

# *How to achieve precision in physics*

## *a case study - mass of the charged pion*

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*PREFACE*

# EXPERIMENT DESIGN

*general considerations*

*think of all you can imagine – there will be more!*

# • MOTIVATION

- Is there an interesting physics question?
- Do we understand preconditions and possible show stoppers?
- Is there an appropriate experimental technique available for the envisaged precision ( $\approx$  **method & statistics**) ?
- Which level of **accuracy** ( $\approx$  **systematics**) is achievable?
- Impact of the expected result?

# • **FIND YOUR LABORATORY**

- **Acquire the pre-experiment level of knowledge**
- **Understanding the laboratory conditions**
- **Is the experiment affordable (money & man power)?**
- **How to get it approved?**

# • EXPERIMENTAL APPROACH

- Planning set-up and experiment
- **Do not underestimate mechanics!**
- How to control an ongoing measurement and gather all necessary information and even more!
- Analysis strategy
- Uncertainties

# • **ASSESSMENT**

- **The result**
- **New physics or experimental aspects**
- **Assessment of limits**
- **Presentation of results**
- **Publication in an appropriate journal**
- **New approaches and outlook**

## *EXAMPLE*

# CHARGED PION MASS

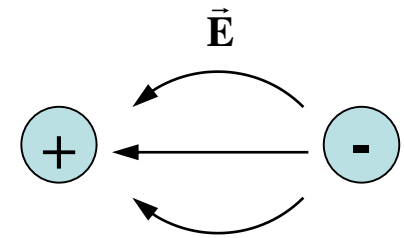
$$\frac{\Delta m_{\pi}}{m_{\pi}} \approx 1 \text{ ppm}$$

$$\tau_{\pi^{\pm}} = 26 \text{ ns}$$

- **MOTIVATION**
- LABORATORY
- EXPERIMENTAL APPROACH
- SOME RESULTS

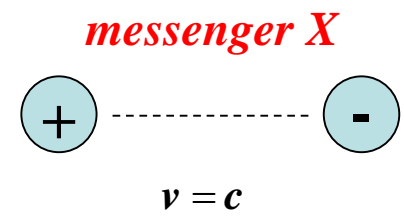


classical: in a field a force acts on a charge



quantum field theory: „virtual“ exchange particle

„exchange“ force



electromagnetic force

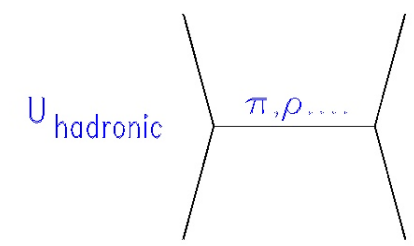
**X = photon ( $\gamma$ )**

strong force

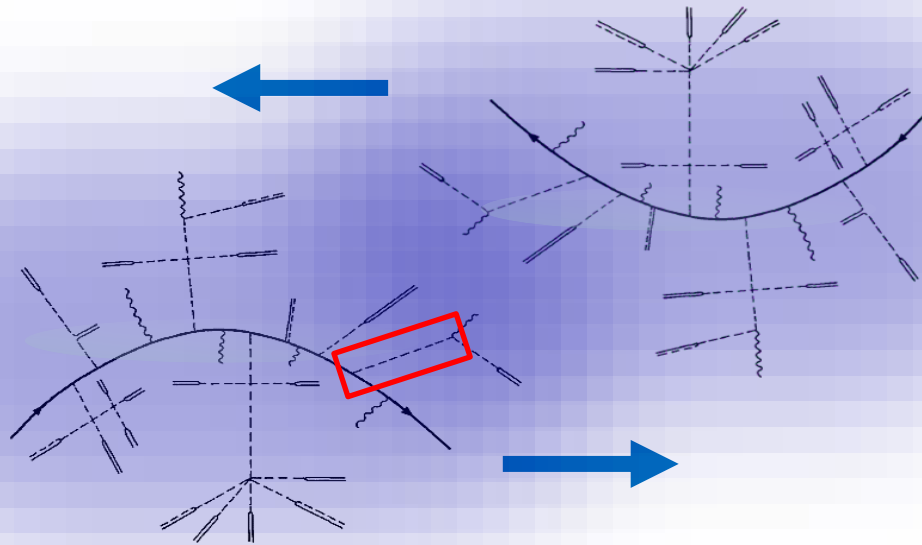
**X = mesons**  $\left( \begin{array}{l} \mathbf{p} \leftrightarrow \mathbf{n}\pi^+ \\ \mathbf{n} \leftrightarrow \mathbf{p}\pi^- \end{array} \right)$

strong-interaction potential U

medium and long range part



# PIONS, NUCLEONS - INTERACTION in terms of QCD



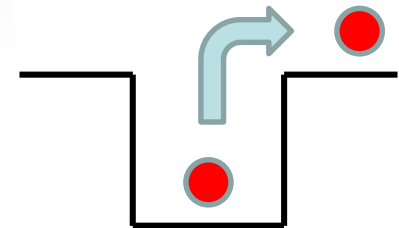
Nucleons with chiral loops,

CHIRAL PERTURBATION THEORY ( $\chi$ PT), ...

--- pions ●

$NN \Leftrightarrow NN$

$NN \Leftrightarrow NN\pi$



*Bring the pion out by inserting energy scattering (production experiments)*

J. Gasser et al., Nucl. Phys. B307, 779 (1988)

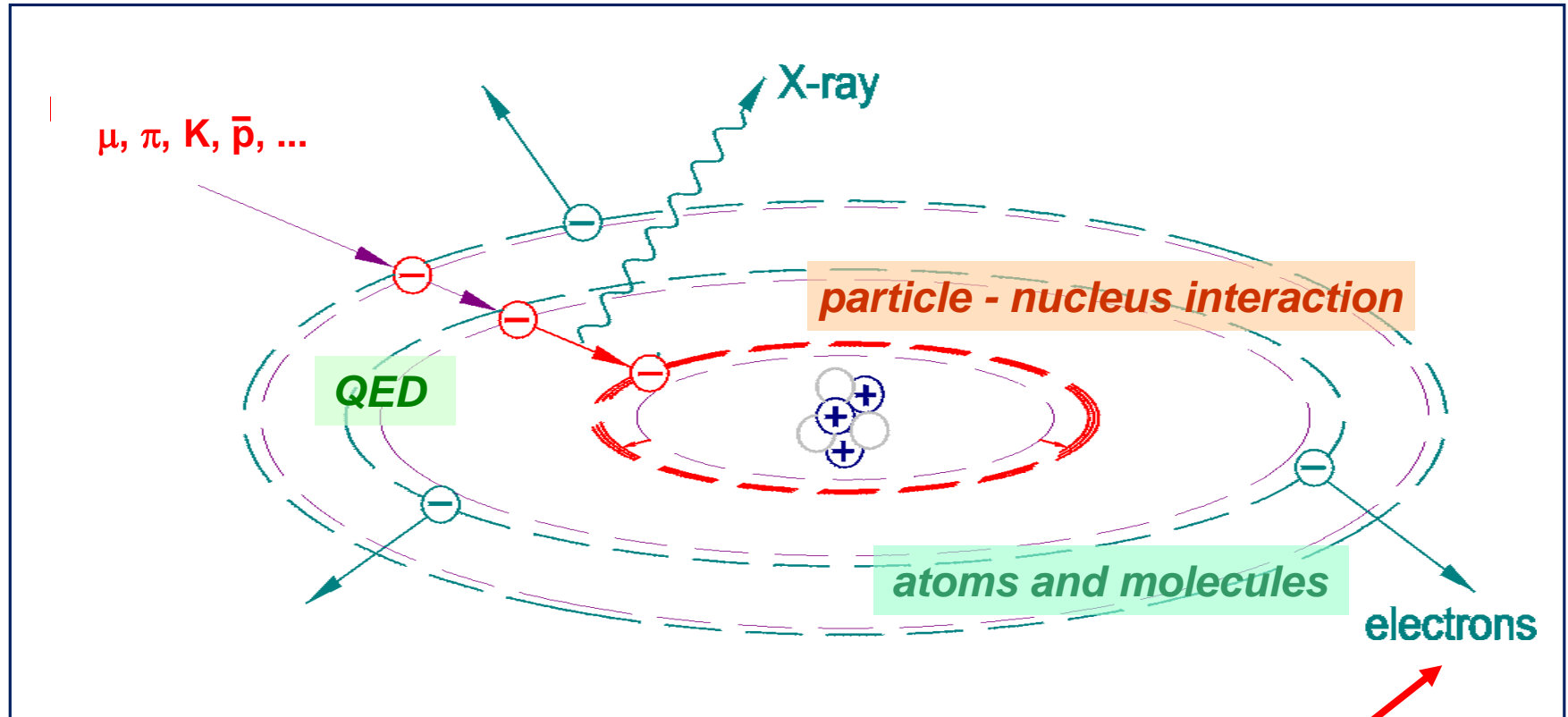
Fig. 1. A typical term in the expansion (3.7) of the nucleon propagator. — nucleon; --- pions; ~ vector current; == axial vector current; - - pseudoscalar density; == scalar density.

