

Sigma term physics

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

by CAS, PIFI



by DFG, SFB 1639



by ERC, EXOTIC



by NRW-FAIR



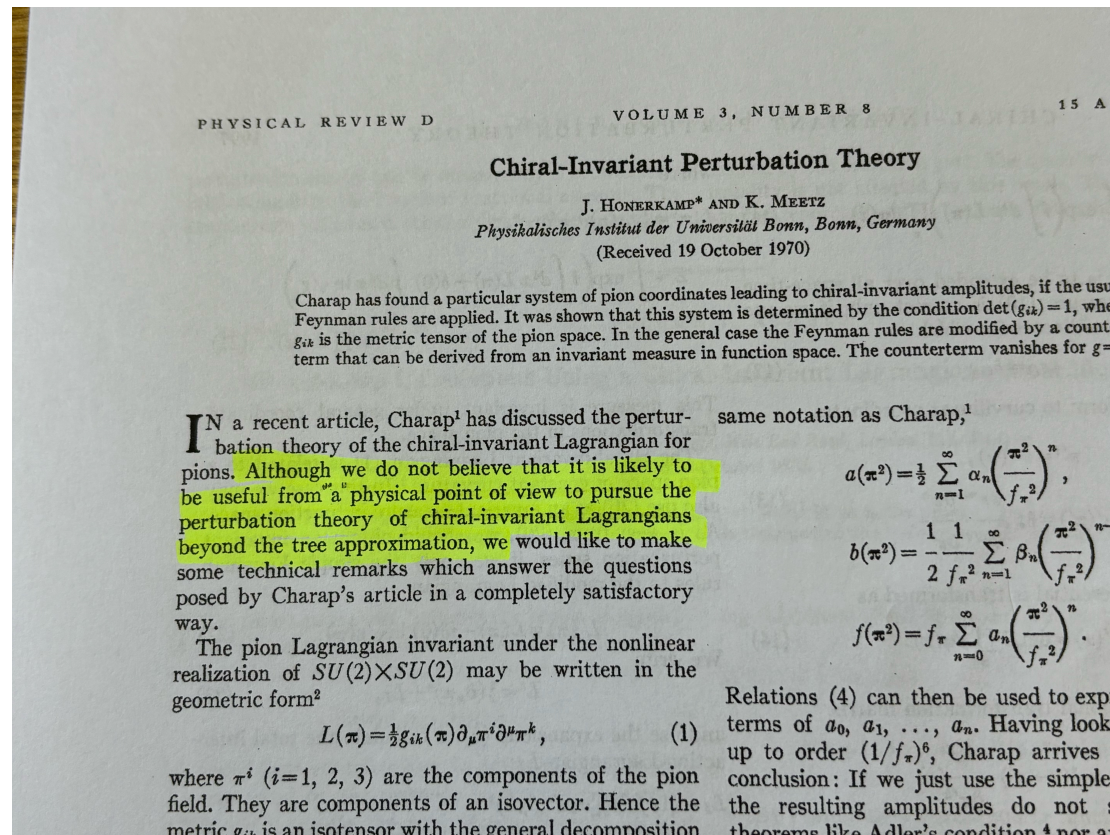
Contents

- Introductory remarks / anecdotes
- Basics of sigma term physics
- Sigma term physics from Bern
- A new look at sigma term physics
- Summary

Introductory remarks

A small anecdote

- The first paper with “chiral perturbation theory” in the title



- Possible consequences:

↪ stop the talk here

↪ don't always listen to Bonn people

Some statistics on Gasser/Leutwyler papers

- What a list of most cited papers! [NB: just the common ones!]

<p>Chiral Perturbation Theory to One Loop</p> <p>J. Gasser (Bern U.), H. Leutwyler (CERN) (Aug, 1983)</p> <p>Published in: <i>Annals Phys.</i> 158 (1984) 142</p> <p>pdf DOI cite claim</p>	#1
<p>Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark</p> <p>J. Gasser (Bern U.), H. Leutwyler (CERN) (Jan, 1984)</p> <p>Published in: <i>Nucl.Phys.B</i> 250 (1985) 465-516</p> <p>DOI cite claim</p>	#2
<p>Quark Masses</p> <p>J. Gasser (Bern U.), H. Leutwyler (Bern U.) (Mar, 1982)</p> <p>Published in: <i>Phys.Rept.</i> 87 (1982) 77-169</p> <p>DOI cite claim</p>	#3
<p>$\pi\pi$ scattering</p> <p>G. Colangelo (Zurich U.), J. Gasser (Bern U.), H. Leutwyler (Bern U.) (Mar, 2001)</p> <p>Published in: <i>Nucl.Phys.B</i> 603 (2001) 125-179 • e-Print: hep-ph/0103088 [hep-ph]</p> <p>pdf DOI cite claim</p>	#4
<p>Chiral Lagrangians for Massive Spin 1 Fields</p> <p>G. Ecker (Vienna U.), J. Gasser (Bern U.), H. Leutwyler (Bern U.), A. Pich (Valencia U.), E. de Rafael (Marseille, CPT and Barcelona U.) (Apr 24, 1989)</p> <p>Published in: <i>Phys.Lett.B</i> 223 (1989) 425-432</p> <p>DOI cite claim</p>	#5
<p>Sigma term update</p> <p>J. Gasser (Bern U.), H. Leutwyler (Bern U.), M.E. Sainio (Helsinki U.) (Sep, 1990)</p> <p>Published in: <i>Phys.Lett.B</i> 253 (1991) 252-259</p> <p>DOI cite claim</p>	#6

“You should always put the
Gasser/Leutwyler papers
under your pillow”

Howard Georgi, Dobogöko, 1991

↪ Much too much good stuff for one talk!

