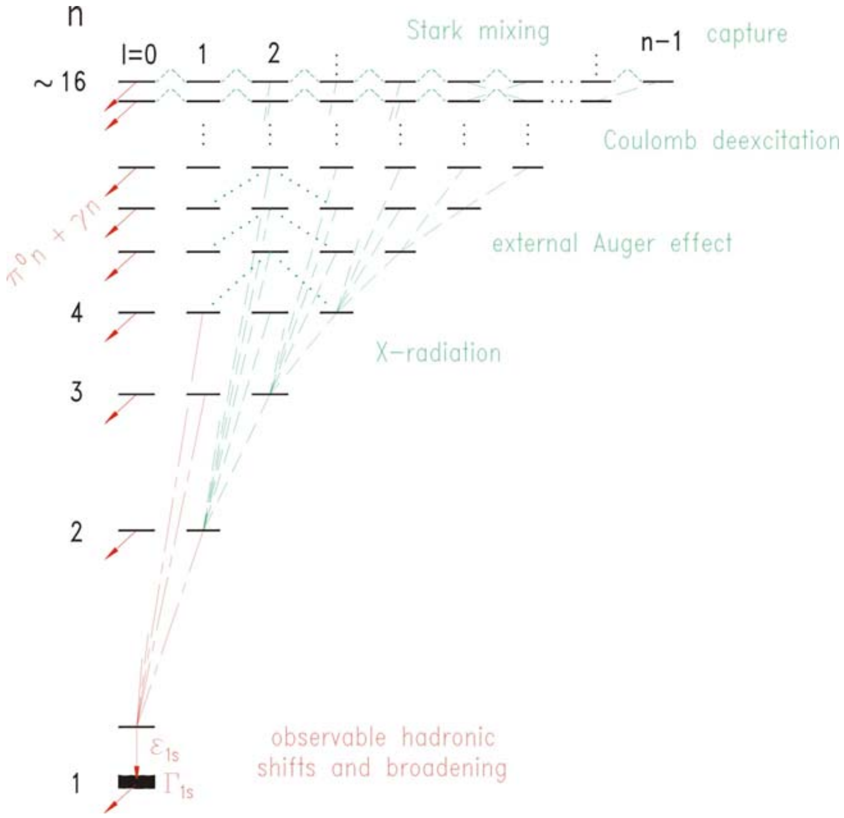
# Pionic hydrogen experiment at PSI: R-98-01.1



## Deser formula

S. Deser et al.. Phys. Rev. 96, 774 (1954)
G. Rasche and W.S. Woolcock, NP A381 405 (1982)

$$\varepsilon_{1s} = -4E_{1s}\frac{1}{r_B}\left(a_{\pi^-p\to\pi^-p}\right)\left(1+\delta_\varepsilon\right)$$

$E_{1s} \approx 7$ eV

$$\Gamma_{1s} = 8E_{1s}\frac{Q_0}{r_B}\left(1+\frac{1}{P}\right)\left(a_{\pi^-p\to\pi^0n}\left(1+\delta_\Gamma\right)\right)^2$$

$\Gamma_{1s} \approx 1$ eV

$E_{1s}$: e.m. binding energy of ground state: 3238 eV
$r_B$ : Bohr radius pionic hydrogen: 222.56 fm
$Q_0$ : kinematic factor: 0.142 fm$^{-1}$
P : Panofsky ratio: 1.546±0.009
$\delta_{\varepsilon,\Gamma}$: e. m. corrections: under debate

**Goals:**

| | | |
|---|---|---|
| $\varepsilon_{1s} \rightarrow$ | $a^+ + a^-$ | 0.2% |
| $\Gamma_{1s} \rightarrow$ | $(a^-)^2$ | 1% |

**Debrecen – Coimbra – Ioannina – Jülich – Leicester – Paris – PSI - Vienna**

H.-Ch.- Schröder et al., Eur. Phys. J. C21,473 (2001): $\varepsilon_{1s}$ =-7.105±0.013$_{\text{stat.}}$±0.034$_{\text{syst.}}$, $\Gamma_{1s}$ =0.868±0.04$_{\text{stat.}}$±0.038$_{\text{syst.}}$eV

# Motivation?

S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)
Y. Tomozawa, Nuovo Cimento A, 707, (1966)

$$a^- = \frac{M_\pi}{8\pi(1+\mu)F_\pi^2}(1+........) \; ; \qquad \mu = \frac{M_\pi}{M_N} \qquad : 79\;[10^{-3}\,m_\pi^{-1}]$$

$$a^+ = 0 \qquad\qquad + ....$$

- Extension of soft pion theorems

  (Goldberger Treiman, Weinberg-Tomozawa, etc.... )

- $\pi$ NN coupling constant

- isospin (non)conservation → D. Gotta's talk

# Higher orders

E. Jenkins and A.V. Manohar, Phys. Lett. B255, 558(1991) : HBCHPT

T. Becher and H. Leutwyler, JHEP0106,017(2001): manifestly Lorentz invariant

**ππ** : expansion to 6th order in chiral dim. (no. of derivatives and/or quark masses)
**πN** : expansion to 4th order in chiral dim.

$$\begin{array}{lcccccc} L_{\pi\pi} = & & L^{(2)}_{\pi\pi} & + & L^{(4)}_{\pi\pi} & + & L^{(6)}_{\pi\pi} \\ & & 2 & & 7 & & 53 \end{array}$$

Number of LEC

$$\begin{array}{lccccccc} L_{\pi N} = & L^{(1)}_{\pi N} & + & L^{(2)}_{\pi N} & + & L^{(3)}_{\pi N} & + & L^{(4)}_{\pi N} \\ & 2 & & 7 & & 23 & & 118 \end{array}$$