# PIONS – LIGHTEST CARRIER of NUCLEAR FORCE

charge	<b>Q</b>	<b>0, ± 1</b>	<b><i>isospin triplet</i></b>
mass	<b>M<sub>π</sub></b>	<b>≈ m<sub>p</sub> / 7 ≈ 270 · m<sub>e</sub></b>	
spin	<b>S</b>	<b>0</b>	
size		0.6 · 10 <sup>-15</sup> m	
life time	<b>τ<sub>0</sub></b>	<b>π<sup>±</sup> 26 · 10<sup>-9</sup> s</b>	<b>m<sub>π±</sub> ≈ 139 MeV/c<sup>2</sup></b>
		<b>π<sup>0</sup> 8 · 10<sup>-17</sup> s</b>	<b>m<sub>π0</sub> ≈ 135 MeV/c<sup>2</sup></b>
decay			
	<b>π<sup>±</sup> → μ<sup>±</sup> ν</b>	<b>limit for the muon neutrino mass m<sub>νμ</sub></b>	<b>(1973: dark matter?)</b>
	<b>π<sup>0</sup> → γ γ</b>	<b>n<sub>colour</sub> = 3!</b>	

- MOTIVATION
- **LABORATORY**
- EXPERIMENTAL APPROACH
- SOME RESULTS

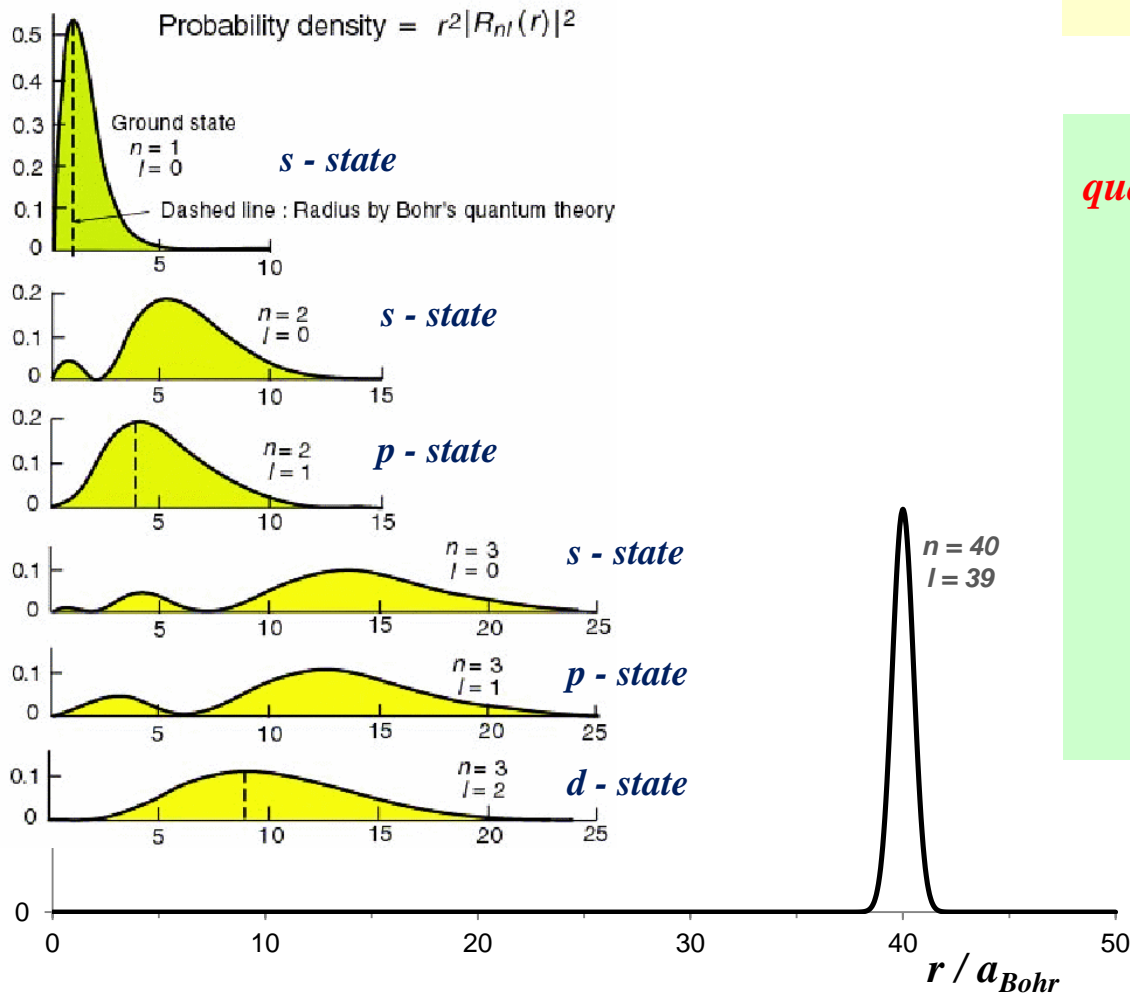
# EXOTIC ATOMS



*emitted by Auger effect !*

# ATOM

$$V_{\text{Coulomb}} = -\frac{Ze^2}{r}$$



*quantisation of action:  $E \cdot t = 2\pi\hbar$*

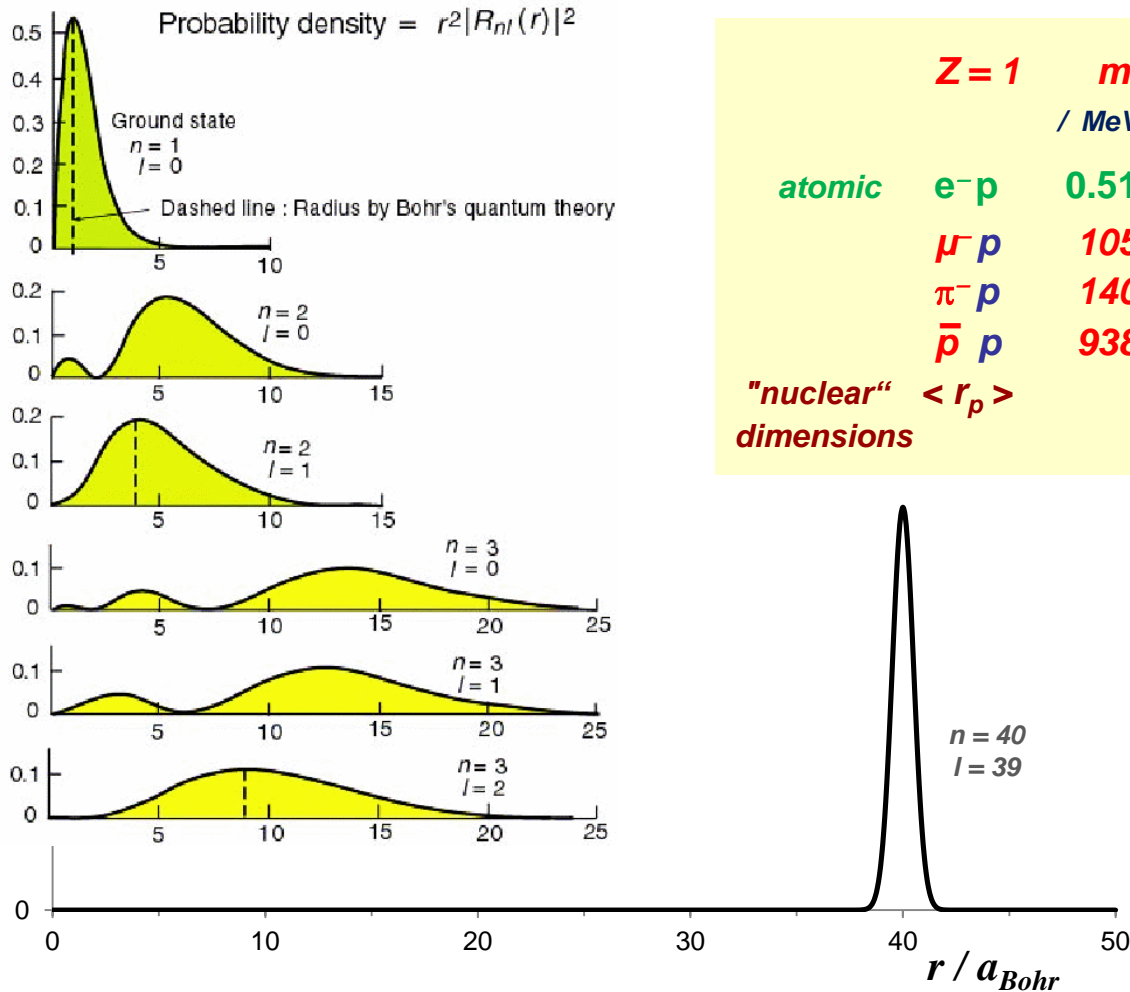
$$a_n = \frac{\hbar c}{m_{\text{red}} c^2 \alpha} \cdot \frac{n^2}{Z}$$

$$a_{\text{Bohr}} = \frac{\hbar c}{m_{\text{red}} c^2 \alpha}$$

$$B_n = -m_{\text{red}} c^2 \alpha^2 \cdot \frac{Z^2}{2n^2}$$

# EXOTIC ATOM

replace *electrons* by *heavier negatively charged particles*



	$Z=1$	$m$ / $\text{MeV}/c^2$	$B_1$ / $\text{keV}$	"Bohr" radius $a_B$ / $\text{fm}$
atomic	$e^- p$	0.511	0.0136	$0.5 \cdot 10^5$
	$\mu^- p$	105	2.6	279
	$\pi^- p$	140	3.2	216
	$\bar{p} p$	938	12.5	58
"nuclear" dimensions	$\langle r_p \rangle$			0.8

$$a_{16}(\pi^-) \approx a_1(e^-)$$

$$a_{40}(\bar{p}) \approx a_1(e^-)$$

# ATOMIC BINDING ENERGY

$$E_B = E_{\text{Coulomb}}$$

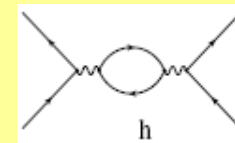
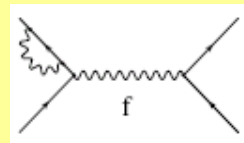
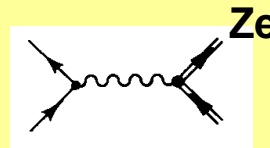
$$+ \Delta E_{\text{QED}}$$

$$+ \Delta E_{\text{screening}}$$

$$+ \Delta E_{\text{finite size}}$$

$$+ \Delta E_{\text{strong interaction}}$$

$$-\frac{Ze}{r}$$



*self energy* + *vakuum polarisation* + *higher orders*

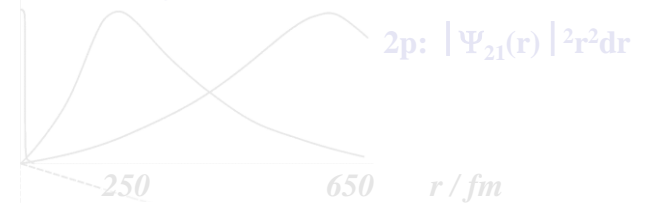
*capture*  $x^- + [A(Z,N)Ze^-] \rightarrow \{[x^-A(Z,N)]ne^-\} + few e^-$

probability density

$$|\Psi_{nl}(r)|^2 r^2 dr$$

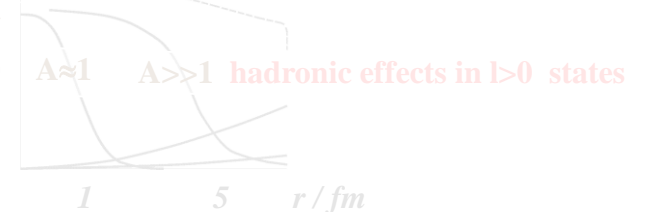
$$1s: |\Psi_{10}(r)|^2 r^2 dr$$

$$2p: |\Psi_{21}(r)|^2 r^2 dr$$



nuclear density

$$\rho(r)$$

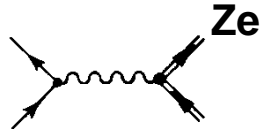


*hadronic effects in l>0 states*

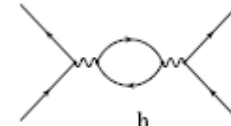
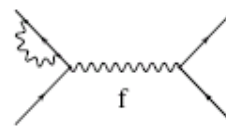