↪ I will focus on #6 and related papers here

Sigma term basics

σ -term basics

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- Scalar form factor of the nucleon (isospin limit $\hat{m} = (m_u + m_d)/2$):

$$\sigma_{\pi N}(t) = \langle N(p') | \hat{m}(\bar{u}u + \bar{d}d) | N(p) \rangle, \quad t = (p' - p)^2$$

- Cheng-Dashen Low-Energy Theorem (LET):

Cheng, Dashen (1971)

$$\bar{D}^+(\nu = 0, t = 2M_\pi^2) = \sigma(2M_\pi^2) + \Delta_R$$

$$\left[\nu = \frac{s-u}{4m_N} \right]$$

- \bar{D}^+ – isospin-even, Born-term subtracted pion-nucleon scattering amplitude

$$\bar{D}^+(0, 2M_\pi^2) = A^+(m_N^2, 2M_\pi^2) - \frac{g_{\pi N}^2}{m_N}$$

↪ best determined from πN data using dispersion relations (unphysical region)

- reminder Δ_R , calculated in CHPT to $\mathcal{O}(p^4)$, no chiral logs

$$\Delta_R \lesssim 2 \text{ MeV}$$

Bernard, Kaiser, UGM (1996)

σ -term basics continued

- Standard decomposition of the σ -term: $\sigma_{\pi N} = \sigma_{\pi N}(0)$

$$\sigma_{\pi N} = \Sigma_d + \Delta_D - \Delta_\sigma - \Delta_R$$

$$\Sigma_d = F_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+) \rightarrow \text{full dispersive analysis}$$

$$\left. \begin{aligned} \Delta_D &= \bar{D}^+(0, 2M_\pi^2) - \Sigma_d \\ \Delta_\sigma &= \sigma(2M_\pi^2) - \sigma_{\pi N} \end{aligned} \right\} \rightarrow \text{dispersive t-channel analysis}$$

- d_{00}^+, d_{01}^+ – subthreshold expansion coefficients (around $\nu = t = 0$)
- Δ_σ is the **scalar form factor of the nucleon**
- Note: strong $\pi\pi$ rescattering in Δ_D and Δ_σ

Role of the pion-nucleon σ -term

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- Nucleon mass from the light u, d quarks

- Scalar couplings of the nucleon:

$$\langle N | m_q \bar{q}q | N \rangle = f_q^N m_N \quad (N = p, n) \\ (q = u, d, s)$$

↪ Dark Matter detection

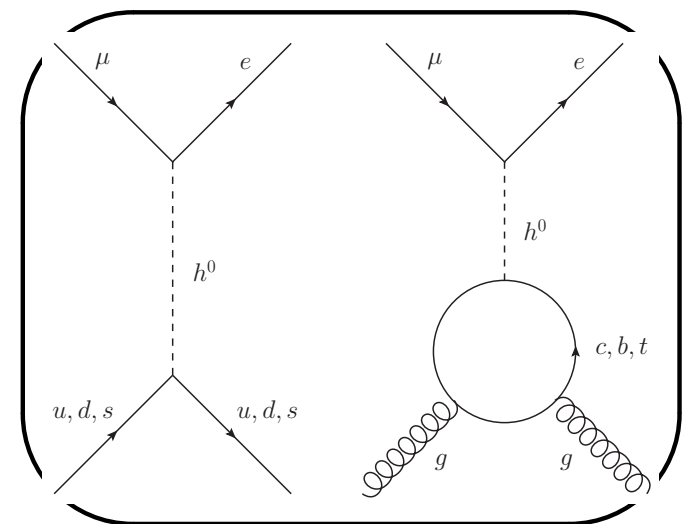
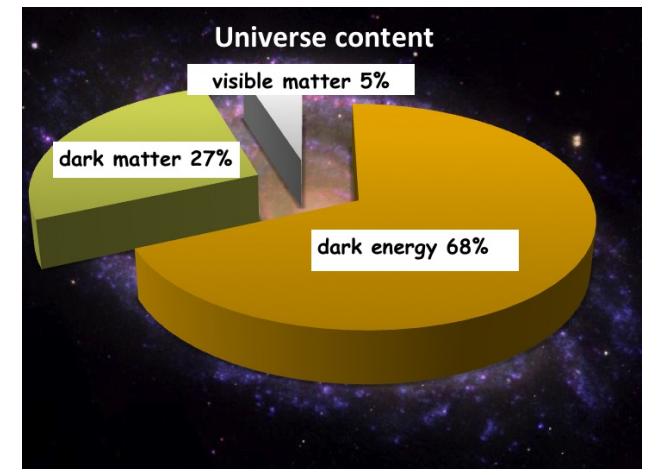
↪ $\mu \rightarrow e$ conversion in nuclei

- Condensates in nuclear matter

$$\frac{\langle \bar{q}q \rangle(\rho)}{\langle 0 | \bar{q}q | 0 \rangle} = 1 - \frac{\rho \sigma_{\pi N}}{F_\pi^2 M_\pi^2} + \dots$$

- CP-violating πN couplings

↪ hadronic EDMs (nucleon, nuclei)



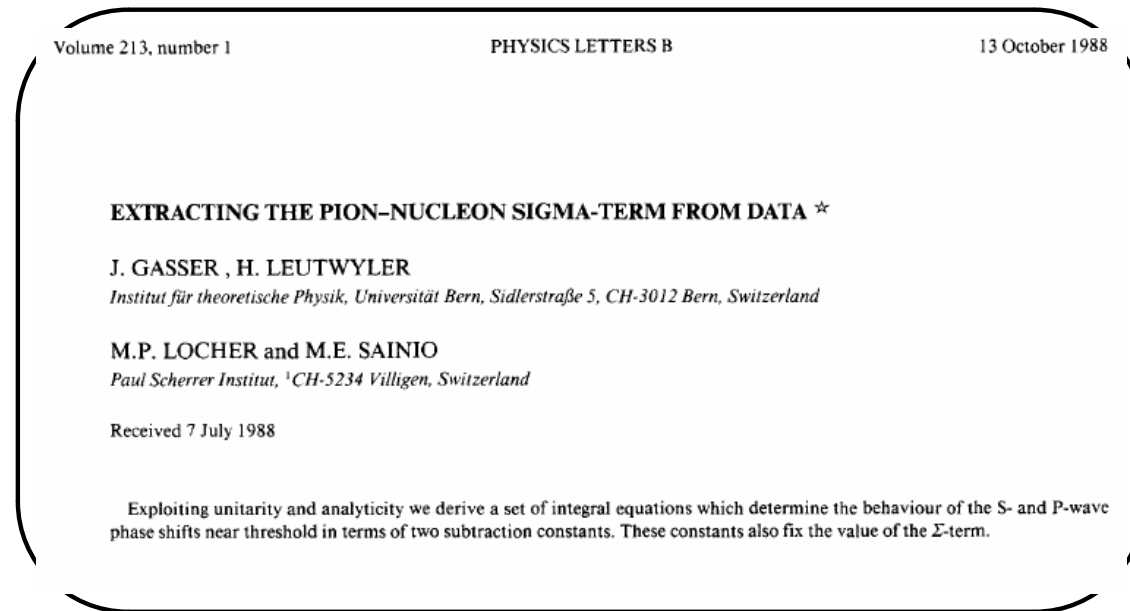
Crivellin, Hoferichter, Procura, ...

Bsaisou, de Vries, UGM, Wirzba, van Kolck, ...