Number of LEC

# An example

N. Fettes and U. G. Meissner, Nucl. Phys. **A676**, 311 (2000)

2. order

3. order

$$a_{0+}^{+} = \frac{M_\pi^2[-g_A^2 + 8m(-2c_1 + c_2 + c_3)]}{16\pi(m + M_\pi)F_\pi^2} + \frac{3g_A^2 m M_\pi^3}{256\pi^2(m + M_\pi)F_\pi^4}$$

4. order

$$- \frac{g_A^2 M_\pi^4}{64\pi(m + M_\pi)m^2 F_\pi^2} - \frac{4M_\pi^4 c_1 c_2}{\pi(m + M_\pi)F_\pi^2} + \frac{2mM_\pi^4 c_1 \ell_3}{\pi(m + M_\pi)F_\pi^4}$$

$$- \frac{g_A M_\pi^4(2\bar{d}_{16} - \bar{d}_{18})}{4\pi(m + M_\pi)F_\pi^2}$$

$$+ \frac{2M_\pi^4 m(2\bar{e}_{14} + 2\bar{e}_{15} + 2\bar{e}_{16} + 2\bar{e}_{19} + 2\bar{e}_{20} + 2\bar{e}_{35} - \bar{e}_{36} - 4\bar{e}_{38})}{\pi(m + M_\pi)F_\pi^2}$$

$$- \frac{M_\pi^4[8 - 3g_A^2 + 2g_A^4 + 4m(2c_1 - c_3)]}{256\pi^3(m + M_\pi)F_\pi^4},$$

1. order

$$a_{0+}^{-} = \frac{mM_\pi}{8\pi(m + M_\pi)F_\pi^2} + \frac{M_\pi^3[g_A^2 + 32m^2(\bar{d}_1 + \bar{d}_2 + \bar{d}_3 + 2\bar{d}_5)]}{32\pi m(m + M_\pi)F_\pi^2}$$

$$+ \frac{M_\pi^3 m}{64\pi^3(m + M_\pi)F_\pi^4},$$

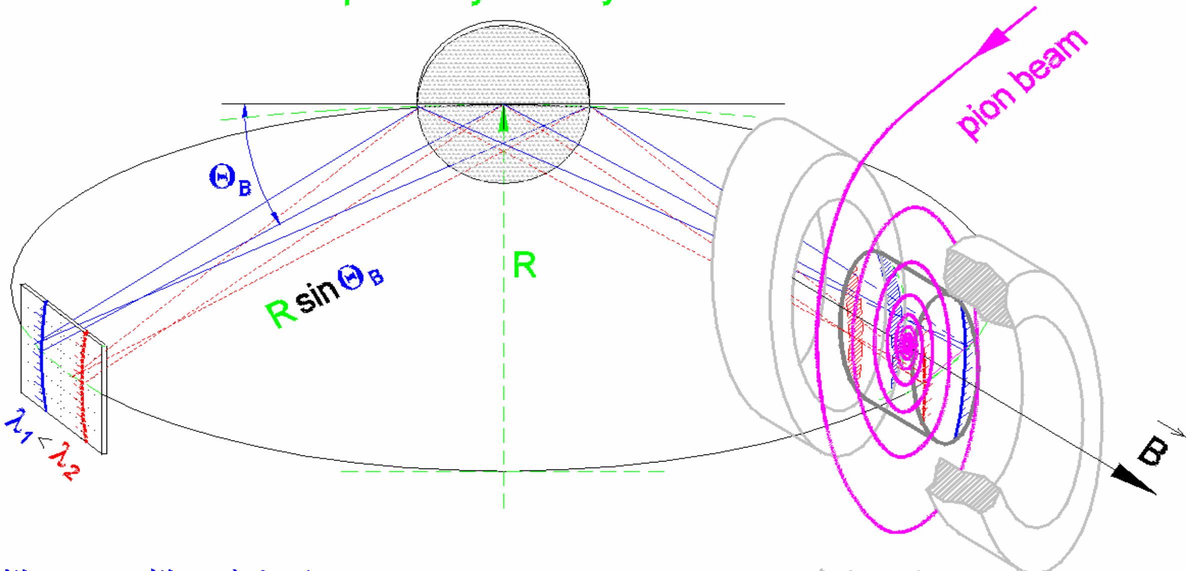
3. order

# PRINCIPLE of SET-UP

**crystal spectrometer**

**highest possible energy resolution**

spherically bent crystal



position-sensitive detector
Charge-Coupled Device (CCD)