# ATOMIC BINDING ENERGY

$$E_B = E_{\text{Coulomb}}$$

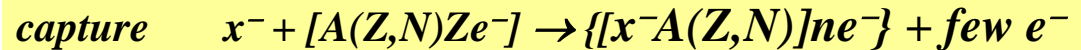
$$-\frac{Ze}{r}$$


$$+ \Delta E_{\text{QED}}$$



*self energy* + *vakuum polarisation* + *higher orders*

$$+ \Delta E_{\text{screening}}$$



$$+ \Delta E_{\text{finite size}}$$

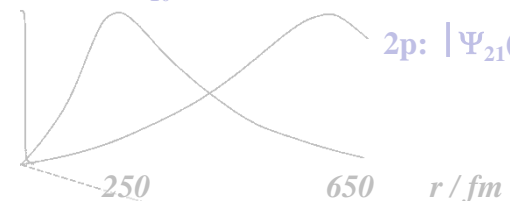
$$+ \Delta E_{\text{strong interaction}}$$

probability density

$$|\Psi_{nl}(r)|^2 r^2 dr$$

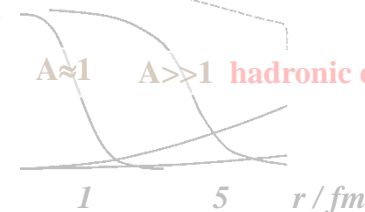
$$1s: |\Psi_{10}(r)|^2 r^2 dr$$

$$2p: |\Psi_{21}(r)|^2 r^2 dr$$



nuclear density

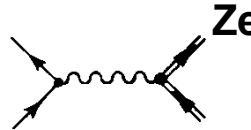
$$\rho(r)$$



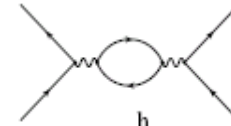
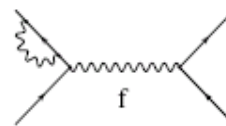
$A \approx 1$   $A \gg 1$  hadronic effects in  $l > 0$  states

# ATOMIC BINDING ENERGY

$$E_B = E_{\text{Coulomb}}$$

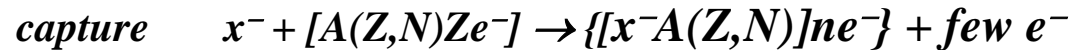
$$-\frac{Ze}{r}$$


$$+ \Delta E_{\text{QED}}$$



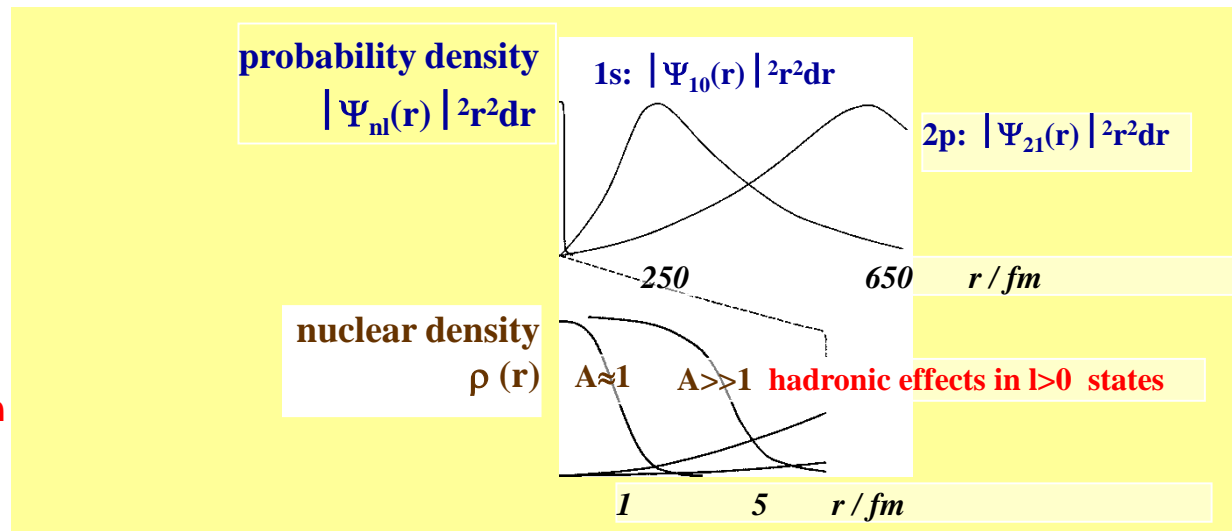
*self energy + vakuum polarisation + higher orders*

$$+ \Delta E_{\text{screening}}$$



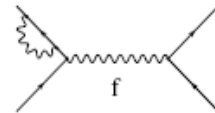
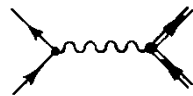
$$+ \Delta E_{\text{finite size}}$$

$$+ \Delta E_{\text{strong interaction}}$$

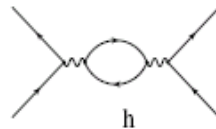


including **STRONG INTERACTION**

$$V_{\text{Coulomb}} = -\frac{Ze^2}{r} + \Delta E_{\text{QED}} + V_{\text{strong interaction}}$$



self energy



vakuum polarisation

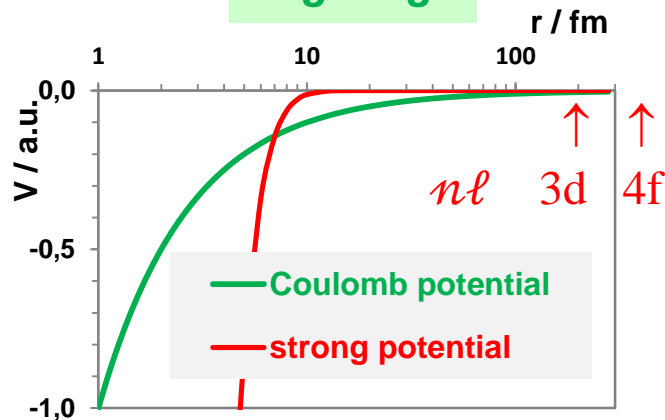
+ higher orders

Yukawa potential

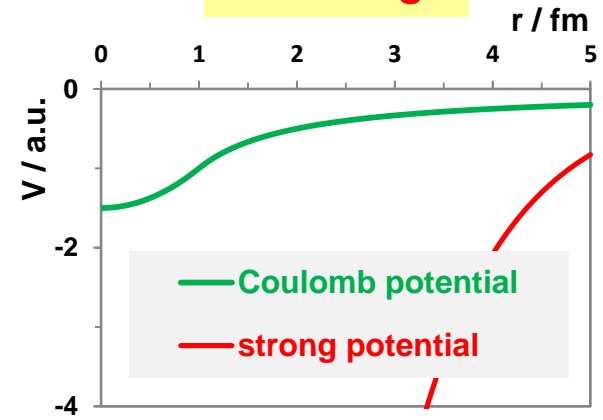
$$V_{\text{strong}} = g^2 \cdot \frac{e^{-\mu r}}{r}$$

