Hypernuclei from the Lattice

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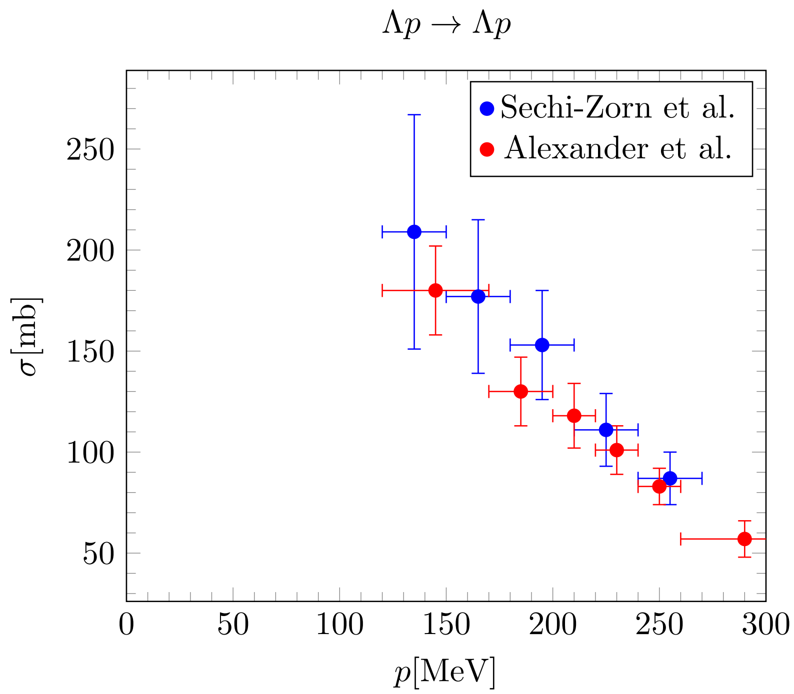
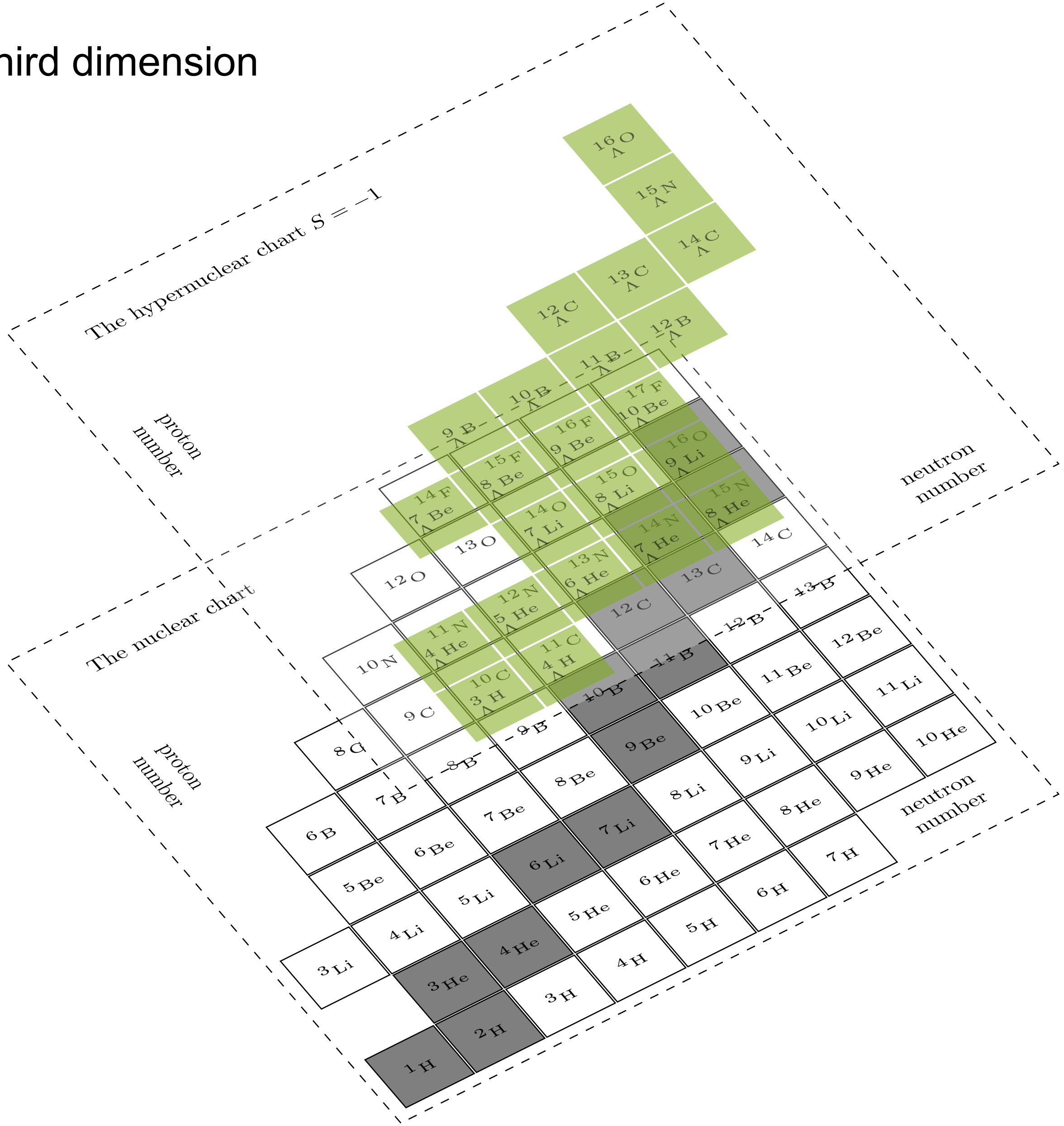
Outline

