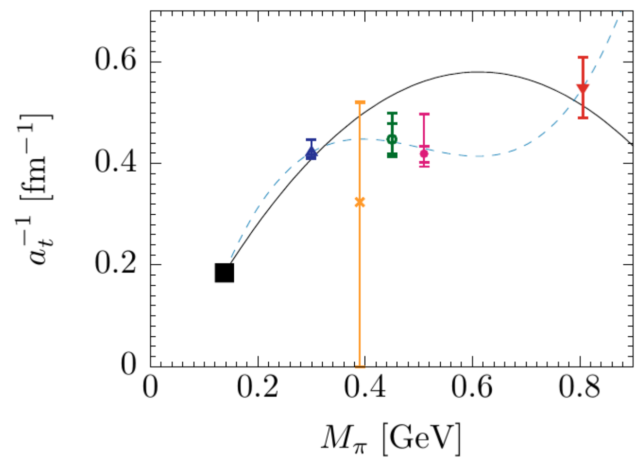
$$x_1 = 0.84(7)\,, \quad x_2 = -0.053(16)$$

$\hookrightarrow$ x_1 and x_2 more precise

$\hookrightarrow$ x_2 now has a definite sign

$\Rightarrow$ update end-of-the-world plot





Beane et al. (2012)
Yamazaki et al. (2015)
Orginos et al. (2015)
Beane et al. (2013)
Yamazaki et al. (2012)

New end-of-the-world plots

Lähde, UGM, Epelbaum, Eur. Phys. J **A 56** (2020) 89

- Constraints now depend on Z, the nucleus and the sign of δm_q

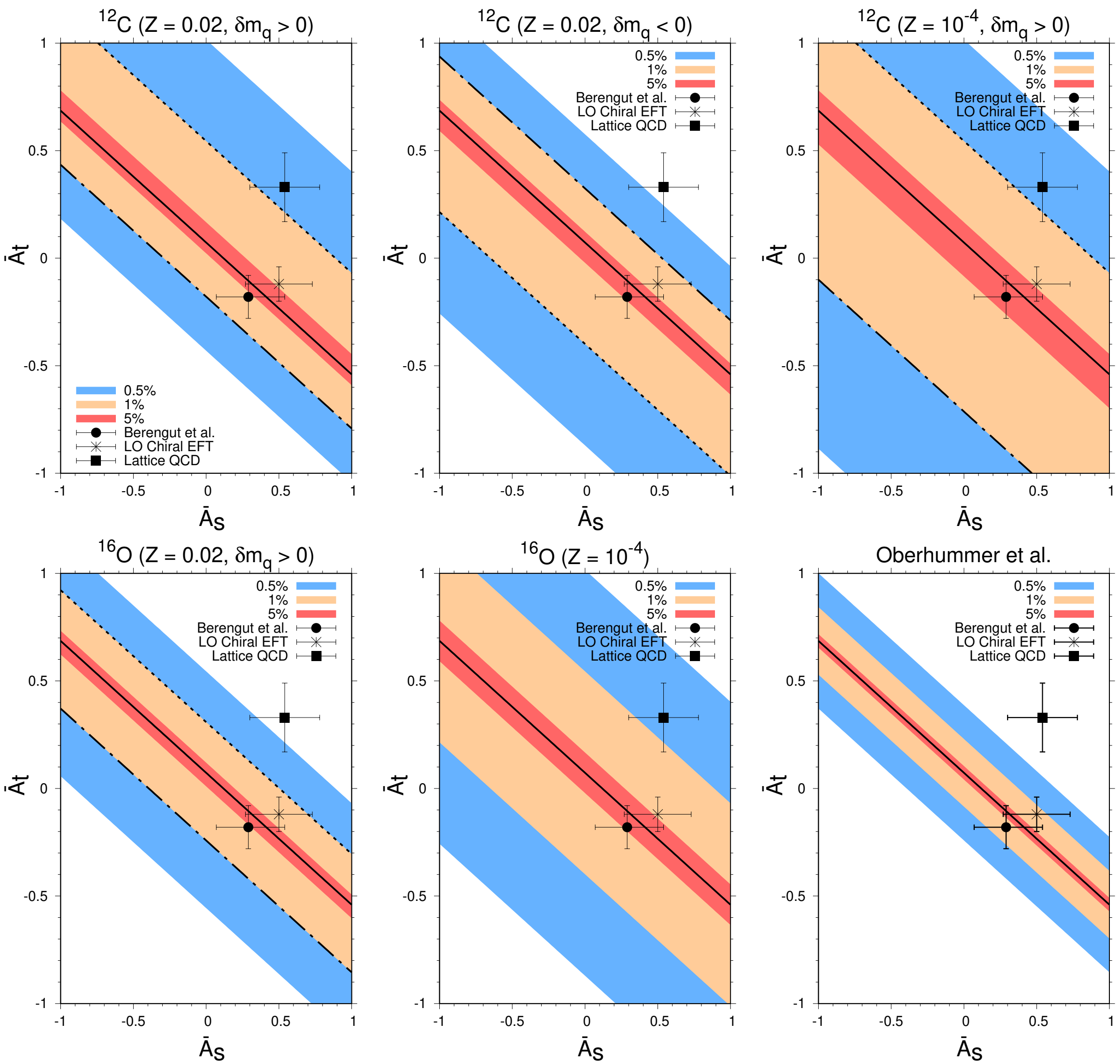
- lattice values for $\bar{A}_{s,t}$:

  The light quark mass is fine-tuned to $\simeq 0.5\,\%$

- chiral EFT values for $\bar{A}_{s,t}$:

  The light quark mass is fine-tuned to $\simeq 1\ldots 5\,\%$

- Bound on $\alpha_{\rm EM}$ softened

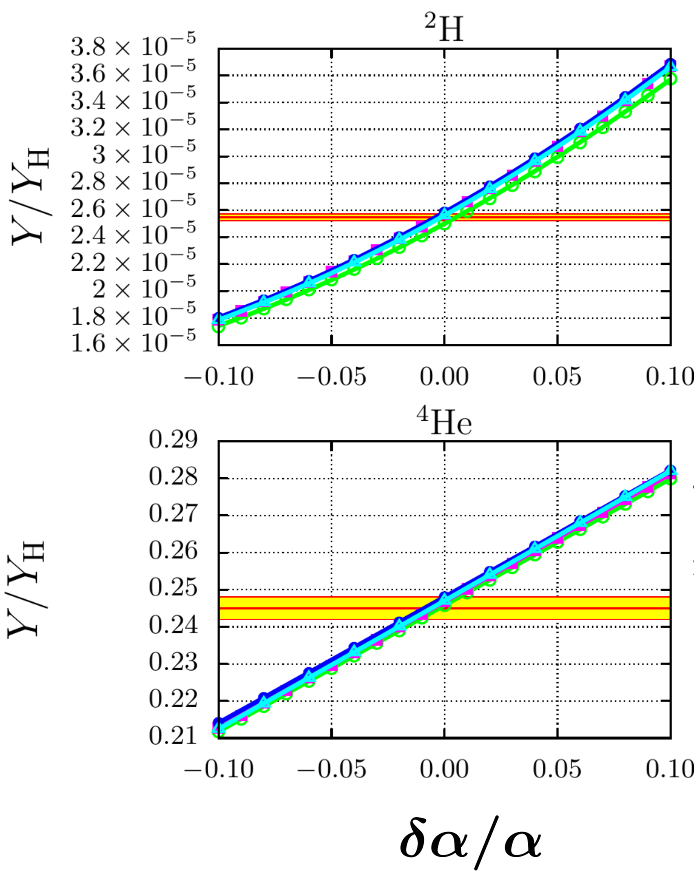


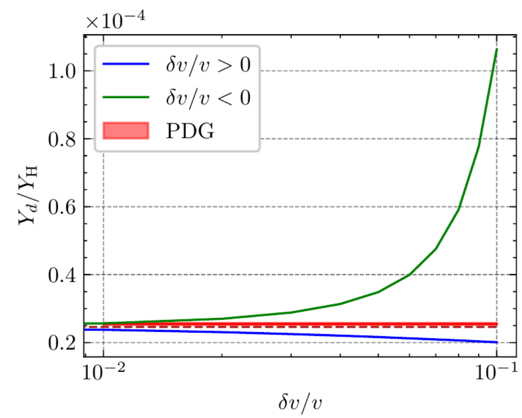
⇒ need better determinations of $\bar{A}_{s,t}$ from lattice QCD with pion masses closer to the physical point!