cyclotron trap
gas cell

**high position resolution**

⇒ **background reduction**

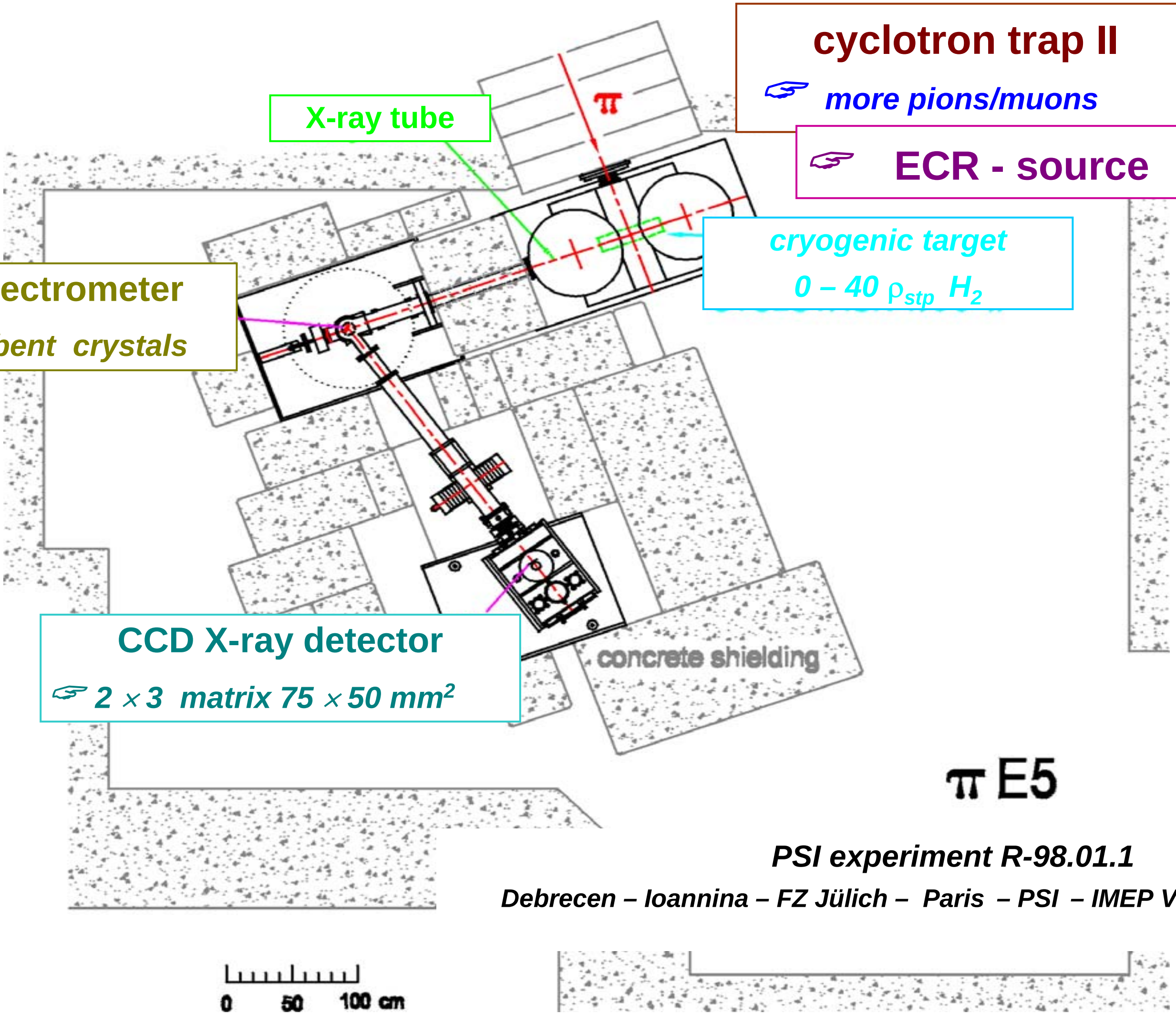
***analysis of hit pattern***

**high stop density**

⇒ **high X - ray line yields**

⇒ **bright X - ray source**

# SET-UP at PSI



**cyclotron trap II**

☞ *more pions/muons*

☞ **ECR - source**

**X-ray tube**

*cryogenic target*

*$0 – 40\ \rho_{stp}\ H_2$*

**crystal spectrometer**

☞ *spherically bent crystals*

**CCD X-ray detector**

☞ *$2 \times 3$ matrix $75 \times 50\ mm^2$*

***PSI experiment R-98.01.1***

***Debrecen – Ioannina – FZ Jülich – Paris – PSI – IMEP Vienna – ETH Zürich***

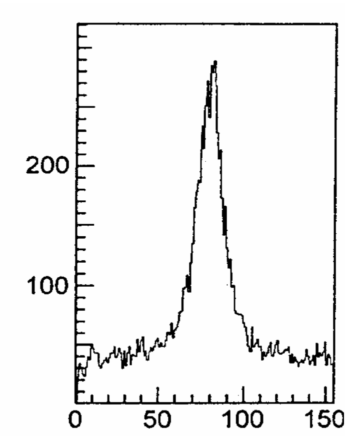
# „Trivial“ difficulties

- Statistics: cyclotron trap (6) + spherically bent crystals (3-4)

  Up to now 68000 events accumulated ( 12 )
- Background : New CCD detectors + much improved shielding (10)

In green: improvement factors compared to PSI experiment ETHZ-Neuchâtel-PSI

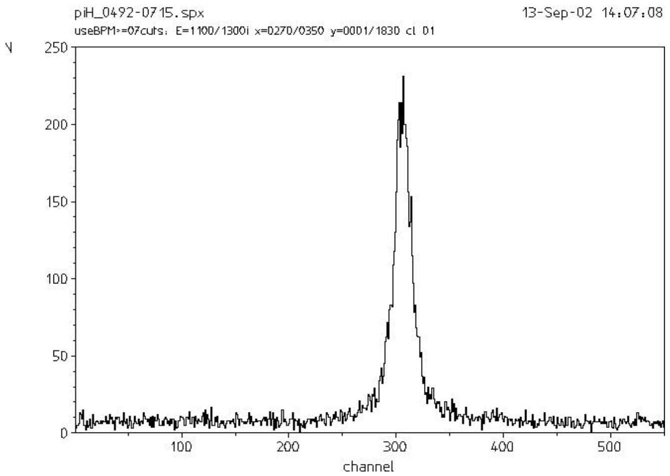
**Small fit range:**
**→ error in Γ**
**(~ 100meV too small)**