- Motivation
- From NLEFT to (Hyper) NLEFT
    - Lattice Interaction
    - Two-body results
    - Inclusion of three-body forces
- Summary and Outlook

Hypernuclear physics in a nutshell

- Strangeness extents the nuclear chart to a third dimension
- Unique opportunity to study the strong force without the Pauli principle
- Typical approach from nuclear physics does not work since two-body data is sparse





- Gateway : Three-Body Systems

Starting point for (Hyper) Nuclear Lattice EFT

very successful nuclear program:

using AFMC and shuttle algorithm

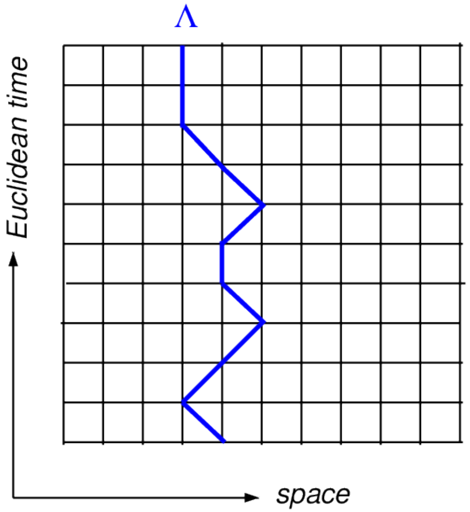
wave function matching to obtain precise results for nuclei and charge radii

AFMC does not converge as good as in a pure nuclear matter simulation

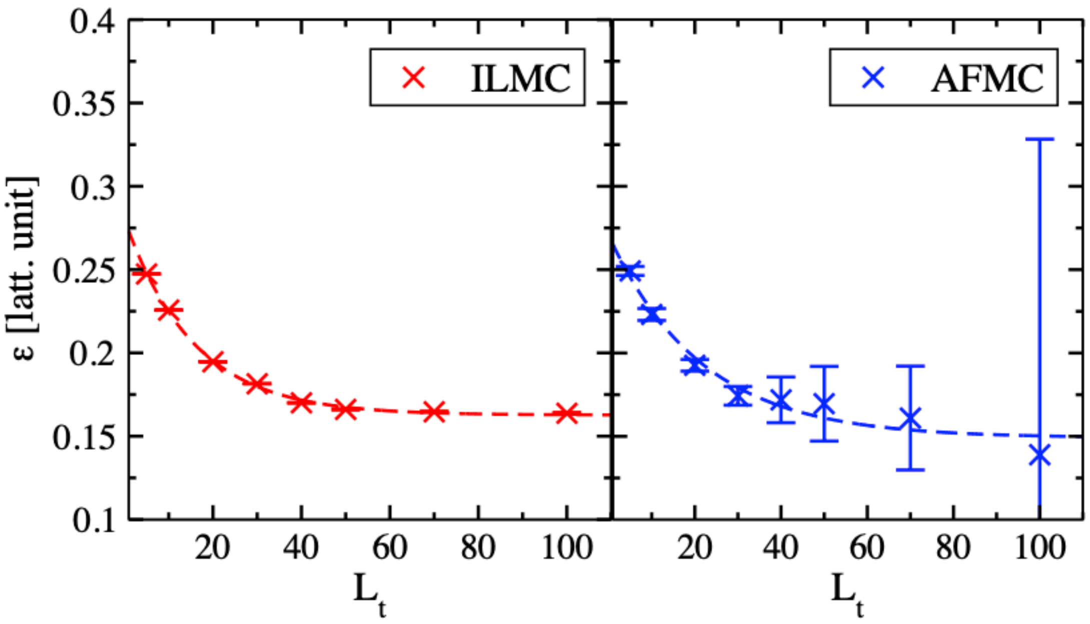
Need to develop a method that threats this impurities more efficient

Treat impurity as worldline:

(S.Bour, D.Lee, H.-W. Hammer, U.-G. Meißner)



(D. Frame, T. A. Lähde, D. Lee, U.-G. Meißner)



Starting point for (Hyper) Nuclear Lattice EFT

- Challenge with IFMC, need to collect millions of worldlines

→ Can we still do hypernuclear calculations with AFMC ?

→ Important for possible applications with many hyperons

- taylor interaction to work non-perturbative with our best NN interaction

→ Evolve together with NN counterparts

Constraints smearing parameters to the NN ones

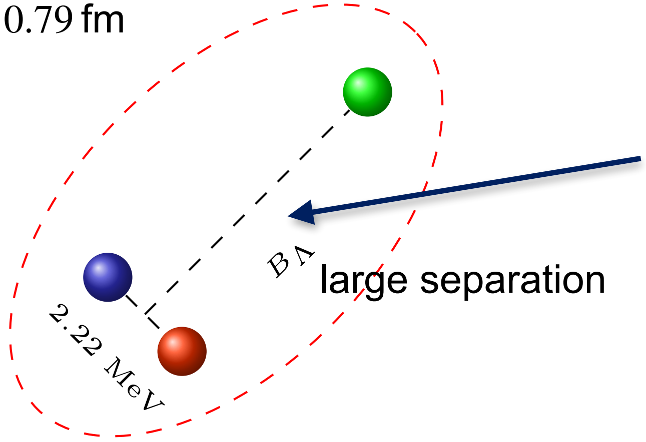
	Phase	
$A = 3$	0.97	
$A = 4$	0.89	
$A = 5$	1	← $\alpha -$ core
$A = 7$	0.92	
$A = 13$	0.97	