$$\mu = \frac{\hbar c}{m_{\pi} c^2}$$

long range

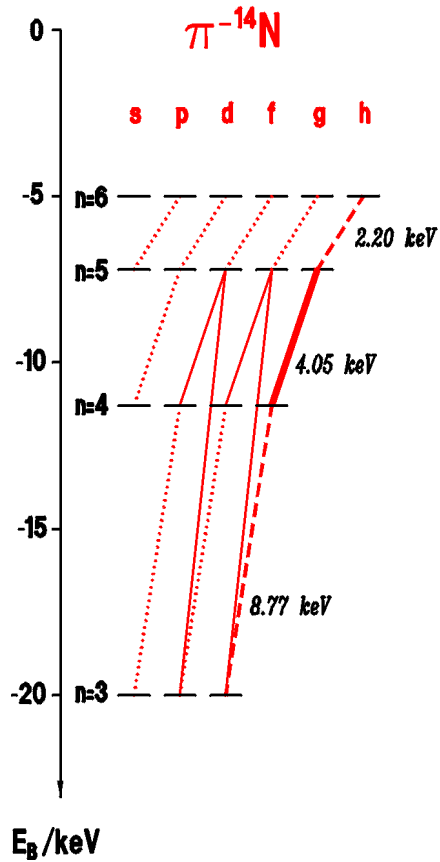


short range



level shift & broadening

# CANDIDATE



*measurement*

$\pi\text{N}(6\text{h}-5\text{g})$

*low energy & yield  $\Leftrightarrow$  absorption losses*

$\pi\text{N}(5\text{g}-4\text{f})$

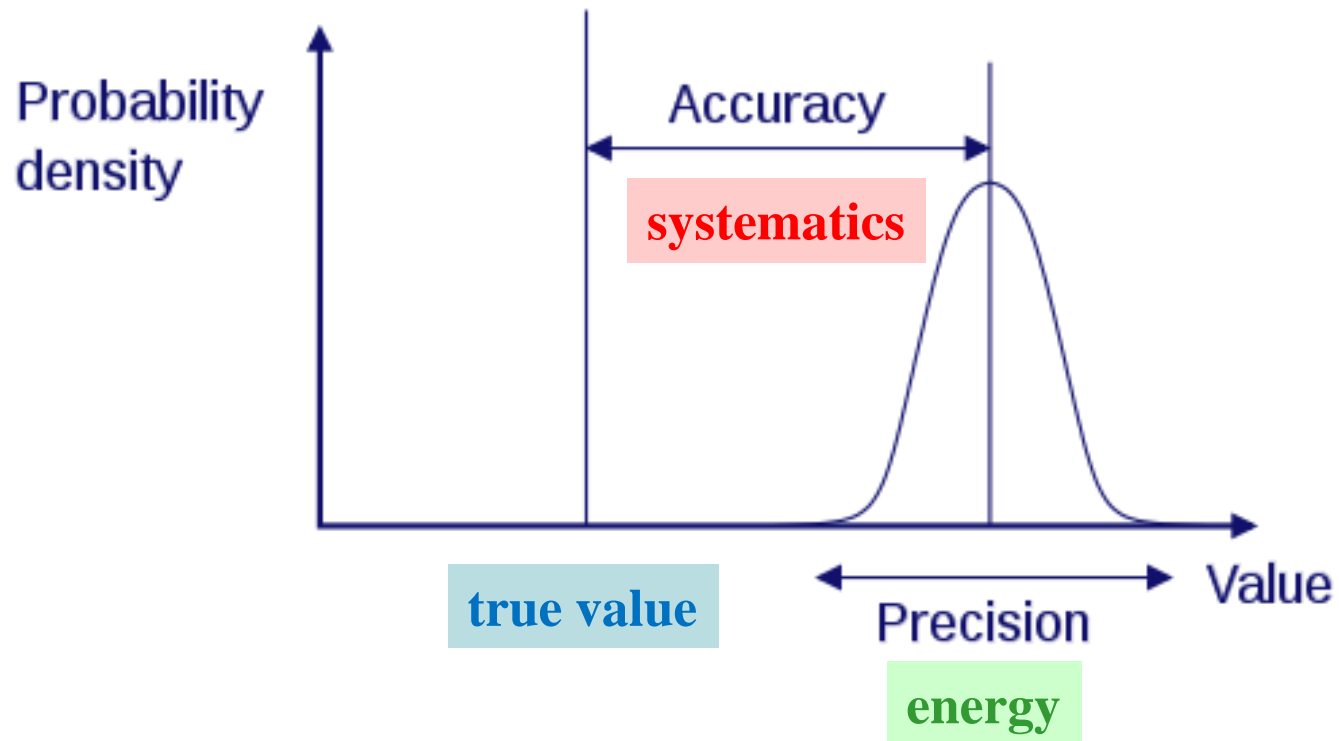
***best!***

$\pi\text{N}(4\text{f}-3\text{d})$

*strong-interaction corrections sizable*

- MOTIVATION
- EXOTIC ATOM
- **EXPERIMENTAL APPROACH**
- SOME RESULTS

# SUMMARY OF ALL EXPERIMENTAL PROBLEMS

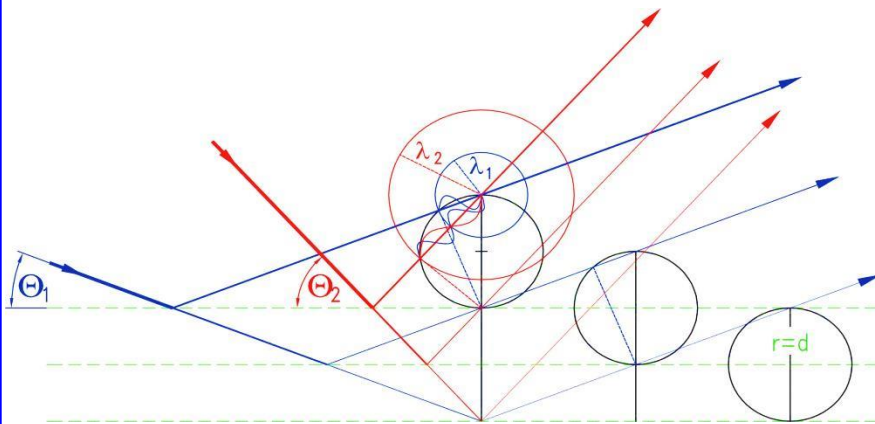


# EXPERIMENT I

*How to achieve the necessary  
precision in the energy determination ?*

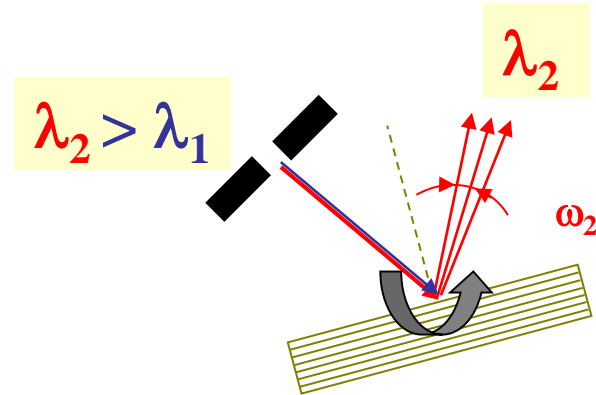
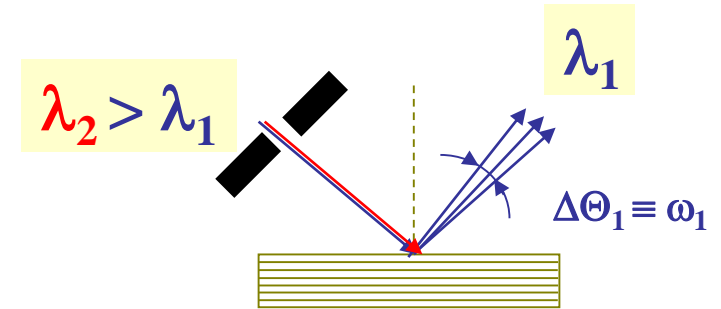
# BRAGG'S LAW $n\lambda = 2d \cdot \sin\theta_B$

**n** order of diffraction  
 **$\lambda$**  wave length  
**d** spacing of diffracting planes  
 **$\theta_B$**  Bragg angle



**$\tau_e$**  extinction length    *coherent reflection*  
 **$\tau_a$**  absorption length    *incoherent*  
*usually*     $\tau_e \ll \tau_a$

**$\omega$**  angular spread of reflection



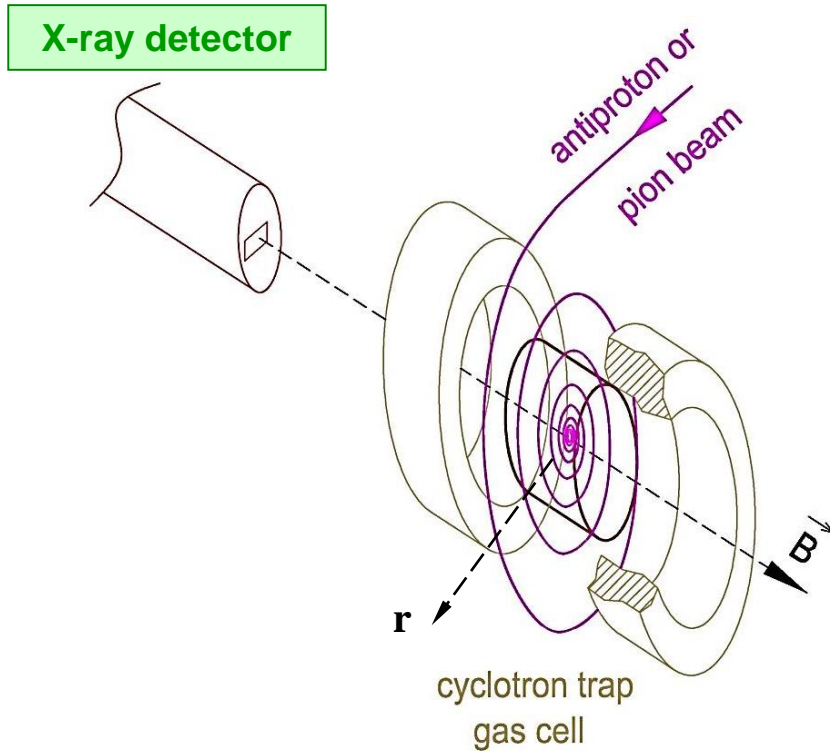


# EXPERIMENT II

*How to produce a suitable X-ray source  
=  
many of exotic atoms (statistics)?*

