PIONIC DEUTERIUM

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for the PIONIC HYDROGEN collaboration

Hadronic Exotic Atoms - Trento, 19.6.2006

PIONIC HYDROGEN

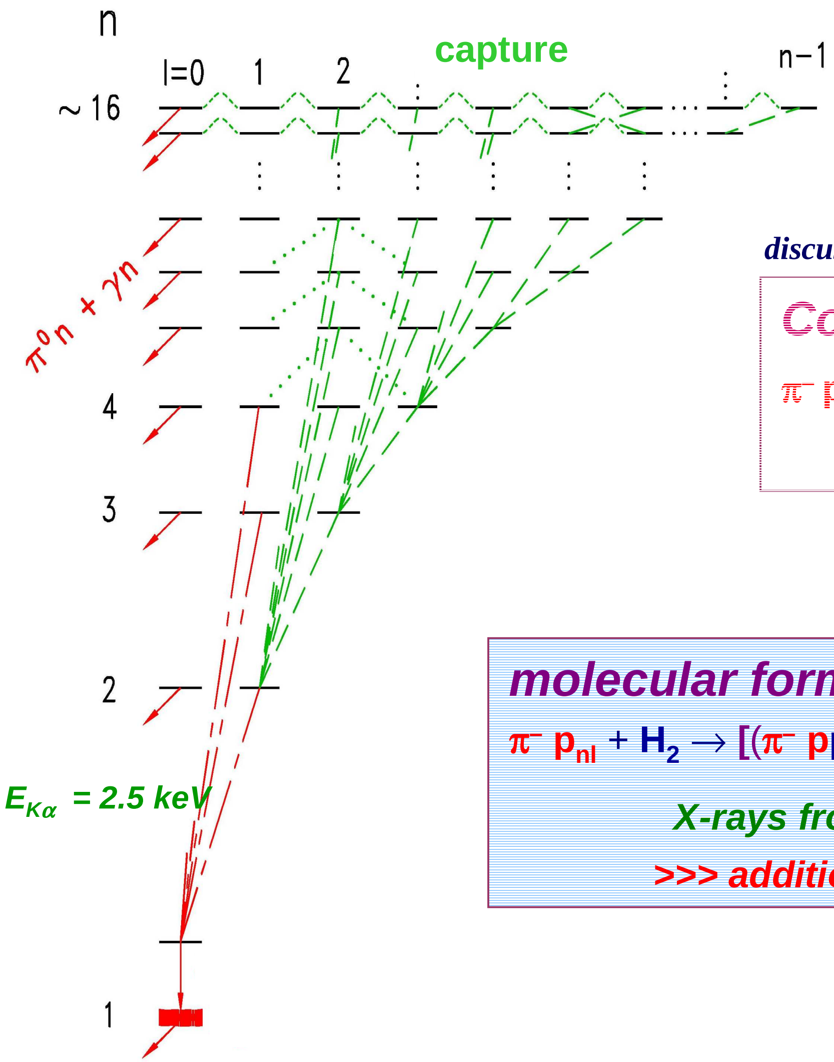
ε_{1s}	πN isospin scattering lengths	$a^+ + a^-$
Γ_{1s}		a^-

PIONIC DEUTERIUM

ε_{1s}	better constraints for	
	πN isospin scattering lengths	a^+ & a^-
	LEC	f_1
Γ_{1s}	pion production at threshold	$\pi NN \leftrightarrow NN$

		$\Delta\varepsilon_{1s}/\varepsilon_{1s}$	$\Delta\Gamma_{1s}/\Gamma_{1s}$
πH	*R-98.01*	***0.2%***	***≈ 2%***
	goal of forthcoming experiment		
πD	*R-06.03*	***3% → < 1%***	***12% → < 4%***

πH - atomic cascade



density dependent effects

discussed in the talk of L. Simons

Coulomb de-excitation

$\pi^- p_{nl}$ + H=H → $\pi^- p_{n'l'}$ + H+H + *kinetic energy*

>>> Doppler broadening ! <<<

molecular formation

$\pi^- p_{nl} + H_2 \rightarrow [(\pi^- pp)_{nj\nu} \cdot p]\, ee_{K\nu}$ *decay by Auger process*

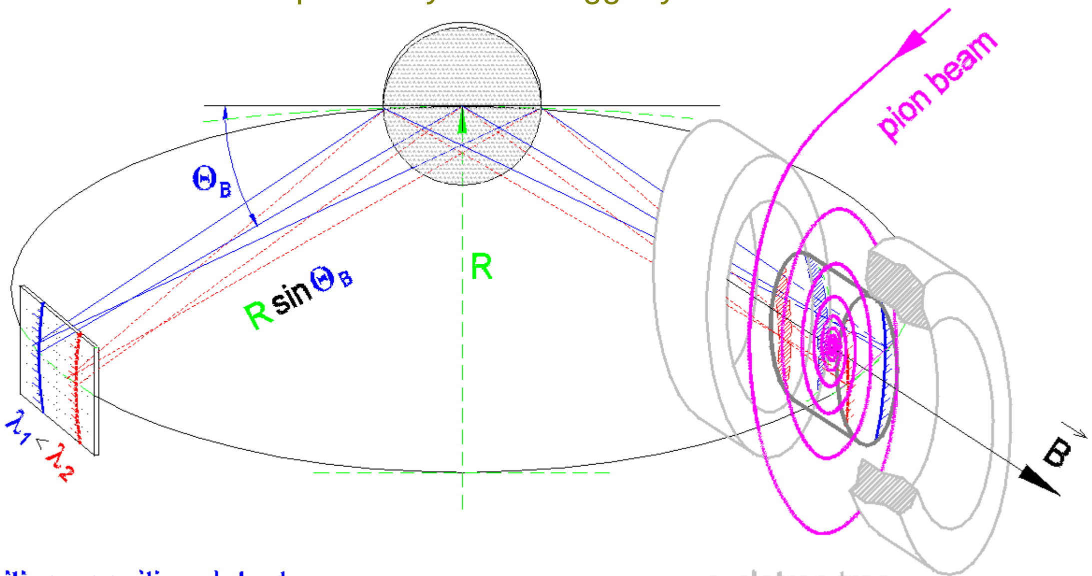
X-rays from molecular states ?

>>> additional energy shift ? <<<

EXPERIMENTAL SET-UP

ultimate energy resolution

spherically bent Bragg crystal



position-sensitive detector
Charge-Coupled Device (CCD)

cyclotron trap
gas cell

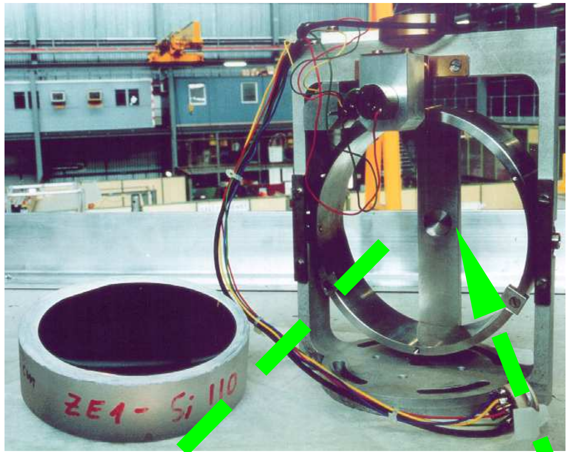
position & energy resolution

⇒ **background reduction**
by analysis of hit pattern

high stop density

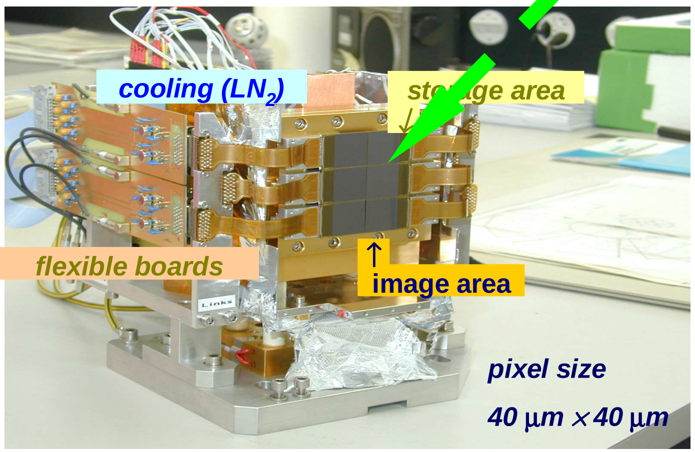
⇒ **high X - ray line yields**
⇒ **bright X - ray source**

