Determination of the hadronic width of the ground state in pionic hydrogen

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Abstract

A dedicated high-precision experiment was performed at the Paul Scherrer Institut remeasuring the hadronic shift (ϵ_{1s}) and width (Γ_{1s}) of the ground state of pionic hydrogen (PSI-Experiment R-98-01). The measured quantities are directly connected to the pion-nucleon isospin scattering lengths. The extracted values can be confronted with recent work of effective field theories defined in the low-energy limit of quantum chromodynamics (QCD) such as, e. g., chiral perturbation theory (ChPT). Moreover, Γ_{1s} is connected to the pion-nucleon coupling constant $f_{\pi N}$. With a precisely known value for $f_{\pi N}$ an accurate determination of the Goldberger-Treiman discrepancy is possible, which constitutes a measure of chiral symmetry breaking. This contribution outlines a method used for an accurate extraction of the hadronic width from the $\pi^- H$ data.

Key words: Pionic hydrogen, Low energy pion nucleon interaction, Pion nucleon coupling constant
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1. Introduction

Negatively charged pions are captured in the Coulomb field of the nucleus into a highly excited atomic state and deexcite to the ground state by various processes comprising the so-called atomic cascade.

Collisional processes, as are elastic and inelastic collisions, external Auger effect and Coulomb deexcitation dominate the pionic-hydrogen cascade. In particular, a detailed understanding of Coulomb deexcitation is of high relevance, since the energy release in this process is converted into kinetic energy of the collision partners, eventually leading to a Doppler broadening of subsequently emitted X-rays. Radiative deexcitation starts

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to become significant only at the final stages of the cascade. The measurement of X-ray transitions to the ground state allows a direct investigation of the strong interaction at threshold by means of the shift and broadening of the atomic $1s$ level. A description of the atomic cascade, including the evolving kinetic energy distribution of the pionic hydrogen atoms, is given by the so-called *extended standard cascade model (ESCM)* [1]. The measurements were performed at the πE5 channel of the Paul Scherrer Institut (PSI) by guiding the 112 MeV/c pion beam to a gas filled cryogenic target by using the cyclotron trap II [2]. X-rays from the $4p \to 1s$, $3p \to 1s$ and $2p \to 1s$ transitions in pionic hydrogen were measured using a high precision Bragg spectrometer, which was equipped with various spherically bent crystals and a large-area position sensitive detector [3].

2. Pion nucleon interaction

The relation between the ground-state broadening Γ_{1s} and the low energy isovector scattering length a^- is given by the Deser-type formula [4, 5] ($\Gamma_{1s} \propto (a^-)^2 (1+\delta_\Gamma)^2$). The correction parameter δ_Γ contains, among others, isospin symmetry breaking effects, calculated within the framework of ChPT [6].
Via the Goldberger-Miyazawa-Oehme (GMO) sum rule [7], the pion-nucleon coupling constant can be obtained from a^-. Finally, $f_{\pi N}$ given, the Goldberger-Treiman discrepancy $\Delta_{\pi N} = 1 - \frac{m_\pi g_A}{\sqrt{16\pi} F_\pi f_{\pi N}}$ is obtained constituting a measure of chiral symmetry breaking. F_π is the pion decay constant and g_A is the axial coupling constant.

3. Extraction of Γ_{1s} from the data

The measured line profile P of the X-ray transitions is a convolution of three functions: $P = L \otimes R \otimes (\sum_i D_i)$. L represents the Lorentz-like hadronic broadening of the ground state in pionic hydrogen. R is the instrument's response function, measured using narrow X-ray transitions produced in highly ionised atoms in an electron cyclotron resonance ion trap (ECRIT) [8]. D_i represents the Doppler broadening, originating from the kinetic energy distribution due to a particular Coulomb transition. It is approximated by rectangular boxes (so-called Doppler boxes). First evidence for Doppler boxes has been observed in neutron time-of-flight spectra in the reaction $\pi^- p \to \pi^0 n$ [9, 10].
A measurement of muonic hydrogen was performed in order to verify the correctness of the Doppler box model for the kinetic energy distribution [11]. Muonic hydrogen is similar to pionic hydrogen, except for the strong interaction and, therefore, the spectrum is solely comprised of the (known) response function R and the Doppler broadenings D_i.
To analyse a line shape measured or modelled as described above, an analysis routine was developed. It allows one to extract the strong interaction width from the measured pionic hydrogen lines by taking into account the spectrometer response function as well as the kinetic energy distribution approximated by Doppler boxes. The strong-interaction width is found from a least-square fit to the data using the program package MINUIT from the CERN program library (Tab. 1). As the background rate is low, i. e., the data points of the background and in the tails of the spectrum follow a Poisson distribution, the analysis routine uses maximum likelihood estimation (MLE).
Maximum likelihood, however, often suffers from biased estimators. Biased means that the mean value of estimates obtained from repeated independent experiments is system-

Year	Γ_{1s}^{fit} [meV]	Bias [meV]	$\Gamma_{1s}^{corrected}$ [meV]	Γ_{1s}^{final} [meV]
2002	811 ± 79	26 ± 1	785 ± 79	765 ± 56
2006	783 ± 79	37 ± 1	746 ± 79	

Table 1
Results for Γ_{1s} for the 2002 and 2006 data for the πH($4p - 1s$) transition. Γ_{1s}^{fit} is the result from a free fit to the data, $\Gamma_{1s}^{corrected}$ is obtained after subtraction of the bias. Γ_{1s}^{final} represents the weighted mean and is considered the final result.

atically different from the true value. The only feasible way to quantify this bias is by means of Monte Carlo studies. The true value for Γ_{1s} is then obtained by the free fit to the data corrected by the corresponding bias (Tab. 1). For a detailed discussion on this issue see, e. g., [12].
Up to now, the bias has been determined for the two $4p \to 1s$ measurements, yielding as *preliminary* result for the ground state width of pionic hydrogen (Tab. 1) [13]:

$$\Gamma_{1s} = 765 \pm 56 \,\text{meV}. \tag{1}$$

4. Conclusions and Outlook

In a high precision experiment at PSI the $4p \to 1s$, $3p \to 1s$ and $2p \to 1s$ X-ray transitions in pionic hydrogen have been measured. Using the $4p \to 1s$ transition, a thorough analysis method has been developed in order to extract the hadronic width from the data and yielding a relative accuracy for Γ_{1s} of 7.3% from the $4p \to 1s$ data. In comparison, the predecessor experiment [14], gave a relative error of 9% for the hadronic width from the measurement of the $3p \to 1s$ transition ($\Gamma_{1s} = 868 \pm 78$). Thus, an improvement is achieved already by using the $4p \to 1s$ transition only. The established analysis procedure will be applied to the $3p \to 1s$ and $2p \to 1s$ data also, aiming at a final accuracy of 2-3% for Γ_{1s}. The same procedure will be used in the analysis of the recently measured $3p \to 1s$ transition in pionic deuterium.

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