8 TH GEORGIAN – GERMAN SCHOOL AND WORKSHOP
IN BASIC SCIENCE (GGSWBS‘ 18)

August 20 – 25, 2018 • Tbilisi, Georgia

MC Simulation in Particle Physics

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Introduction. Basic principles of experiment.

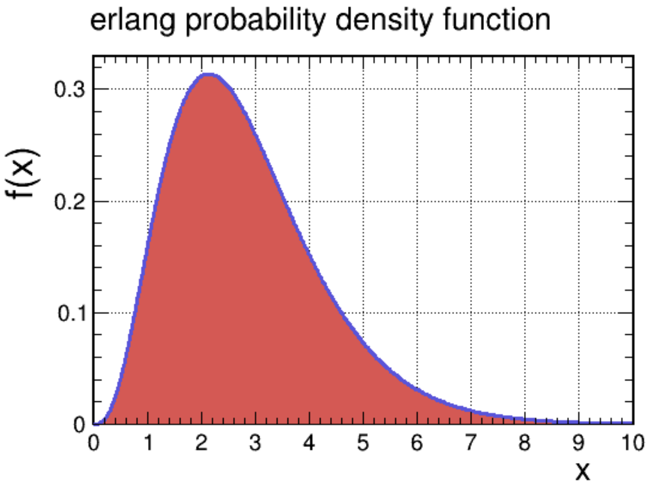
All measured quantities are random numbers (e.g. c, electron mass, proton mass, Avogadro number, etc), but not π, e, Euler constant !

In general the measured physical parameter is presented as an evaluation:
$X \pm \sigma$, where σ denotes the measurement uncertainty.

σ is more important than X value itself, because it defines our confidence in the result.

Random variable/ random number

– is not a single number but a set of numbers; (the set is called a sampling)
– all random numbers have a probability distribution function and corresponding probability density function (pdf);
– most frequently random numbers usually are statistically independent and distributed with the same pdf (iid);
– time series are statistically dependent random samples (except the white noise);



$$\int_{-\infty}^{+\infty} f(x)dx = 1; \quad f(x) \geqslant 0;$$

$$p(x) = f(x)dx$$

Random variables

– statistically independent;
– time series (statistically dependent), autocorrelation;
– correlated with some unknown factors;
– randomness comes also from a measurement process;

Monte Carlo simulation method is a numerical statistical sampling method.

Random variables distribution functions

- uniform (0,1) – is used to generate variables with any pdf
- gaus – most important and used pdf
- exponential – describes the radioactive decay, flux attenuation
- gamma –
- Poisson – almost all counting values
- Student's – mean value differences,...

and many others

Truly random variables: electronics white noise, radioactive decay, cosmic ray arrival, thermal noise,

Computer generates pseudo random numbers. This means that the sequence of these numbers is always the same and is defined by the algorithm so is deterministic. The period is restricted by maximal integer used. Actually they are not random.

Simulation task

– model development: i.e. problem definition, collection of available data, mathematical description, coding;