# CYCLOTRON TRAP

concentrates particles



“wind up” range curve

in a (weakly) focusing magnetic field

$$n = -\frac{\frac{\partial B}{\partial r}}{\frac{B}{r}} < 1 \quad \text{field index}$$

increase in stop density

compared to a linear stop arrangement

pions (PSI)  $\times 200$

antiprotons (LEAR)  $\times 10^6$

⇒ high X - ray line yields

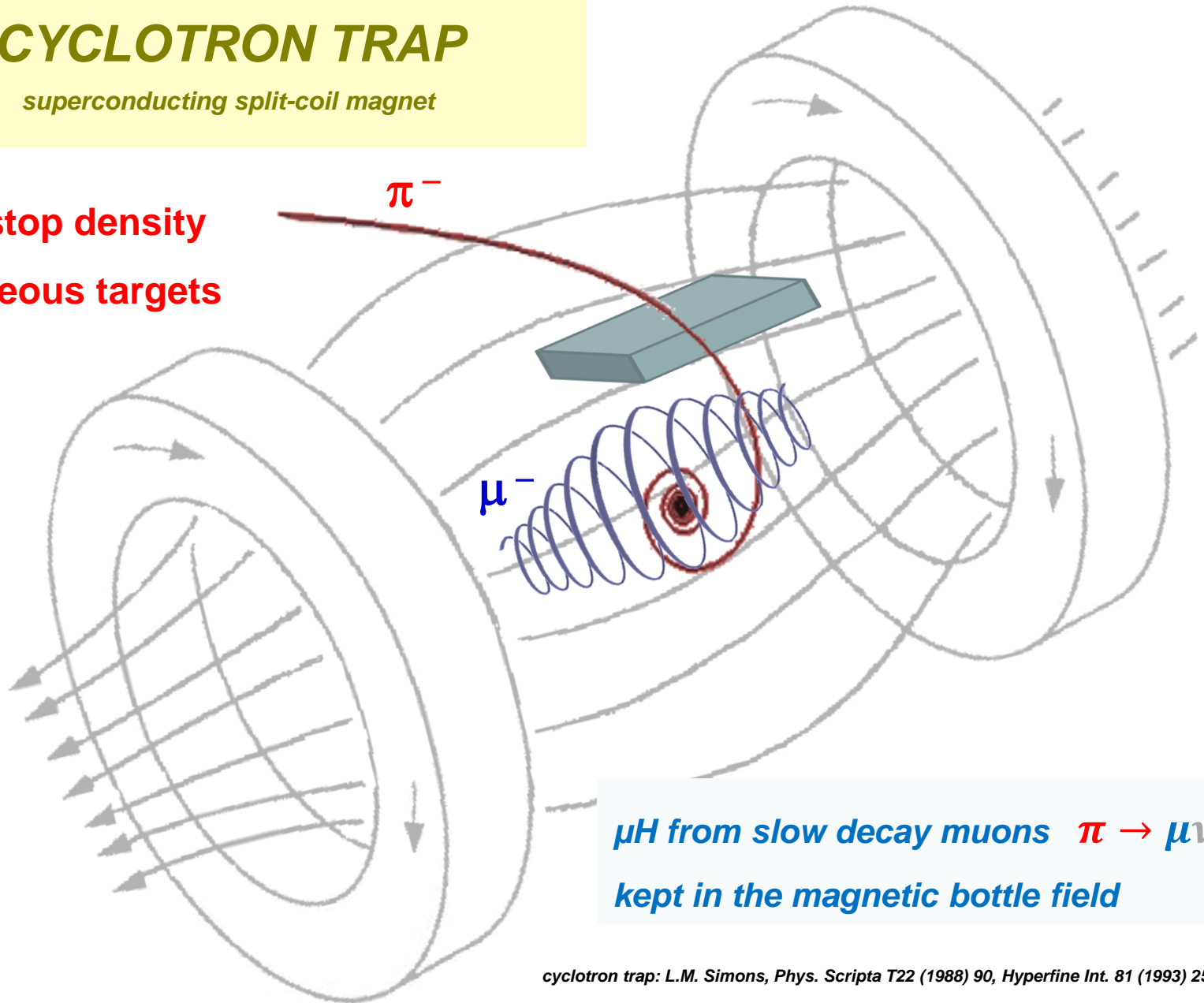
⇒ bright X - ray source

L. Simons, *Physica Scripta* 90 (1988), *Hyperfine Int.* 81 (1993) 253

# CYCLOTRON TRAP

superconducting split-coil magnet

high stop density  
in gaseous targets



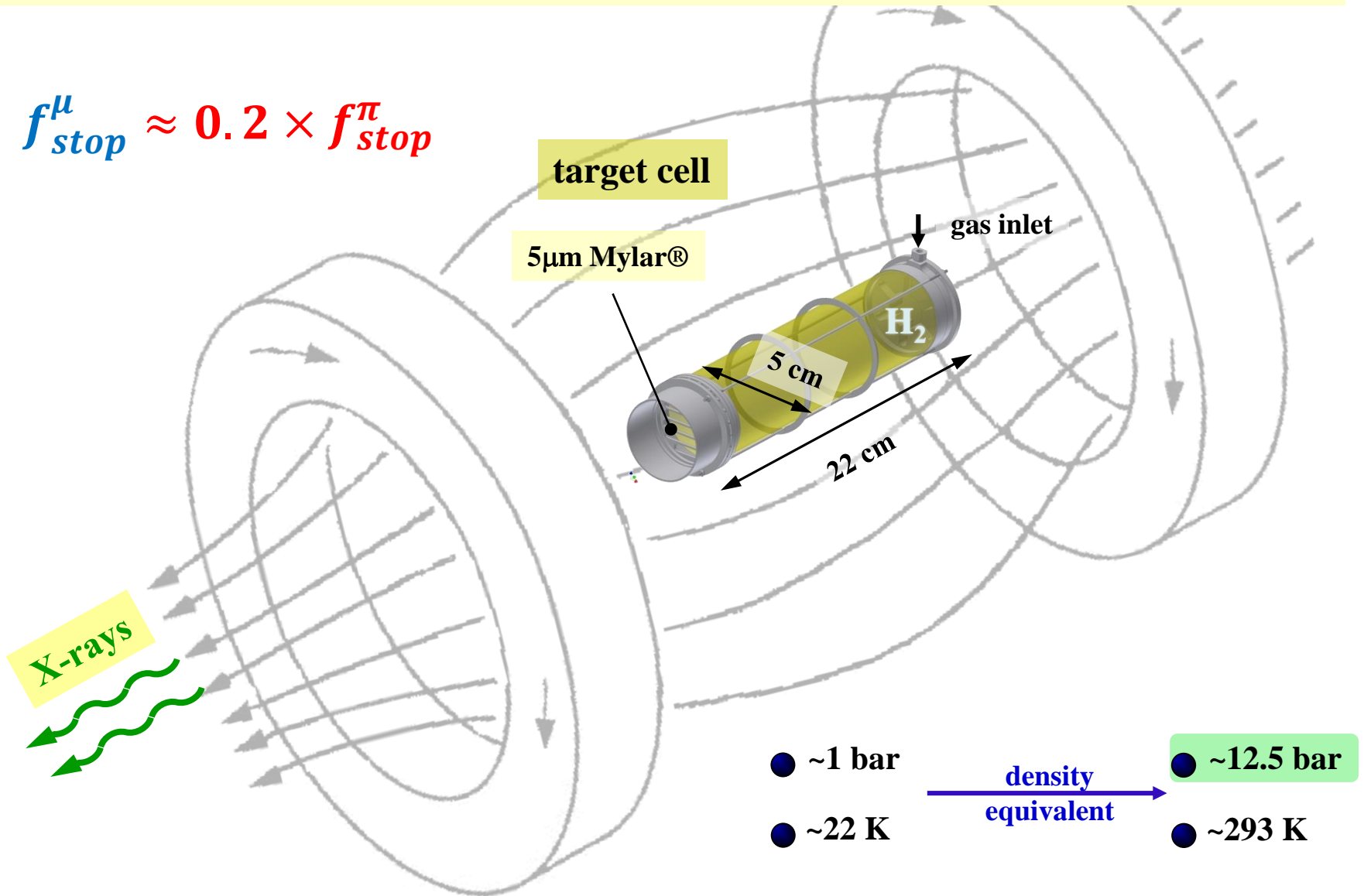
$\mu H$  from slow decay muons  $\pi \rightarrow \mu \nu$   
kept in the magnetic bottle field

cyclotron trap: L.M. Simons, Phys. Scripta T22 (1988) 90, Hyperfine Int. 81 (1993) 253

# CYCLOTRON TRAP

pion and muon setup

$$f_{\text{stop}}^{\mu} \approx 0.2 \times f_{\text{stop}}^{\pi}$$



# EXPERIMENT III

*How to bring it together?*

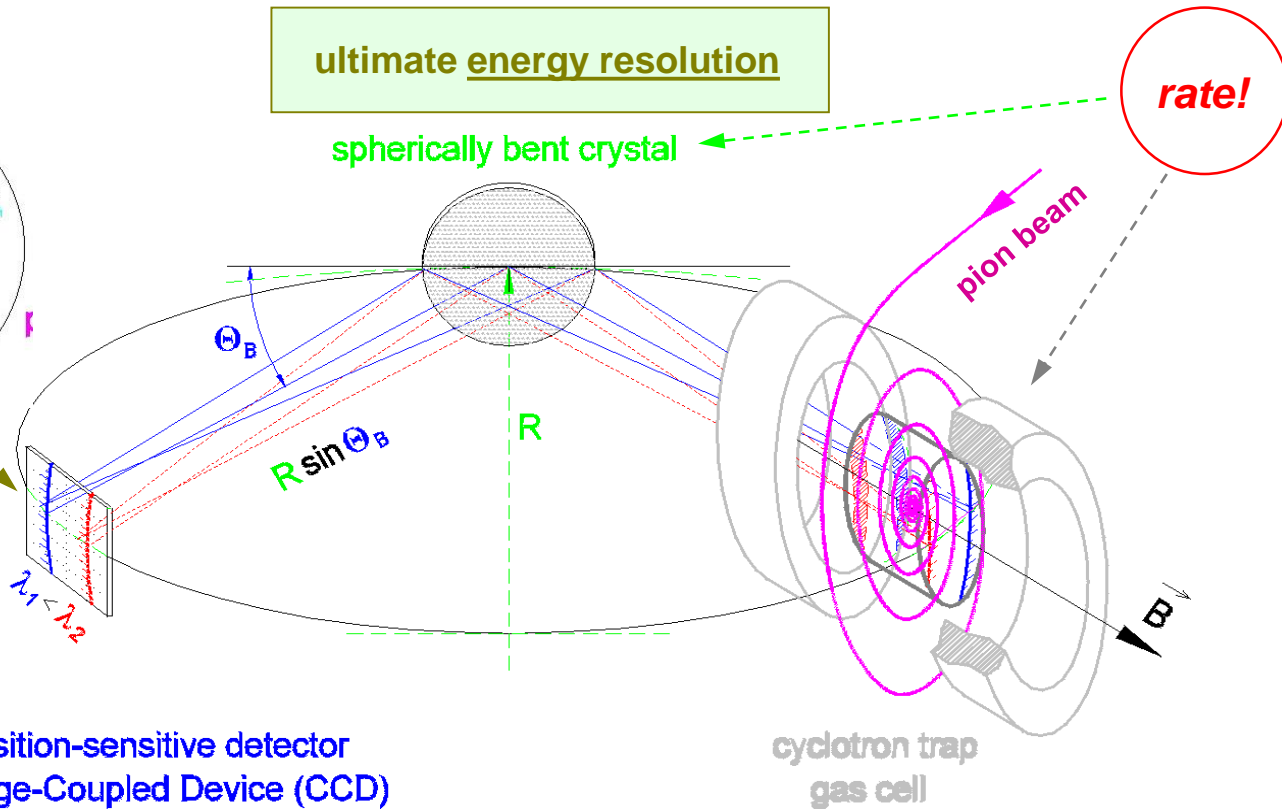
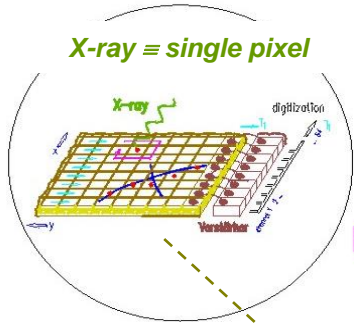
# JOHANN-TYPE SET-UP

ultimate energy resolution

spherically bent crystal

rate!