Sigma term physics from Bern

The fundamental papers

- Three very important papers in this field



The fundamental papers

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- Three very important papers in this field

Volume 213, number 1

PHYSICS LETTERS B

13 October 1988

EXTRACTING THE PION-NUCLEON SIGMA-TERM FROM DATA ☆

J. GASSER, H. LEUTWYLER

Institut für theoretische Physik, Universität Bern, Sidlerstraße 5, CH-3012 Bern, Switzerland

M.P. LOCHER and M.E. SAINIO

Paul Scherrer Institut, CH-5234 Villigen S

Received 7 July 1988

Exploiting unitarity and analyticity we determine the pion-nucleon sigma term in terms of two s

Volume 253, number 1,2

PHYSICS LETTERS B

3 January 1991

Sigma-term update ☆

J. Gasser, H. Leutwyler

Institute for Theoretical Physics, University of Bern, Sidlerstraße 5, CH-3012 Bern, Switzerland

and

M.E. Sainio

Research Institute for Theoretical Physics, University of Helsinki, Siltavuorenpenger 20C, SF-00170 Helsinki, Finland

Received 24 September 1990

The determination of the σ -term from πN scattering is critically examined. The currently available data indicate $\sigma \simeq 45$ MeV. Low-energy precision measurements are needed to clarify discrepancies in experimental data and to reduce the uncertainty in the strangeness content for which we obtain $y \simeq 0.2$.

The fundamental papers

- Three very important papers in this field

Volume 213, number 1

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EXTRACTING THE PION-NUCLEON

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Received 7 July 1988

Exploiting unitarity and analyticity we calculate the pion-nucleon phase shifts near threshold in terms of two parameters.

Volume 253, number 1,2

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3 January 1991

Sigma-term update ☆

J. Gasser, H. Leutwyler

Institute for Theoretical Physics, University of Bern, Sidlerstraße 5, CH-3012 Bern, Switzerland

and

M.E. Sainio

Research Institute for Theoretical Physics, University of Helsinki, SF-00014 Helsinki, Finland

Received 24 September 1990

The determination of the σ -term from low-energy precision measurements of the nucleon strangeness content for which we obtain

Volume 253, number 1,2

PHYSICS LETTERS B

3 January 1991

Form factor of the σ -term ☆

J. Gasser, H. Leutwyler

Institute for Theoretical Physics, University of Bern, Sidlerstraße 5, CH-3012 Bern, Switzerland

and

M.E. Sainio

Research Institute for Theoretical Physics, University of Helsinki, Siltavuorenpenger 20C, SF-00170 Helsinki, Finland

Received 24 September 1990

We use dispersion relations to calculate the nucleon form factor of the scalar operator $\bar{u}u + \bar{d}d$ and show that the corresponding mean square radius is remarkably large.

Deep new insights

- Most important results of this work:

- Upgrade of the KH80 value for Σ PLB 213 (1988) 85

$$\hookrightarrow \Sigma = F_\pi^2 \bar{D}^+|_{s=m_N^2, t=2M_\pi^2} = 56 \pm 2 \text{ MeV [from } 64 \pm 8 \text{ MeV]}$$

- Upgrade of the value for $\sigma_{\pi N}$ - the standard for decades PLB 253 (1991) 252

$$\hookrightarrow \sigma_{\pi N} \simeq 45 \text{ MeV [errors discussed in the paper]}$$

$$\hookrightarrow \text{strangeness content } y \simeq 0.2$$

$$\text{based on } \sigma_0 = 35 \pm 5 \text{ MeV Gasser, Ann. Phys. (NY) 136 (1981) 62}$$

- Detailed analysis of the scalar form factor of the nucleon PLB 253 (1991) 260

$$\hookrightarrow \langle r^2 \rangle_N^S \simeq 1.6 \text{ fm}^2 \text{ much bigger than the proton charge radius, } \langle r^2 \rangle_p^E = 0.71 \text{ fm}^2$$

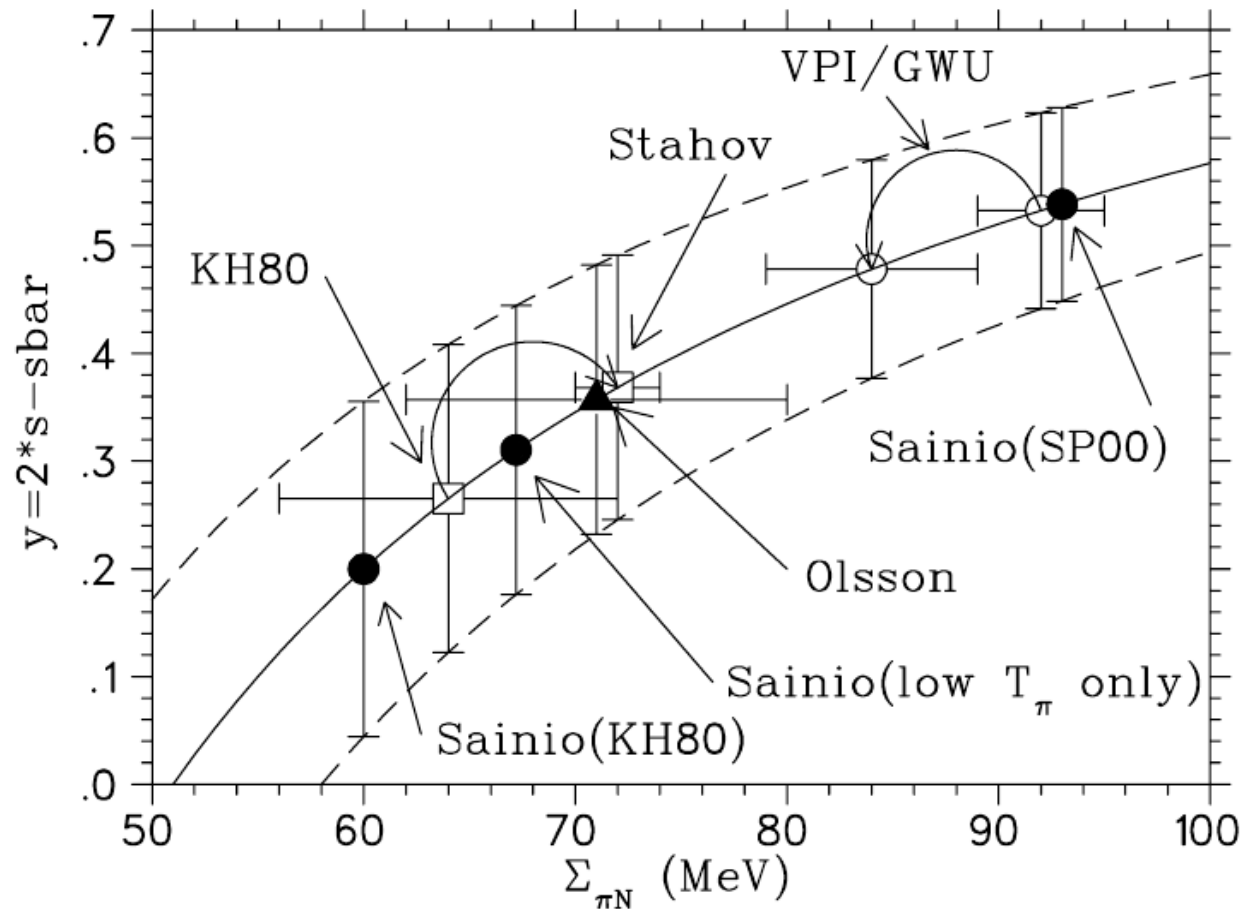
$$\hookrightarrow \text{strong pionic rescattering in } \Delta_\sigma \text{ and } \Delta_D \text{ cancels}$$