**← Width: 12 eV →**

↑

***ETHZ-Neuchâtel-PSI***



**← Width: 72 eV →**

↑

***R98-01.1***

**From Monte Carlo simulation:**
**Fit error for this spectrum is**
**12%**

**From Monte Carlo simulations:**
**Fit error is**
**5 %**

# Less trivial : line shape

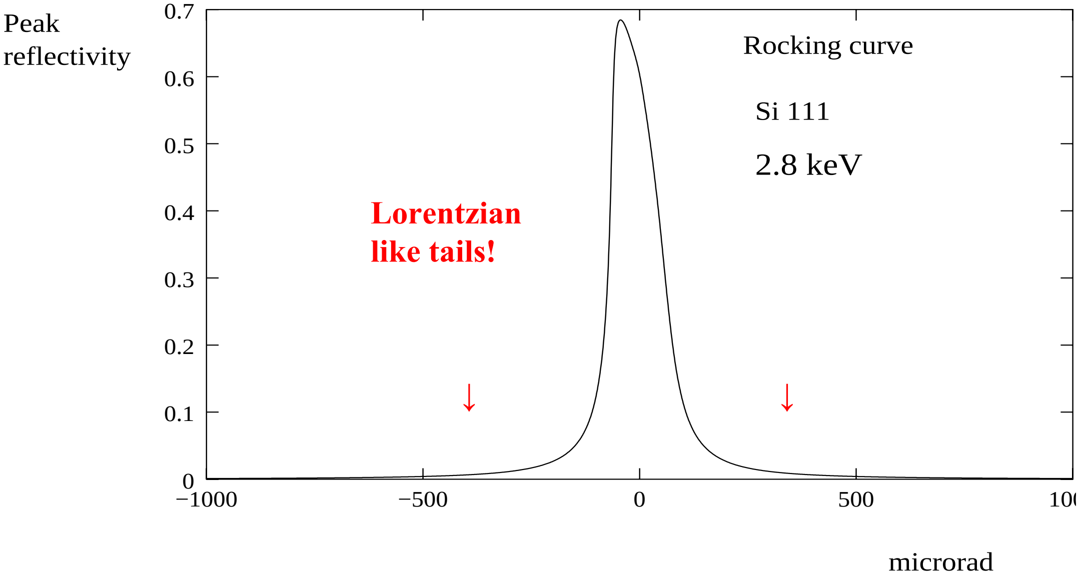
There are Lorentzian tails in the response function.

$\Delta\Gamma/\Gamma < 1\%$ requires a good knowledge of the resp.fct.:

Needed: > 30000 events + „no“ background.

Previous experiment (calibration πBe-1400 cts): $\Delta\Gamma/\Gamma = 8\%$

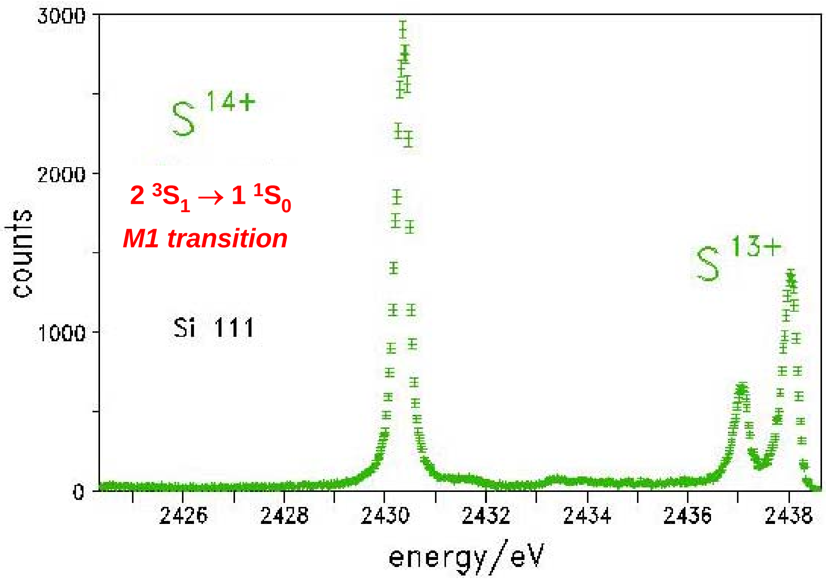
Present experiment: First round ( πC-4500cts): $\Delta\Gamma/\Gamma = 3.5\%$