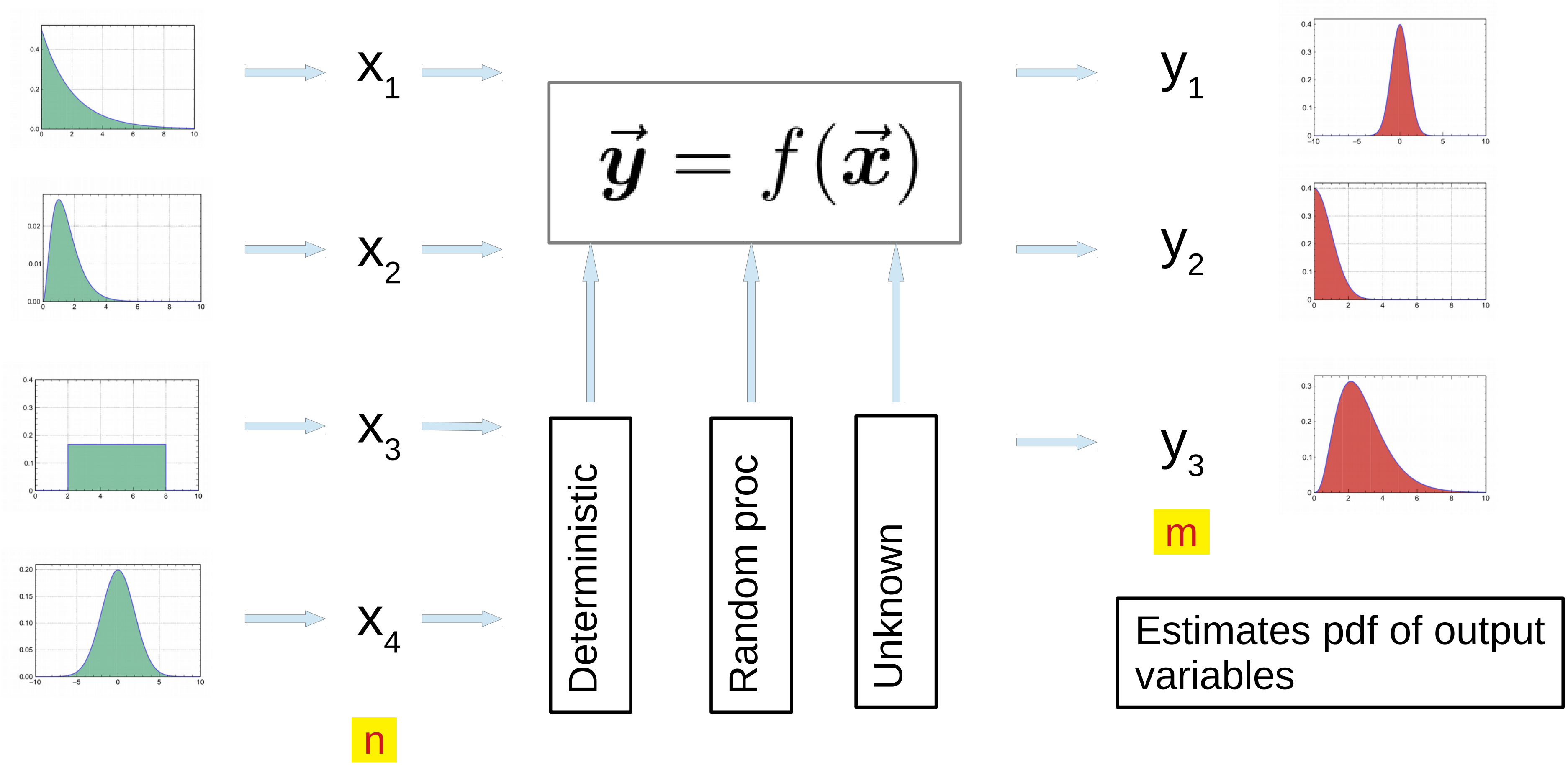
– model validation: quantitative comparison to existing data, noncontroversal;

– simulation;

– analysis;

Simulation in particle physics

A model



Simulation in particle physics

Deterministic model.

System development in time or its behavior for infinite number of events is the same and completely is defined by the model.

So the input multivariate state uniquely defines the output. e.g. accelerator beam development in time (mathematically it is a solution of differential equation).

Randomized model.

The system is undergone unpredictable influence of intrinsic or environmental factors

Most complicated case, but MC method is the only method to get an appropriate solution for this case

System with unknown behavior.

When input and corresponding output is available, but mechanism is unknown.

An Artificial Neural Network is used to replace model. ANN is a nonparametric nonlinear model, nevertheless useful. <u>This issue is out of the following discussion.</u>

Deterministic model

Examples:

– solution of complex differential equation in aerodynamics;

– systems that evolve over time;

– thermodynamics

Example of deterministic model

proton sychrotron longitudinal phase space development during acceleration and its dependence on the accelerator parameters.

$$\frac{d}{dt}(\phi - \phi_s) = \frac{\omega_s^2 \eta}{\beta_s^2 E_s} \frac{\delta E}{\omega_s}$$

$$\frac{d}{dt}\left(\frac{\delta E}{\omega_s}\right) = \frac{eV_o}{2\pi h}(\sin\phi - \sin\phi_s)$$

Diff. Equations describe a longitudinal motion of protons in a synchrotron. Two variable describing a particle Longitudinal phase space are

δE – energy gain, and

φ – phase

$$\phi_s = asin\left(\frac{\dot{B}\,\rho_m\,C_o\,\beta}{eV_o}\right).$$

Accelerating phase advance and the bending magnetic filed are functionally connected.

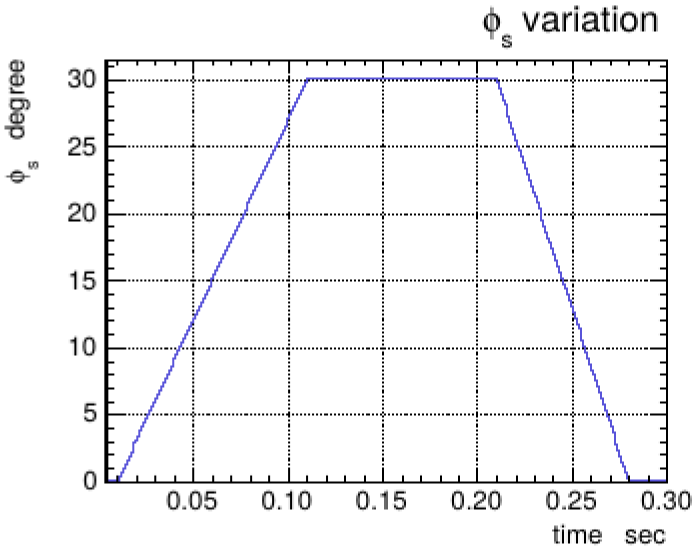


Figure 1: ϕ_s variation in time. It is used as an independent variable.