Dark ages

15

- Then, chaos emerged [using $\sigma_0 = 36 \pm 7$ MeV]

Borasoy, UGM, Ann. Phys. **254** (1997) 192



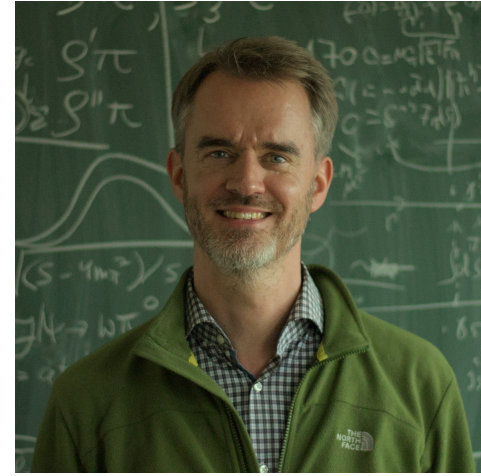
UGM, Smith, Chiral Dynamics 2000, hep-ph/0011277

A new look at sigma term physics

Overcoming the dark ages

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- Step 1: assemble a bunch of fearless knights



Christoph Ditsche Martin Hoferichter Jacobo Ruiz de Elvira Bastian Kubis

- Step 2: lay out the framework (RS equations) and gather novel input
 - ↪ fantastic work at PSI on pionic atoms (hydrogen, deuterium)
 - ↪ EFT framework allows to extract the pertinent scattering lengths
 - ↪ new determinations of the pion-nucleon coupling constant (GMO sum rule, NN scattering)
 - ↪ include isospin breaking corrections to the CD LET and elsewhere

Timeline of our investigations

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- 2012 – Roy-Steiner equations for pion-nucleon scattering, JHEP **06** (2012) 043
- 2012 – Dispersive analysis of the scalar form factor of the nucleon, JHEP **06** (2012) 063
- 2015 – High-Precision Determination of the Pion-Nucleon σ Term from Roy-Steiner Equations, Phys. Rev. Lett. **115** (2015) 092301
- 2015 – Matching pion-nucleon Roy-Steiner equations to chiral perturbation theory, Phys. Rev. Lett. **115** (2015) 192301
- 2016 – Roy-Steiner-equation analysis of pion-nucleon scattering, Phys. Rept. **625** (2016) 1
- 2016 – Remarks on the pion-nucleon σ -term, Phys. Lett. B **760** (2016) 74
- 2016 – On the $\pi\pi$ continuum in the nucleon form factors and the proton radius puzzle, Eur. Phys. J. A **52** (2016) 331 [w/ H.-W. Hammer]
- 2017 – Reconciling threshold and subthreshold expansions for pion-nucleon scattering, Phys. Lett. B **770** (2017) 27 [w/ D. Siemens, E. Epelbaum, H. Krebs]
- 2018 – Extracting the σ -term from low-energy pion-nucleon scattering, J. Phys. G **45** (2018) 024001
- 2023 – On the role of isospin violation in the pion-nucleon σ -term, Phys. Lett. B **843** (2023) 138001
- 2024 – Nucleon resonance parameters from Roy-Steiner equations, Phys. Lett. B **853** (2024) 138698

↔ discuss assorted hi-lites

- RS equations have a limited range of validity:

$$\sqrt{s} \leq \sqrt{s_m} = 1.38 \text{ GeV},$$

$$\sqrt{t} \leq \sqrt{t_m} = 2.00 \text{ GeV}$$

- Input/constraints

S- and P-waves above the
matching point ($s > s_m, t > t_m$)

Inelasticities

Higher waves (D-, F-, ...)

Scattering lengths from hadronic atoms

- Output

S- and P-wave phase shifts below the
matching point ($s \leq s_m, t \leq t_m$)