X-ray  $\equiv$  single pixel



position-sensitive detector  
Charge-Coupled Device (CCD)

cyclotron trap  
gas cell

position & energy resolution

$\Rightarrow$  background reduction I  
by analysis of hit pattern

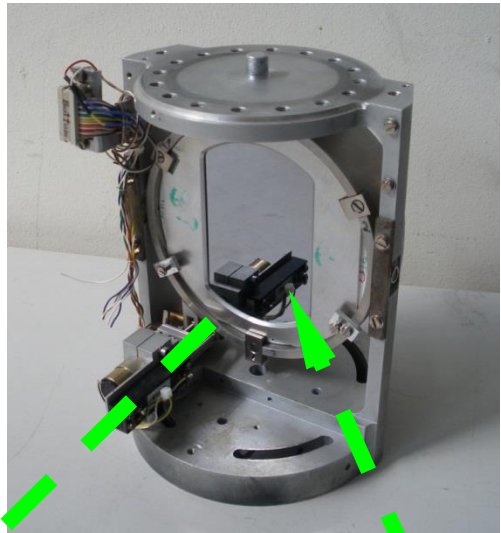
high stop density

$\Rightarrow$  high X - ray line yields  
 $\Rightarrow$  bright X - ray source

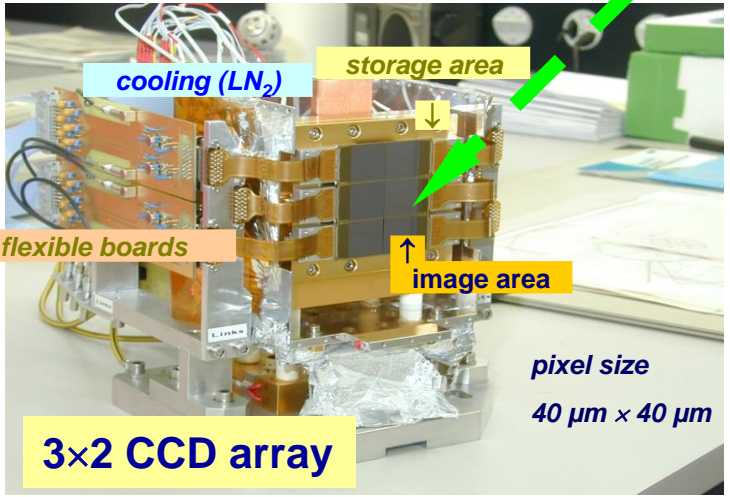
# BRAGG CRYSTAL

Si 111

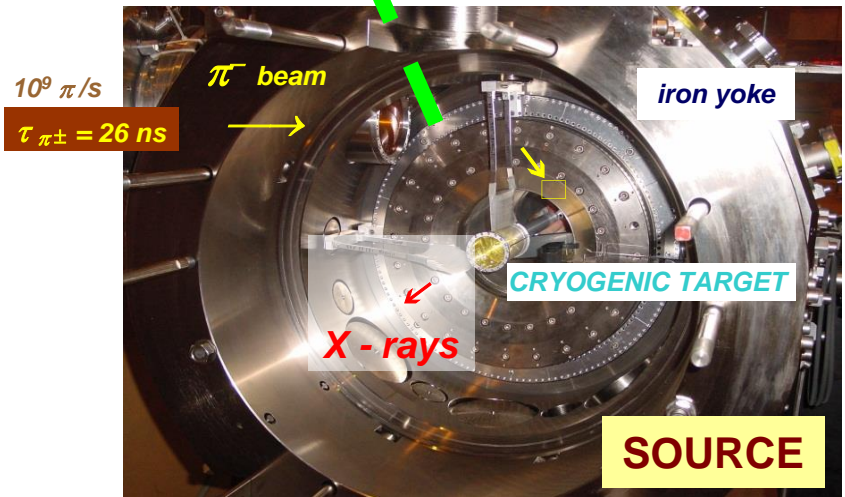
spherically curved  
 $R = 3\text{ m}$   
 $\Phi = 10\text{ cm}$



## Large - Area Focal Plane Detector



## CYCLOTRON TRAP one coil removed



N. Nelms et al., Nucl. Instr. Meth 484 (2002) 419

# DETECTOR *crystal spectrometer* Large - Area Focal Plane Detector

CCD: charge-coupled device

pixel distance

manufacturer

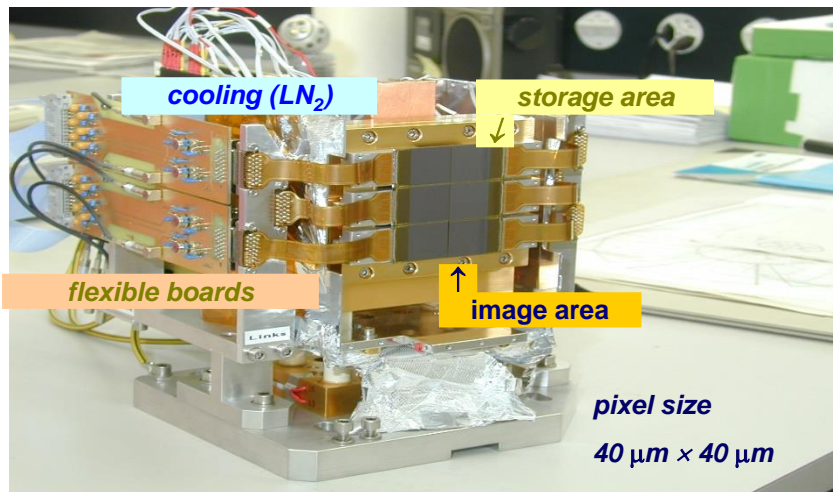
@ 20°C 40.0  $\mu\text{m}$   $\pm$  0.17 nm

@ -100°C 39.9775  $\mu\text{m}$   $\pm$  0.6 nm

P. Indelicato et al., Rev. Sc. Instr. 77 (2006) 043107

$\Delta \rightarrow$  4.2ppm of  $M_\pi$

2  $\times$  3 array of 24 mm  $\times$  24 mm devices



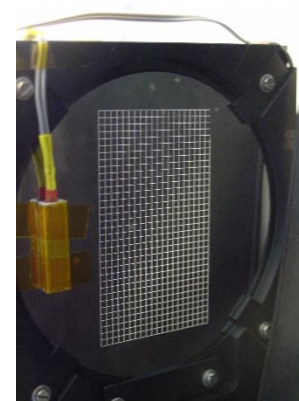
N. Nelms et al., Nucl. Instr. Meth 484 (2002) 419

1. try wire eroded mask

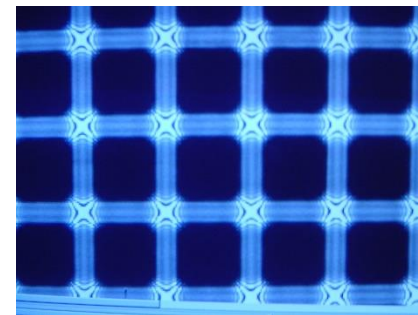
- gap  $\equiv$

- pixel size ?

2. try nano mask (C. David LNS/PSI)



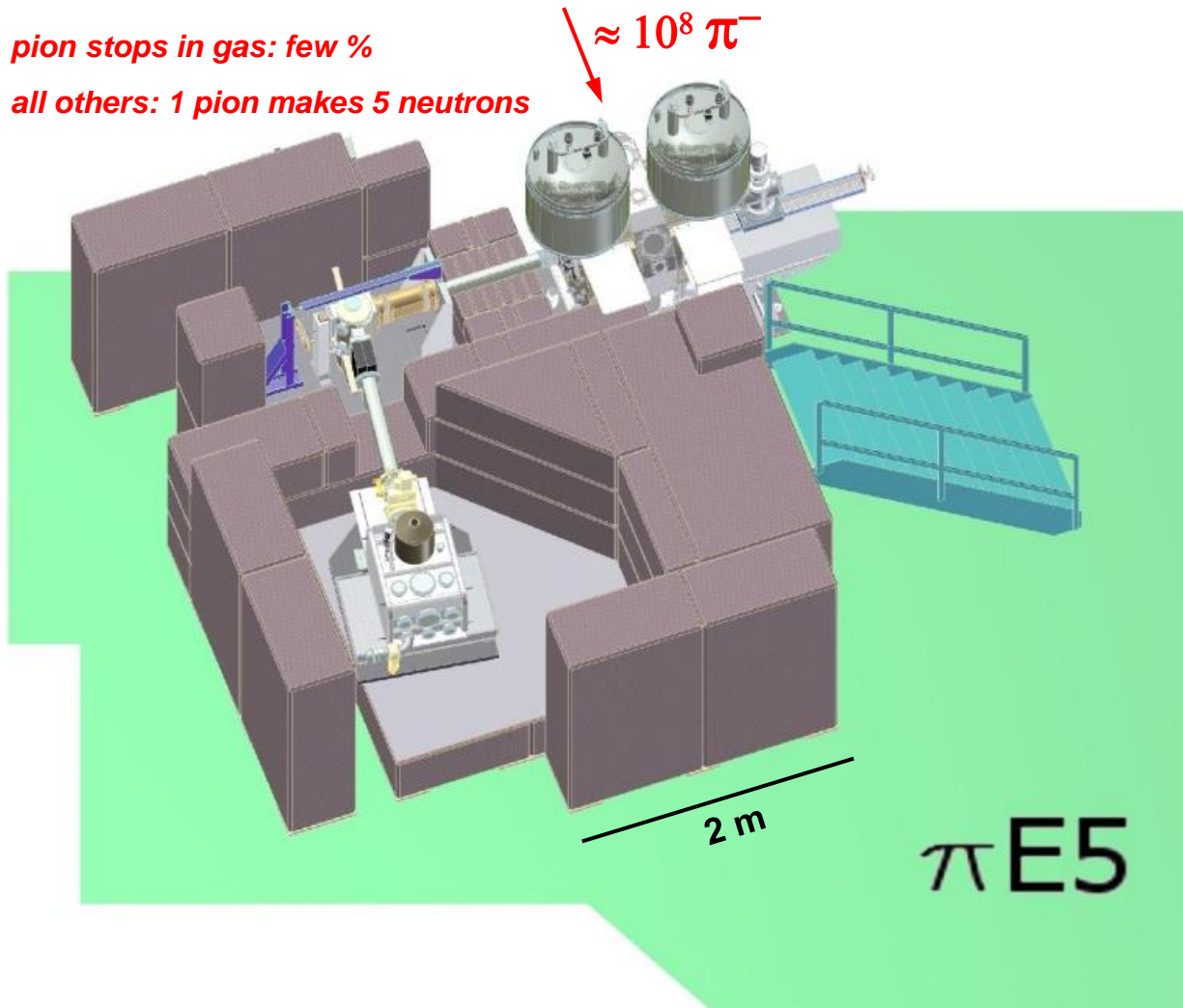
illuminated  
by light source  
at 6 m distance  
 $T = 20^\circ\text{C}$