Spherically curved Bragg crystal



R = 3 m

Large - Area Focal Plane Detector



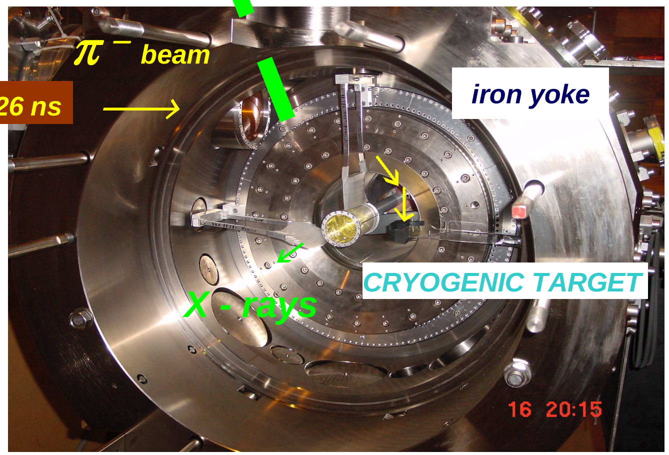
N. Nelms et al., Nucl. Instr. Meth 484 (2002) 419

CYCLOTRON TRAP

one coil removed

10^9 π/s

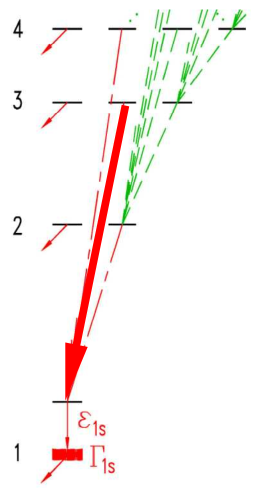
$\tau_{\pi\pm}$ = 26 ns



L. M. Simons, Hyperfine Interactions 81 (1993) 253

ε_{1s}

πH(3p - 1s) transition energy



density dependence ?

MOLECULAR FORMATION

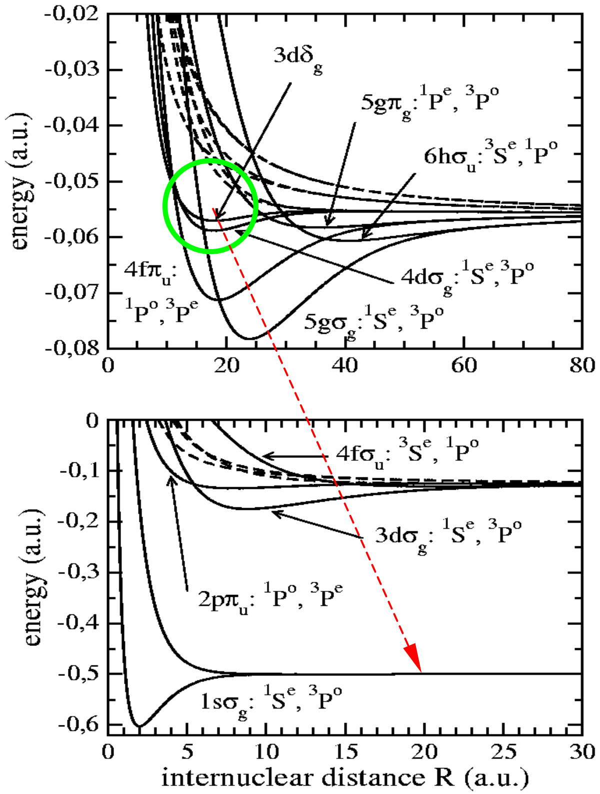
"Vesman" mechanism for excited states: $\pi p_{nl} + H_2 \rightarrow [(\pi pp)_{nJv} \cdot p]\, ee_{K\nu}$

experiment: *muon-catalysed fusion, μH*

X-ray transitions from molecular states ?

N=3

N=1, 2



consequences for πH (np → 1s) transitions

$E_X \rightarrow E_X - \Delta E$ **?**

(are there) bound states below ***dissociation limit of 4.5 eV ?***

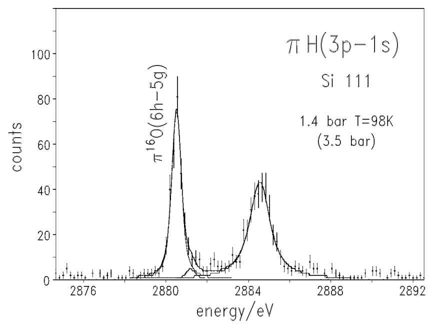
Jonsell, Froelich and Wallenius *for* ***n=1,2,3***
Phys. Rev A 59 (1999) 3440

	ppμ	*ddμ*
$\Gamma_{X\text{-}ray} / \Gamma_{total}$	≈ 0.03	≈ 1

Lindroth, Wallenius and Jonsell
Phys. Rev A 68 (2003) 032502

Kilic, Karr and Hilico
Phys. Rev A 70 (2004) 042506

πH(3p - 1s) - density dependence



mixture H_2 / $^{16}O_2$

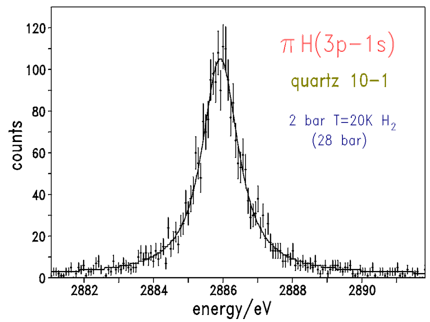
(98%/2%)

1.2 bar @ T = 85K

≈ 4 bar equivalent density

πH / πO

energy calibration simultaneously



H_2

2 bar @ T = 20K

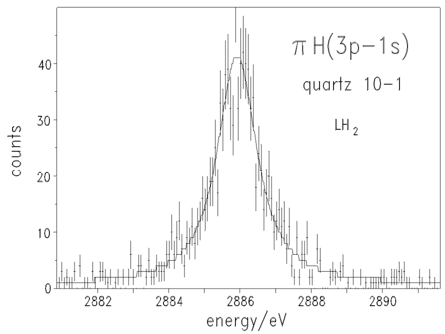
≈ 28.5 bar equivalent density

alternately **πH / πO**

mixture ^4He / $^{16}O_2$ / $^{18}O_2$

(≈ 80%/10%/10%)