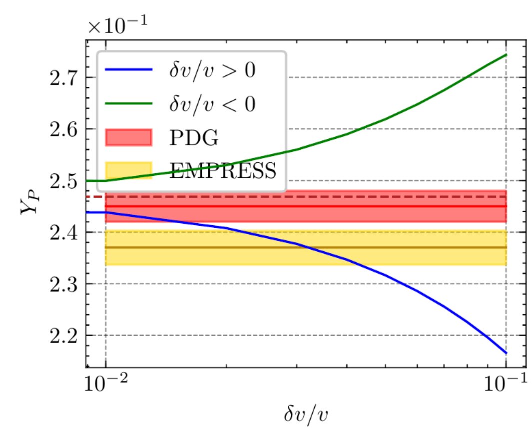
Bounds from the Big Bang

UGM, Metsch, Meyer, Eur. Phys. J **A 59** (2023) 223; Meyer, UGM, JHEP **06** (2024) 074

- Apply the same machinery to element production in the Big Bang
  - variations of $\alpha_{\rm EM}$
  - variations of the Higgs VEV







$\hookrightarrow$ stronger bounds, in particular from Y_d

Summary & outlook

Nuclear lattice EFT: a new quantum many-body approach

$\longrightarrow$ based on the successful continuum nuclear chiral EFT

$\longrightarrow$ efficiently combined with stochastic (Monte Carlo) simulations

$\hookrightarrow$ a number of highly visible results already obtained

$\hookrightarrow$ much more to come (neutron stars, weak decays, hypernuclei, ...)

$\hookrightarrow$ amenable to quantum computing

$\Rightarrow$ **Nuclear physics is a rich & fascinating field**

SPARES

Remarks on Wigner's SU(4) symmetry

- Wigner SU(4) spin-isospin symmetry in the context of pionless nuclear EFT

  ↪ large scattering lengths — Mehen, Stewart, Wise, Phys. Rev. Lett. **83** (1999) 931

- Wigner SU(4) spin-isospin symmetry is particularly beneficial for NLEFT

  ↪ suppression of sign oscillations — Chen, Lee, Schäfer, Phys. Rev. Lett. **93** (2004) 242302

  ↪ provides a very much improved LO action when smearing is included

  Lu, Li, Elhatisari, Lee, Epelbaum, UGM, Phys. Lett. B **797** (2019) 134863

- Initimately related to α-clustering in nuclei

  ↪ cluster states in ^{12}C like the famous Hoyle state

  Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501

  ↪ nuclear physics is close to a quantum phase transition

  Elhatisari et al., Phys. Rev. Lett. **117** (2016) 132501

Wigner's SU(4) symmetry and the carbon spectrum

- Study of the spectrum of ^{12}C Shen, Lähde, Lee, UGM, Eur. Phys.J. A **57** (2021) 276

  $\hookrightarrow$ spin-orbit splittings are known to be weak
  Hayes, Navratil, Vary, Phys. Rev. Lett. **91** (2003) 012502 Johnson, Phys. Rev. C **91** (2015) 034313

  $\hookrightarrow$ start with cluster and shell-model configurations $\longrightarrow$ next slide

- Locally and non-locally smeared SU(4) invariant interaction:

$$V = C_2 \sum_{\mathbf{n}',\mathbf{n},\mathbf{n}''} : \rho_{\rm NL}(\mathbf{n}') f_{s_{\rm L}}(\mathbf{n}'-\mathbf{n}) f_{s_{\rm L}}(\mathbf{n}-\mathbf{n}'') \rho_{\rm NL}(\mathbf{n}'') : , \quad f_{s_{\rm L}}(\mathbf{n}) = \begin{cases} 1, & |\mathbf{n}| = 0, \\ s_{\rm L}, & |\mathbf{n}| = 1, \\ 0, & \text{otherwise} \end{cases}$$

$$\rho_{\rm NL}(\mathbf{n}) = a^\dagger_{\rm NL}(\mathbf{n}) a_{\rm NL}(\mathbf{n})$$

$$a^{(\dagger)}_{\rm NL}(\mathbf{n}) = a^{(\dagger)}(\mathbf{n}) + s_{\rm NL} \sum_{|\mathbf{n}'|=1} a^{(\dagger)}(\mathbf{n}+\mathbf{n}') , \quad s_{\rm NL} = 0.2$$

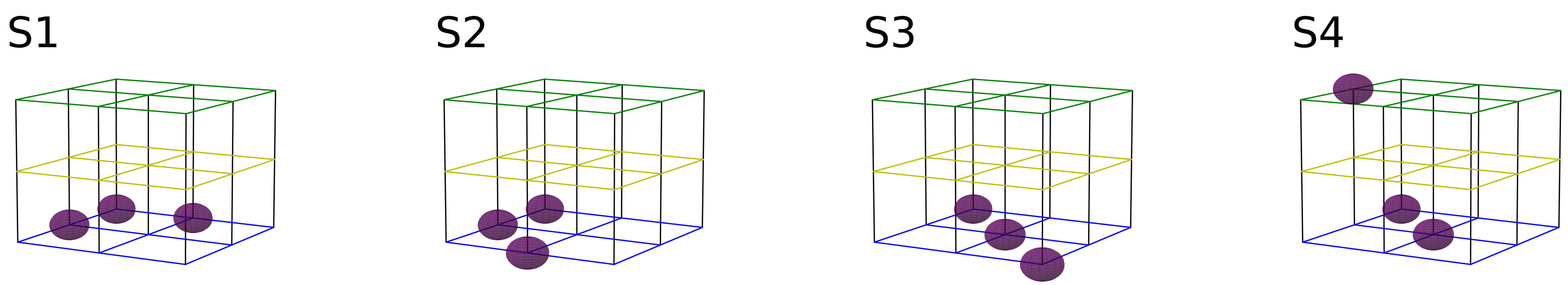
  $\hookrightarrow$ only two adjustable parameters (C_2, s_L) fitted to $E_{^4\rm He}$ & $E_{^{12}\rm C}$