$L = 12 \quad Lt = 500$

this is very promising,
for larger hypernuclei

Construction of a first Lattice ΛN interaction

$\sqrt{\langle r^2_{\Lambda d}\rangle} \approx 10.79$ fm



Emulsion:

$B_\Lambda = 0.130 \pm 0.050$ MeV Juric 1973

Heavy Ion:

$B_\Lambda = 0.406 \pm 0.120$ MeV Star 2020

$B_\Lambda = 0.102 \pm 0.063$ MeV Alice 2023

World Average:

$B_\Lambda = 0.164 \pm 0.043$ MeV Mainz 2025

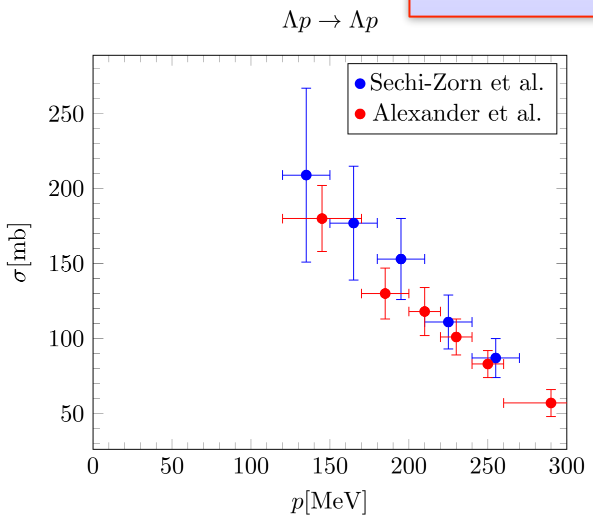
Shallow S-Wave State

$$J^P = \frac{1}{2}^+$$

Distinguishable

$$I = 0 \Rightarrow \frac{1}{\sqrt{2}}(pn - np)\Lambda$$

Combine 2-Body data with hypertriton in exact calculation



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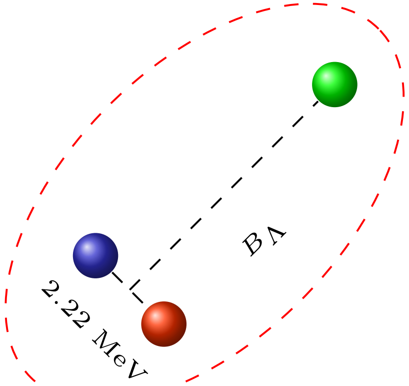
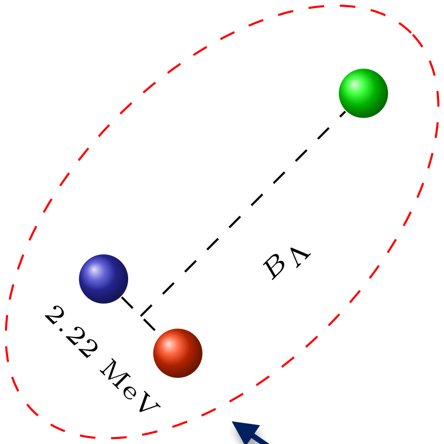


Construction of a first Lattice ΛN interaction



large separation → Large box sizes needed

GPU Lanczos Code to fix 2-body forces

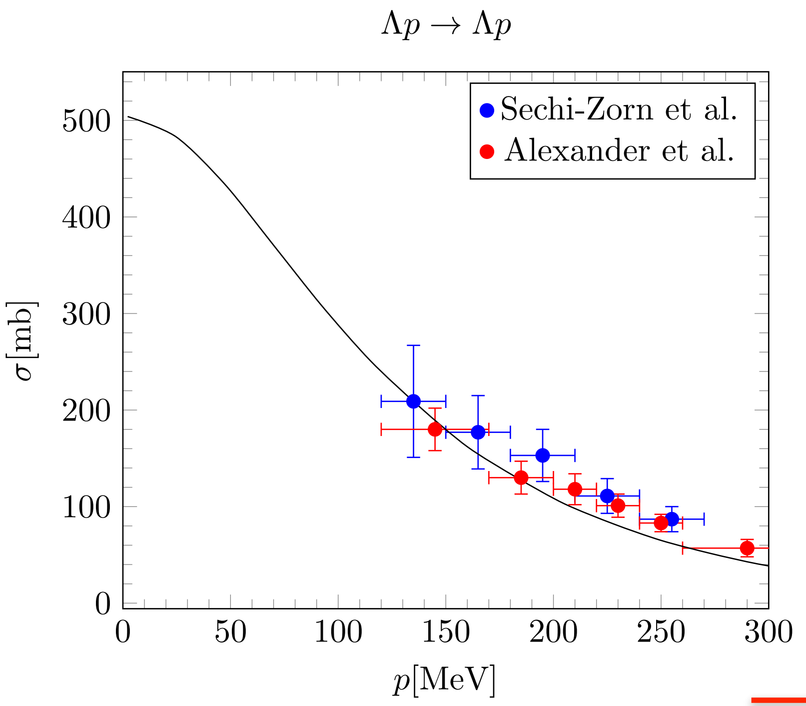
→ Allows L = 23.76 fm

Tightly bound compared to Λ separation

Use simpler nuclear interaction

Non-pertubative LO interaction → $^3S_1$ / $^1S_0$ interaction

Construction of a first Lattice ΛN interaction


$\Lambda p \to \Lambda p$

Best SMS N^2LO interaction

(Haidenbauer et al.)