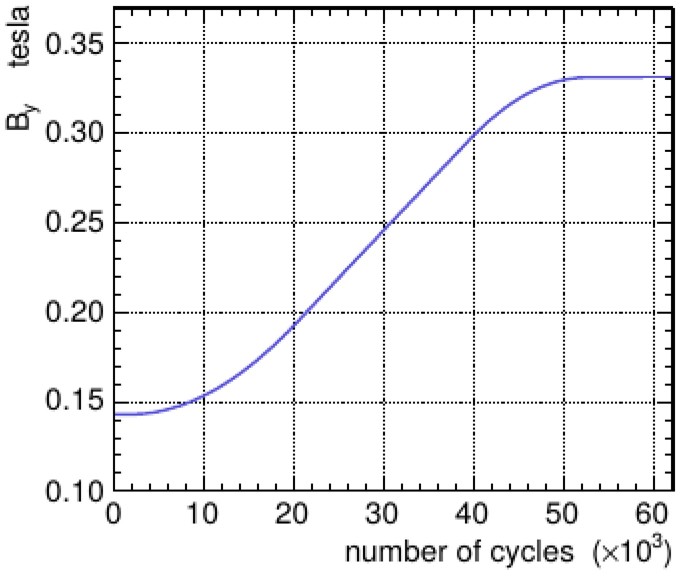
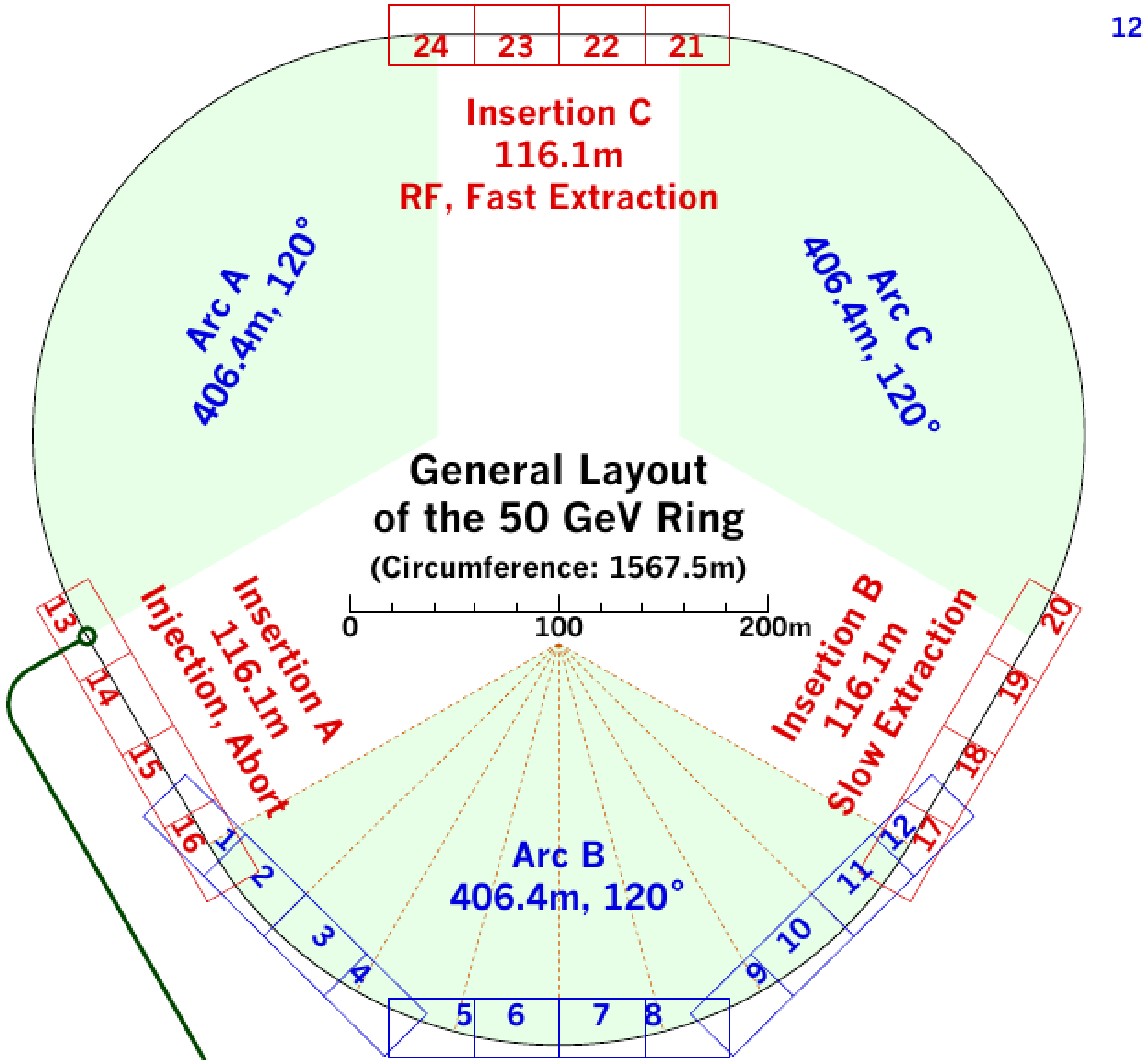


Figure 2: Magnet field strength evolution during acceleration.