2 bar @ T = 86K

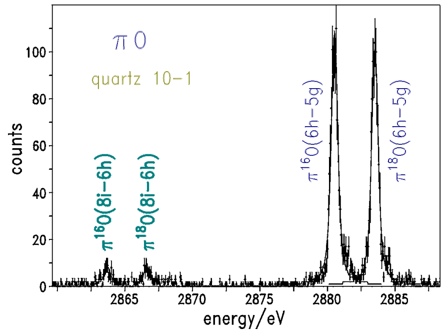


H_2

1 bar @ T = 17K

LH_2

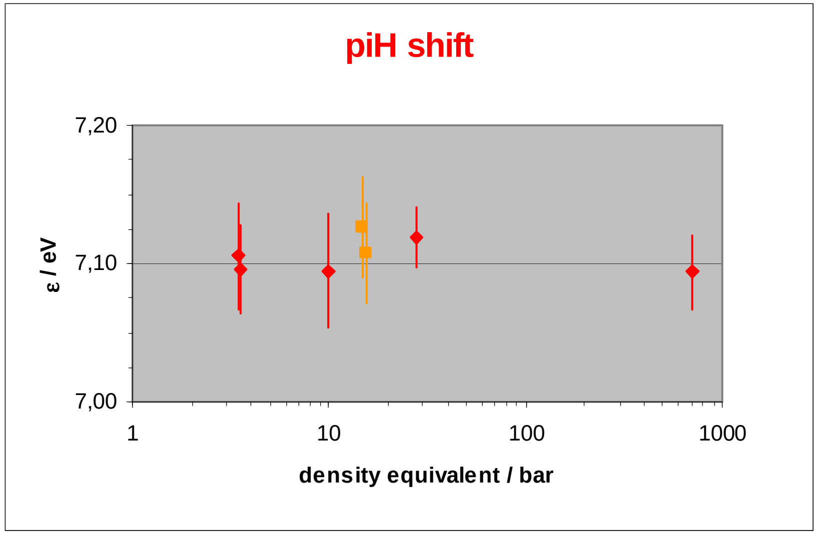
first time



πH(3p-1s) energy

<u>no</u> density dependence identified

$\Rightarrow$ *"no" X-ray transitions from molecular states*



previous experiment – <u>Ar Kα</u>

ETHZ-PSI H.-Ch.Schröder et al. Eur.Phys.J.C 1(2001)473

R-98.01

***Maik Hennebach,** thesis Cologne 2003*

$\varepsilon_{1s} = + 7.120 \pm 0.008 \pm 0.009$ eV

↑

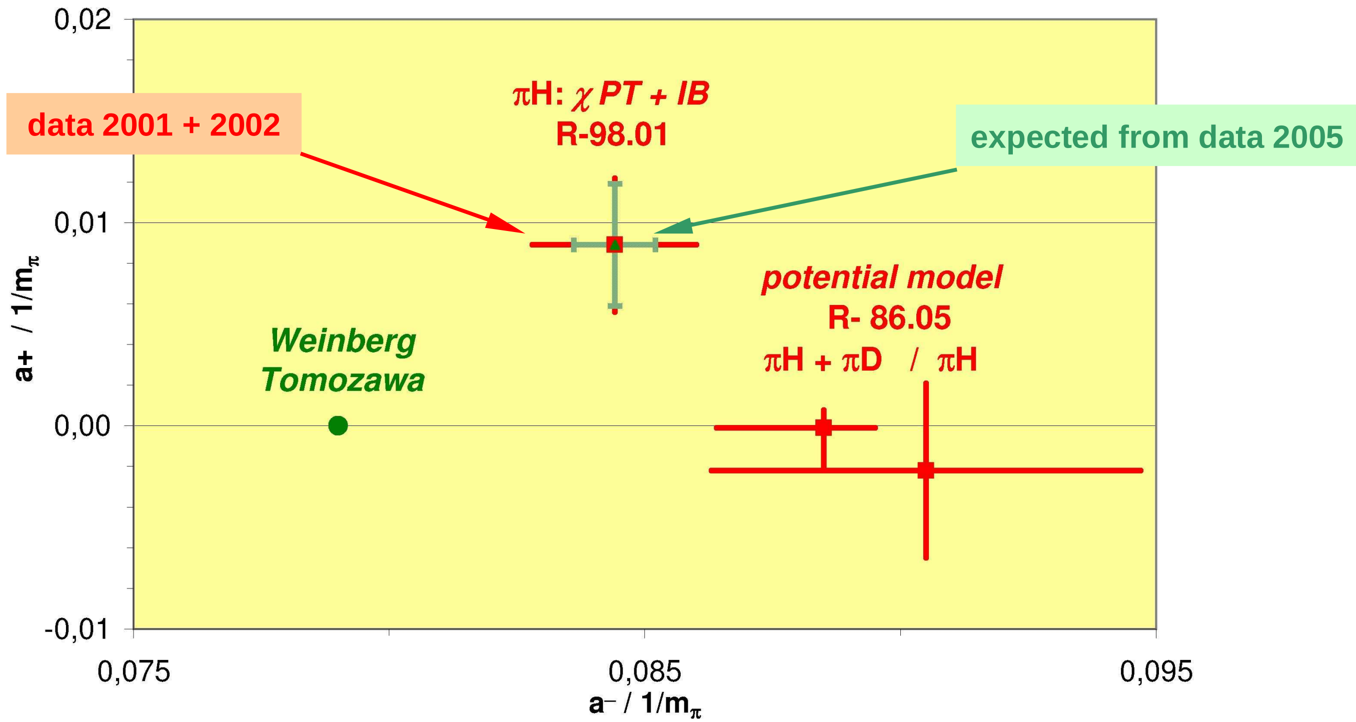
$\Delta E_{QED} = \pm 0.006$ eV !

new calculation $\Rightarrow \Delta E_{QED} = \pm 0.001$ eV !

P. Indelicato, priv. comm.

$$\varepsilon_{1s} = + 7.120 \pm 0.008 \pm 0.006 \text{ eV} \quad (\pm 0.2\%)$$

πN scattering lengths $a^{\pm}$



$\varepsilon_{1s} \propto [\, a^{+} + a^{-}]\,(1 + \delta_{\varepsilon})$ $\quad \delta_{\varepsilon} = -7.2 \pm 2.9\ \%$ *J. Gasser et al., Eur. Phys. J. C 26 (2003) 13*

$\Gamma_{1s} \propto [\, a^{-} \,(1 + \delta_{\Gamma})\,]^{2}$ $\quad \delta_{\Gamma} = +\,0.6 \pm 0.2\ \%$ *P. Zemp, thesis University of Bern 2004*

πH: χPT theory 3rd order

πH - hadronic shift ε_{1s}
&
πN s-wave isospin scattering lengths

Deser formula → *incl. Coulomb - strong-int. interference*

$$\epsilon_{1s} = -2\alpha^3 \mu_c^2 \mathcal{A}\left(1 - 2\alpha\mu_c (\ln\alpha - 1)\mathcal{A}\right) + \cdots$$

Trueman (1961), …

Ericson et al. recently

2nd order χPT

$$\mathcal{A} = a_{0+}^{+} + a_{0+}^{-} + \epsilon$$
$$= \frac{1}{8\pi(m_p + M_{\pi^+})F_\pi^2}$$
$$\times \left\{ m_p M_{\pi^+} - \frac{g_A^2 m_p M_{\pi^+}^2}{m_n + m_p + M_{\pi^+}} + m_p\left(-8c_1 M_{\pi^0}^2 + 4(c_2 + c_3)M_{\pi^+}^2 - 4e^2 f_1 - e^2 f_2\right) \right\},$$

$O(\delta^2)$ in $\delta = q$,
$\alpha = 1/137$,
$(m_d - m_u)$

LECs c_1, f_1, f_2

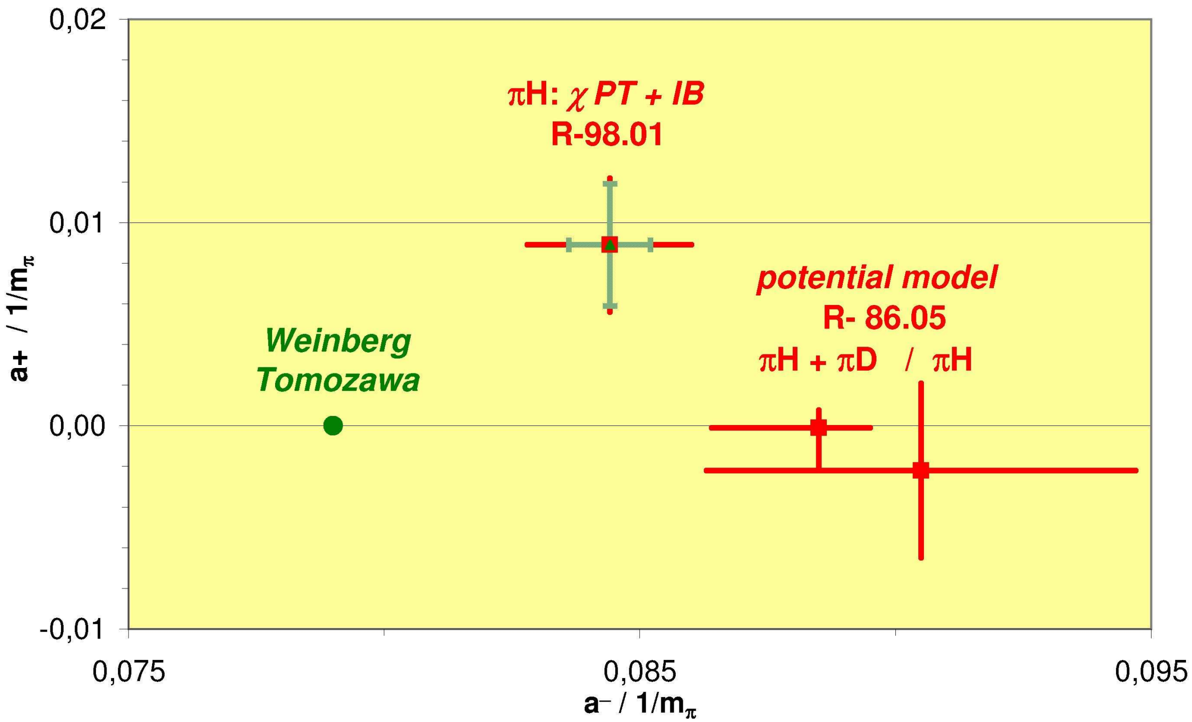
contribute to isospin breaking in $O(\delta^2)$

estimate $|f_1| \leq 1.4\ \mathrm{GeV}^{-1}$

V.E. Lyubovitskij & A. Rusetsky, Phys. Lett. B 494 (2000) 9

V.E. Lyubovitskij et al., Phys. Lett. B 520 (2001) 204

πN scattering lengths $a^{\pm}$



$\delta_\varepsilon = -7.2 \pm 2.9\ \%$ uncertainty: LECs c_1, f_1, f_2 + higher orders

$\pm 1.9\ \%$ from f_1

improvement on c_1 new πN phase shifts

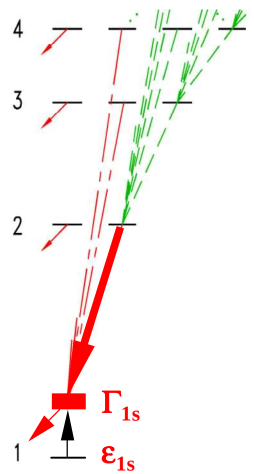
f_2 $pn \rightarrow \pi^0 d$ forward/backward asmmetry

4^{th} order

f_1 ?

ε_{1s}

$\pi D(3p - 1s)$ *transition energy*



density dependence ?