  $\hookrightarrow$ investigate the spectrum for $a = 1.64$ fm and $a = 1.97$ fm

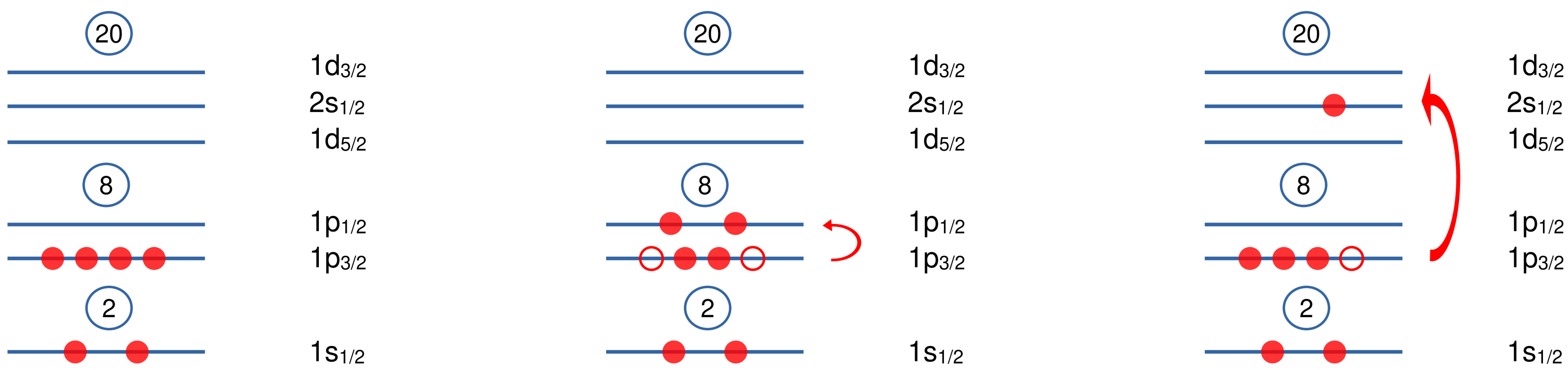
Configurations

- Cluster and shell model configurations

Gaussian wave packets
$w = 1.7 - 2.1$ fm



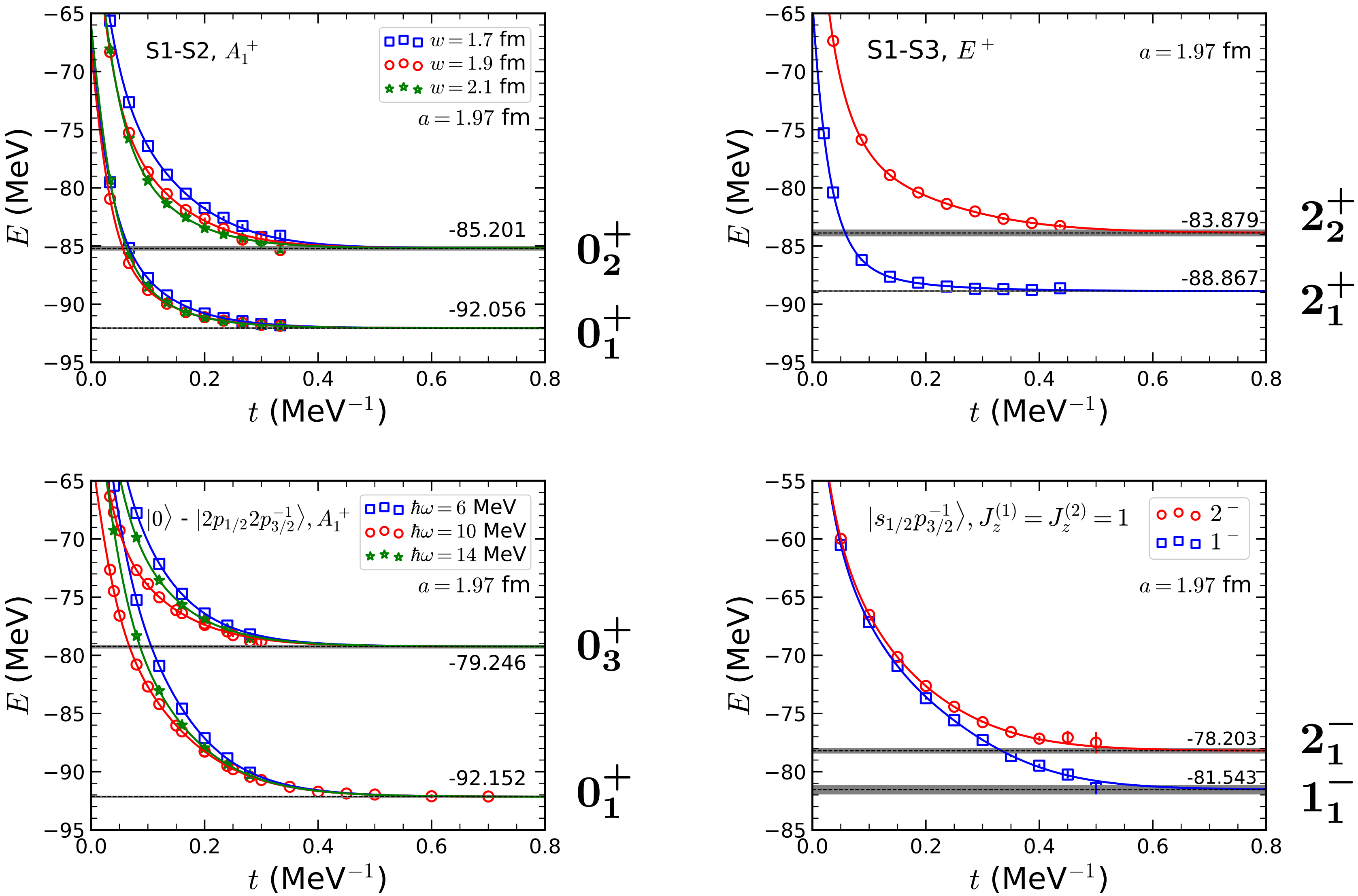
— isoscele right triangle — “bent-arm” shape — linear diagonal chain — acute isoscele triangle



— ground state $|0\rangle$ — $2p$-$2h$ state, $J_z = 0$ — $1p$-$1h$ state, $J_z^{(1)} = J_z^{(2)} = 1$

Transient energies

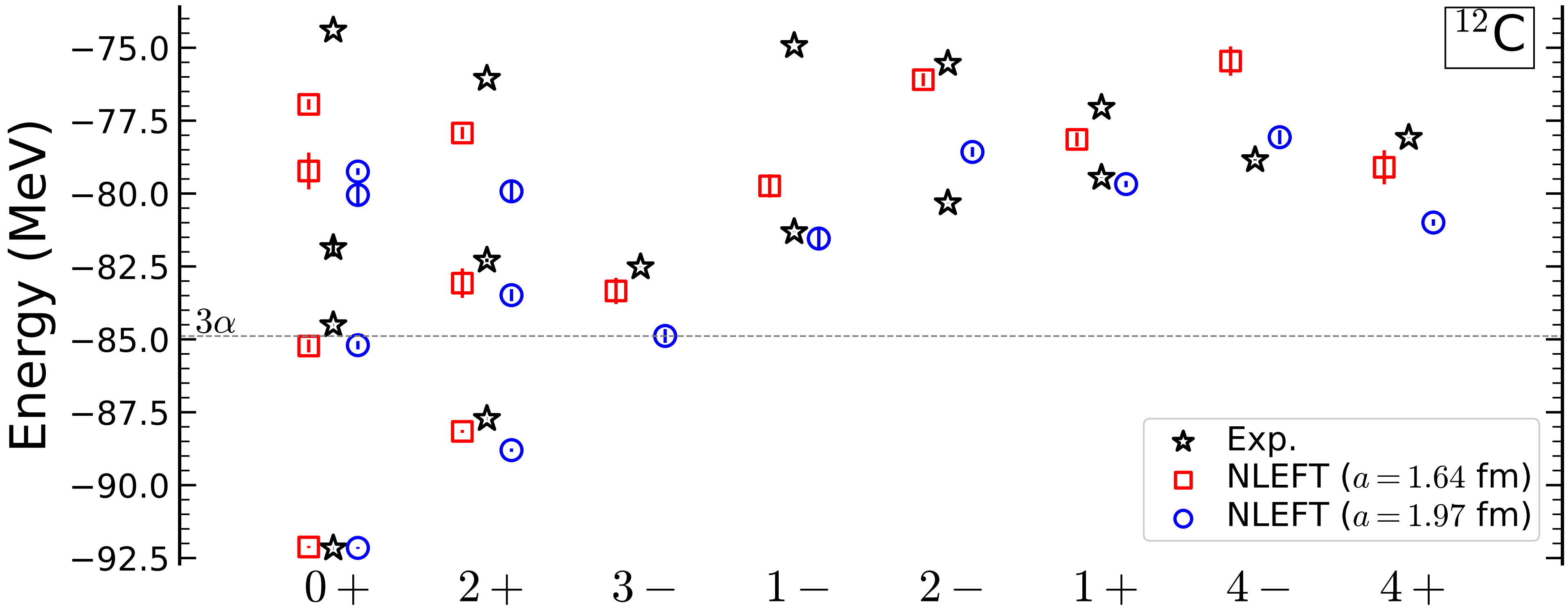
- Transient energies from cluster and shell-model configurations



Spectrum of ^{12}C

Shen, Lähde, Lee, UGM, Eur. Phys.J. A **57** (2021) 276 [arXiv:2106.04834]

- Amazingly precise description $\rightarrow$ great starting point



Gives all states, other than NCSM

$\rightarrow$ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

A closer look at the spectrum of ^{12}C

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777

- Include also 3NFs: $V = \frac{C_2}{2!}\sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2 + \frac{C_3}{3!}\sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^3$

- Fit the four parameters:

  C_2, C_3 – ground state energies of ^{4}He and ^{12}C

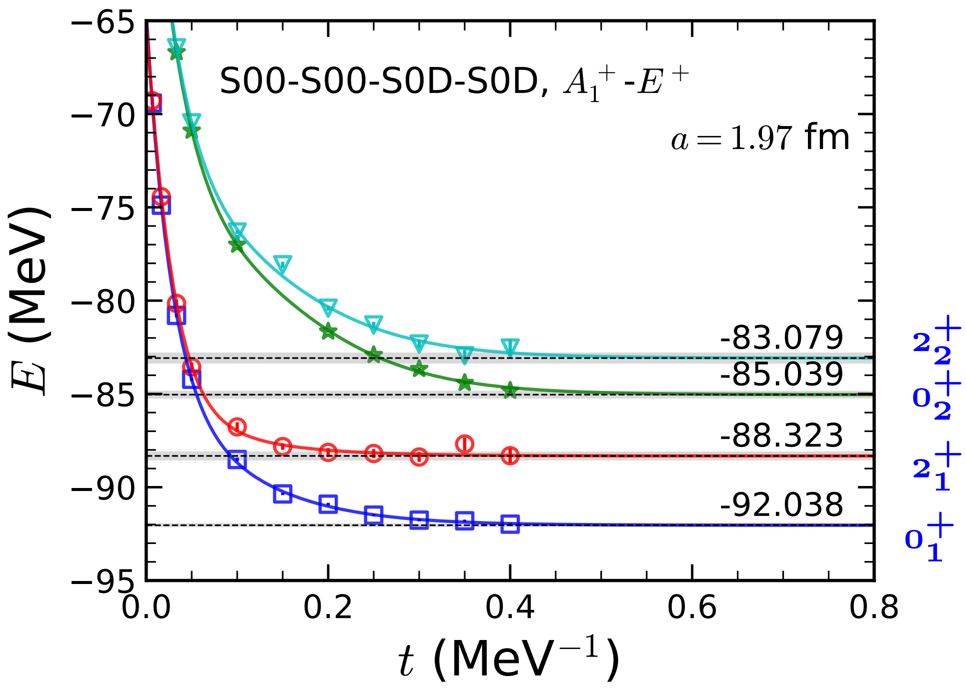
  $s_{\rm L}$ – radius of ^{12}C around 2.4 fm

  $s_{\rm NL}$ – best overall description of the transition rates

- Calculation of em transitions

  requires coupled-channel approach

  e.g. 0^+ and 2^+ states



The hidden spin-isospin exchange symmetry

Nucleon-nucleon interaction in large-N_C

Kaplan, Savage, Phys. Lett. **365B** (1996) 244; Kaplan, Manohar, Phys. Rev. **C 56** (1997) 96

- Performing the large-N_C analysis:

$$V^{\rm 2N}_{{\rm large}-N_c} = V_C + W_S\, \vec\sigma_1\cdot\vec\sigma_2\vec\tau_1\cdot\vec\tau_2 + W_T\, S_{12}\vec\tau_1\cdot\vec\tau_2 + \ldots$$