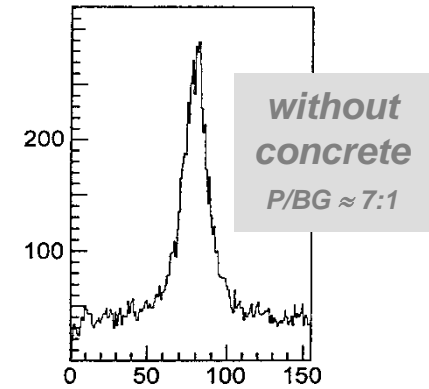
@ -100°C HOR 39.9802 $\pm$ 0.0026  $\mu\text{m}$   
VER 39.9794 $\pm$ 0.0022  $\mu\text{m}$



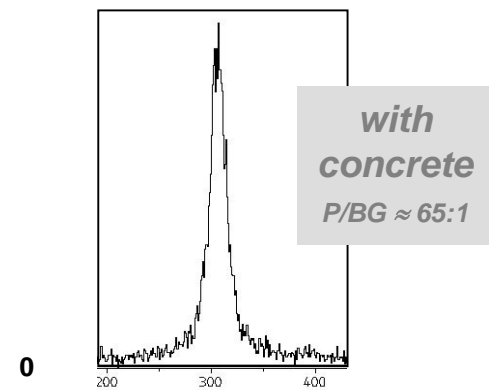
# SET-UP at the pion factory of Paul-Scherrer-Institute (Switzerland)



*pionic hydrogen*

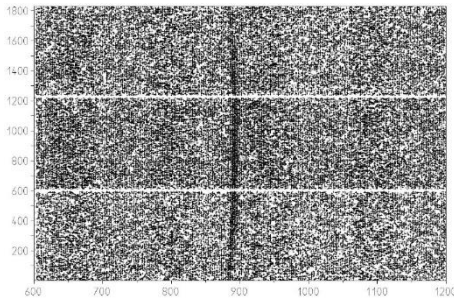


*peak/background x 10*

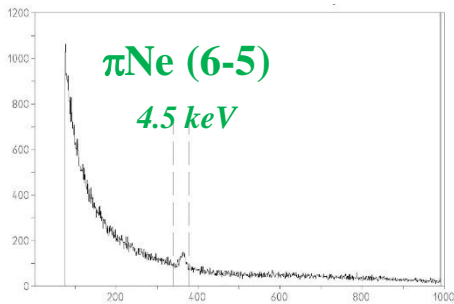


background reduction II

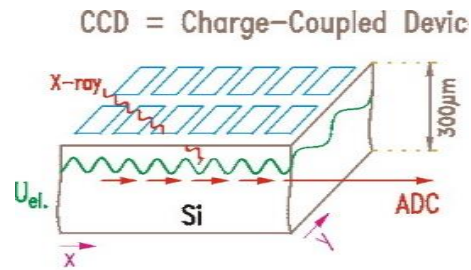
# SPECIAL DEMANDS FOR EXOTIC ATOMS



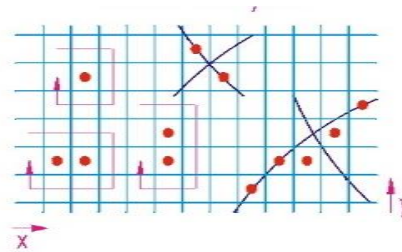
projection  
onto axis of dispersion



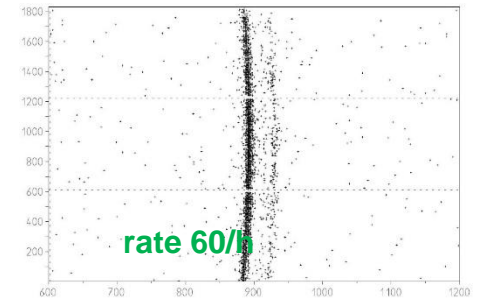
→ coordinate x or energy



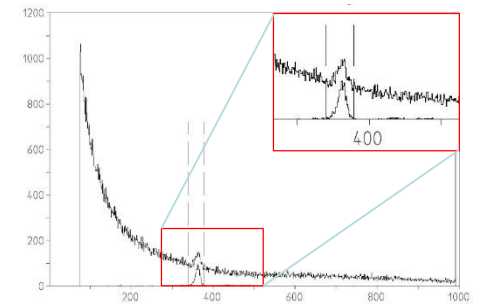
$\Delta E/E$  like Si(Li)