$a_s = -2.80$ fm $r_s = 2.89$ fm

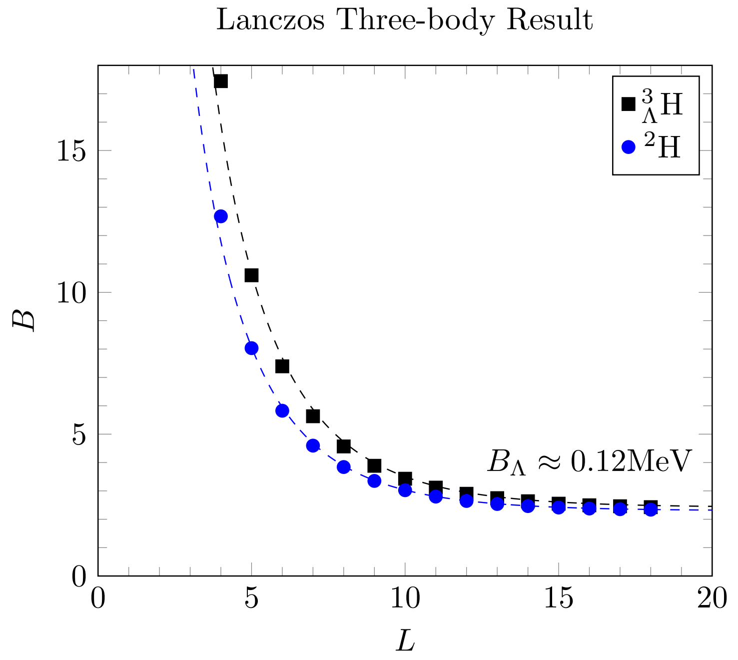
$a_t = -1.58$ fm $r_t = 3.09$ fm

This interaction

$a_s = -2.89$ fm $r_s = 3.28$ fm

$a_t = -1.60$ fm $r_t = 3.94$ fm

Phase shift similar to $p \sim 60$ MeV


Lanczos Three-body Result

$$E(L) = E_{L\to\infty} + \frac{A}{L} e^{-\frac{L}{L_0}}$$

$\approx$ Emulsion

$$B^{\Lambda}_{L\to\infty} = (90+30)\ \text{keV} \approx 120\ \text{keV}$$

2-Body

GIR corrections

This is different from the previous talk

Results: Two Body interaction (L=12 l.u.) (light nuclei)

During Evolution:

Spin-averaged Interaction:

$$C = \frac{3\,{}^3S_1 + {}^1S_0}{4}$$

Perturbative part:

Spin-dependent Interaction:

$$C_S = \frac{{}^3S_1 - {}^1S_0}{4}$$

Nuclear Interaction:

N^3LO interaction, same as for WFM results

Results: Two Body	**Experiment**
$B_\Lambda({}^3_\Lambda\text{H}) = 0.38 \pm 0.08$ MeV	0.164 ± 0.43 MeV

→ Box effect, consistent with exact L=12 result

$B_\Lambda({}^4_\Lambda\text{H}^{0^+}) = 2.11 \pm 0.18$ MeV	2.169 ± 0.042 MeV
$B_\Lambda({}^4_\Lambda\text{H}^{1^+}) = 1.23 \pm 0.18$ MeV	1.081 ± 0.042 MeV

→ Splitting quite good, missing 0.2 MeV

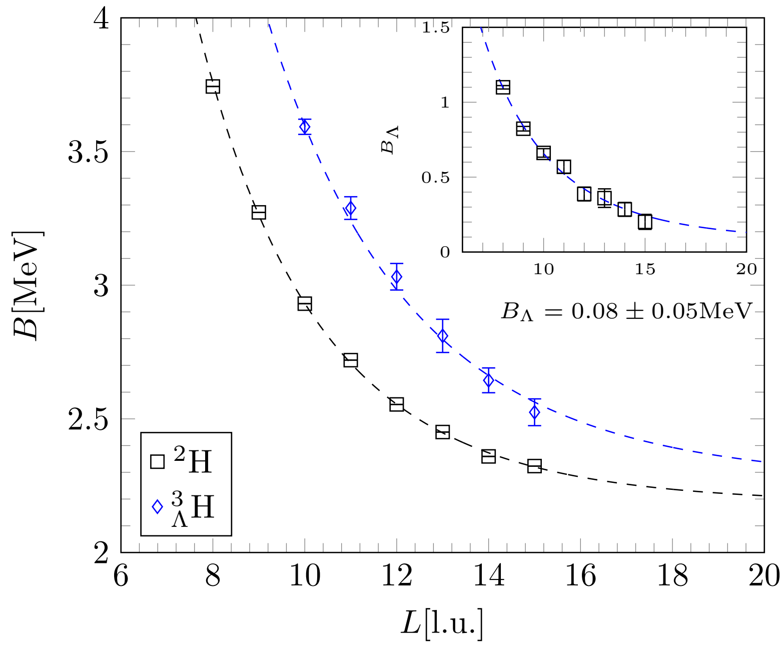
$B_\Lambda({}^5_\Lambda\text{He}) = 3.51 \pm 0.12$ MeV	3.102 ± 0.03 MeV