- Leading terms are $\sim N_C$
- First corrections are $1/N_C^2$ suppressed, fairly strong even for $N_C = 3$
- Velocity-dependent corrections can be incorporated
- Based on spin-isospin exchange symmetry of the nucleon w.f. $d_\uparrow \leftrightarrow u_\downarrow$ or on the nucleon level $n_\uparrow \leftrightarrow p_\downarrow$
- Constraints on 3NFs: Phillips, Schat, PRC **88** (2013) 034002; Epelbaum et al., EPJA **51** (2015) 26

Hidden spin-isospin symmetry: Basic ideas

Lee, Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, UGM, Phys. Rev. Lett. **127** (2021) 062501 [2010.09420 [nucl-th]]

- $V^{\rm 2N}_{{\rm large}-N_c}$ is not renomalization group invariant: $\dfrac{dV_\mu(p,p')}{d\mu} \neq 0$

  $\simeq$ implicit setting of a preferred renormalization/resolution scale

- How does this happen?
  - **high energies**: corrections to the nucleon w.f. are $\sim v^2$
    $\to$ these high-energy modes must be $\mathcal{O}(1/N_C^2)$ in our low-energy EFT
    $\to$ momentum resolution scale $\Lambda \sim m_N/N_C \sim \mathcal{O}(1)$
    $\to$ consistent with the cutoff in a Δless th'y $\sim \sqrt{2m_N(m_\Delta - m_N)}$
  - **low energies:** the resolution scale must be large enough,
    so that orbital angular momentum and spin are fully resolved
    $\to$ as nucleon size is independent of N_C, so should be Λ $\surd$

- as will be shown, the optimal scale (where corrections are $\sim 1/N_C^2$) is:

$$\Lambda_{{\rm large}-N_c} \simeq 500\,{\rm MeV}$$

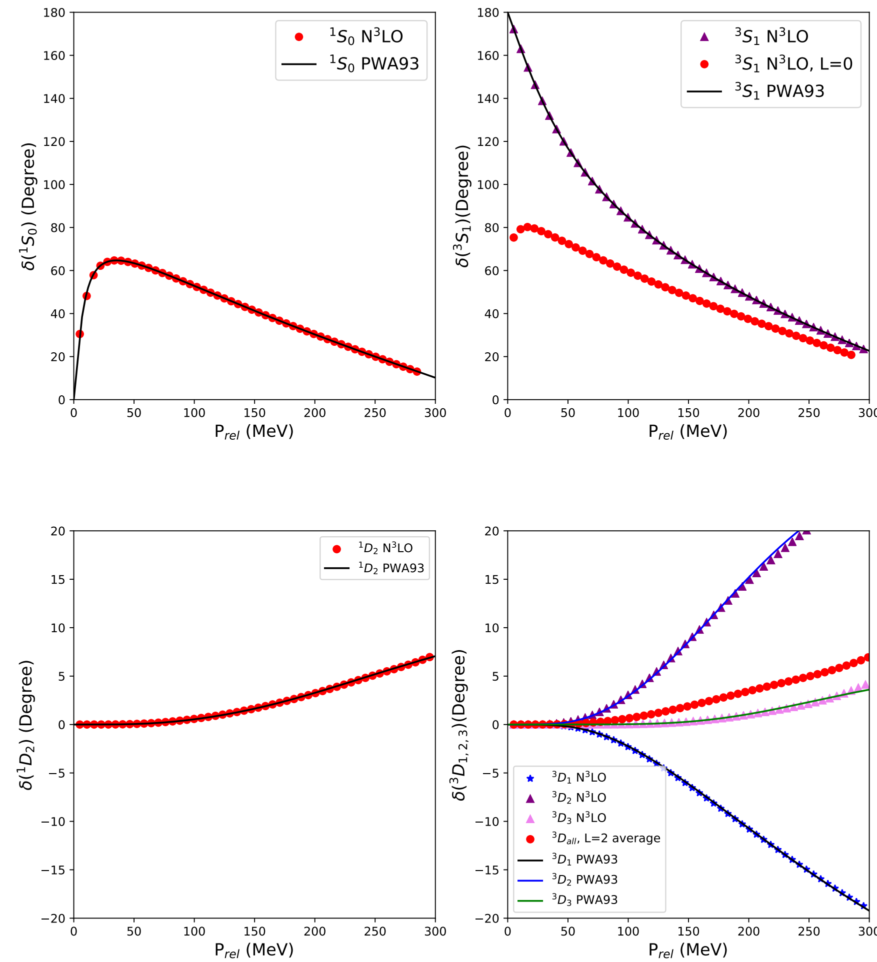
Nucleon-nucleon phase shifts – lattice

Lee, Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, UGM,
Phys. Rev. Lett. **127** (2021) 062501 [2010.09420 [nucl-th]]

- Use N3LO action (w/ TPE absorbed in contact interactions) at $a = 1.32$ fm

  $\hookrightarrow \Lambda = \pi/a = 470$ MeV

- compare $S = 0, T = 1$ w/ $S = 1, T = 0$
- S-waves: switch off the tensor force in $^3S_1$
- D-waves: average the spin-triplet channel



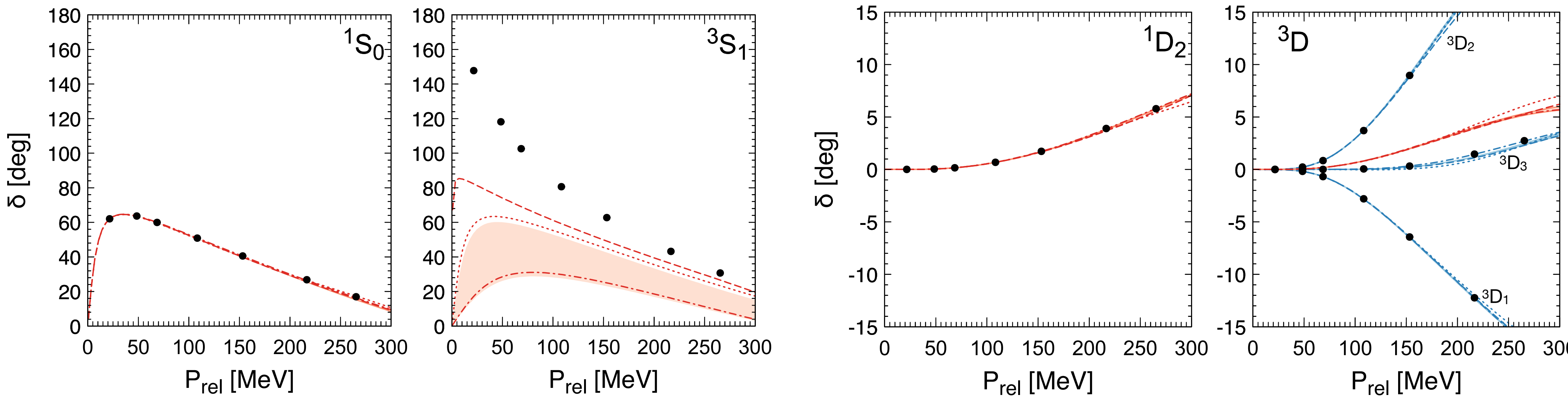
- NLEFT low-energy constants

ch., order	LEC (l.u.)	ch., order	LEC (l.u.)
$^1\mathrm{S}_0, Q^0$	1.45(5)	$^3\mathrm{S}_1, Q^0$	1.56(3)
$^1\mathrm{S}_0, Q^2$	−0.47(3)	$^3\mathrm{S}_1, Q^2$	−0.53(1)
$^1\mathrm{S}_0, Q^4$	0.13(1)	$^3\mathrm{S}_1, Q^4$	0.12(1)
$^1\mathrm{D}_2, Q^4$	−0.088(1)	$^3\mathrm{D}_{\mathrm{all}}, Q^4$	−0.070(2)

$\Rightarrow$ works pretty well

Nucleon-nucleon phase shifts – continuum

- Consider various (chiral) continuum potentials $\longrightarrow$ also works $\surd$



IDAHO N3LO — Entem, Machleidt, PRC **68** (2003) 041001

IDAHO N4LO ($\Lambda = 500$ MeV) — Entem, Machleidt, Nosyk PRC **96** (2017) 024004

CD-Bonn — Machleidt, PRC **63** (2001) 024001

Bochum N4$^+$LO ($\Lambda = 400 - 550$ MeV) — Reinert, Krebs, Epelbaum, EPJA **54** (2018) 86

Nijmegen PWA — Wiringa, Stoks, Schiavilla, PRC **51** (1995) 38

Two-nucleon matrix elements

- Consider the ME between any two-nucleon states A and B. Both have total spin S and total isospin T. Then (for isospin-inv. H):

$$M(S,T) = \frac{1}{2S+1}\sum_{S_z=-S}^{S}\langle A;S,S_z;T,T_z|H|B;S,S_z;T,T_z\rangle$$

- Spin-isospin exchange symmetry: $\boxed{M(S,T) = M(T,S)}$

- Ex: ^{30}P has 1 proton + 1 neutron in the $1s_{1/2}$ orbitals (minimal shell model)

$\to$ if spin-isospin exchange symmetry were exact, the $S=0, T=1$ & $S=1, T=0$ states should be degenerate

- Data: The 1^+ g.s. is 0.677 MeV below the 0^+ excited state ($E_{g.s.} \simeq 220$ MeV)

$\to$ fairly good agreement, consistent w/ $1/N_C^2$ corrections

$\to$ explanation: interactions of the np pair with the ^{28}Si core are suppressing spatial correlations of the 1^+ w.f. caused by the tensor interaction

Two-nucleon matrix elements in the s-d shell

- Test the spin-isospin echange symmetry for general two-body MEs 1s-0d shell
- Use the spin-tensor analysis developed by Kirson, Brown et al.