Subthreshold parameters

→ Pion-nucleon σ -term

→ LECs of pion-nucleon CHPT

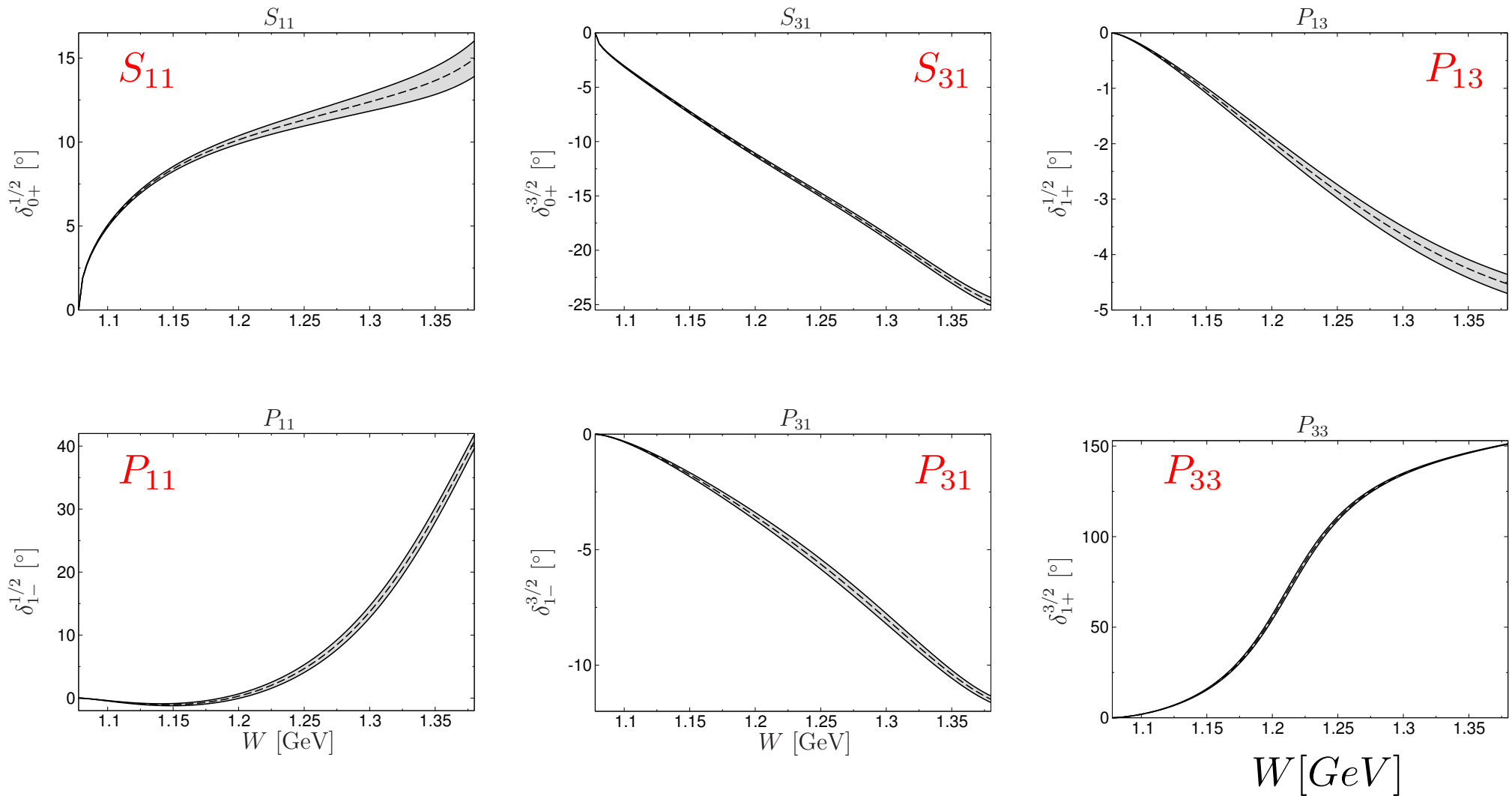
→ N form factor spectral functions

→ Resonance parameters

- πN input from SAID/GWU, $\pi\pi$ input from Bern and Madrid/Cracow
- important check: recover KH80 phases with appropriate input
- much effort in the error analysis → first time for a dispersive analysis of πN scattering

Phase shifts

- S- and P-waves up to the matching point [Notation: $L_2 I_s 2J$]



Threshold parameters

- Threshold parameters from:

$$\text{Re} f_{\ell\pm}^I(s) = q^{2\ell} \left(a_{\ell\pm}^I + b_{\ell\pm}^I q^2 + \dots \right)$$

	RS	KH80
$a_{0+}^{1/2}$	169.8 ± 2.0	173 ± 3
$a_{0+}^{3/2}$	-86.3 ± 1.8	-101 ± 4
$a_{1+}^{1/2}$	-29.4 ± 1.0	-30 ± 2
$a_{1+}^{3/2}$	211.5 ± 2.8	214 ± 2
$a_{1-}^{1/2}$	-70.7 ± 4.1	-81 ± 2
$a_{1-}^{3/2}$	-41.0 ± 1.1	-45 ± 2
$b_{0+}^{1/2}$	-35.2 ± 2.2	-18 ± 12
$b_{0+}^{3/2}$	-49.8 ± 1.1	-58 ± 9

	RS	KH80
a_{0+}^+	-0.9 ± 1.4	-9.7 ± 1.7
a_{0+}^-	85.4 ± 0.9	91.3 ± 1.7
a_{1+}^+	131.2 ± 1.7	132.7 ± 1.3
a_{1+}^-	-80.3 ± 1.1	-81.3 ± 1.0
a_{1-}^+	-50.9 ± 1.9	-56.7 ± 1.3
a_{1-}^-	-9.9 ± 1.2	-11.7 ± 1.0
b_{0+}^+	-45.0 ± 1.0	-44.3 ± 6.7
b_{0+}^-	4.9 ± 0.8	13.3 ± 6.0

- In units of $10^{-3}/M_\pi$ or $10^{-3}/M_\pi^3$, respectively
- Most striking difference to KH80: S-wave scattering lengths! (here: input)
- Sanity check: re-analyse πN data w/o hadronic atoms input \rightarrow consistent!

Results for the σ -term

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- Basic formula:

$$\sigma_{\pi N} = F_{\pi}^2(d_{00}^+ + 2M_{\pi}^2 d_{01}^+) + \Delta_D - \Delta_{\sigma} - \Delta_R$$

- Subthreshold parameters: $d_{00}^+ = -1.36(3)M_{\pi}^{-1}$, $d_{01}^+ = 1.16(3)M_{\pi}^{-3}$

- $\Delta_D - \Delta_{\sigma} = (1.8 \pm 0.2) \text{ MeV}$

Hoferichter, Ditsche, Kubis, UGM (2012)

- $\Delta_R \lesssim 2 \text{ MeV}$

Bernard, Kaiser, UGM (1996)

- Isospin breaking in the CD theorem shifts $\sigma_{\pi N}$ by $+3.0 \text{ MeV}$

$$\sigma_{\pi N} = (59.1 \pm 1.9_{\text{RS}} \pm 3.0_{\text{LET}}) \text{ MeV} = (59.1 \pm 3.5) \text{ MeV}$$

[Q: can we recover $\sigma_{\pi N} = 45 \text{ MeV}$? \rightarrow next slide]

- Update due to final results of pionic hydrogen

Hirtl et al., Eur. Phys. J. A **57** (2021) 70

$$\sigma_{\pi N} = (59.0 \pm 3.5) \text{ MeV}$$

Comparison to GLS

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- Why does this value is so different from GLS 1991?
- GLS 1991 relies on the Karlsruhe-Helsinki PWA KH80
- Different input:

	$g_{\pi N}^2/4\pi$	$a_{0+}^{1/2} [10^{-3}/M_\pi]$	$a_{0+}^{3/2} [10^{-3}/M_\pi]$	$\sigma_{\pi N} [\text{MeV}]$
GLS 1991	14.28	173(3)	-101(4)	45
RS 2023	13.7(2)	169.8(2.0)	-86.3(1.8)	59.0(3.5)

- RS equations with KH80 input $\hookrightarrow \boxed{\sigma_{\pi N} = 46 \text{ MeV}} \checkmark$

\hookrightarrow KH80 is internally consistent but at odds with modern a_0^I determinations