Deser formula

$$\epsilon_{1s} + i\,\Gamma_{1s}/2 = -\,(2\alpha^3 m_{red}^2 c^4/\hbar c)\cdot a_{\pi d}$$ + *Coulomb corrections*

$$\Re\, a_{\pi D} = a_{\pi^- p} + a_{\pi^- n} + \textit{corrections}$$

$$= a^+ + \textit{corrections}$$

corrections are large

single + multiple scattering
absorption

constraint für $a^{\pm}$

$$\Im\, a_{\pi d} \propto (\Gamma_{\pi^- d \to nn} + \Gamma_{\pi^- d \to nn\gamma})$$

access to $\pi NN_{I=0} \leftrightarrow NN_{I=1}$ reaction

Deser formula

$$\epsilon_{1s} + i\,\Gamma_{1s}/2 = -\,(2\alpha^3 m_{red}^2 c^4/\hbar c)\cdot a_{\pi d}$$ **+ *Coulomb corrections***

Single + multiple scattering

$$\Re a_{\pi d} = S + D + \cdots$$

from $\pi D\ \varepsilon_{1s}$

$$\begin{aligned}
&\frac{1+m_\pi/M}{1+m_\pi/M_d}(a_{\pi^- p}+a_{\pi^- n}) \\
&+\ 2\frac{(1+m_\pi/M)^2}{1+m_\pi/M_d}\left[\left(\frac{(a_{\pi^- p}+a_{\pi^- n})}{2}\right)^2 - 2\left(\frac{(a_{\pi^- p}-a_{\pi^- n})}{2}\right)^2\right] < 1/r > \\
&+\cdots
\end{aligned}$$

from $\pi H\ \Gamma_{1s}$

$$\begin{aligned}
= &\ 2\frac{1+m_\pi/M}{1+m_\pi/M_d}a^+ \\
&+\ 2\frac{(1+m_\pi/M)^2}{1+m_\pi/M_d}\left[\left(\frac{a^+}{2}\right)^2 - 2\left(\frac{a^-}{2}\right)^2\right] < 1/r > \\
&+\cdots \qquad (11)
\end{aligned}$$

πD wave function

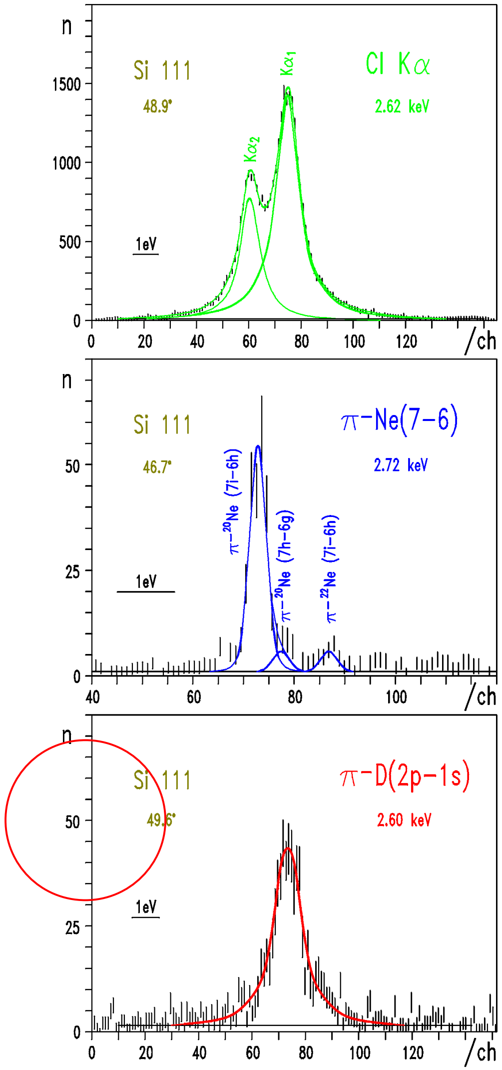
energy calibration I

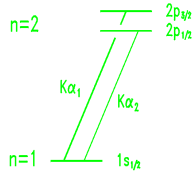
response function I

strong interaction

$\varepsilon_{1s} = -\,2.469 \pm 0.055$ eV
$\Gamma_{1s} = \;\; 1.093 \pm 0.129$ eV

P. Hauser et al., PR C 58 (1998)R1869

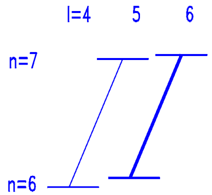




Cl Kα

2.62 keV

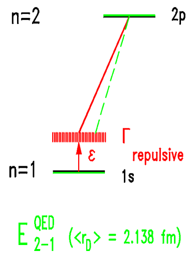
15 min



πNe(7-6)

2.72 keV

12 h

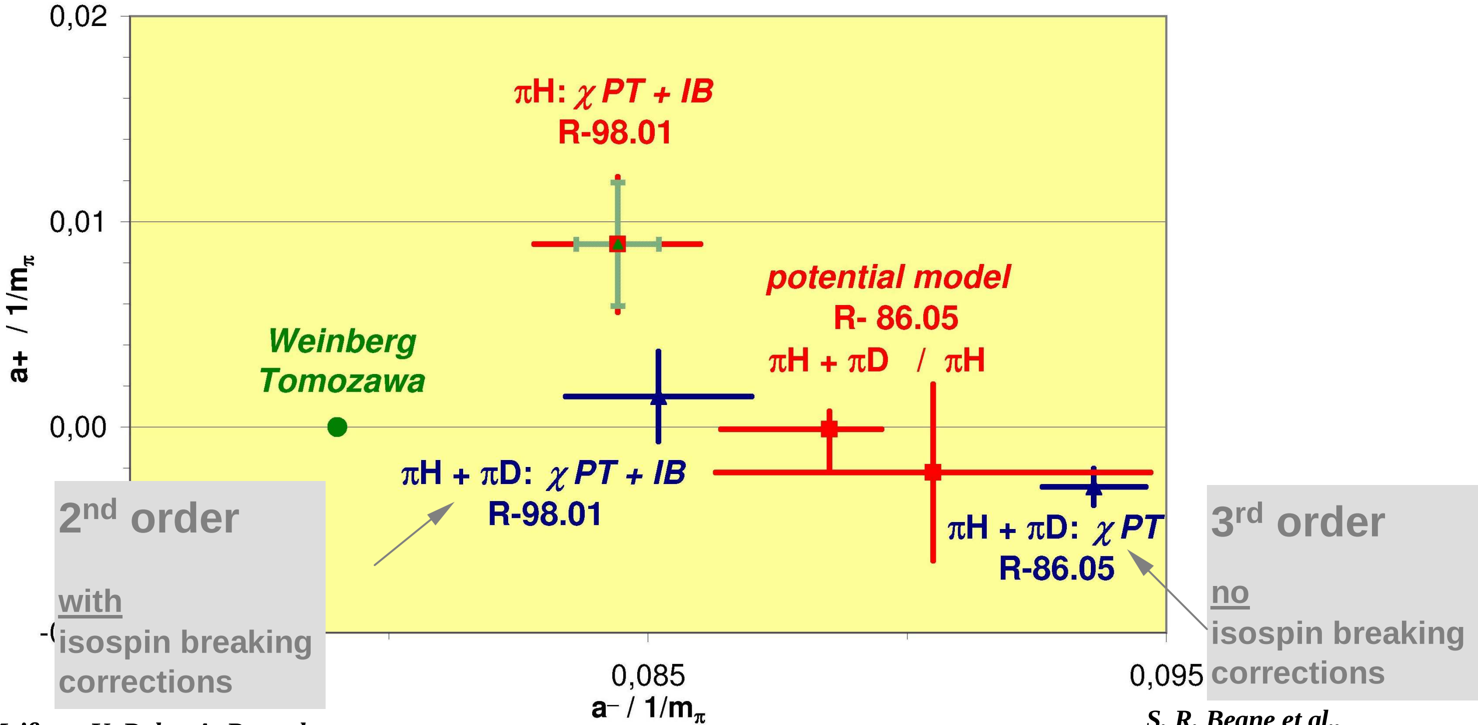


πD(2p-1s)

2.60 keV

15 h

πN scattering lengths $a^{\pm}$



U.-G. Meißner, U. Raha, A. Rusetsky
arXiv:nucl-th/0512035

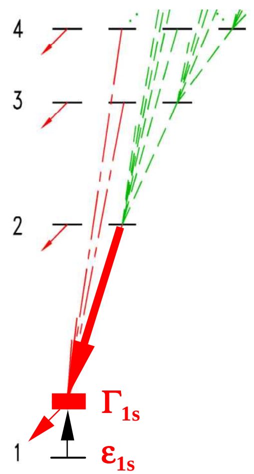
S. R. Beane et al.,
Nucl. Phys. A 720 (2003) 399

1. $a^{\pm}$ from πH (ε_{1s}, Γ_{1s}) and πD (ε_{1s}) <u>must</u> fit !