Kirson, PLB **47** (1973) 110; Brown et al., JPhysG **11** (1985) 1191; Ann. Phys. **182** (1988) 191

- Seven two-body MEs for $(S,T) = (1,0)$ and $(S,T) = (0,1)$

ME	L_1	L_2	L_3	L_4	L_{12}	L_{34}
1	2	2	2	2	0	0
2	2	2	2	2	2	2
3	2	2	2	2	4	4
4	2	2	2	0	2	2
5	2	2	0	0	0	0
6	2	0	2	0	2	2
7	0	0	0	0	0	0

L_1, L_2: orbital angular momenta of the outgoing orbitals of A

L_{12}: total angular momentum of state A

L_3, L_4: orbital angular momenta of the outgoing orbitals of B

L_{34}: total angular momentum of state A

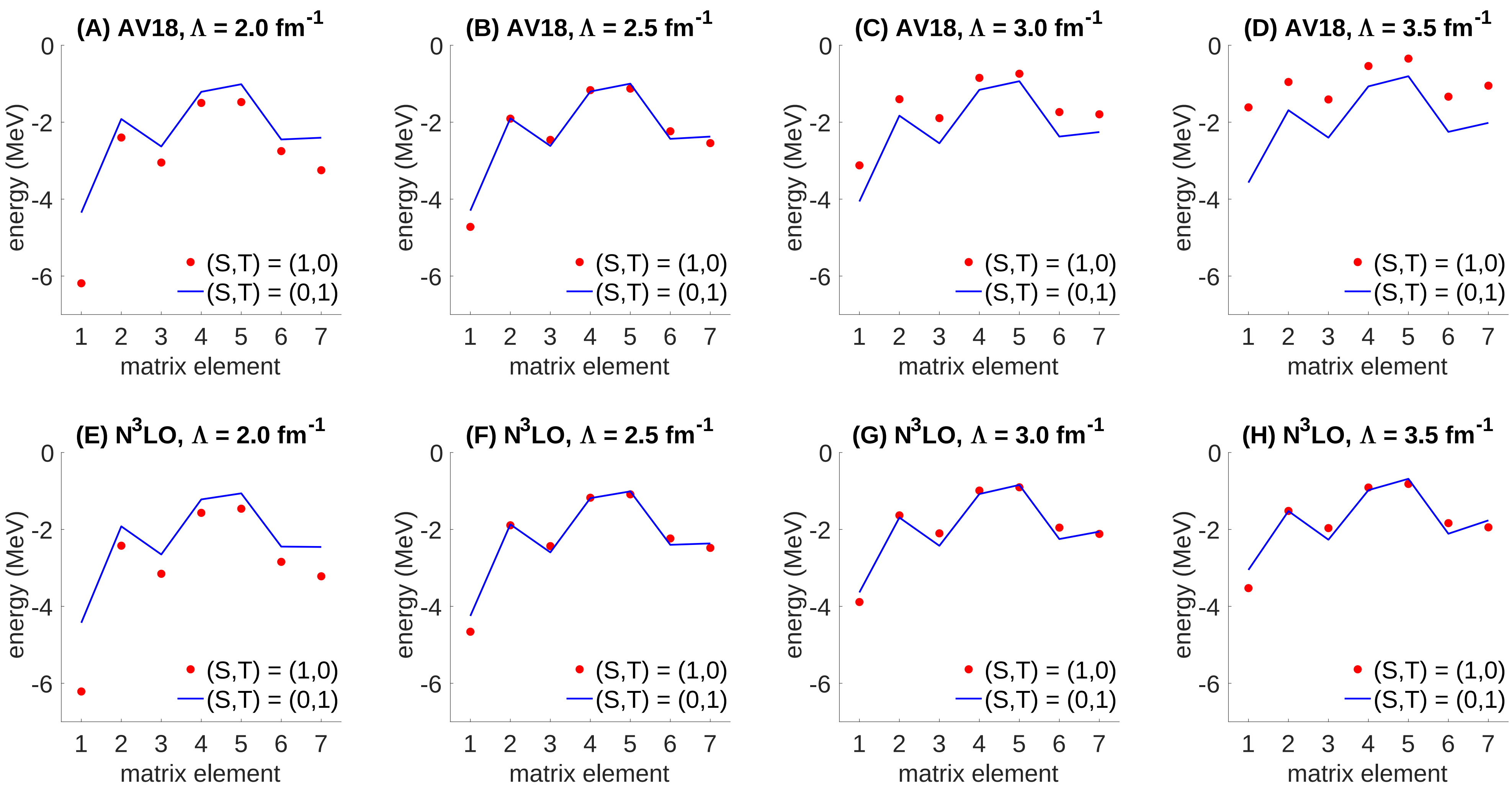
ME 7 corresponds to the $1s_{1/2}$ orbitals discussed before

set $L_Z = (L_{12})_z = (L_{34})_z$, average over L_z

$\rightarrow$ Work out $M(S,T)$ for various forces at $\Lambda = 2.0, 2.5, 3.0, 3.5$ fm^{-1}

Two-nucleon matrix elements in the s-d shell

- Results for the AV18 and N3LO chiral potentials



Two-nucleon matrix elements: Conclusions

- As anticipated:
  - The optimal resolution scale is obviously $\Lambda \sim 500$ MeV
  - For $\Lambda < \Lambda_{\mathrm{large}-N_c}$, the $(S,T) = (1,0)$ channel is more attractive
  - For $\Lambda > \Lambda_{\mathrm{large}-N_c}$, the $(S,T) = (0,1)$ channel is more attractive
  - These results do not depend on the type of interaction, while AV18 is local, chiral N3LO has some non-locality (and similar for more modern interactions like chiral N4^{+}LO)

$\hookrightarrow$ consistent with the results for NN scattering

$\Rightarrow$ **Validates Weinberg's power counting!** $\surd$

Three-nucleon forces

- Leading central three-nucleon force at the optimal resolution scale:

$$\begin{aligned} V^{\rm 3N}_{{\rm large}-N_c} &= V_C^{\rm 3N} + [(\vec\sigma_1\times\vec\sigma_2)\cdot\vec\sigma_3][(\vec\tau_1\times\vec\tau_2)\cdot\vec\tau_3]W^{\rm 3N}_{123} \\ &+ \vec\sigma_1\cdot\vec\sigma_2\vec\tau_1\cdot\vec\tau_2 W^{\rm 3N}_{12} + \vec\sigma_2\cdot\vec\sigma_3\vec\tau_2\cdot\vec\tau_3 W^{\rm 3N}_{23} \\ &+ \vec\sigma_3\cdot\vec\sigma_1\vec\tau_3\cdot\vec\tau_1 W^{\rm 3N}_{31} + \ldots, \end{aligned}$$

- Subleading central 3N interactions are of size $1/N_C$, of type

$$\vec\sigma_1\cdot\vec\sigma_2[(\vec\tau_1\times\vec\tau_2)\cdot\vec\tau_3]\,, \qquad [(\vec\sigma_1\times\vec\sigma_2)\cdot\vec\sigma_3]\vec\tau_1\cdot\vec\tau_2$$

$\Rightarrow$ helps in constraining the many short-range three-nucleon interactions that appear at higher orders in chiral EFT

- The spin-isospin exchange symmetry of the leading interactions also severely limits the isospin-dependent contributions of the 3N interactions to the nuclear EoS

$\Rightarrow$ relevant for calculations of the nuclear symmetry energy and its density dependence in dense nuclear matter