- On the technical side: GLS use forward DR to extract d_{00}^+ and d_{01}^+

Isospin corrections and lattice QCD results

- Isospin-breaking in $\sigma_{\pi N}$ [$\Delta_\pi = M_{\pi^\pm}^2 - M_{\pi^0}^2$]:

$$\sigma_{\pi N} = \Sigma_d + \Delta_D - \Delta_\sigma - \Delta_R + \underbrace{\frac{81g_A^2 M_{\pi^\pm} \Delta_\pi}{256\pi F_\pi^2}}_{+3.4 \text{ MeV}} + \underbrace{\frac{e^2}{2} F_\pi^2 (4f_1 + f_2)}_{(-0.4 \pm 2.2) \text{ MeV}}$$

- We define isospin amplitudes by the $\pi^+ p$ channel, LQCD uses M_{π^0} to define the σ -term: $\bar{\sigma}_{\pi N}$

- Our analysis: $\Delta\sigma = \sigma_{\pi N} - \bar{\sigma}_{\pi N} = 3.1(5) \text{ MeV}$

↪ not small, must be accounted for

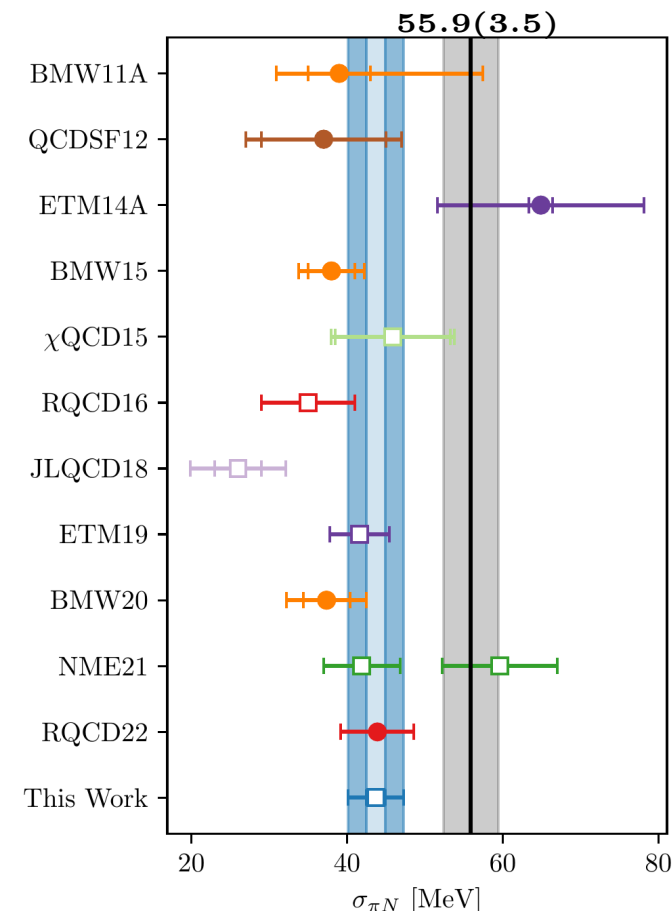
- Central value built into the recent Mainz analysis

Agadjanov et al., Phys. Rev. Lett. **131** (2023) 261902

↪ discrepancies remain!

↪ excited state contaminations?

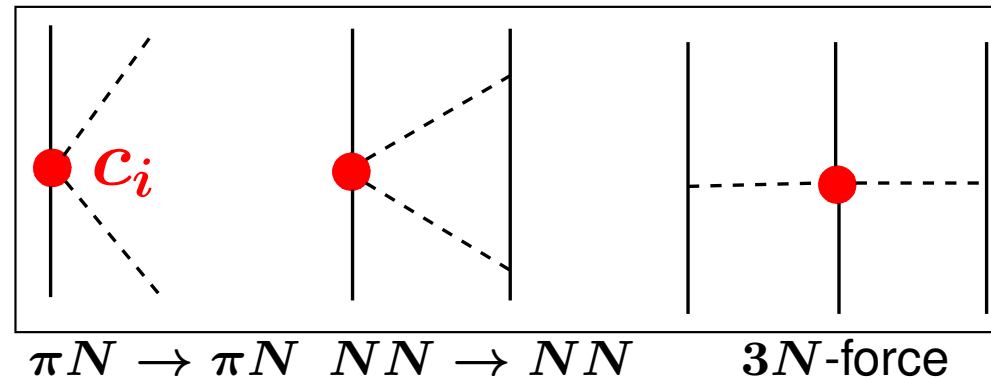
↪ status of these not conclusive at present



Chiral symmetry in nuclear interactions

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- Chiral symmetry breaking in QCD relates *many* processes Weinberg
- One of the best examples: dimension-two vertices from the effective πN Lagrangian ($\sim c_i$)



- Strategy: Fix in $\pi N \rightarrow \pi N$ and then explore in few-nucleon interactions

LEC	RS	KGE 2012	UGM 2005
c_1 [GeV ⁻¹]	-1.11 ± 0.03	$-1.13 \dots -0.75$	$-0.9^{+0.2}_{-0.5}$
c_2 [GeV ⁻¹]	3.13 ± 0.03	$3.49 \dots 3.69$	3.3 ± 0.2
c_3 [GeV ⁻¹]	-5.61 ± 0.06	$-5.51 \dots -4.77$	$-4.7^{+1.2}_{-1.0}$
c_4 [GeV ⁻¹]	4.26 ± 0.04	$3.34 \dots 3.71$	$-3.5^{+0.5}_{-0.2}$

Krebs, Gasparyan, Epelbaum, Phys. Rev. C **85** (2012) 054006; UGM, PoS LAT2005 (2006) 009

The proton charge radius

- Much ado about the so-called proton radius puzzle [μH versus ep]