→ Smaller overbinding compared to other LO calculations

$B_\Lambda({}^7_\Lambda\text{Li}) = 5.68 \pm 0.96$ MeV	5.619 ± 0.06 MeV

→ Typically overbound by ~1 MeV in LO calculations

Box Size effects:



Proper extrapolation:

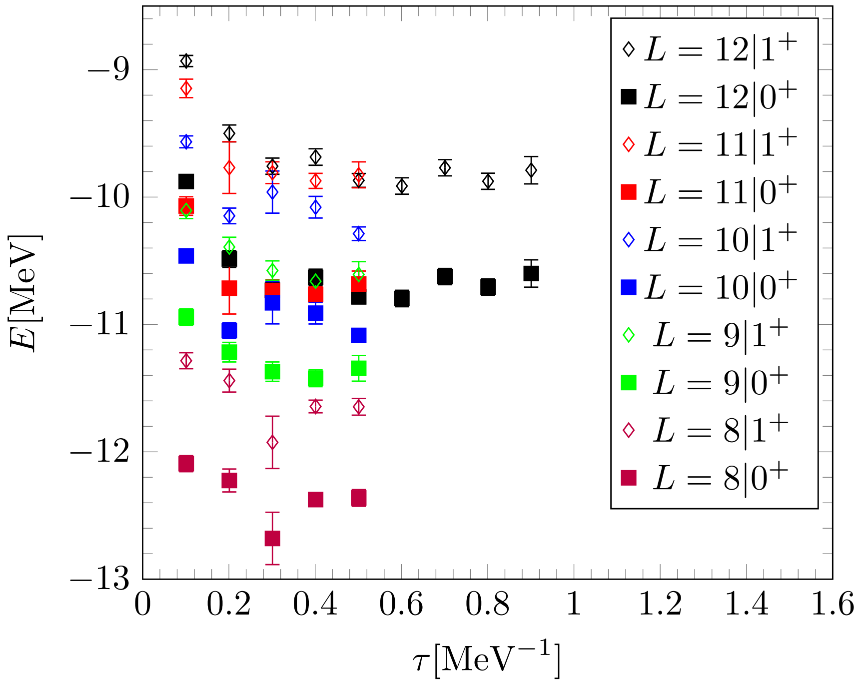
$B_\Lambda(^3_\Lambda\text{H}) = 0.08 \pm 0.05$ MeV

Experiment

0.164 ± 0.43 MeV

→ Result consistent with newest experimental data

- Hypertriton has large Λ separation of ~ 10.6 fm
- Second larges system is the 4-body system



→ L = 12 shows converged results

→ L = 12 Box is big enough for all other hypernuclei

→ Finite Box under control

Results: Two Body interaction, further analysis

Missing ~ 0.2 MeV in A=4 systems

A=5 system only slightly overbound

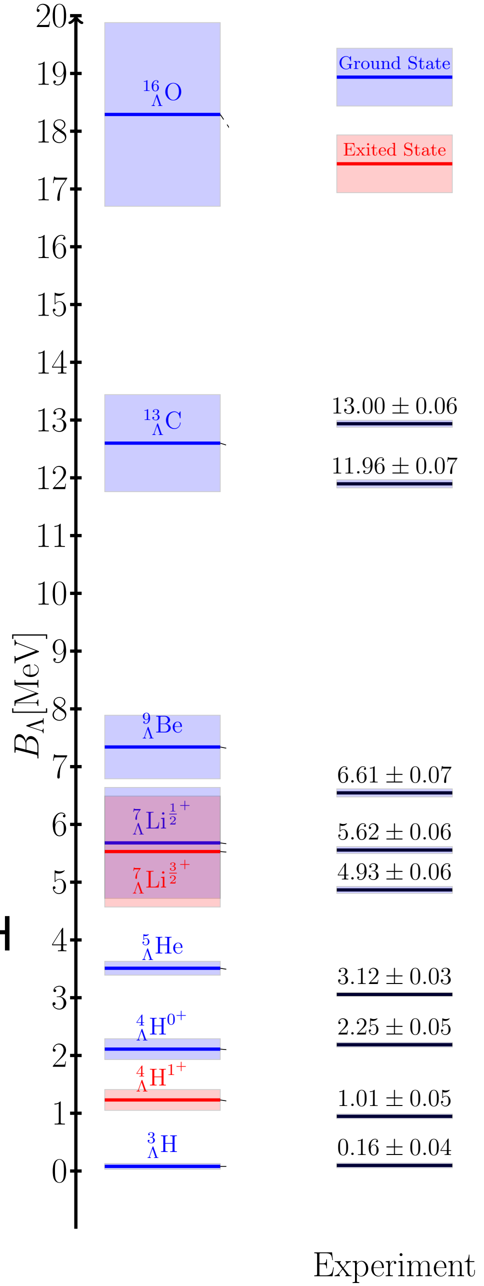
Less Attraction

Modify C

Fix ${}^{5}_{\Lambda}$He

Underbinding of ${}^{4}_{\Lambda}$H

Can three-body forces help us here?