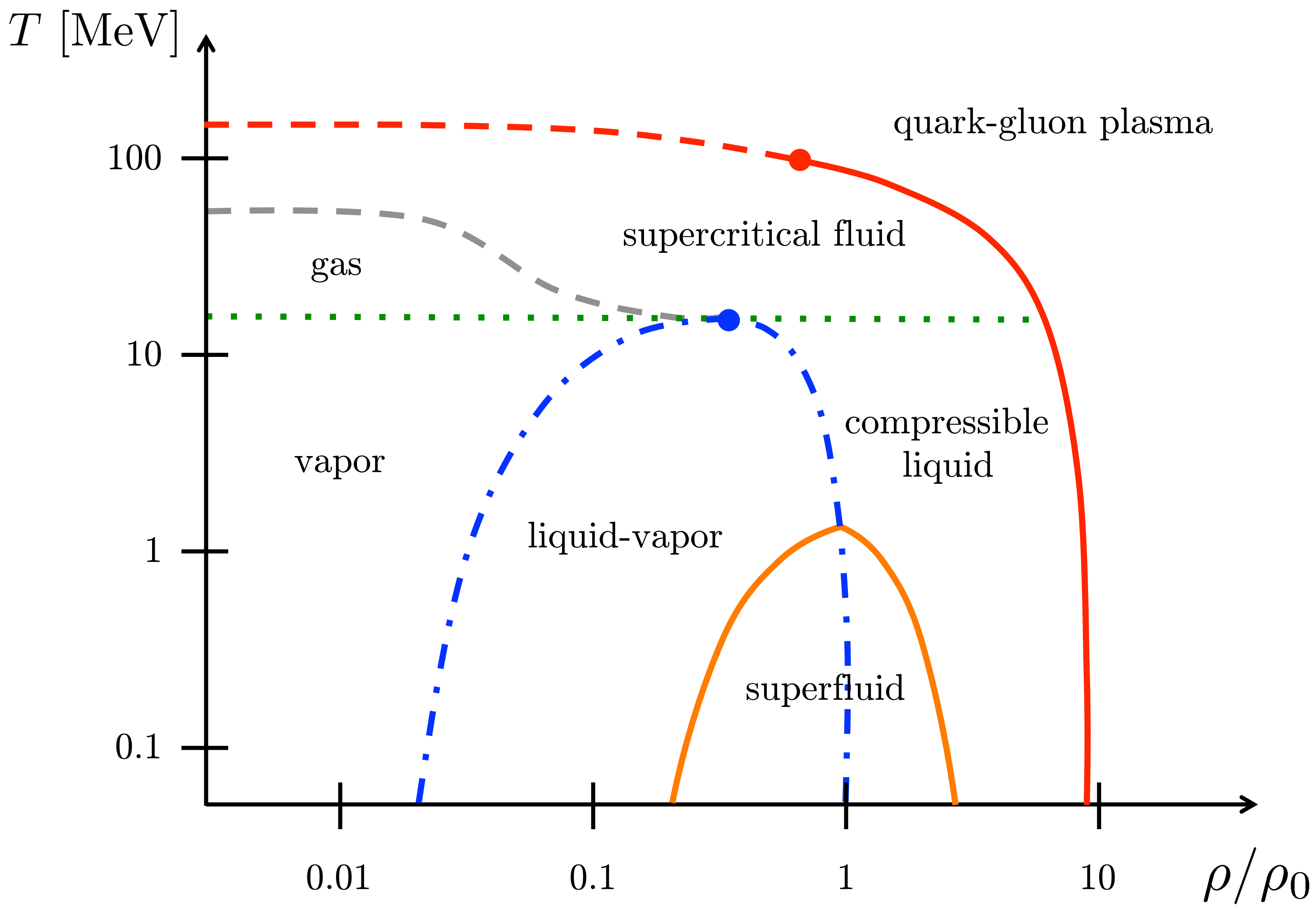
Ab Initio Nuclear Thermodynamics

B. N. Lu, N. Li, S. Elhatisari, D. Lee, J. Drut, T. Lähde, E. Epelbaum, UGM,
Phys. Rev. Lett. **125** (2020) 192502 [arXiv:1912.05105]

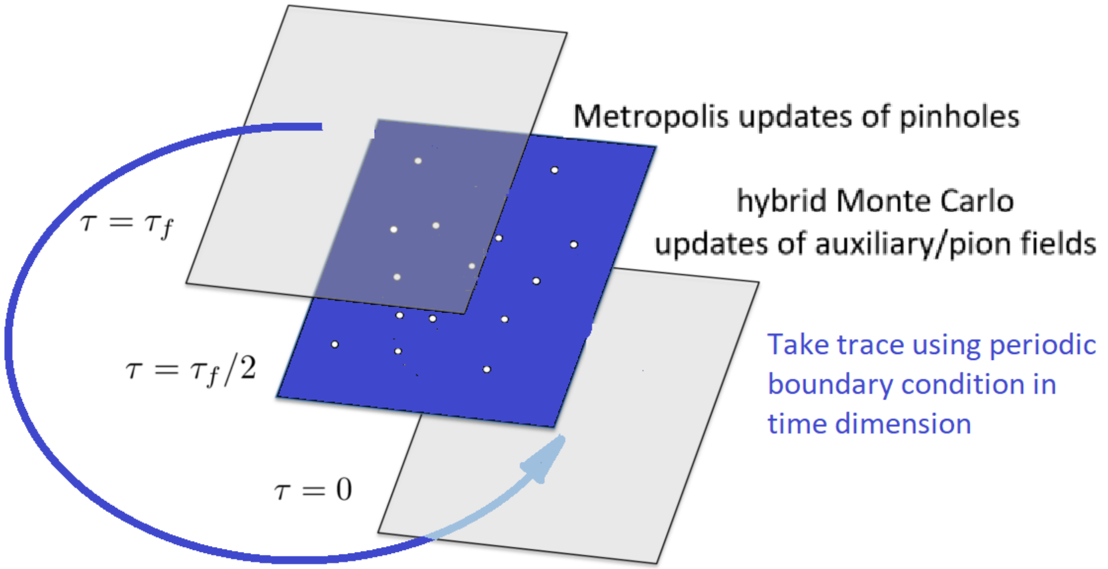
Phase diagram of strongly interacting matter

- Sketch of the phase diagram of strongly interacting matter

Fig. courtesy B.-N. Lu



Pinhole trace algorithm (PTA)



- The pinhole states span the whole A-body Hilbert space
- The canonical partition function can be expressed using pinholes:

$$Z_A = \mathrm{Tr}_A\left[\exp(-\beta H)\right]\,,\ \beta = 1/T$$
$$= \sum_{n_1,\cdots,n_A} \int \mathcal{D}s\mathcal{D}\pi \langle n_1,\cdots,n_A|\exp[-\beta H(s,\pi)]|n_1,\cdots,n_A\rangle$$

- allows to study: liquid-gas phase transition → this talk

  thermodynamics of finite nuclei

  thermal dissociation of hot nuclei

  cluster yields of dissociating nuclei

New paradigm for nuclear thermodynamics

- The PTA allows for simulations with fixed neutron & proton numbers at non-zero T

$\hookrightarrow$ thousands to millions times faster than existing codes using the grand-canonical ensemble ($t_{\rm CPU} \sim VN^2$ vs. $t_{\rm CPU} \sim V^3N^2$)

- Only a mild sign problem $\to$ pinholes are dynamically driven to form pairs

- Typical simulation parameters:

up to $N = 144$ nucleons in volumes $L^3 = 4^3, 5^3, 6^3$

$\hookrightarrow$ densities from 0.008 fm^{-3} ... 0.20 fm^{-3}

$a = 1.32$ fm $\to \Lambda = \pi/a = 470$ MeV , $a_t \simeq 0.1$ fm

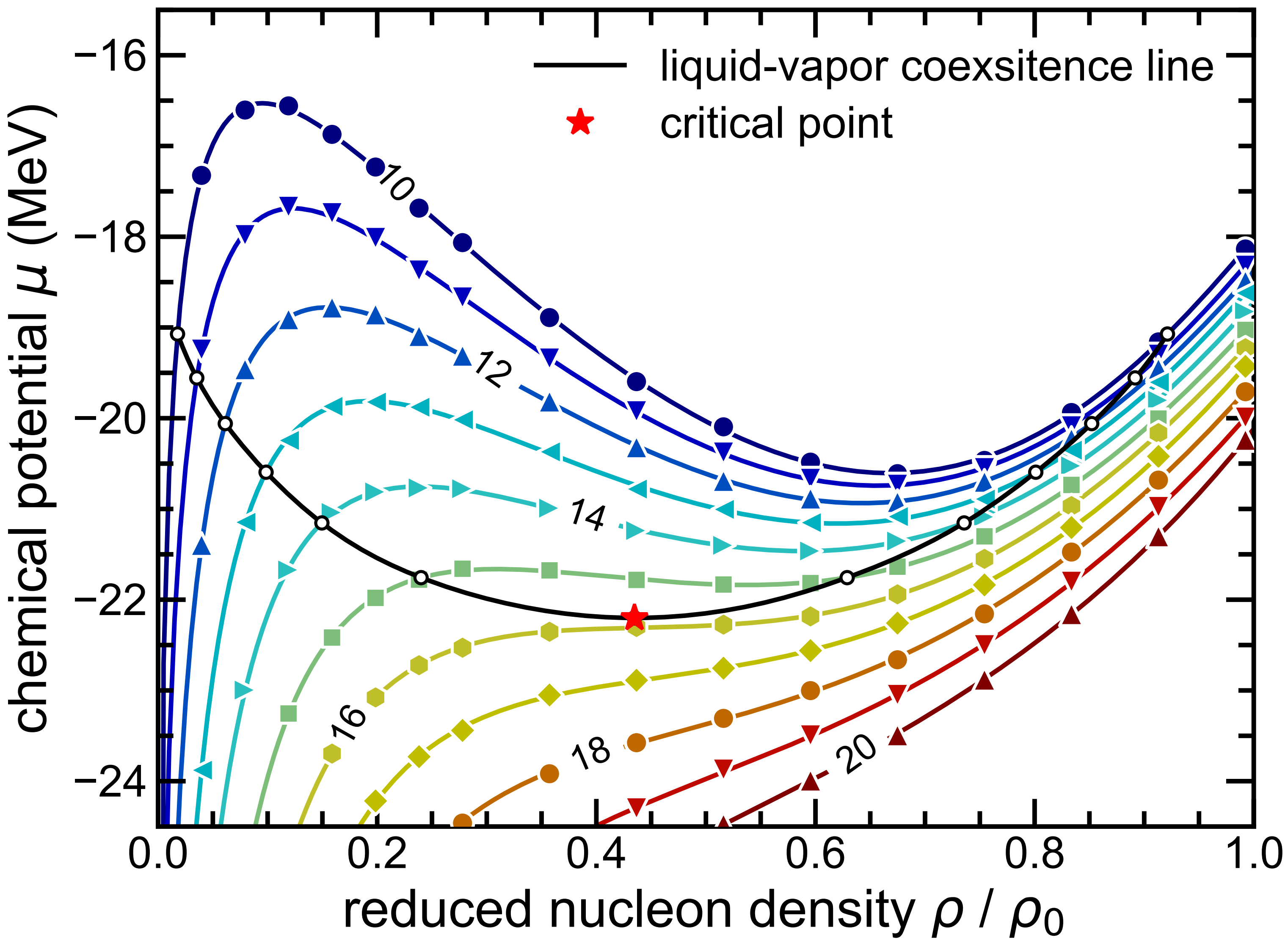
consider $T = 10 \ldots 20$ MeV

- use twisted bc's, average over twist angles $\to$ acceleration to the td limit

- very favorable scaling for generating config's: $\Delta t \sim N^2 L^3$

Chemical potential

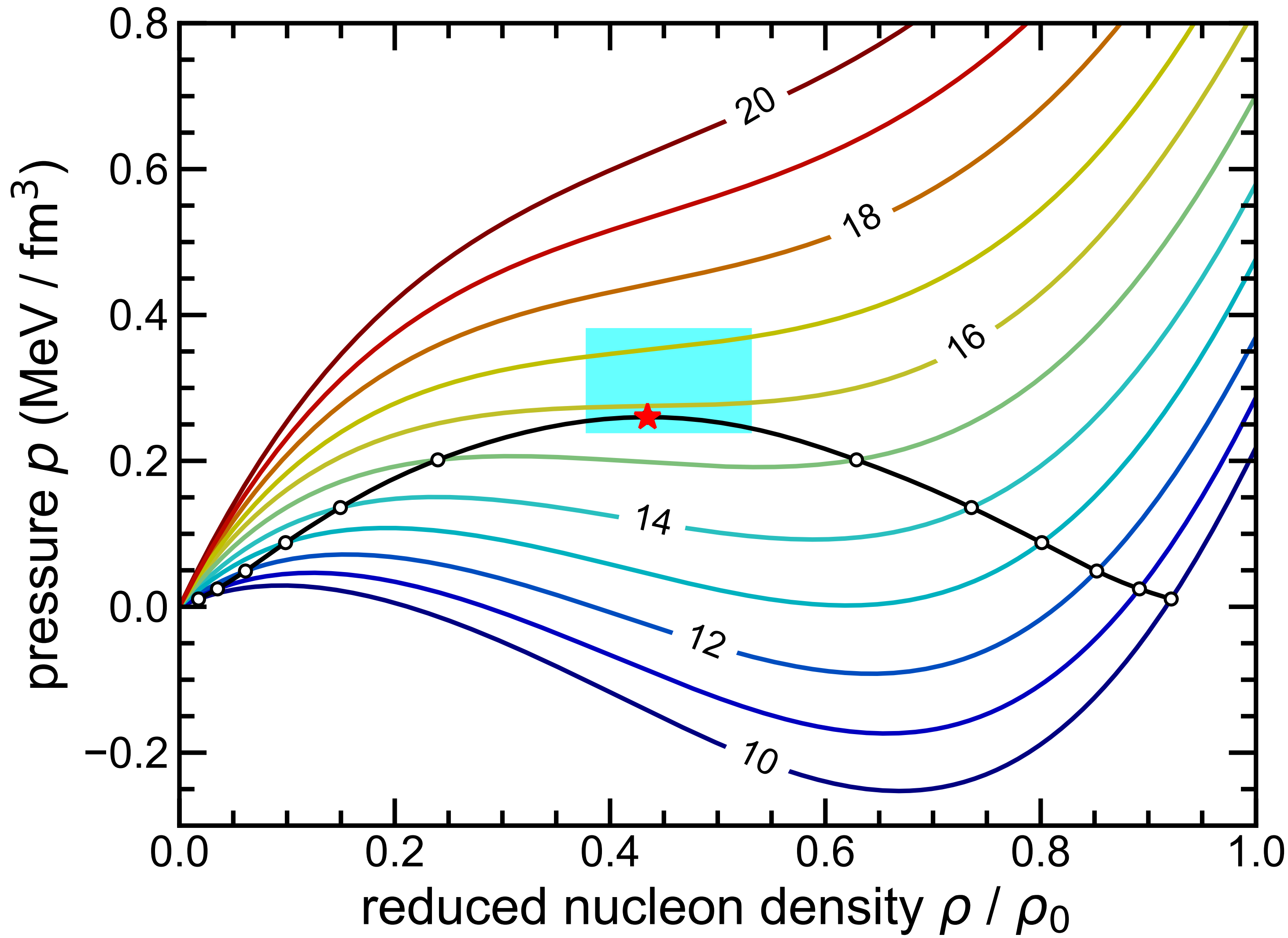
- Calculated from the free energy: $\mu = (F(N+1) - F(N-1))/2$



at very low densities like the ideal gas $\mu \propto \log(\rho)$

Equation of state

- Calculated by integrating: $dP = \rho\, d\mu$
- Crtitical point: $T_c = 15.8(1.6)$ MeV, $P_c = 0.26(3)$ MeV/fm^3, $\rho_c = 0.089(18)$ fm^{-3}



Experiment: $T_c = 15.0(3)$ MeV, $P_c = 0.31(7)$ MeV/fm^3, $\rho_c = 0.06(2)$ fm^{-3}

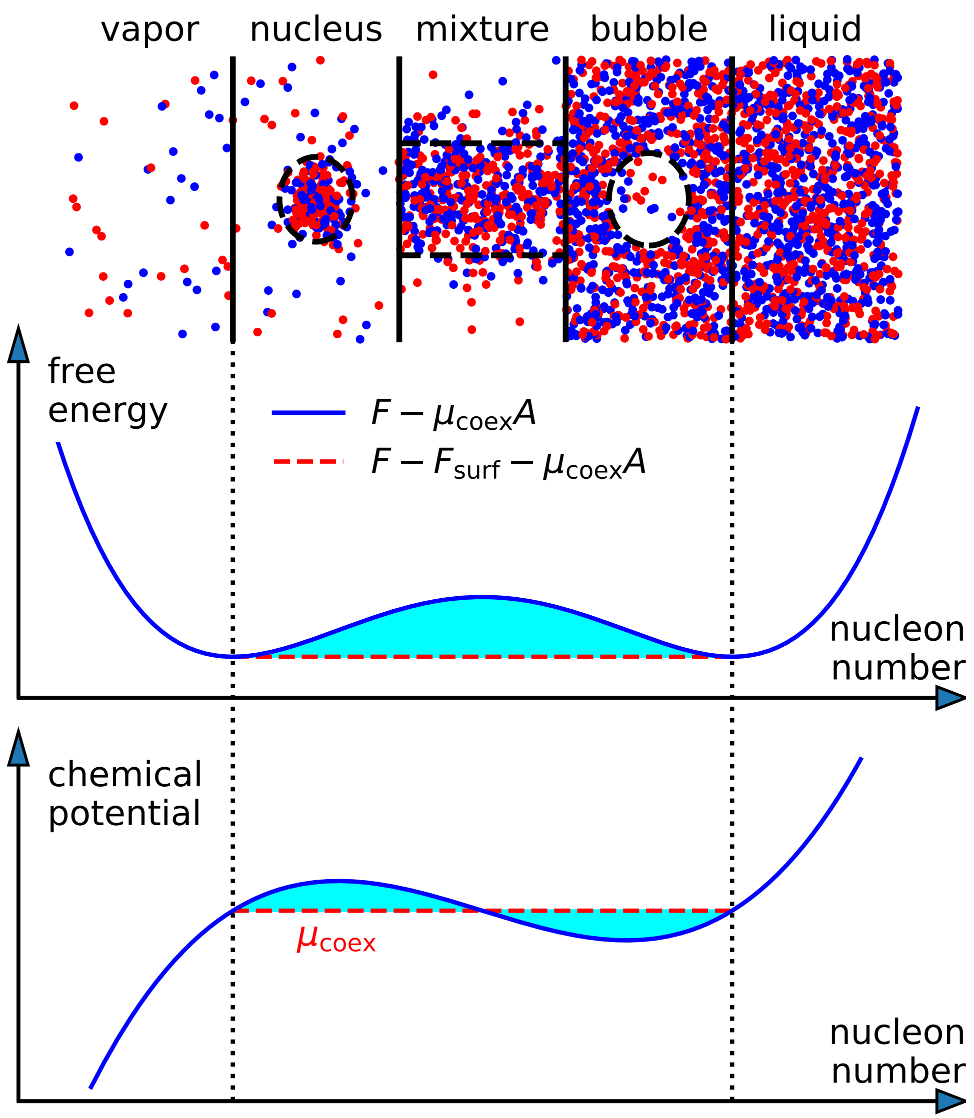
Vapor-liquid phase transition

- Vapor-liquid phase transition
  in a finite volume V & $T < T_c$

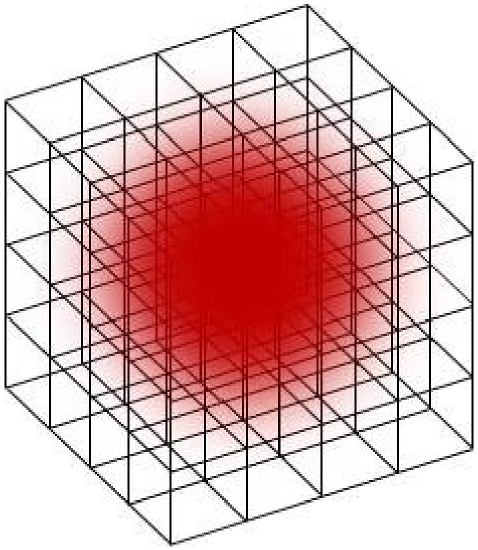
- the most probable configuration
  for different nucleon number A