2. correlated fit (a^{+}, a^{-}, f_1)

Γ_{1s}

$\pi D(3p - 1s)$ transition energy



to be corrected for Doppler broadening

Deser formula

$$\epsilon_{1s} + i\,\Gamma_{1s}/2 = -\,(2\alpha^3 m_{red}^2 c^4/\hbar c)\cdot a_{\pi d}$$ + Coulomb corrections

$$\Re\, a_{\pi D} = a_{\pi^- p} + a_{\pi^- n} + \text{corrections}$$
$$= a^+ + \text{corrections}$$

corrections are large

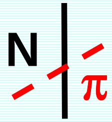
single + multiple scattering
absorption

constraint für $a^{\pm}$

$$\Im\, a_{\pi d} \propto (\Gamma_{\pi^- d \to nn} + \Gamma_{\pi^- d \to nn\gamma})$$

access to $\pi NN_{I=0} \leftrightarrow NN_{I=1}$ reaction

origin of Γ_{1s}

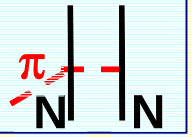
CEX scattering 

πH *scattering* $\pi^{-}p \rightarrow \pi^{0}n + n\gamma$

radiative capture

BR are well known

πD *absorption* $\pi^{-}d \rightarrow nn + nn\gamma$

„true" absorption 

πNN threshold parameter α

charge symmetry — *detailled balance*

$$\sigma_{\pi^- d \to nn} \leftrightarrow \sigma_{\pi^+ d \to pp} \leftrightarrow \sigma_{pp \to \pi^+ d}$$

$NN \quad {}^3S_1(I=0) \to {}^3P_1(I=1)$

πD atom

$$\Im a_{\pi D} \propto \Gamma_{\pi^- d \to nn} + \Gamma_{\pi^- d \to nn\gamma}$$

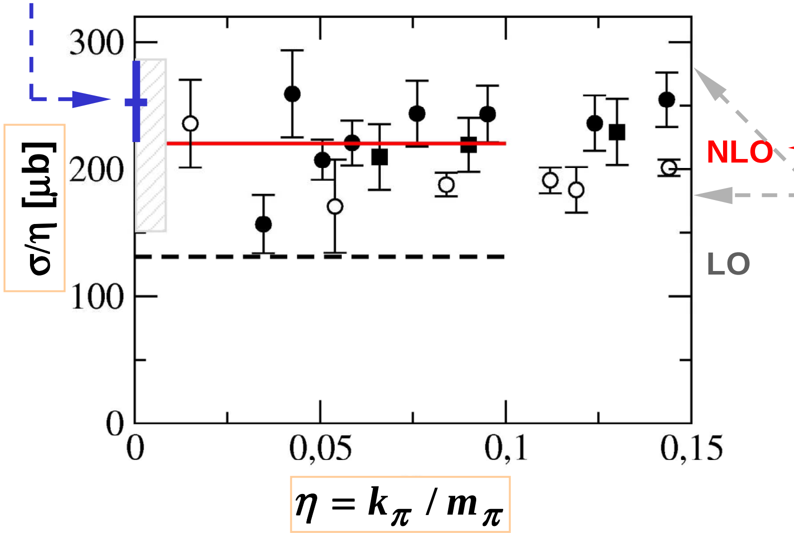
$$= \frac{1}{6\pi} m_p \cdot \alpha$$

J. Hüfner, Phys. Rep. 21 (1975) 1

π production

$$\sigma_{pp \to \pi^+ d} \to \alpha\, C_0^2\, \eta + \beta\, C_1^2\, \eta^3$$

extrapolation to threshold



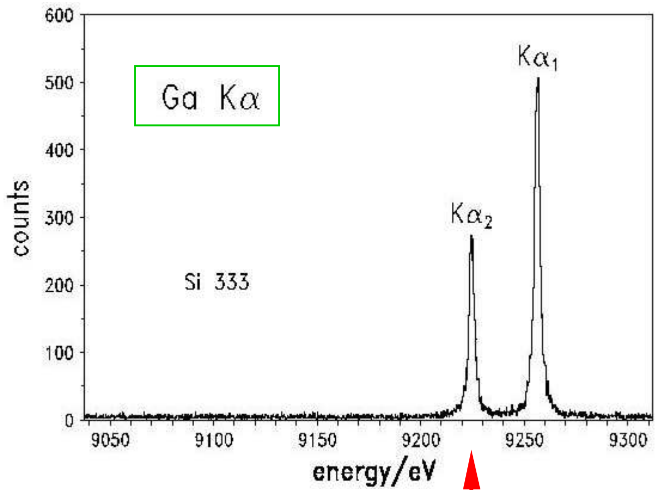
χPT

at present $\Delta\alpha / \alpha \approx 30\%$

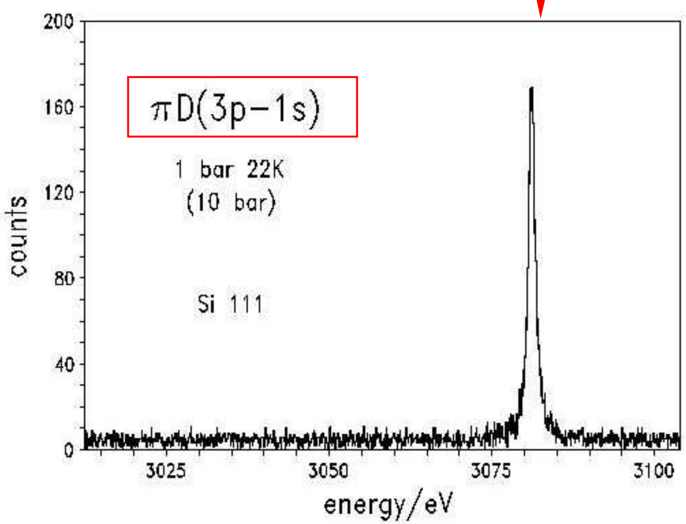
V. Lensky et al., nucl-th/0511054,2005

→ few %

Monte-Carlo simulations



same Bragg angle: 40°



energy calibration ***3. order reflection***

Ga $K\alpha_1$ ***9257.67 ± 0.066 eV***

$K\alpha_2$ ***9224.84 ± 0.027 eV***

2001: πO(6h-5g)

crystal response ***Ar*** $^{16+}$ ***M1*** ***3104 eV***

πD(3p-1s) at 3 densities

foreseen 3.5 bar, 28 bar and LD_2

count rate 2000- 5000 /week (500 Cb)

similar to 2001

Coulomb de-excitation like in πH

OUTLOOK

PIONIC TRITIUM & HELIUM

EXPERIMENT

	ε_{1s}	Γ_{1s}		Γ_{2p}
π^3He	32 ± 3 eV	28 ± 7 eV	a	1.6 ± 0.8 meV *
	34 ± 4 eV	36 ± 7 eV	b	
πT		2.0 ± 0.4 eV +		
π^4He	- 75.7 ± 2.0 eV	45 ± 3 eV	c	0.7 ± 0.3 meV * 2.2 ± 0.3 meV **

a I.Schwanner et al., NP A 412 (1984) 253, measurement in 1979)

b G. R. Mason et al., NP A340 (1980) 240

c G. Backenstoss et al., NP A 232 (1974) 519

\+ Werntz et al. predicted from radiative capture $\pi T \rightarrow \gamma nnn$

* deduced from fit to K yields

** Mol. Ion f. cascade model (input ε_{1s} , Γ_{1s})

Theory T/^{3}He comparison: Baru, Haidenbauer, Hanhart, Niskanen, Eur. Phys. J. A 16 (2003) 437

PION - NUCLEAR SCATTERING LENGTH

	$\Re\, a_{\pi A}$	$\Im\, a_{\pi A}$
p	+ 0.0883 ± 0.0008	
n	- 0.0907 ± 0.0016	
d	- 0.0261 ± 0.0005	- 0.0063 ± 0.0007
^{3}He	+ 0.043 ± 0.004	0.019 ± 0.005
^{4}He	- 0.098 ± 0.003	0.030 ± 0.002

Elementary reactions

$$\pi^-_{1s}\, pp \rightarrow pn \qquad {}^1S_0(I=1) \rightarrow {}^3P_0(I=1) \qquad g_1$$

$$\pi^-_{1s}\, pn \rightarrow nn \qquad {}^3S_1(I=0) \rightarrow {}^3P_1(I=1) \qquad g_0$$