Structure of contact three-body forces

(Petschauer et al.)

$$V_{ct}^{\Lambda NN} = C_1(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(3+\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3) \longleftarrow I = 1$$

$$+C_2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$

$$+C_3(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$

$I = 0$ → Can influence hypertriton

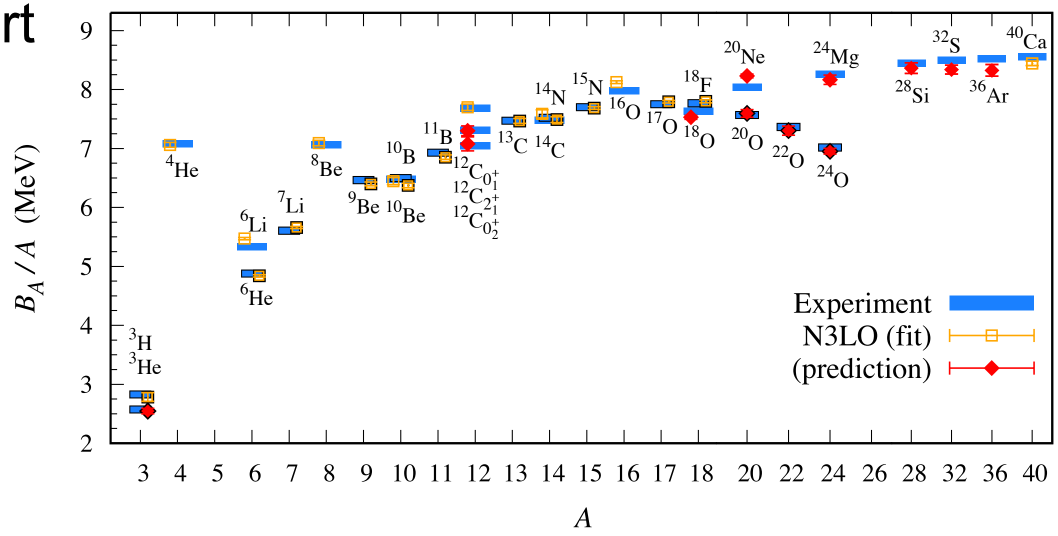
C_2 only interaction that depends on Λ spin → ${}^4\mathrm{H}_\Lambda^{0^+} - {}^4\mathrm{H}_\Lambda^{1^+}$ Splitting

C_1, C_3 are iso/spin interchanged to each other → Might not split for small Hypernuclei

→ Effectively N3LO χEFT(NN) + LO $\not\pi$EFT(YN)

Results: Fitting 3-Body forces

Nuclear 3-Body Forces are fitted as part of the WFM interaction



(Elhatisari et al.)

- Use similar classes of non-local as well as local smeared YNN Forces

- Leads to a total of 343 (7 each) combination of YNN forces

- Use Root Mean Square Deviation to select TBF :

$$\text{RMSD}(S) = \sqrt{\frac{1}{M_S}\sum_{i\in S}\left(\frac{{}^iB^c_\Lambda - {}^iB^{exp}_\Lambda}{{}^iB^{exp}_\Lambda}\right)^2}$$

- Without TBF RMSD(S) = $18.4\,\%$

First approach Decouplet Saturation

$$
\begin{aligned}
V_{ct}^{\Lambda NN} &= C_1(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(3+\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3) && \longrightarrow \quad C_1\\
&+C_2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3) && \longrightarrow \quad C_2 = 0\\
&+C_3(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3) && \longrightarrow \quad C_1
\end{aligned}
$$

Enforce same smearing

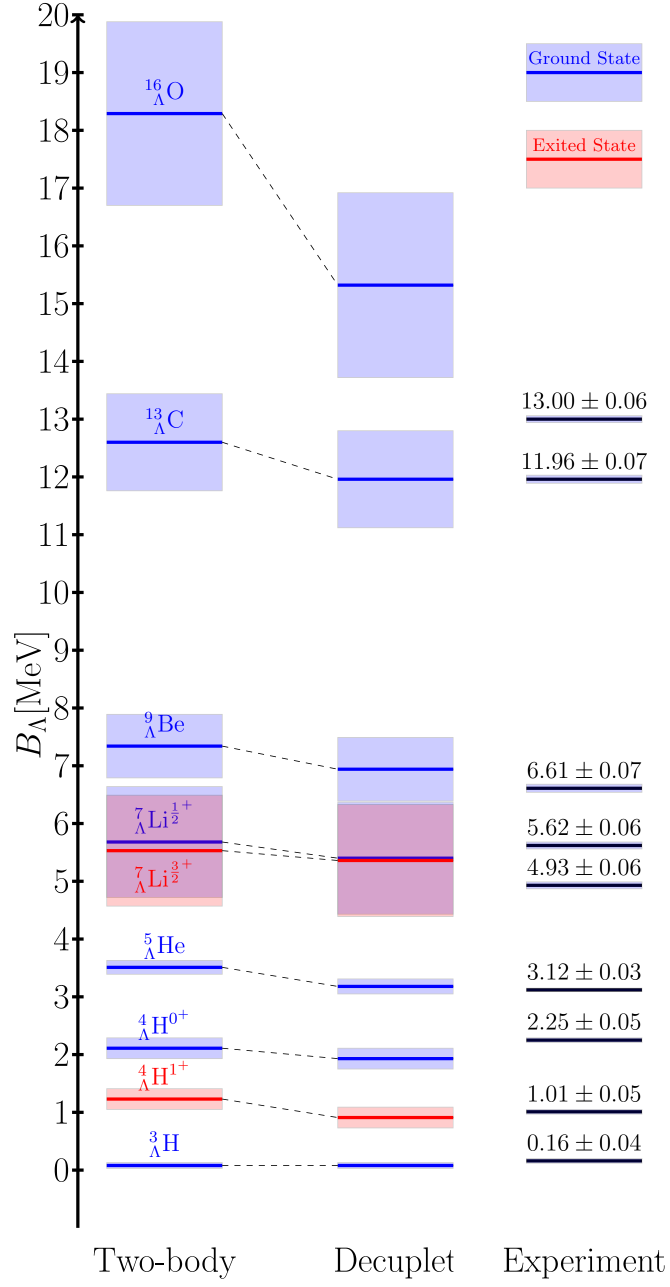
Also include excited state of ${}^{7}_{\Lambda}$Li and ${}^{9}_{\Lambda}$Be, ${}^{13}_{\Lambda}$C, ${}^{16}_{\Lambda}$O, so far all overbound