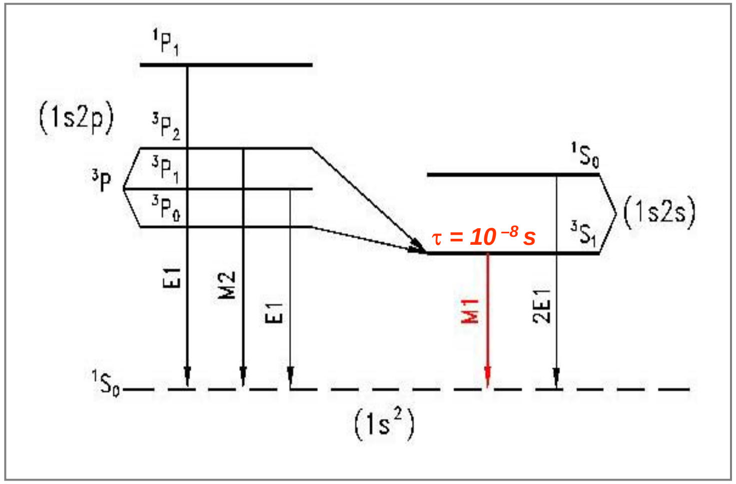


# ECRIT measurements 2004

***M1 transitions*** **in He-like**

**S ↔ πH(2p-1s)**
**Cl ↔ πH(3p-1s)**
**Ar ↔ πH(4p-1s)**

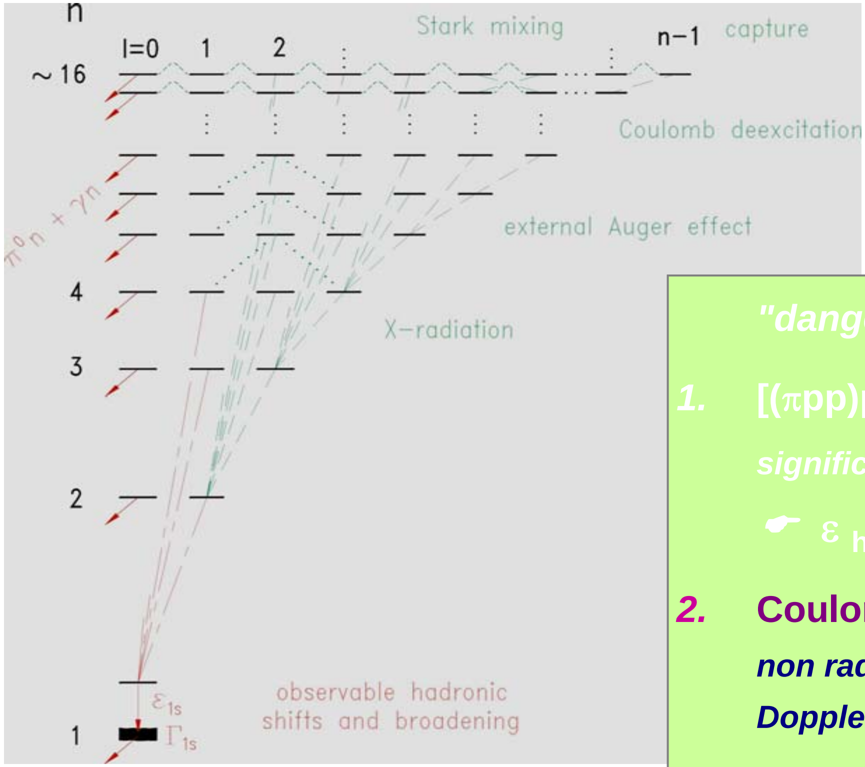




***30000 events in line ↔ tails can be fixed with sufficient accuracy***

# Show stopper: πp NOT isolated

Theoretical input: V.M. Markushin PSI
T. S. Jensen PSI/Paris
V. Popov, V. Pomerant‘sev
Moscow State University



***"dangerous" processes***

1. **[(πpp)p]ee – molecule formation („DH“) ?**

   ***significant radiative decay modes ?***

   ☛ $\varepsilon_{\text{had}}$

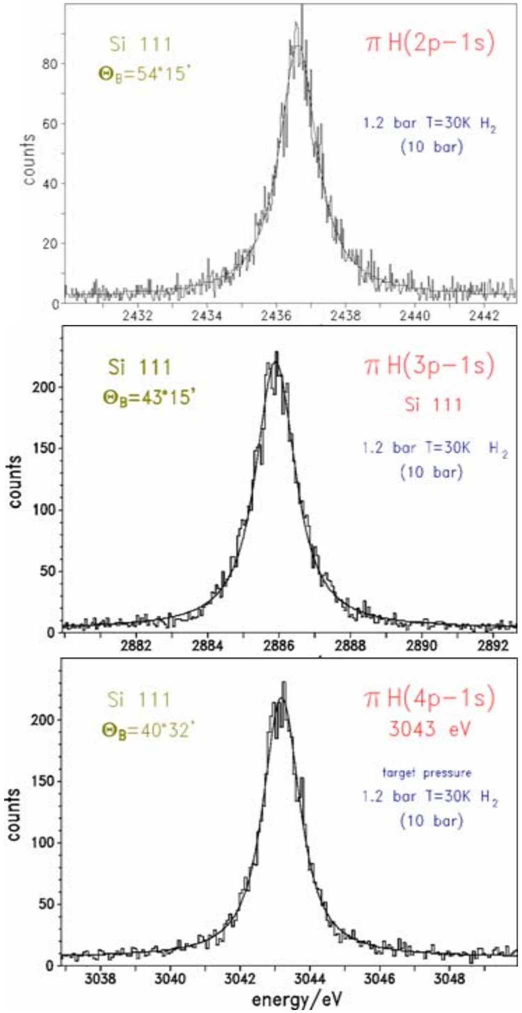
2. **Coulomb - de-excitation !**

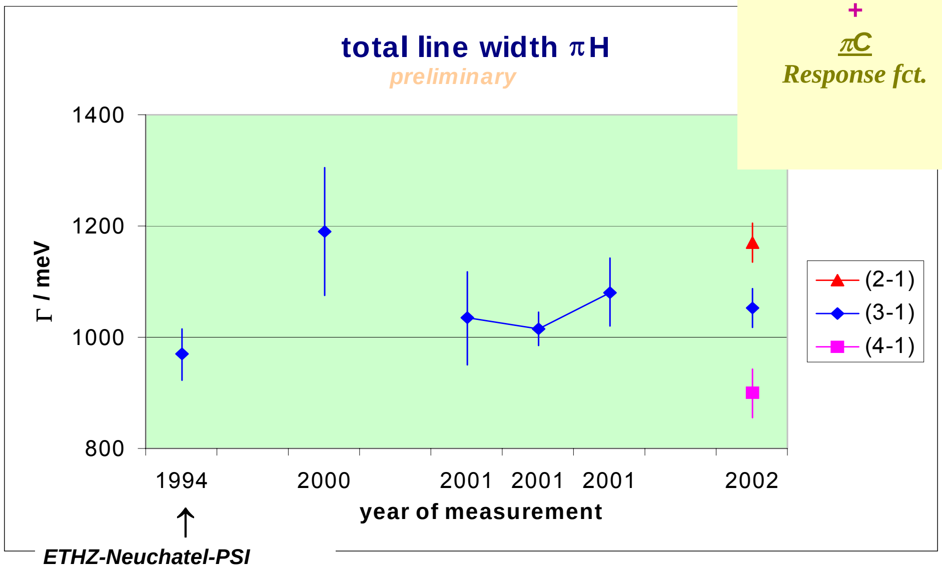
   ***non radiative process $n_i \rightarrow n_f$ + kinetic energy***

   ***Doppler broadening***

   ☛ $\Gamma_{\text{had}}$

# Stategy of width measurement: different initial states





***not***
***corrected for***
***Coulomb***
***de-excitation***
**+**
*πC*
*Response fct.*

**$\Gamma_{1s} < 850$ meV**

***Maik Hennebach**, thesis Cologne 2003*

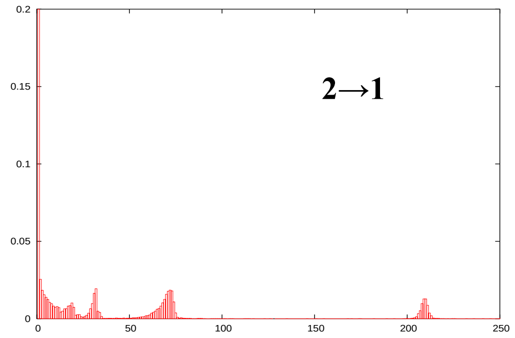
***PEAK-TO-BACKGROUND ratio improved by one order of magnitude !***