- the free energy

- chemical potential $\mu = \partial F/\partial A$



CENTER-of-MASS PROBLEM



- AFQMC calculations involve states that are superpositions of many different center-of-mass (com) positions

$$Z_A(\tau) = \langle \Psi_A(\tau) | \Psi_A(\tau) \rangle$$

$$|\Psi_A(\tau)\rangle = \exp(-H\tau/2)|\Psi_A\rangle$$

- but: translational invariance requires summation over all transitions

$$Z_A(\tau) = \sum_{i_{\rm com}, j_{\rm com}} \langle \Psi_A(\tau, i_{\rm com}) | \Psi_A(\tau, j_{\rm com}) \rangle \,, \;\; {\rm com} = {\rm mod}((i_{\rm com} - j_{\rm com}), L)$$

$i_{\rm com}$ $(j_{\rm com})$ = position of the center-of-mass in the final (initial) state

→ density distributions of nucleons can not be computed directly, only moments

→ need to overcome this deficieny

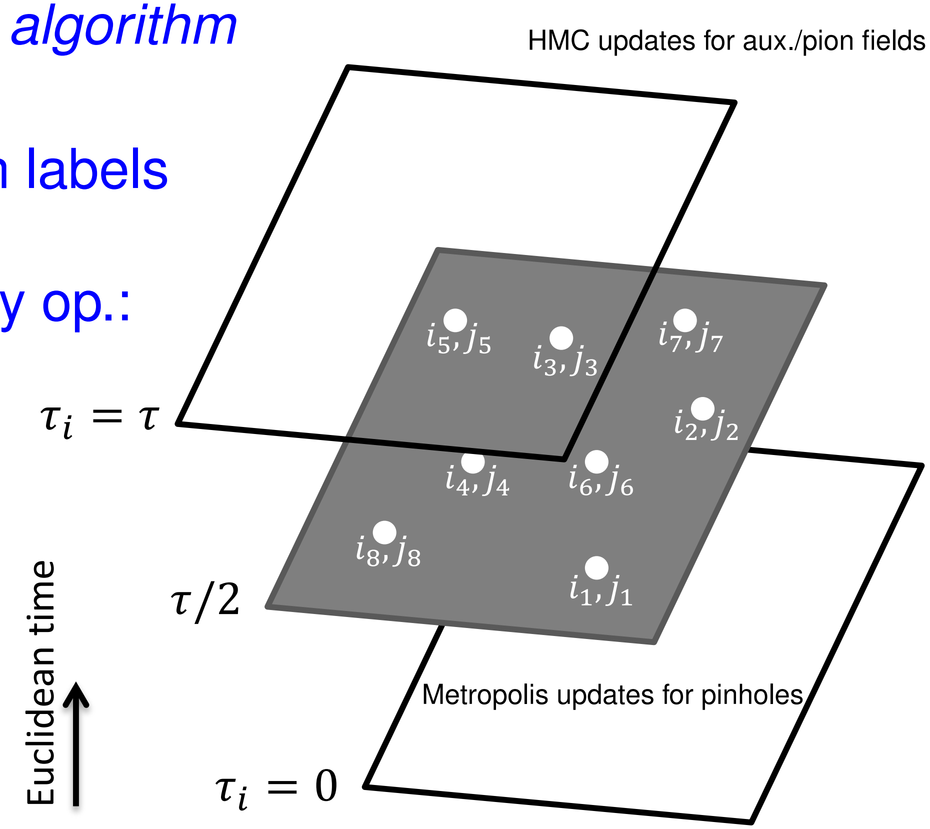
PINHOLE ALGORITHM

- Solution to the CM-problem:

  track the individual nucleons using the *pinhole algorithm*

- Insert a screen with pinholes with spin & isospin labels that allows nucleons with corresponding spin & isospin to pass = insertion of the A-body density op.:

$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A) \\ =:\rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$



- MC sampling of the amplitude:

$$A_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A,L_t) \\ = \langle\Psi_A(\tau/2)|\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A)|\Psi_A(\tau/2)\rangle$$

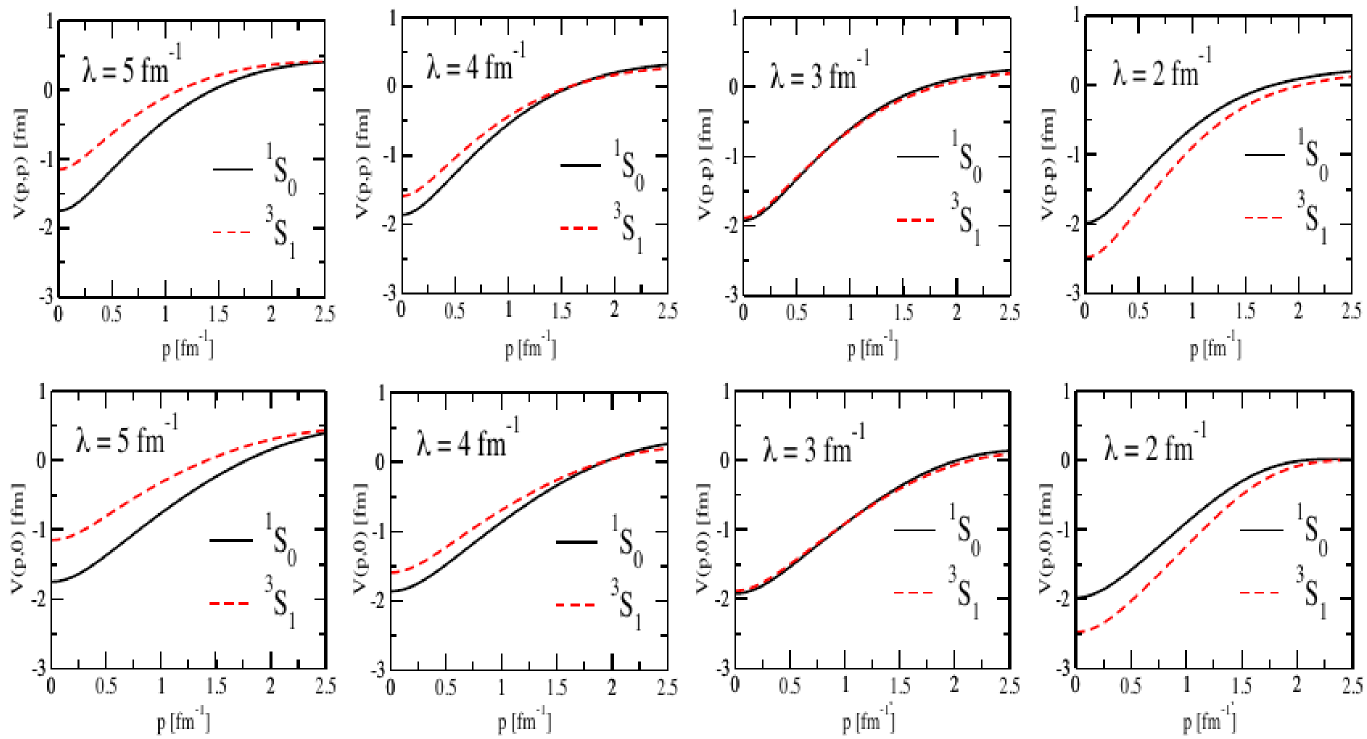
- Allows to measure proton and neutron distributions

- Resolution scale $\sim a/A$ as cm position $\mathbf{r}_{\rm cm}$ is an integer $\mathbf{n}_{\rm cm}$ times a/A

Similarity renormalization group studies

Timoteo, Szpigel, Ruiz Arriola, Phys. Rev. C **86** (2012) 034002

- Investigation of Wigner SU(4) symmetry using the SRG, use AV18:



- At the scale $\lambda_{\rm Wigner} \simeq 3\,{\rm fm}^{-1}$ one has $V_{^1S_0,{\rm Wigner}}(p',p) \approx V_{^3S_1,{\rm Wigner}}(p',p)$