„πD“

$$ {}^1S_0(I=1) \rightarrow {}^3P_0(I=1) \qquad g_1$$

Isospin decomposition

$$\frac{\Gamma(\pi^- pn \rightarrow nn)}{\Gamma(\pi^- pp \rightarrow pn)} = \frac{\frac{1}{4}\Gamma_{11} + \frac{1}{2}\Gamma_{01}}{\frac{1}{4}\Gamma_{11} + \frac{1}{6}\Gamma_{01}} \qquad \Gamma_{if}$$

$$\pi_{ns} = \frac{\frac{1}{4}\Gamma_{11} + \frac{1}{2}\Gamma_{01}}{\frac{1}{4}\Gamma_{11}}$$

$$\pi_{np} = \frac{\frac{1}{2}\Gamma_{01}}{\frac{1}{6}\Gamma_{10}}$$

NN pairs

	$pp\ {}^1S_0(I=1)$	$pn\ {}^1S_0(I=1)$	$pn\ {}^3S_1(I=0)$
D	$-$	$-$	1
T	$-$	$\frac{1}{2}$	$\frac{3}{2}$
^3He	1	$\frac{1}{2}$	$\frac{3}{2}$
^4He	1	1	3

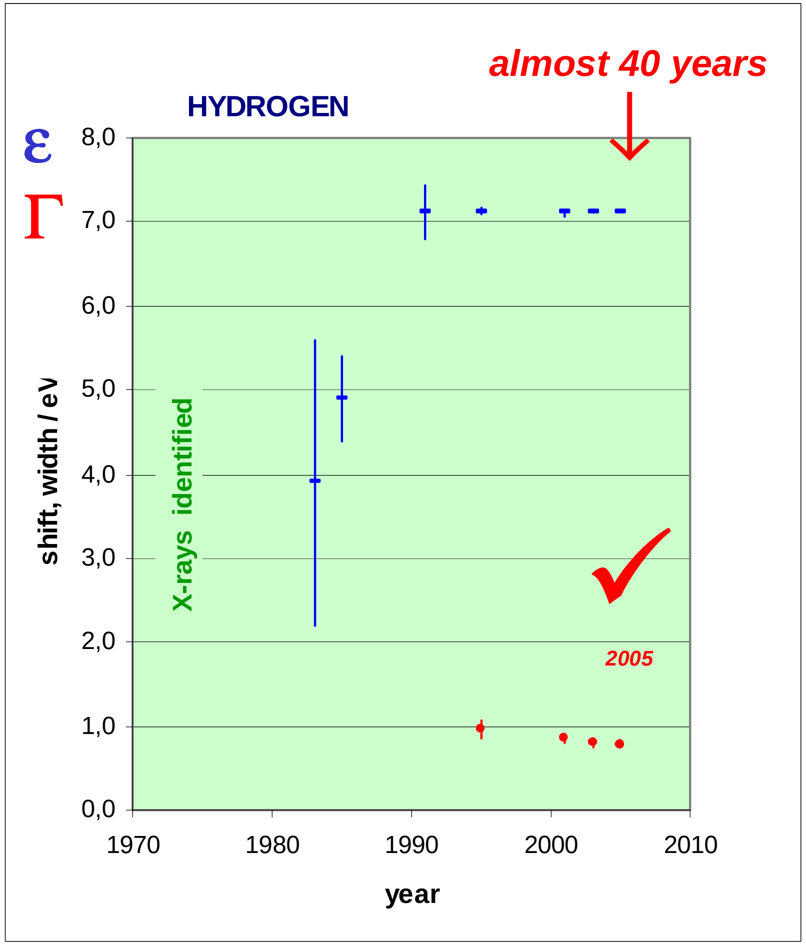
$$\Gamma = \Gamma_{true\ absorption} + \Gamma_{charge\ exchange} + \Gamma_{radiative\ capture}$$

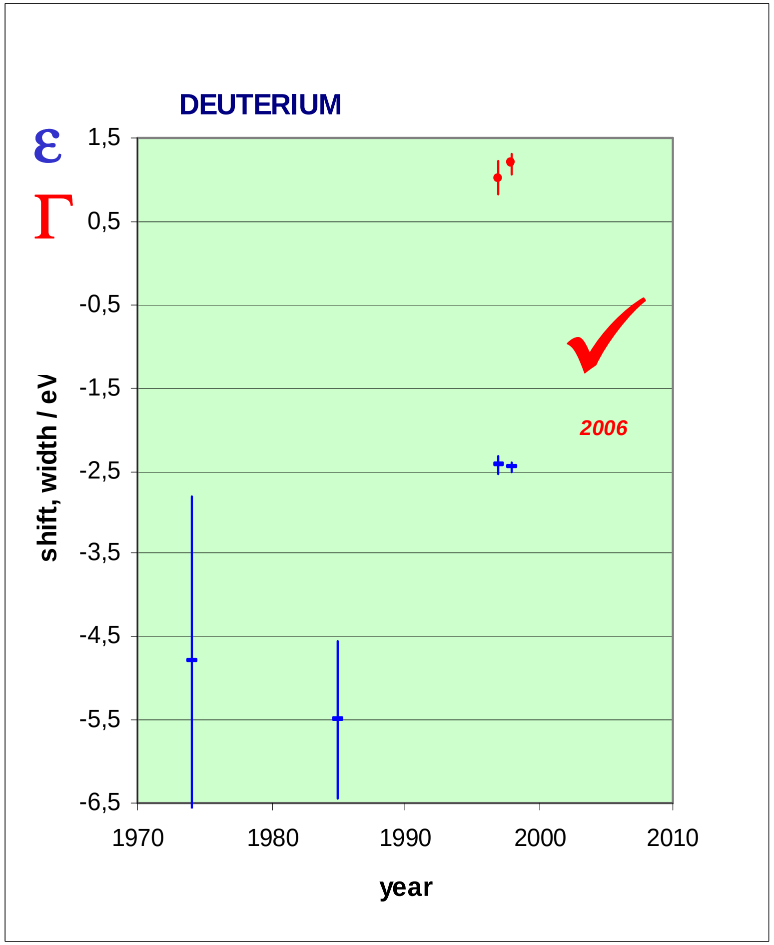
Experiment

	$\left(\frac{nn}{pn}\right)_{ns}$	$\left(\frac{nn}{pn}\right)_{np}$
^3He	6.3 ± 1.1	<18
^4He	$12.9^{+2.3}_{-4.6}$	$-$

SUMMARY

PIONIC HYDROGEN - HISTORY

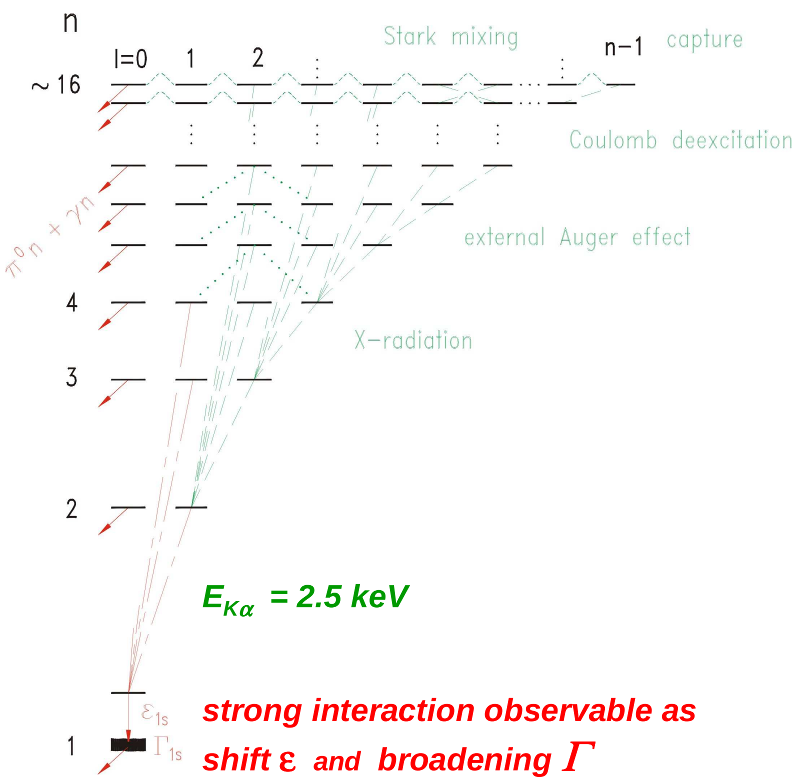




π^3He ⇔ π^3He, π^4He ?