# Kinetic energy distributions (Cascade theory)

**Fit results without Doppler effect**

**Fit results including boxes with FREE weights**

**Si 111 10b**
**1170±60 meV**

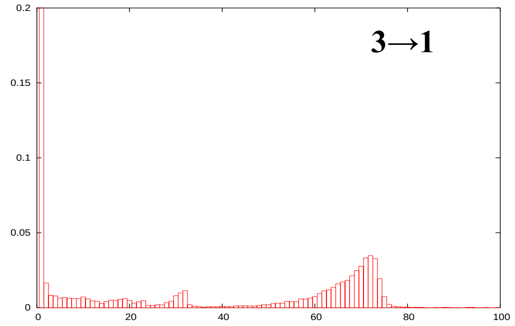


**250 eV**

**Boxes:**
**0-2 eV**
**2-20 eV**
**29-33 eV**
**65-75 eV**
**210-220 eV**

**907±34 meV**

**Si 111 10b**
**1053± 40 meV**

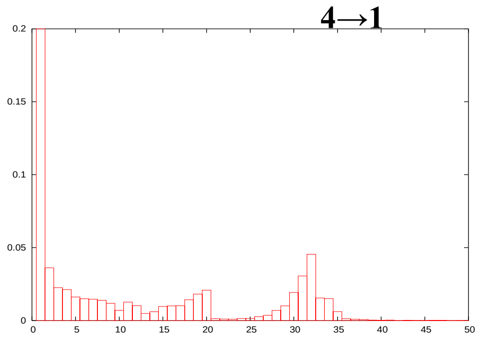


**100 eV**

**Boxes:**
**0-2 eV**
**2-20 eV**
**29-33 eV**
**65-75 eV**

**775±40 meV**

**Si 111 10b**
**899±50 meV**



**50 eV**

**Boxes**
**0-2eV**
**3-15 eV**
**27-34 eV**

**812±60 meV**

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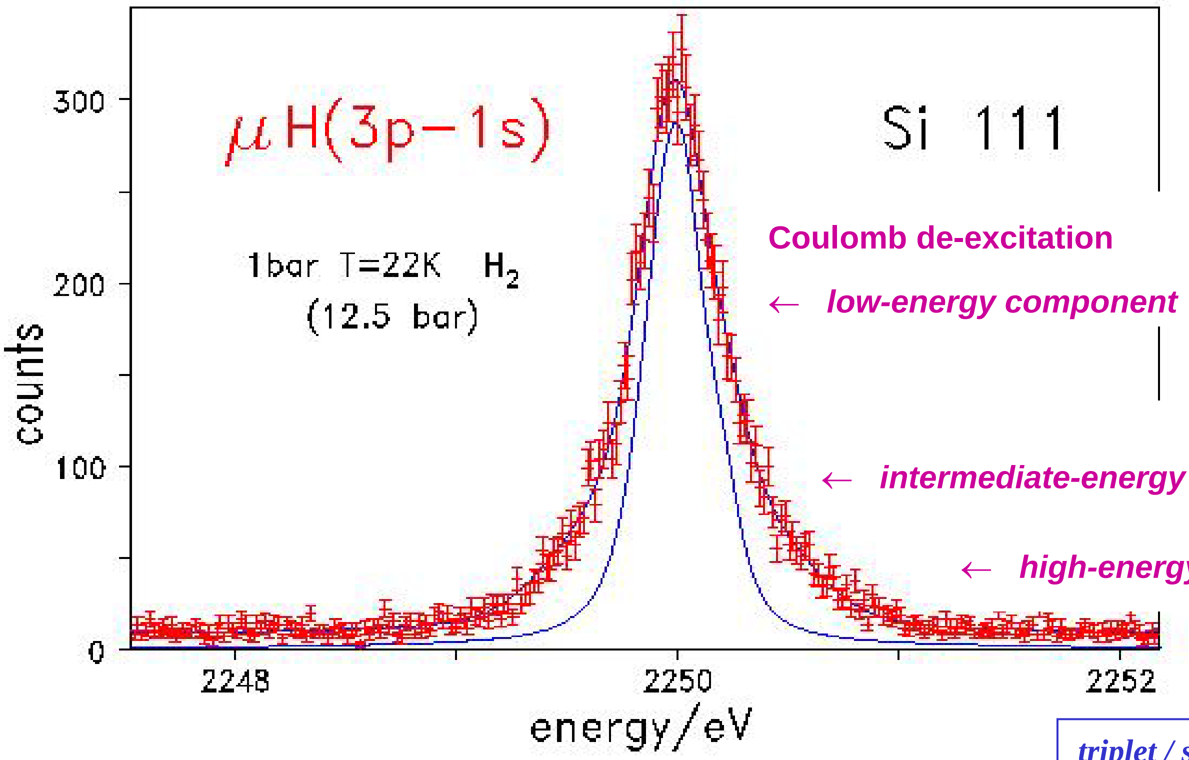
**$\Gamma_{1s} \approx$ 823±19 meV (2.3%)**