JPARC MR parameters

Parameter	symbol	unit	value	comments
MR circumference	C_o	m	1567.5	
MR equilibrium radius	R_o	m	249.475	$= C_o/2\pi$
MR effective radius	ρ_m	m	89.381	$= 96\,L_m/2\pi$
Bending magnet length	L_m	m	5.85	
Bending factor	$B\,\rho_m$	$T\,m$	12.7584	
Transition gamma	γ_t		-31.6	transition at $28\,GeV$
Longitudinal emittance (inj.)	ϵ	$eV\,s$	10.75	
Longitudinal emittance (extr.)	ϵ	$eV\,s$	10.75	
$\Delta p/p$ at injection	$\pm\Delta p/p$	%	± 0.67	
ϑ for the matched beam	$\pm\vartheta$	deg	± 8.5	full width $\simeq 250$
Max value of synchronous phase ϕ_s	ϕ_s^{max}	deg	30	
Base harmonic number	h		9	$\omega_s = h\,\Omega_s$
Base harmonic peak RF voltage	V_o	kV	280	$6\,cavities\ (47\,kV)$
Second harmonic number	h_1		18	
Second harmonic peak RF voltage	V_1	kV	140	$3\,cavities\ (47\,kV)$
Injection kinetic energy	E_o	GeV	3	
synchrotron tune $(3\,GeV)$	ν_s		0.0025	
synchrotron tune $(8\,GeV)$	ν_s		0.0006	
synchrotron tune $(50\,GeV)$	ν_s		0.0001	



24 23 22 21
Insertion C
116.1m
RF, Fast Extraction
Arc A
406.4m, 120°
Arc C
406.4m, 120°
General Layout
of the 50 GeV Ring
(Circumference: 1567.5m)
0 100 200m
Insertion A
116.1m
Injection, Abort
Insertion B
116.1m
Slow Extraction
Arc B
406.4m, 120°
13 14 15 16 17 18 19 20
1 2 3 4 5 6 7 8 9 10 11 12