↪ never really existed in DR analyses

↪ always the 2π continuum

à la Frazer-Fulco included PRL 2 (1959) 365

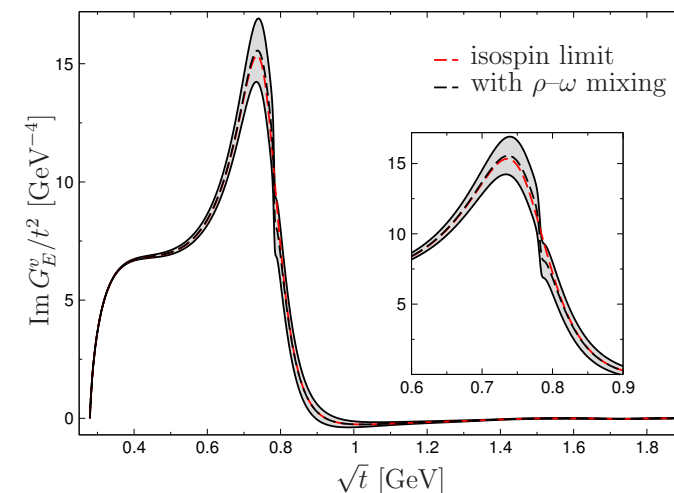
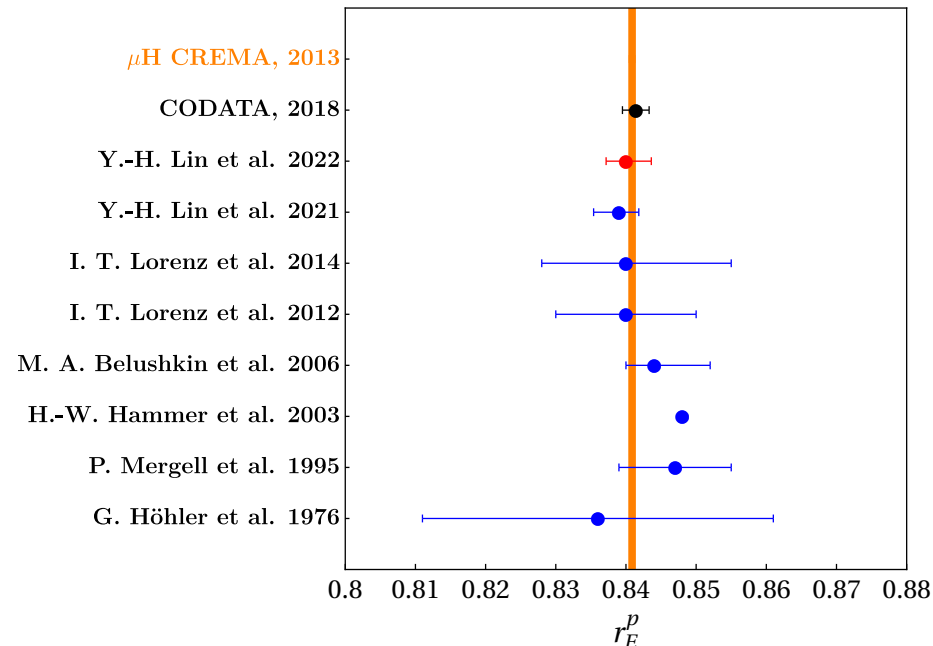
- Updated in 2016 using the RS partial waves $f_{\pm}^1(t)$ and modern data on the pion vector form factor

$$\text{Im } G_E^V(t) = \frac{q_t^3}{m_N \sqrt{t}} (F_{\pi}^V(t))^* f_{+}^1(t) \theta(t - t_{\pi})$$

- Use DR and fit to all space- and timelike XS data including 2γ -exchange corrections

$$r_E^p = 0.840(3)_{\text{sta}}(2)_{\text{sys}}$$

Lin, Hammer, UGM, PRL 128 (2022) 052002



- The pion-nucleon interaction is one of the most studied processes in strong interaction physics/chiral dynamics → more Bernese footprints

ANNALS OF PHYSICS **136**, 62–112 (1981)

Hadron Masses and the Sigma Commutator in Light of Chiral Perturbation Theory*

J. GASSER

*Institute for Theoretical Physics,
University of Berne, Sidlerstrasse 5, CH-3012 Berne, Switzerland*

Eur. Phys. J. C **9**, 643–671 (1999)
Digital Object Identifier (DOI) 10.1007/s100529900055

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Baryon chiral perturbation theory in manifestly Lorentz invariant form

T. Becher, H. Leutwyler

Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland



RECEIVED: June 5, 2001, ACCEPTED: June 6, 2001

Low energy analysis of $\pi N \rightarrow \pi N$

Thomas Becher

*Newman Laboratory of Nuclear Studies, Cornell University
Ithaca, NY 14853, USA*

E-mail: becher@mail.lns.cornell.edu

Heinrich Leutwyler

*Institute for Theoretical Physics, University of Bern
Sidlerstr. 5, CH-3012 Bern, Switzerland*

E-mail: leutwyler@itp.unibe.ch

- The pion-nucleon interaction is one of the most studied processes in strong interaction physics/chiral dynamics → more Bernese footprints

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JHEP
HYPER VERSION

RECEIVED: June 5, 2001, ACCEPTED: June 6, 2001

Low energy analysis of $\pi N \rightarrow \pi N$

Thomas Becher

*Newman Laboratory of Nuclear Studies, Cornell University
Ithaca, NY 14853, USA*

E-mail: becher@mail.lns.cornell.edu

Heinrich Leutwyler

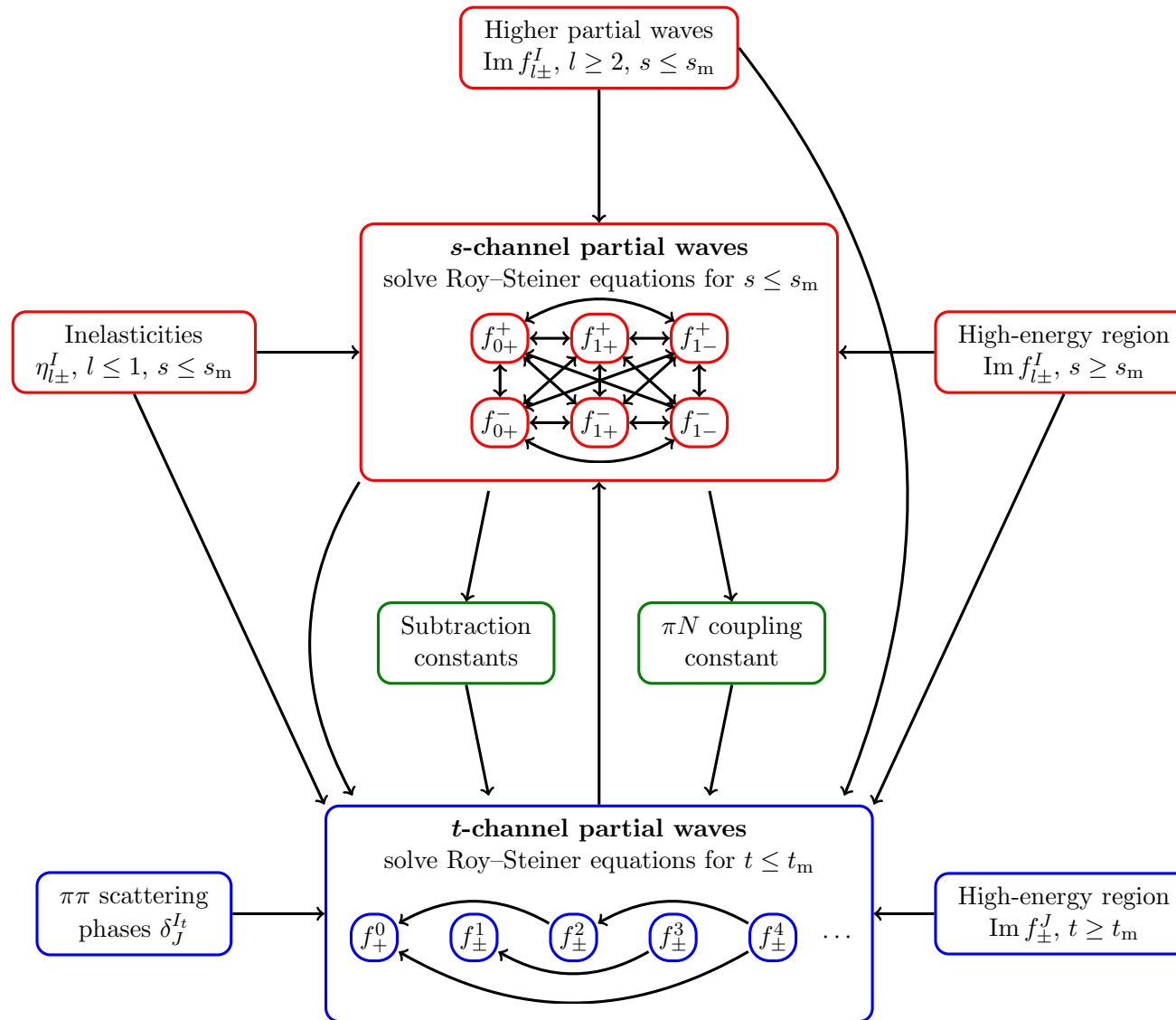
*Institute for Theoretical Physics, University of Bern
Sidlerstr. 5, CH-3012 Bern, Switzerland*