# MUONIC HYDROGEN RUN



**Coulomb de-excitation**

*← low-energy component*

*← intermediate-energy component*

*← high-energy component*

***-----   crystal response ECRIT 2004***

*triplet / singlet = 3.02±0.2*

***HFS splitting: 191±2 meV***
***(cf. 184.6 meV)***

# Situation

$\varepsilon_{1s}$ : Extraction of scattering lengths dominated by

*$\chi$PT correction* $\delta_\varepsilon = (-7.2 \pm 2.9)\%$ *

*J. Gasser et al.,Eur. Phys. J. C 26, 13 (2003)*

$$a^+ + a^- = (93.2 \pm 2.9)\ [10^{-3} m_\pi^{-1}]$$

New result: $\Gamma_{1s} = 823 \pm 19$ meV (2.3%) *preliminary*

*$\chi$PT correction* $\delta_\Gamma = (0.6 \pm 0.2)\%$ *

*P. Zemp, hadatom05*

$$a^- = (86.4 \ {}^{+0.099}_{-1.02}\ )\ [10^{-3} m_\pi^{-1}]$$

$$a^+ = (6.8 \pm 3.1)\ [10^{-3} m_\pi^{-1}]$$

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*T.E.O. Ericson et al.: Phys. Lett. B **594**, 76 (2004): $\delta_\varepsilon = -0.62(29)\%$, $\delta_\Gamma = 1.02(23)\%$

PAUL SCHERRER INSTITUT

# $a^+$ and $a^-$ in units of $[10^{-3} m_\pi^{-1}]$

**Experiment R98-01.1:**

**$a^+ = +(6.8 \pm 3.1)$ $a^- = (86.4 \pm 1)$**

| | | $a^+$ | $a^-$ |
|---|---|---|---|
| W - T | | 0 | 79 |
| HBChPT $O(p^2)$ | Moj(98) | -18 | 76.8 |
| HBChPT $O(p^3)$ | Moj(98) | $-9 \pm 12$ | $93.5 \pm 14$ |
| HBChPT $O(p^4)$ I | Fet(00) | -9.6 | 90.29 |
| HBChPT $O(p^4)$ II | Fet(00) | +4.5 | 77.03 |
| HBChPT $O(p^4)$ III | Fet(00) | +2.7 | 86.7 |
| RChPT $O(p^4)$ | Bec(01) | -8.4 ↔ -13.1 | 91.41 |

| | |
|---|---|
| Moj(98): M. Mojzis, Eur. Phys. J. C2, 181 (1998) | Koch,Pietarinen |
| Fet(00): N. Fettes and U.-G. Meissner, Nucl. Phys. A676, 311 (2000) | KA85, Matsinos, VPI/GW98 |
| Bec(01): T. Becher and H. Leutwyler, JHEP 0106,017 (2001) | KA85 |

# Pion nucleon coupling constant

## GMO sum rule

M.L. Goldberger, H. Miyazawa, R. Oehme
Phys. Rev. 99, 986 (1955)

$$(1+\frac{m_\pi}{M})\frac{a^-}{m_\pi}=\frac{2f_{\pi N}^2}{m_\pi^2-(m_\pi^2/2M)^2}+\frac{1}{2\pi^2}\underbrace{\int_0^\infty\frac{\sigma^{tot}_{\pi^-p}(k_\pi)-\sigma^{tot}_{\pi^+p}(k_\pi)}{2\omega(k_\pi)}dk_\pi}_{J}$$

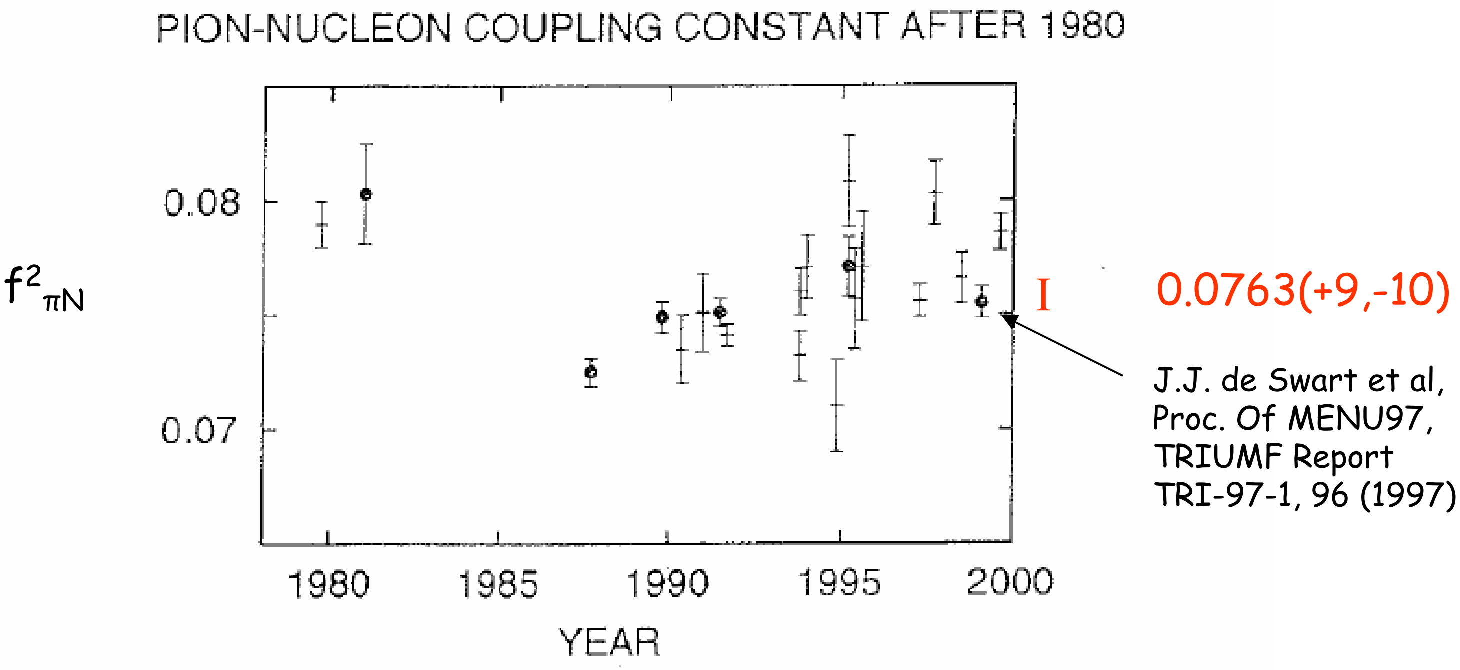
J = -(1.082±0.032)mb T. E. O. Ericson et al. Phys. Rev. C 66, 014005(2002)