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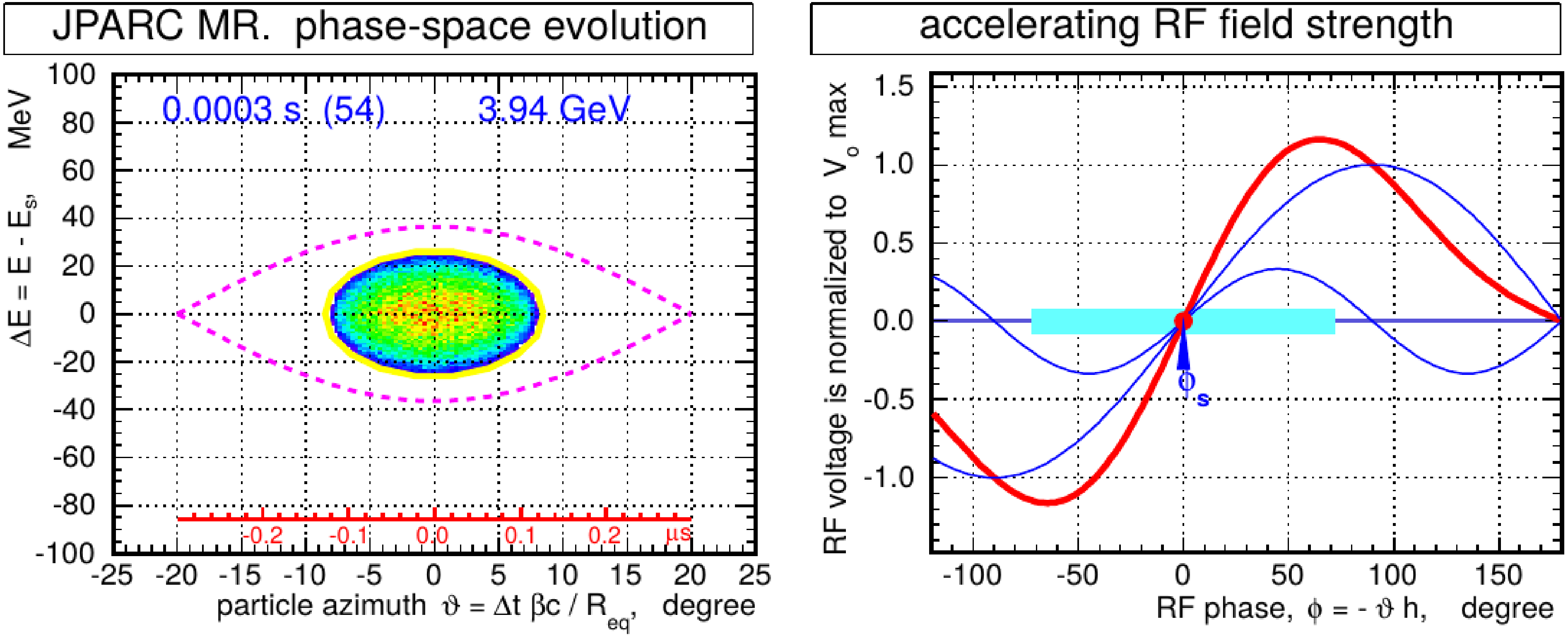