APPENDICES

PIONIC HYDROGEN - πN scattering at „rest“



$E_{K\alpha}$ = 2.5 keV

strong interaction observable as shift ε and broadening Γ

ε_{1s} + 7 eV attractive

Γ_{1s} 1 eV

2 isospin scattering length

$a^{\pm} = a_{\pi^- p \to \pi^- p} \pm a_{\pi^+ p \to \pi^+ p}$

isospin invariance: $m_u = m_d$

$a_{\pi^- p \to \pi^- p} + a_{\pi^+ p \to \pi^+ p} = -\sqrt{2}\, a_{\pi^- p \to \pi^0 n}$

THEORY

$\varepsilon_{1s} \propto a_{\pi^- p \to \pi^- p}$

$\propto (a^+ + a^-)\cdot(1 + \delta_\varepsilon)$

$\Gamma_{1s} \propto (1+1/P)\cdot(a_{\pi^- p \to \pi^0 n})^2$

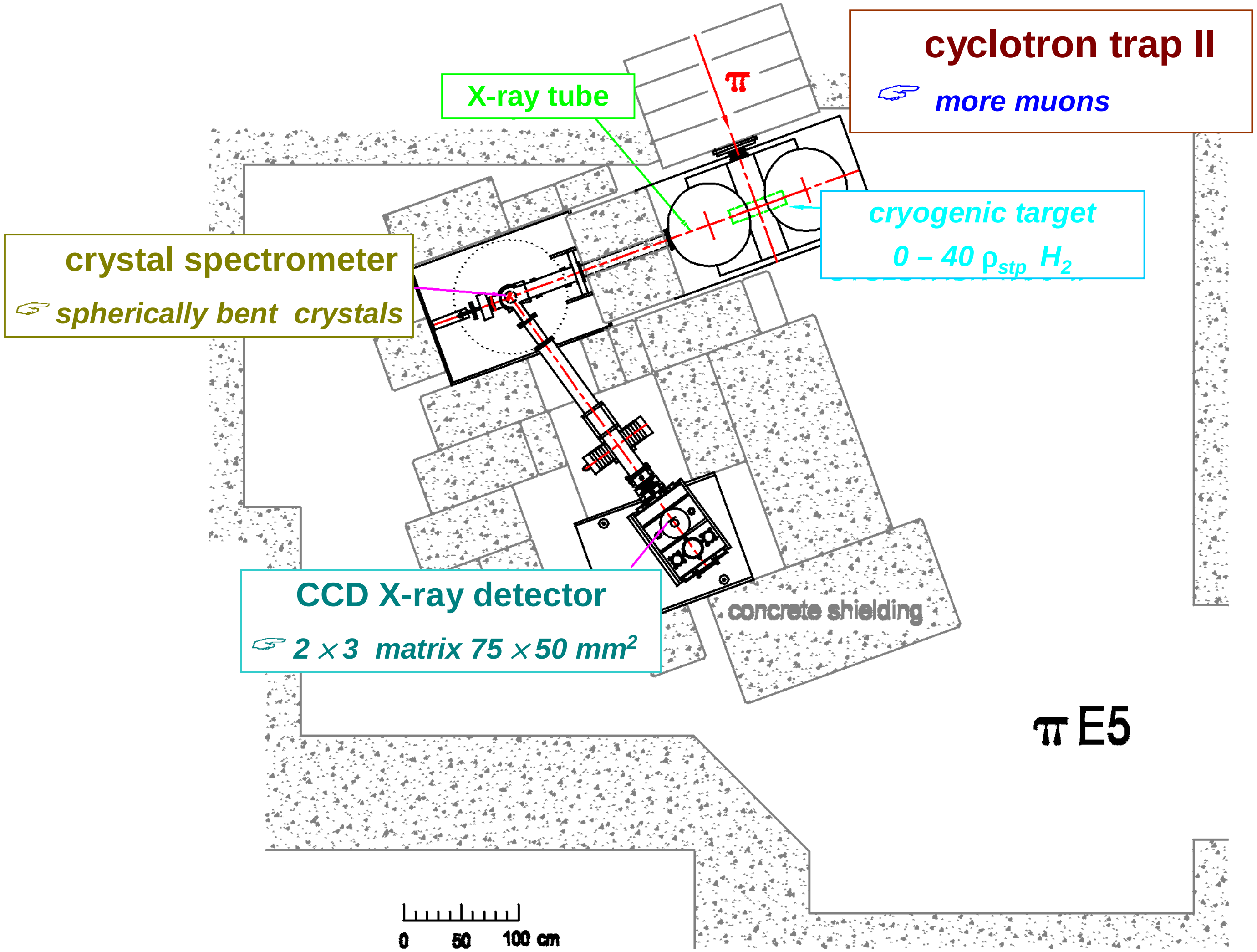
$\propto (1+1/P)\cdot(a^-(1 + \delta_\Gamma))^2$

PANOFSKY ratio P

$\pi^- p \to \pi^0 n / \pi^- p \to \gamma n = 1.546 \pm 0.009$

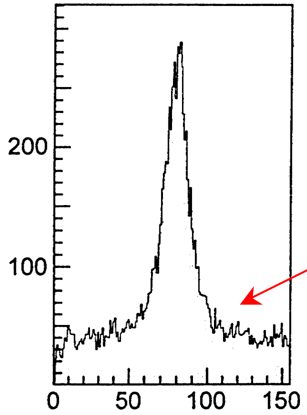
J. Spuller et al., Phys. Lett. 67 B (1977) 479

SET-UP at PSI

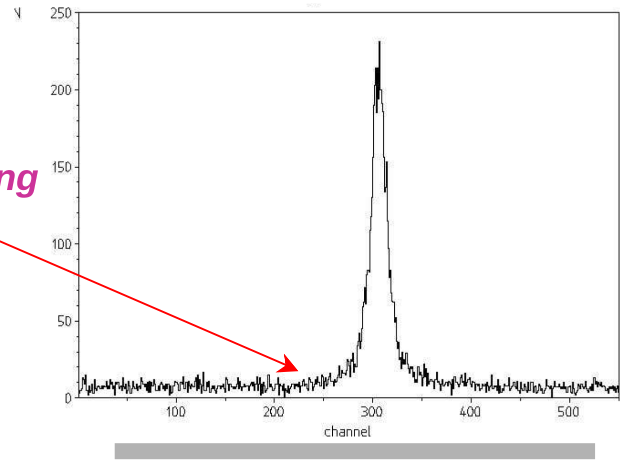

cyclotron trap II
☞ more muons
X-ray tube
π
cryogenic target
0 – 40 ρ_stp H_2
crystal spectrometer
☞ spherically bent crystals
CCD X-ray detector
☞ 2 × 3 matrix 75 × 50 mm²
concrete shielding
π E5
0
50
100 cm

PEAK / BACKGROUND *and* FIT INTERVAL !

massive concrete shielding
+
large area X-ray detector



Lorentz tails
Doppler broadening



↑
previous experiment

↑
new experiment

PEAK-TO-BACKGROUND ratio improved by one order of magnitude !

Connection to perturbation theory

scattering lengths

LO: current algebra
Weinberg,Tomozawa:
chiral limit $m_{quark} \to 0$

$$\left.\begin{array}{l} a^{+} = 0 \\ a^{-} = -0.079 / m_{\pi} \\ f^{2}_{\pi N} = \dfrac{m^{2}_{\pi}}{4} \dfrac{g^{2}_{A}}{F^{2}_{\pi}} = 0.072 \end{array}\right\} + \textit{higher orders } \chi PT$$

Goldberger-Treiman relation
πN coupling constant $f^2_{\pi N}$

Sigma term
explicit chiral symmetry breaking

$$\sigma_N = \frac{m_u + m_d}{4M} \langle p | \bar{u}u + \bar{d}d | p \rangle \leftrightarrow \langle p | \bar{s}s | p \rangle \text{ contents}$$

πN sigma-term σ_N

$$\left(1+\frac{m_\pi}{M}\right) a^{+} = \frac{m^2_\pi}{4\pi f^2_\pi} \cdot \left(\frac{\sigma_N}{m^2_\pi} + d - \frac{g^2_A}{4M_N} \right)$$

πN coupling constant $f_{\pi N}$

$$\left(1+\frac{m_\pi}{M}\right) \frac{a^{-}}{m_\pi} = \frac{2 f^2_{\pi N}}{m^2_\pi - (m^2_\pi / 2M)^2} + \frac{1}{2\pi^2} \int_0^\infty \frac{\sigma^{tot}_{\pi^- p}(k_\pi) - \sigma^{tot}_{\pi^+ p}(k_\pi)}{2\omega(k_\pi)} dk_\pi$$

Goldberger- Miyazawa-Oehme
(GMO)
sum rule

$\Delta \int \approx 1\%$

Formulae πH

Coulomb corrections

$$\epsilon_{1s} = -2\alpha^3\mu_r^2(a^+ + a^-)(1+\delta_\epsilon)$$

$$\epsilon_{1s} = -2\alpha^3\mu_c^2\mathcal{A}(1 - 2\alpha\mu_c(\ln\alpha - 1)\mathcal{A}) + \cdots$$

CHPT 2. order

V.E. Lyubovitskij & A. Rusetsky, Phys. Lett. B 494 (2000) 9

$$\mathcal{A} = a_{0+}^+ + a_{0+}^- + \epsilon = \frac{1}{8\pi(m_p + M_{\pi^+})F_\pi^2} \times \left\{ m_p M_{\pi^+} - \frac{g_A^2 m_p M_{\pi^+}^2}{m_n + m_p + M_{\pi^+}} + m_p\left(-8c_1 M_{\pi^0}^2 + 4(c_2+c_3)M_{\pi^+}^2 - 4e^2 f_1 - e^2 f_2\right)\right\},$$