E-mail: leutwyler@itp.unibe.ch

Congrats to Jürg and Heiri for their well-deserved award!

Spares

Solution strategy



- Variation of the input:

use KH80 input instead of GWU/SAID (higher PWs, inelasticities) → small effect

very small effect from s-channel PWs with $\ell > 5$

small effect from the S-wave extrapolation for $t > 1.3$ GeV

negligible effect of the ρ' and the ρ''

very significant effect of the D-waves (esp. $f_2(1270)$)

F-waves shown to be negligible

- Other sources of uncertainty:

statistical errors (shallow fit minima)

matching conditions (close to W_m) [no error on SAID, use smoothed KH80]

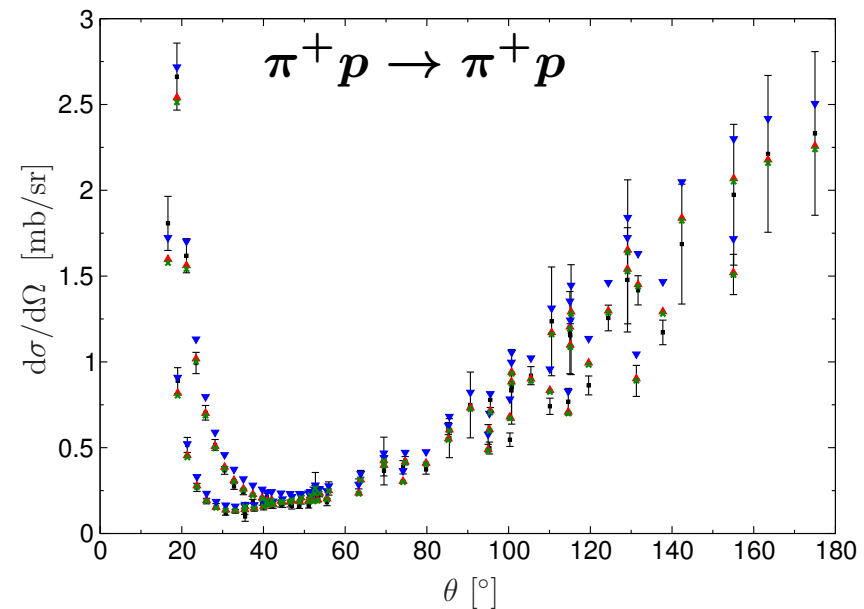
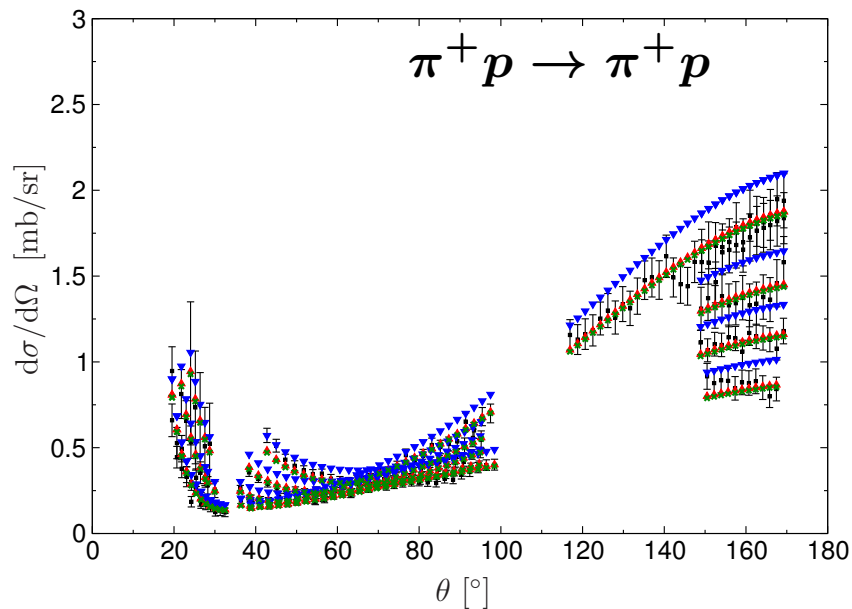
scattering lengths errors (important for $\sigma_{\pi N}$)

⇒ First time this has been achieved in a dispersive analysis of πN scattering!

Sanity check: No hadronic atom input

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- Fit to the pion-nucleon data base (GWU), electromagnetic corrections to the data à la Tromberg et al and treat normalizations of the data as fit parameters
- Fit to low-energy data based on the RS representation that are dominated from the scattering lengths (up to $T_{\pi}^{\max} = 33 - 55$ MeV)



black: data with readjusted norm

green triangles: new RS solution

red triangles: RS solution with hadronic atom input

blue triangles: KH80

$$\hookrightarrow \boxed{\sigma_{\pi N} = 58(5) \text{ MeV}} \quad \checkmark$$