	Fit to all $A \geq 4$	Fit to only A=4/5
RMSD(S)	$9.2\% - 14.6\,\%$	$9.3\% - 14.7\,\%$

Since $C_2 = 0$:
splitting remains untouched

Improvement due to an overall downwards shift

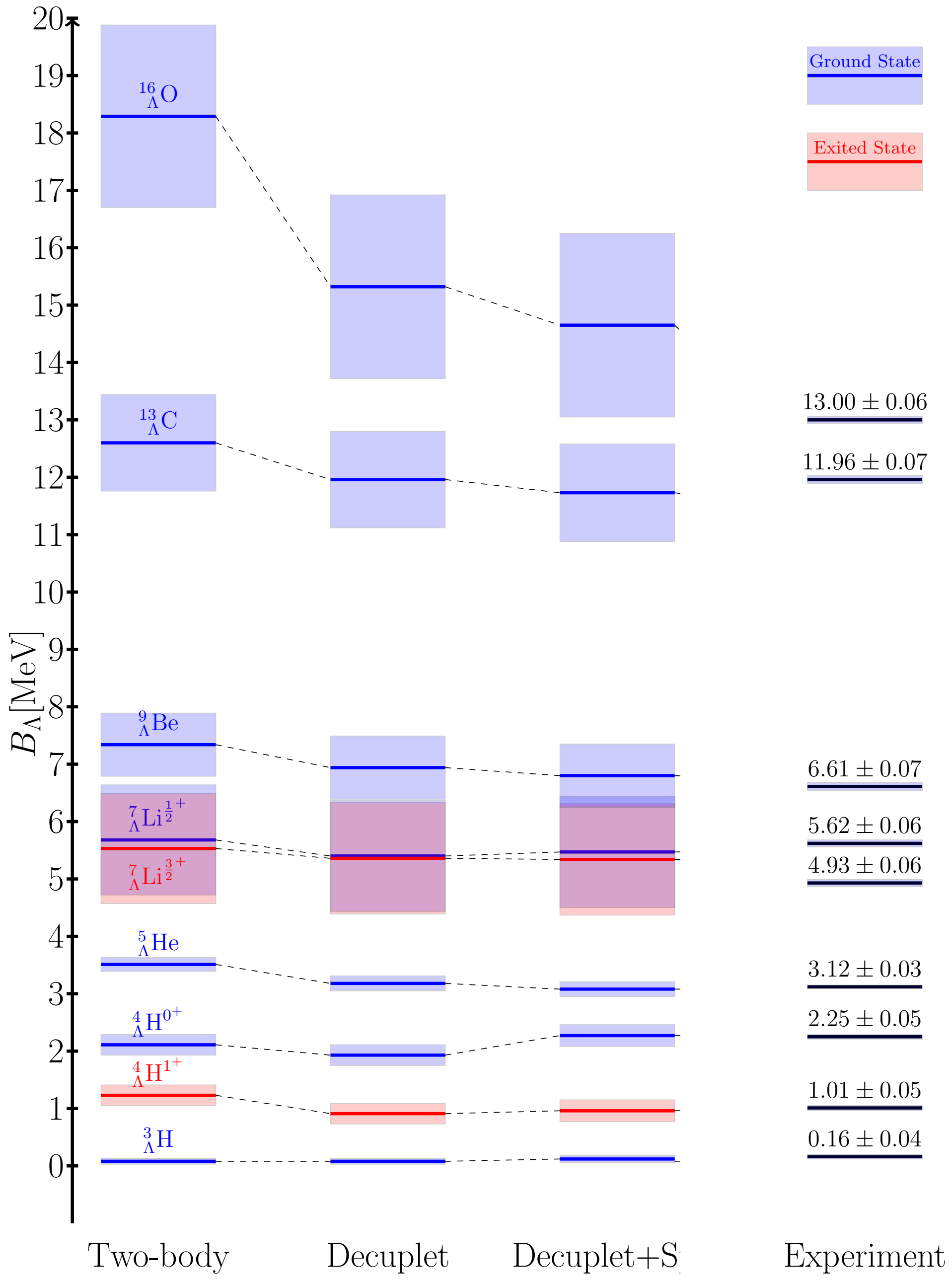
How does this translate to Binding energies?



- Overall downwards shift in energies
- Underbinding of the 4-body systems
- Splitting in 4/7-body system is still the same
- Naiv improvment:

$$
\begin{aligned}
V_{ct}^{\Lambda NN} &= C_1(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(3+\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3) && \longrightarrow \quad C_1\\
&+C_2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3) && \longrightarrow \quad C_2\neq 0\\
&+C_3(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3) && \longrightarrow \quad C_1
\end{aligned}
$$

Decouplet + Spin dependent force?