CHPT 3. order $\delta_\epsilon = -7.2 \pm 2.9\ \%$ *J. Gasser et al., Eur. Phys. J. C 26 (2003) 13*

$$\Gamma_{1s} = 8\,\alpha^3 M_r^2\, p_0^\star \left(1 + \frac{1}{P}\right) [a_{0+}^-(1+\delta_\Gamma)]^2$$

$\delta_\Gamma = (0.6 \pm 0.2) \times 10^{-2}$ [CHPT, leading order]

$$\Gamma_{1s} = 8\,\alpha^3 M_r^2\, p_1^\star \left(1 + \frac{1}{P}\right) \mathcal{A}^2\ (1 + K + \delta_\epsilon^{\text{vac}})$$

$$K = 4M_r\,\alpha\,(1 - \ln\alpha)(a_{0+}^+ + a_{0+}^-) + 2M_r(M_\Sigma - \bar{M}_\Sigma)(a_{0+}^+)^2$$

$$\mathcal{A} = a_{0+}^- + \epsilon$$

P. Zemp, thesis University of Bern 2004

$$\epsilon = -\frac{1}{16F_\pi^2\pi M_\Sigma}\left[2e^2 f_2 F_\pi^2 m_p + g_A^2(M_{\pi^+}^2 - M_{\pi^0}^2)\right] + \mathcal{O}(p^3)$$

Formulae πD

U.-G. Meißner, U. Raha, A. Rusetsky, arXiv:nucl-th/0512035

$$\epsilon_{1s}^{d} = -2\alpha^3\mu_d^2\,\mathrm{Re}\,a_{\pi d}$$ **+ *Coulomb corrections***

$$\mathrm{Re}\,a_{\pi d}^{\mathrm{exp}} = -(0.0261 \pm 0.0005) M_\pi^{-1}$$

P. Hauser et al., PR C 58 (1998)R1869

$$\begin{aligned}\mathrm{Re}\,\tilde{a}_{\pi d} = {} & 2\,\frac{1+\mu}{1+\mu/2}\,a^+ \\ & + 2\,\frac{(1+\mu)^2}{1+\mu/2}\,\left((a^+)^2 - 2(a^-)^2\right)\frac{1}{2\pi^2}\left\langle\frac{1}{\mathbf{q}^2}\right\rangle_{\mathrm{wf}} \\ & + 2\,\frac{(1+\mu)^3}{1+\mu/2}\,\left((a^+)^3 - 2(a^-)^2(a^+ - a^-)\right)\frac{1}{4\pi}\left\langle\frac{1}{|\mathbf{q}|}\right\rangle_{\mathrm{wf}} \\ & + a_{\mathrm{boost}} + \cdots , \qquad (3)\end{aligned}$$

40%!

$$\begin{aligned}\mathrm{Re}\,a_{\pi d} &= \mathrm{Re}\,\tilde{a}_{\pi d} + \Delta a_{\pi d}, \\ \Delta a_{\pi d} &= A_1\alpha + A_2(m_d - m_u) + O(\delta^2),\end{aligned}$$

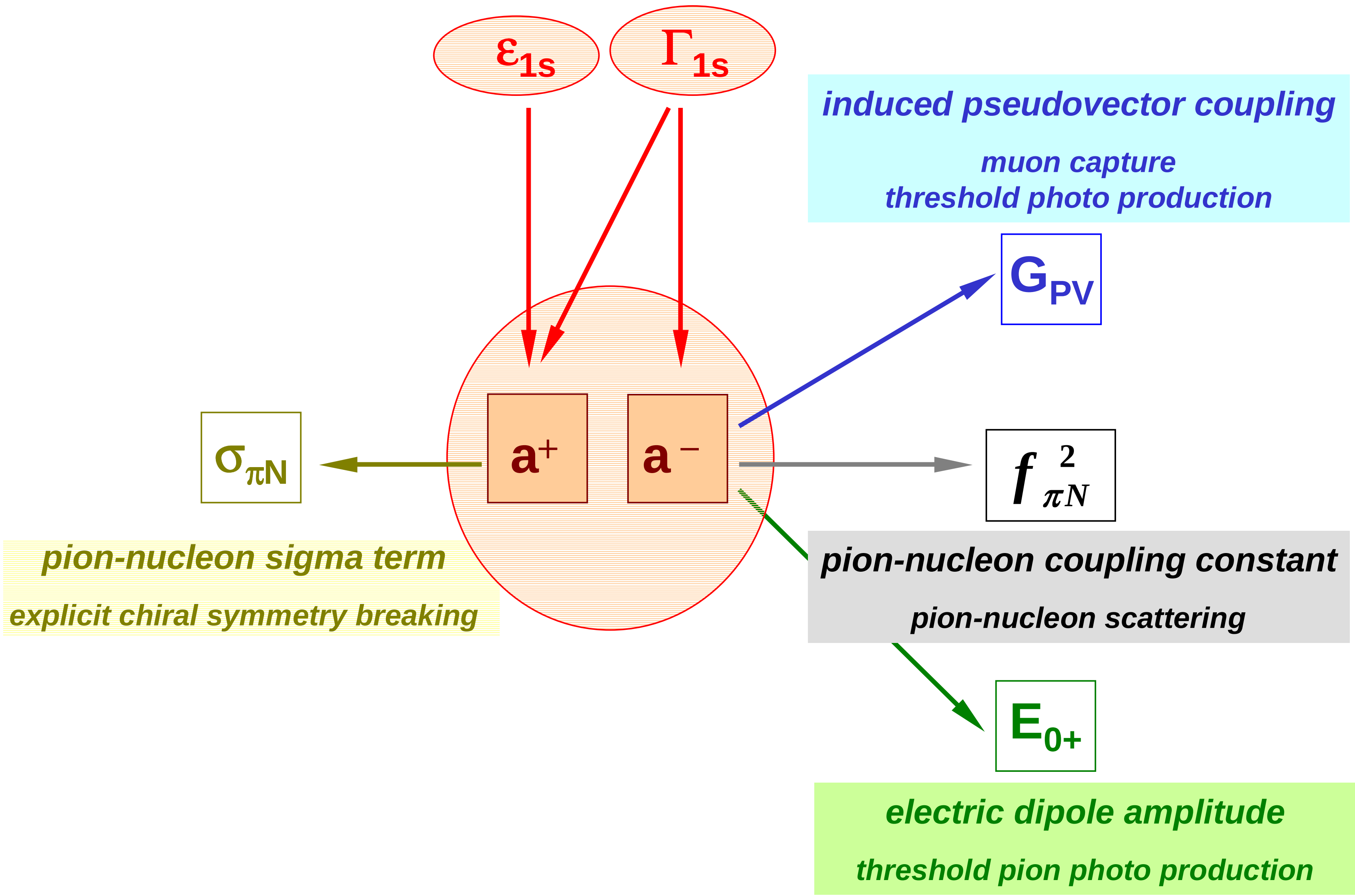
$$\Delta a_{\pi d}^{\mathrm{LO}} = -(0.0110^{+0.0081}_{-0.0058})\,M_\pi^{-1}$$

$$\begin{aligned}\Delta a_{\pi d} &= \Delta a_{\pi d}^{\mathrm{LO}} + O(p^3) \\ \Delta a_{\pi d}^{\mathrm{LO}} &= (4\pi(1+\mu/2))^{-1}(\delta T_p + \delta T_n)\end{aligned}$$

$$\begin{aligned}\delta T_p &= \frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2}\,c_1 - \frac{e^2}{2}(4f_1 + f_2) + O(p^3) \\ \delta T_n &= \frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2}\,c_1 - \frac{e^2}{2}(4f_1 - f_2) + O(p^3), \\ \delta T_x &= -\sqrt{2}\left(\frac{g_A^2(M_\pi^2 - M_{\pi^0}^2)}{4m_pF_\pi^2} + \frac{e^2 f_2}{2}\right) + O(p^3),\end{aligned}$$

$$\begin{aligned}a^+ &= (0.0015 \pm 0.0022)\,M_\pi^{-1} \\ a^- &= (0.0852 \pm 0.0018)\,M_\pi^{-1}\end{aligned}$$

$$f_1 = -2.1^{+3.2}_{-2.2}\ \mathrm{GeV}^{-1}$$


ε_{1s}
Γ_{1s}
induced pseudovector coupling
muon capture
threshold photo production
G_{PV}
$\sigma_{\pi N}$
a^+
a^-
$f^2_{\pi N}$
pion-nucleon sigma term
explicit chiral symmetry breaking
pion-nucleon coupling constant
pion-nucleon scattering
E_{0+}
electric dipole amplitude
threshold pion photo production