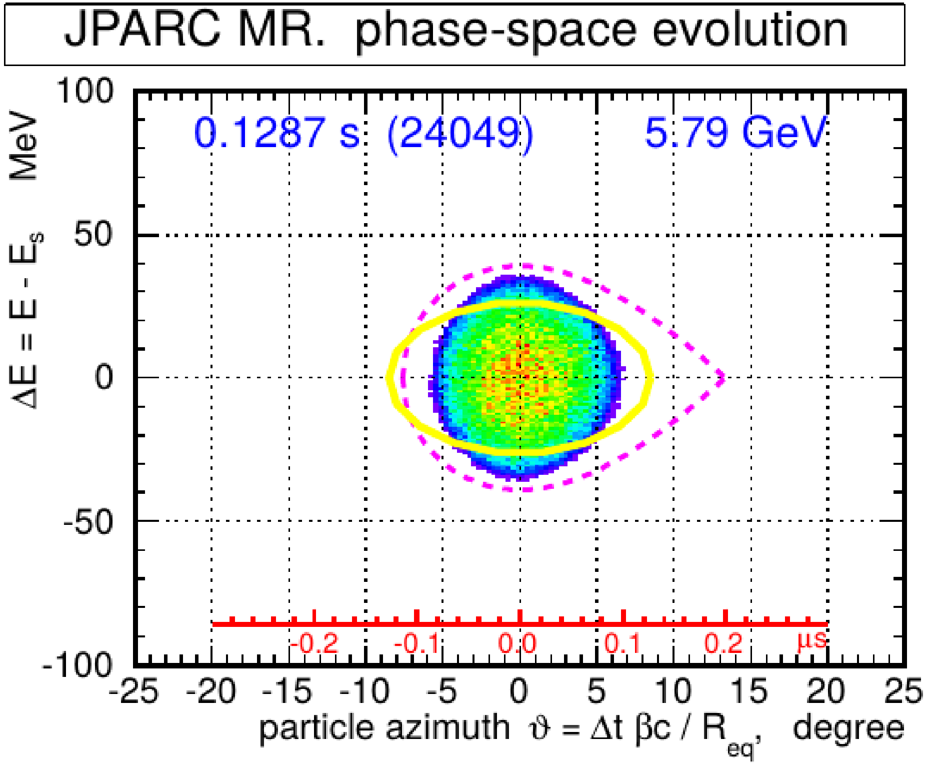
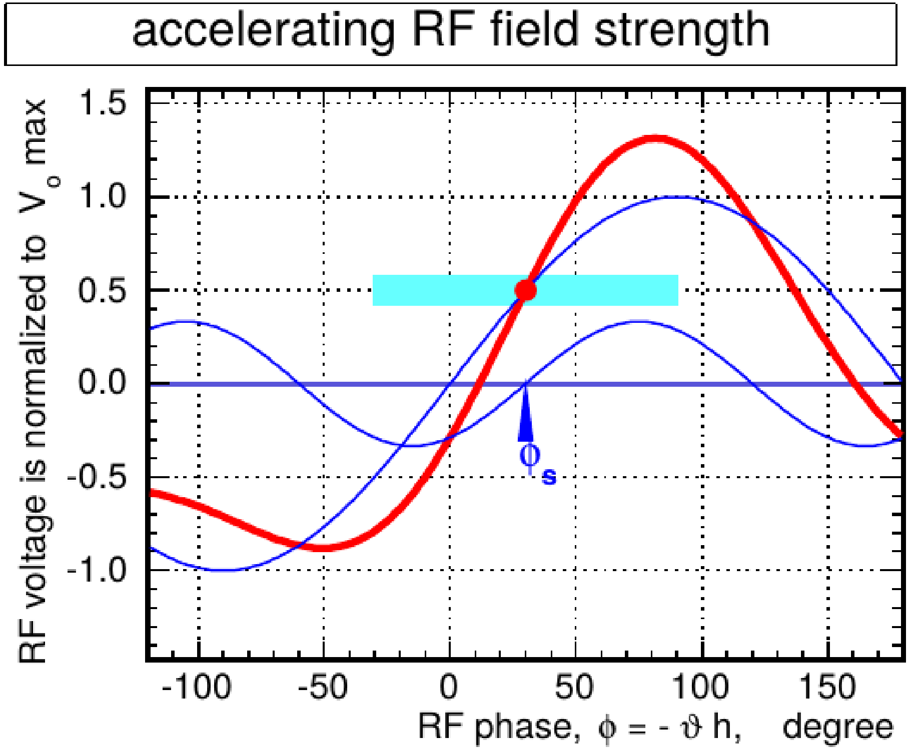
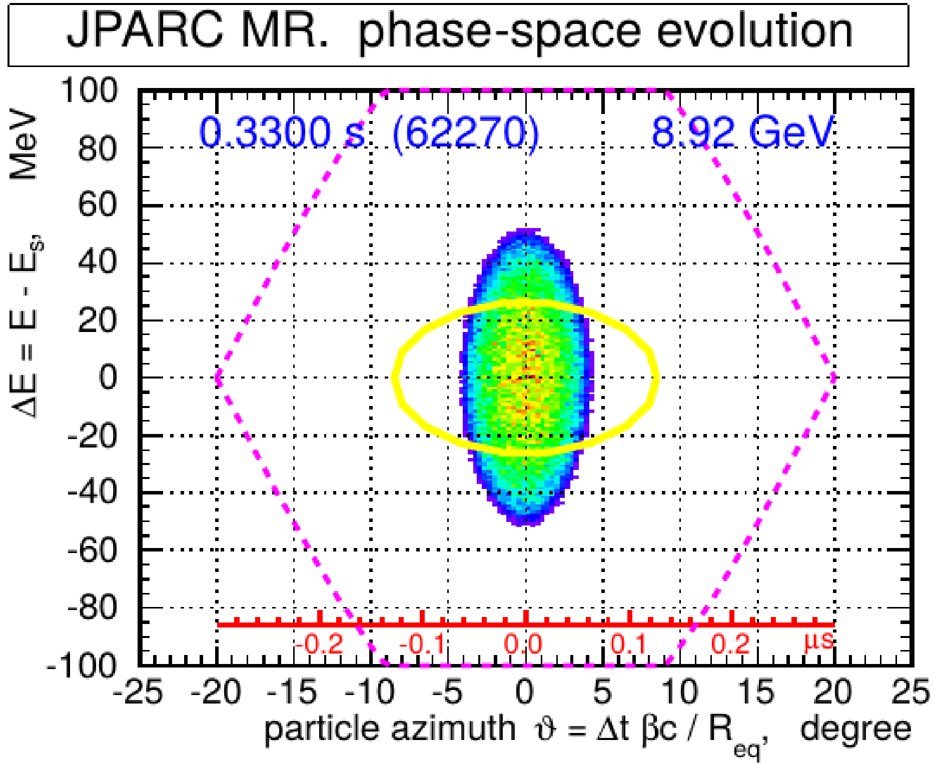