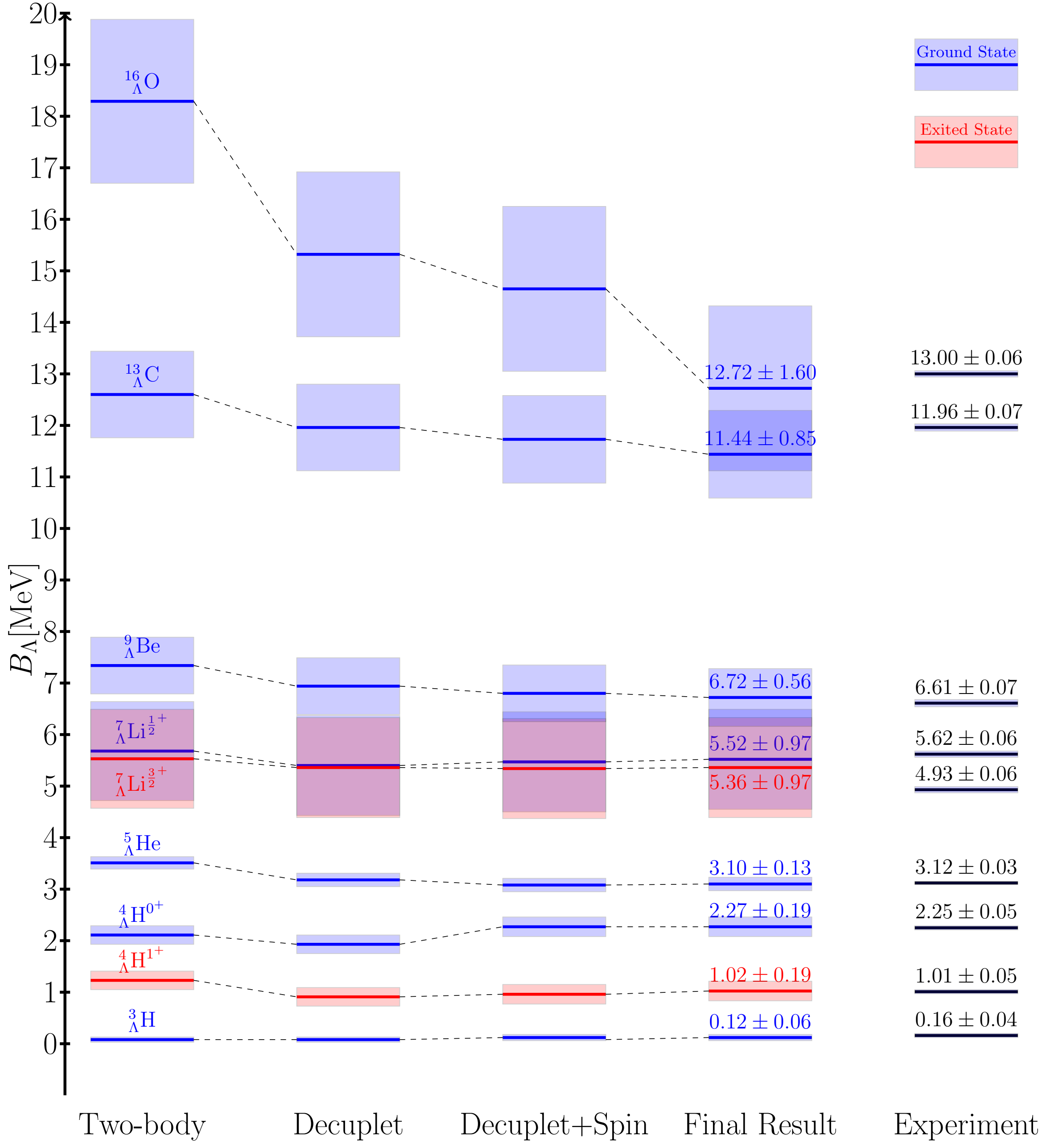
- 49 different combinations of forces
- If fitted to the 4/5 body system 21 improve the overall description

	Fit to all $A \geq 4$	Fit to only A=4/5
Best:	$5.3\,\%$	$5.7\,\%$

Weakly smeared forces outperform Stronger smeared forces

Original selection of smearing parameter is sufficient

Final result



- All 343 different combinations

Fit to all $A \geq 4$	Fit to only A=4/5
$3.6\,\%$	$3.7\,\%$

- All hypernuclei are consistent
- Good splitting in the 4 body system
- Best results have only local smearing, V_2 is always unsmeared

Possible Paths to improvement

- Go to higher orders in the two-body interaction

→ Typical LO problems go away in other methods

- Include two-pion exchange/pion exchange 3B forces

→ Long-Range behaviour of the interaction

- fit two-body forces with better nuclear interaction

→ Removes any dependence of the NN Force on the YN Force

- Improve statistics in the NN part of the hypernuclei

→ Main uncertainty from sampling of the NN part of the nucleus

Summary and Outlook

Good Results for light hypernuclei nuclei A=3-16
with $N^3LO(NN)$ and $LO(YN)$ interaction

Method scales with A, straightforward application to the whole hypernuclear chart

Many possible path ways to improve the results

Calculate the hypernuclear chart

Many excited states in A=7/9 hypernuclei