$f^2_{\pi N}$ = 0.5712 $a^-[m_\pi]$ +0.02488 J[mb$^{-1}$] = 0.0763(+9,-10)

$g_{\pi NN}$ = 47.66 $f_{\pi N}$ = 13.165(+0.077,-0.087)

[ $g^2_{\pi NN}/4\pi=13.79\pm^{0.164}_{0.180}$ ]

From M. Sainio's talk, Meson Nucleon99 ZUOZ, πN Newsletter 15, 156 (1999)



0.0763(+9,-10)

J.J. de Swart et al,
Proc. Of MENU97,
TRIUMF Report
TRI-97-1, 96 (1997)

**Figure 2** The values of the pion-nucleon coupling constant $f^2$ after 1980 until the prêsent. Neutral pion couplings are denoted by the solid dots, the remaining points refer to charged pion couplings or charge independent determinations

# Recent publications

VPI-GWU Analysis (FA02) http://gwdac.phys.gwu.edu/
R. Arndt et al. Phys. Rev. **C69**, 035213 (2004) GMO **NOT** used!!!

$a^{-}$ = (88.3±0.47) [$10^{-3}\ m_\pi^{-1}$] ; $g_{\pi NN}$ = 13.145±0.048 (constrained by earlier PSI result)
No constraints: $g_{\pi NN}$ = 13.08; claimed to be robust

D, V. Bugg, Eur. Phys. J **C33**, 505 (2004) GMO **NOT** used

$a^{-}$ = (85.0 (Fit I)to 86.6 (Fit II)) [$10^{-3}\ m_\pi^{-1}$] ; $g_{\pi NN}$ : 13.09 to13.168

T.E.O. Ericson et al. Phys. Lett. **B 594**,76 (2004)
Analysis of earlier PSI result, GMO sum rule **USED**

$a^{-}$ = (88.39±0.3) [$10^{-3}\ m_\pi^{-1}$] $g_{\pi NN}$ = 13.28±0.08

M. Döring et al. (nucl-th/0402086): $a^{-}$ = (88.1±0.48) [$10^{-3}\ m_\pi^{-1}$]

# So what?

## I. Goldberger Treiman relation:

$$g_{\pi NN} = M_N G_A / F_\pi (1+\Delta_{GT})\ ;\ \Delta GT \approx m_q$$

$\Delta_{GT} = c\, M^2_\pi + O(M^2_\pi)$, $c \approx 1/\mathrm{GeV}^2$; T. Becher, hep-ph/0206165

$$\Downarrow$$

$$\boxed{\Delta_{GT}^{theor} \approx 2\%}$$

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R98-01.1: $g_{\pi NN} \approx 13.165$; together with $M_N G_A / F_\pi \approx 12.9$

$$\Downarrow$$

$\Delta_{GT}^{exp.} = 2.05(+0.60,-0.67)\%$ $\longrightarrow$ values for LEC's

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## II. Induced pseudoscalar coupling constant $g_P$ (muon capture: least well known)

$$g_P = \frac{2m_\mu g_{\pi NN} F_\pi}{m_\pi^2 + 0.88 m_\mu^2} - \frac{1}{3} g_A m_\mu m_N < r_A >^2$$

V. Bernard, N. Kaiser, U.-G. Meissner, Phys. Rev.D **50**,6899 (1994)
N. Kaiser, Phys. Rev. *C* **67**, 027002 (2003)

$\langle r_A \rangle^2$: axial radius of nucleon (0.44±0.02) fm$^2$

With $g_{\pi NN}$ from R98-01.1:

$g_P = 8.3 \pm 0.07$

# III. S-wave electric dipole multipoles E in charged pion photoproduction: Corrections to LET (Kroll-Ruderman) to O ($m_\pi^3$)

V. Bernard, N. Kaiser, U.-G. Meissner, Phys. Lett. **B383**, 116 (1996) „BKM"
V. Bernard, Proc. of Chiral Dynamics 1997, Mainz, Springer Lecture notes
in Physics 513, (Springer, Berlin, 1998) hep-ph/9710430

With $g_{\pi NN}$ = 13.165 : Values in units of $[10^{-3} m_\pi^{-1}]$

| | LET | BKM | DA* | Experiment |
|---|---|---|---|---|
| $E^{thr}_{0+}$ | 27.3 | 27.9±0.6 | 27.99 | $28.06 \pm 0.27_{stat.} \pm 0.45_{syst.}$ ** |
| $E^{thr}_{0+}$ $(\gamma p \rightarrow \pi^+ n)$ $(\gamma n \rightarrow \pi^- p)$ | -31.4 | - 32.4 ±0.6 | -31.7 | -31.5±0.8 *** |

* „Dispersion theor. analysis" O. Hanstein et al, Phys. Lett. **B399**, 13 (1999)

** E. Korkmaz et al., Phys. Rev. Lett. 18, 3609 (1999)

*** M.A. Kovash et al., πN Newsletter 12, 55 (1997)

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Independently: $(a^-)^2 = q/k_o \, P \, |(E^{thr}_{0+}(\pi^- p))|^2$ ; $q, k_0$ CMS momenta of γ, π

With R98-01.1 value of $a^-$: $E^{thr}_{0+}(\pi^- p) = -32.46 \pm 0.39 \, [10^{-3} m_\pi^{-1}]$

# Last remarks

HBChPT & RCHPT: predictive power restricted by poor knowledge of LEC
But: who knows? (Lattice calculations)

Special predictions and comparison of related quantities deserve an effort to improve on $a^-$

Using GMO sum rule with value J= -1.032±0.032 mb is supported

An increase in accuracy by factor ~ 5 in $a^-$ requires an improved understanding of cascade processes (may be worthwile for other experiments as well).