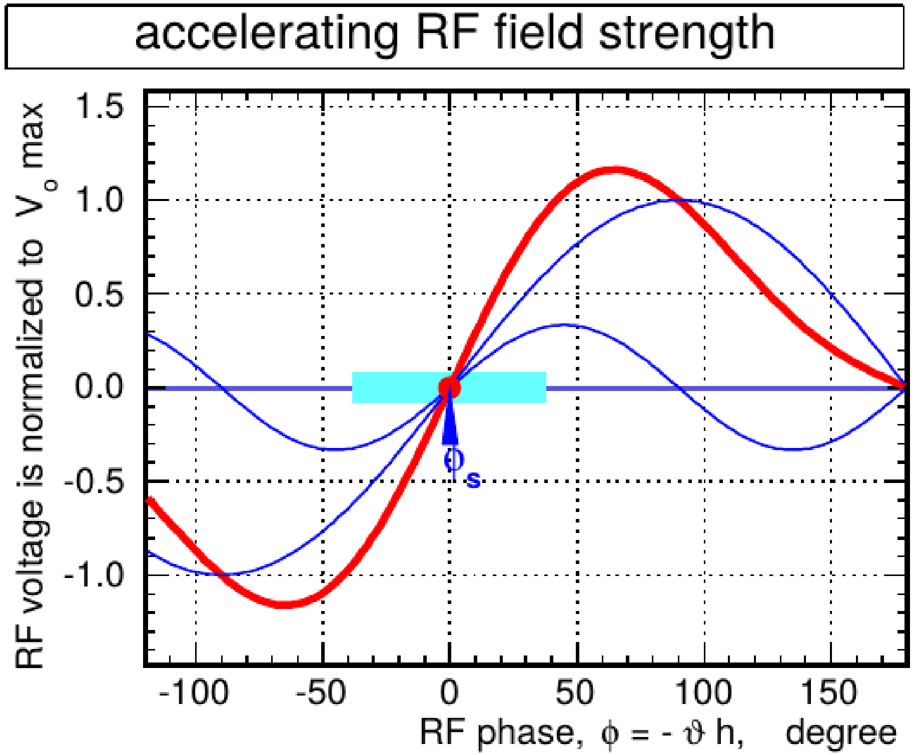
Simulation in particle physics





Simulation in particle physics





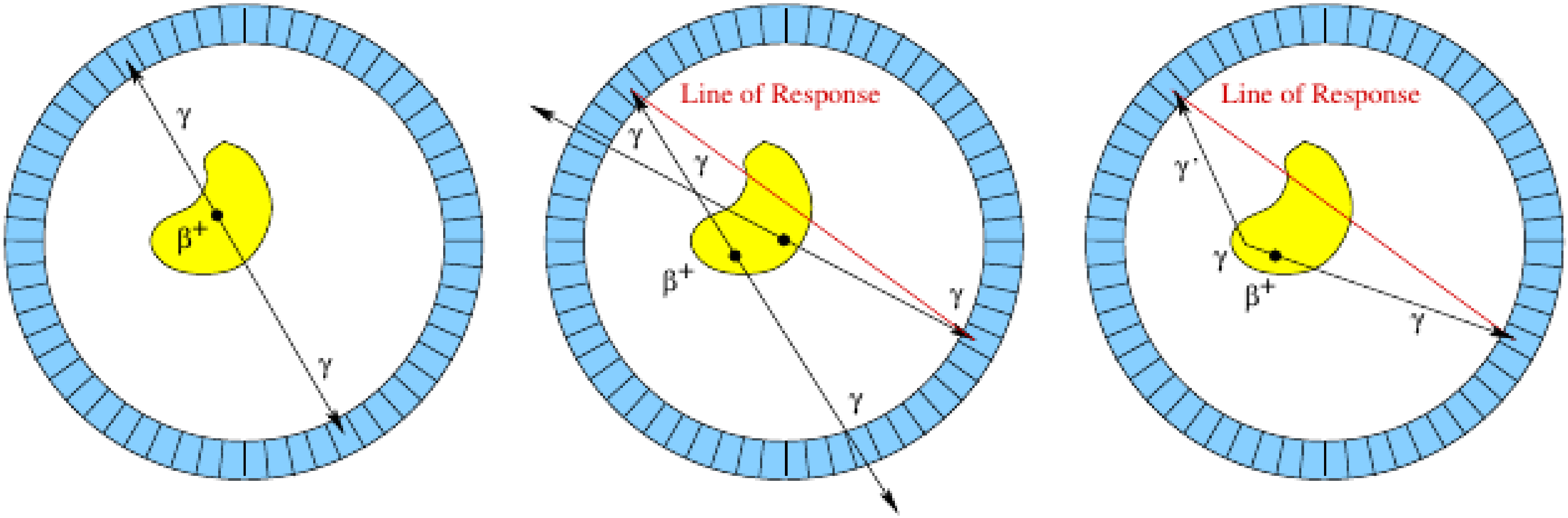
Example of random model

Natural random physical processes simulation for PET scanner performance estimate.

A specialized code: GATE (based on Geant4)

The example simulation code is written using Geant4 libraries

PET scanner. How it works

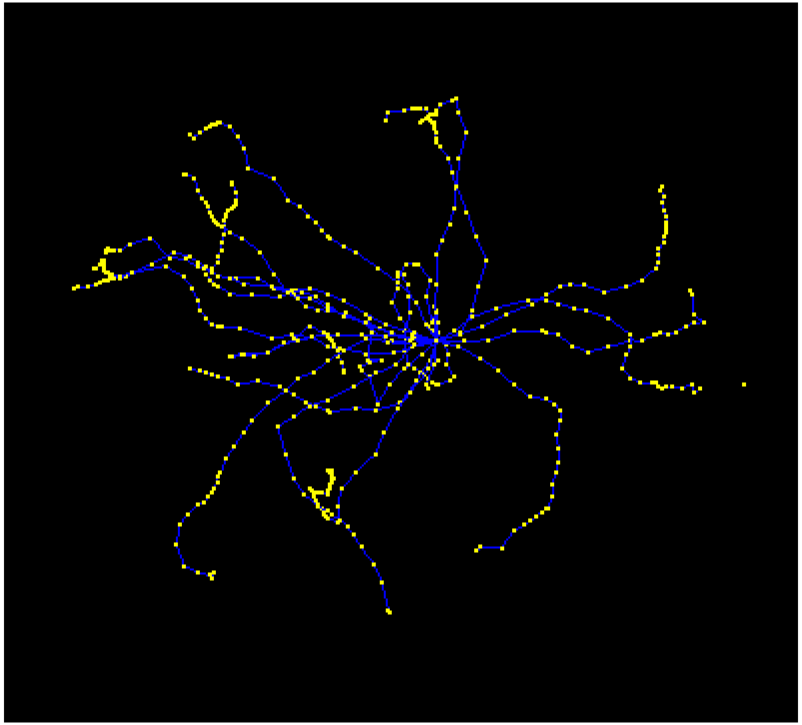


Simulation in particle physics

radionuclide	$T_{1/2}$	E_{max} (MeV)	R_{max}	R_{rms}	β^+ branching ratio
^{11}C	20.4 min	0.96	3.9	0.4	99%
^{13}N	9.97 min	1.20	5.1	0.6	100%
^{15}O	122 s	1.73	8.0	0.9	100%
^{18}F	109.8 min	0.63	2.3	0.2	97%
^{22}Na	2.60 y	0.55	15	1.6	98%
^{62}Cu	9.74 min	2.93	2.0	0.2	19%
^{64}Cu	12.7 h	0.65	20	3.3	56%
^{68}Ga	67.6 min	1.89	9.0	1.2	88%
^{76}Br	16.2 h	Various	19	3.2	54%
^{82}Rb	1.27 min	2.60, 3.38	18	2.6	95%
^{86}Y	14.7 h	1.4	6.0	0.7	32%
^{124}I	4.17 d	1.53, 2.14	7.0	0.8	22%

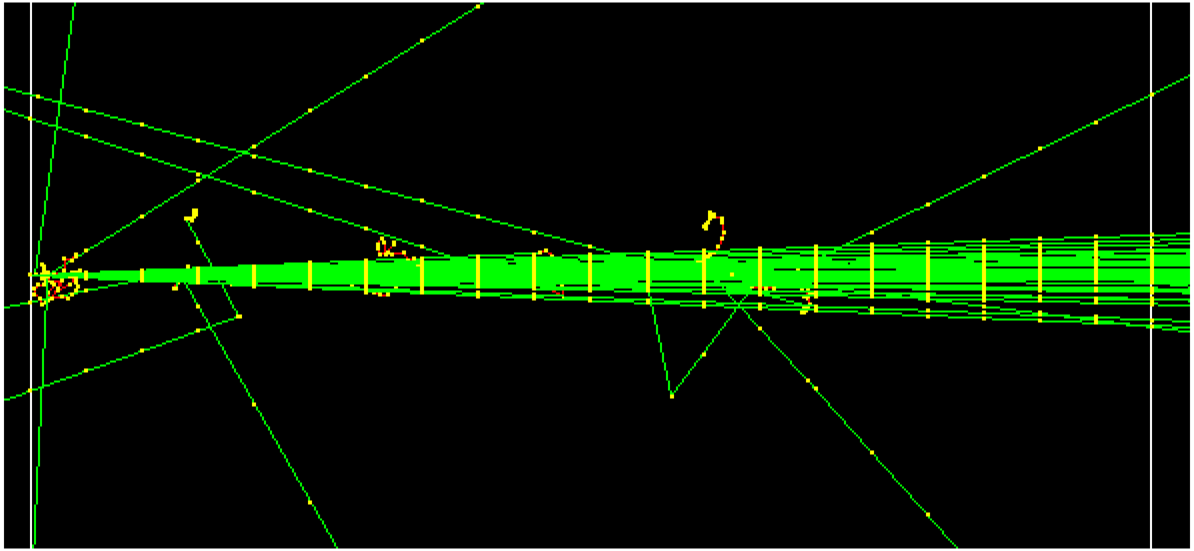
Simulation in particle physics

Positrons with energy of 0.511 MeV are generated at position (0,0,0) in water



Simulation in particle physics

γ- quanta with energy of 0.511 MeV generated at the entry point of a volume



Processes of gamma interaction with medium

Mainly the following random processes affect gamma passage through medium, reducing its energy and changing direction.

– Compton scattering;

– photoeffect;

– pair production: $\gamma \rightarrow e^{+}e^{-}$ when $E>2m_e$;

Processes of e+/e- interaction with medium

These processes randomize the energy and angle (phase space variables)

– elastics and inelastic interactions;

– brehmstralung (gamma radiation due to acceleration);

– multiple scattering (parameterized as continuous process);

– ionization energy losses (parameterized as continuous process) ;

(the list is nearly correct for all charged particles)

A medium description in Geant4

The medium is described with:

– effective A, atomic number;
– effective Z number;
– mass density;

Particles interaction with medium needs a cross-section (probability) data base for all elements and for all particles. All the our knowledge about the cross-sections are coded in Geant4. Some of them are parameterized, or interpolated, sometimes theoretical model are used, some rare cases are unknown. User can apply his own model.

advantages of MC simulation

– simulation is much cheaper than blind “test measurements”. So the method is most useful when the experiment outcome is hard to obtain;

– simulation results are more “realistic” than a mathematical model, because it can account fluctuations, random factors;

– simulation makes possible to test a ranges of input parameters to perform “what if” analysis, which is impossible to do in real experiment;

– when input state parameters are independent, simulation can be done in parallel, Independently;

– simulation makes possible to estimate probabilities of future events, e.g. in econometric/financial forecasting;

Drawbacks of MC simulation

– to have an adequate outcomes it is necessary to account in model all factors influencing outcomes. In reality it is impossible;

– simulation does not guarantee that the model is good;

– there is no way to prove reliability of result;

– building a model can take a great deal of time;

– complex problems need significant computational resources;

– does not account human factor;

Thank you