cluster analysis



projection  
onto axis of dispersion

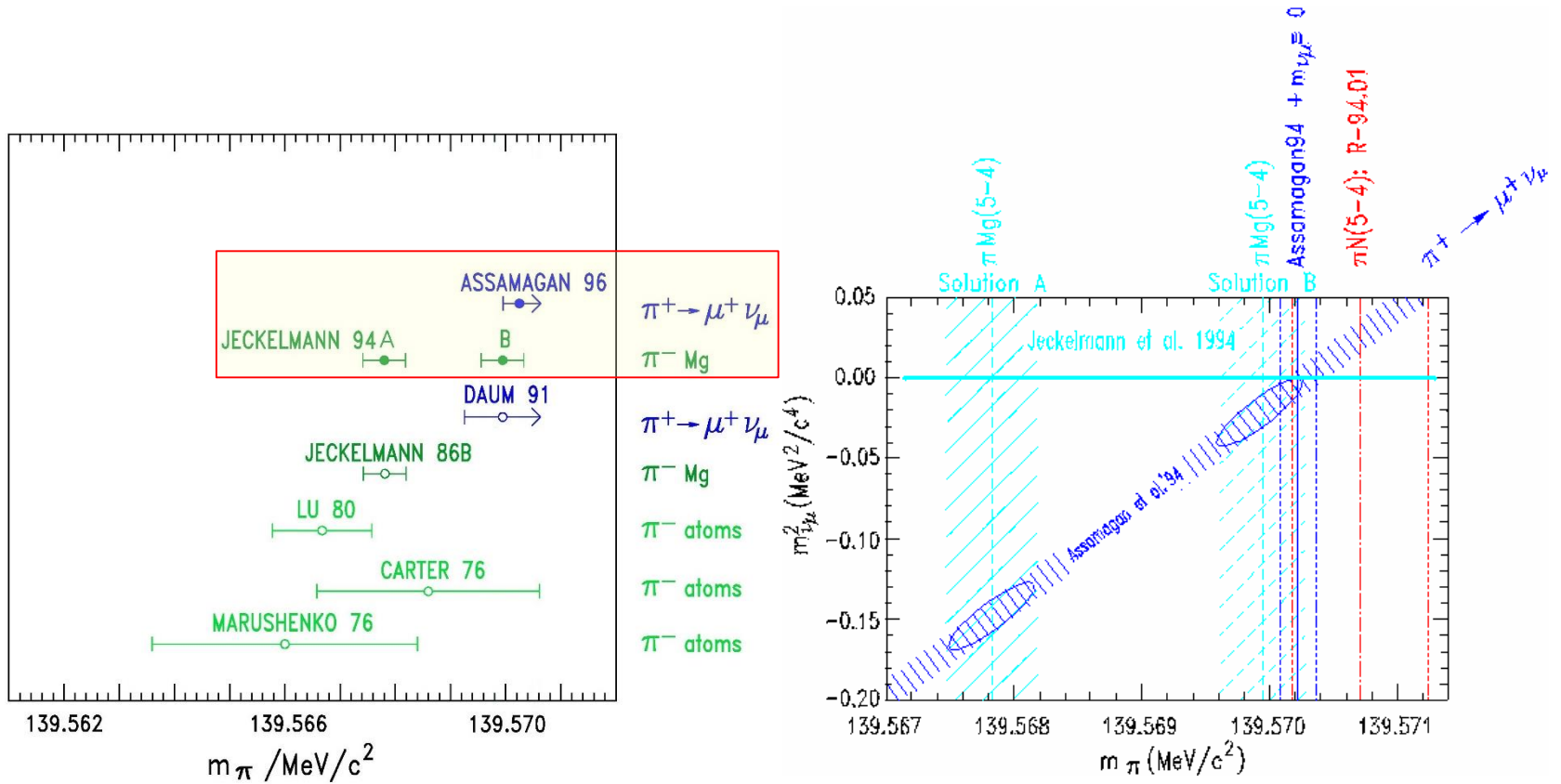


→ coordinate x or energy

# EXPERIMENT IV

***LET'S DO IT!***

# STARTING POINT - *two solutions for $M_\pi$*



$A \Rightarrow m_{\nu_\mu}^2 < 0!$

# $\pi^- \text{Mg} (4f - 3d)$

$$E_x = 25.9 \text{ keV}$$

measurement

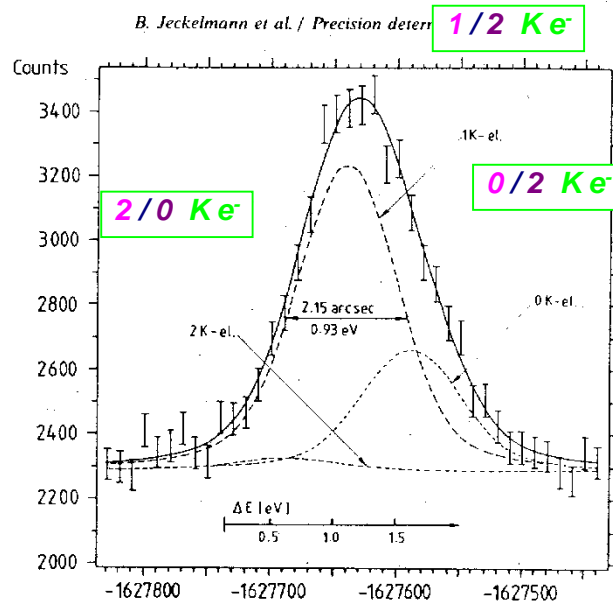
DuMont (transmission-type) crystal spectrometer

**Mg solid state target** - refilling of electrons

B. Jeckelmann, et al., Phys. Rev. Lett. 56 (1986) 1444.

B. Jeckelmann, et al., Nucl. Phys. A 457 (1986) 709.

B. Jeckelmann, P.F.A. Goudsmit, H.J. Leisi, Phys. Lett. B 335 (1994) 326.



$$\Delta E_{\text{exp}} / E = 3 \text{ ppm}$$

but

*linewidth* > *resolution!*

1 or 2 K electrons?

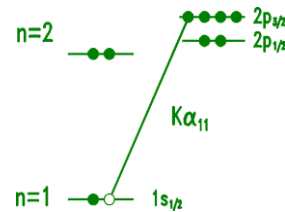
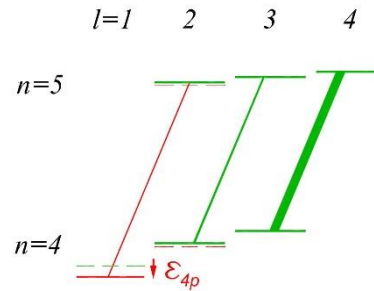
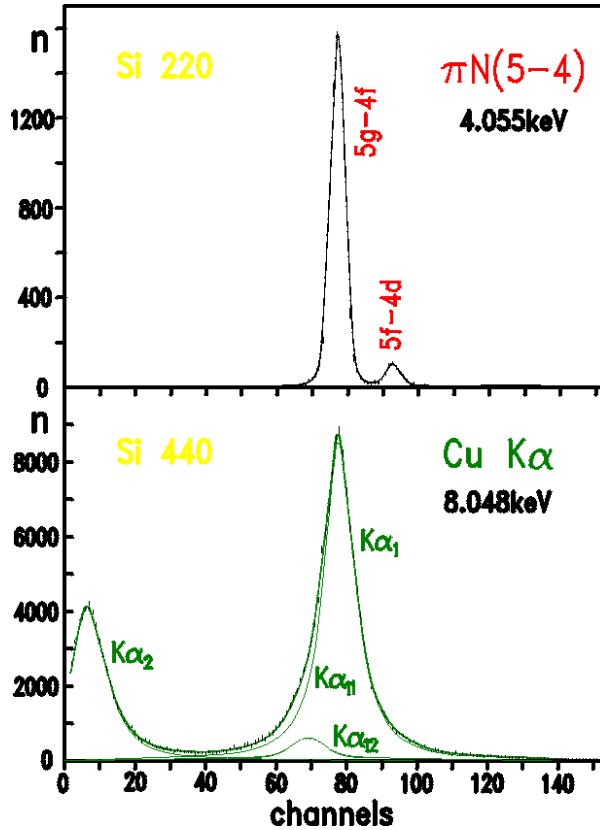
2 solutions A & B:  $\Delta^{AB} = 15 \text{ ppm}$

$$A \Rightarrow m_{\nu\mu}^2 < 0!$$

*interpretation A / interpretation B*

# FIRST STEP - How to get rid of the electrons?

**$N_2$  gas target** – no electrons because no refilling



energy calibration

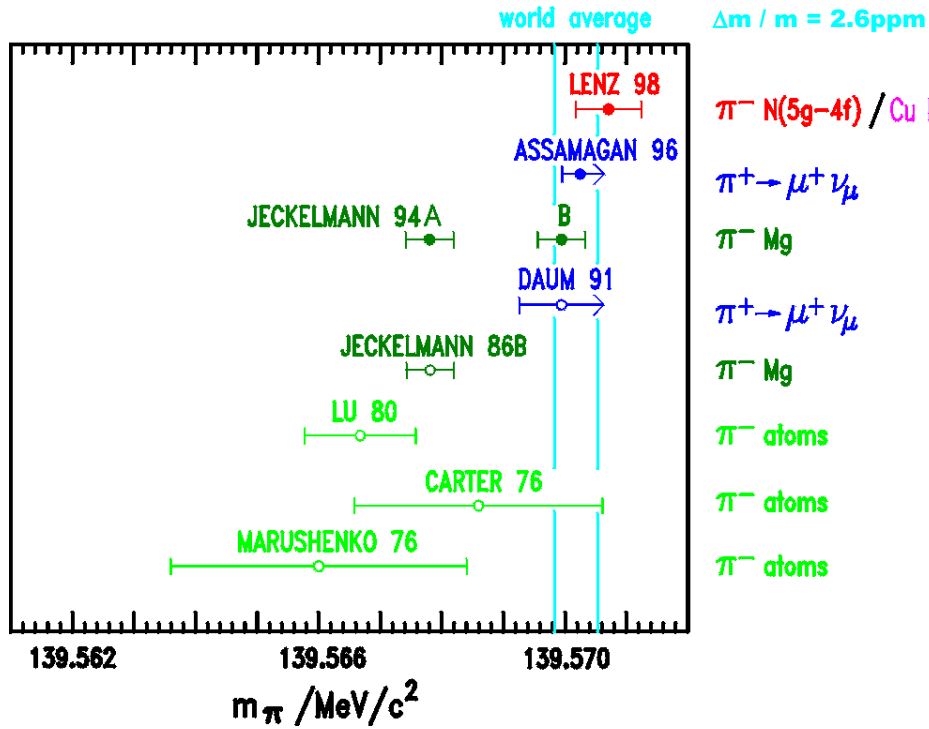
*Cu K  $\alpha$  line shape*

from Deutsch et al. PR A 51 (1995) 283

## Problems

- large natural line width
- multiple ionisation  $\Rightarrow$  satellite lines

# RESULT



$\pi^- N(5g-4f) / \text{Cu } K\alpha_1$

$$\Delta m_\pi / m_\pi = 4 \text{ ppm}$$

$\pi^+ \rightarrow \mu^+ \nu_\mu$

**15 ppm discrepancy removed**

$\pi^- \text{ Mg}$

S. Lenz et al. PL B 416 (1998) 50

$\pi^+ \rightarrow \mu^+ \nu_\mu$

$\pi^- \text{ Mg}$

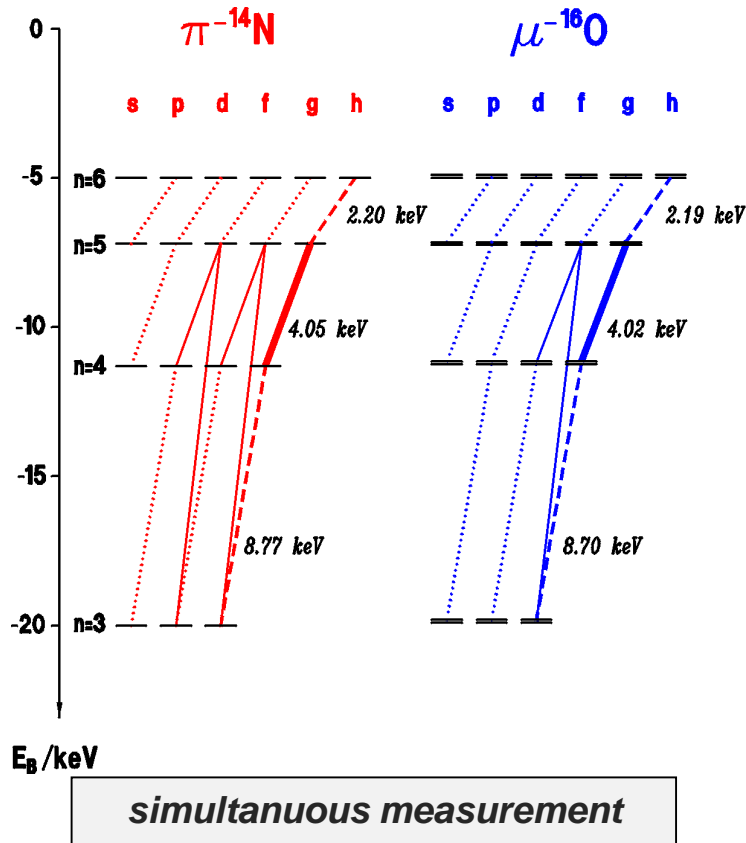
$\pi^- \text{ atoms}$

$\pi^- \text{ atoms}$

$\pi^- \text{ atoms}$

## SECOND STEP - How to improve the calibration standard?

### Energy calibration with muonic atom



- point like Coulomb potential

- no electron screening

- $$\frac{E_{\mu\text{O}(5g-4f)}}{E_{\pi\text{N}(5g-4f)}} = \frac{m_{\mu}}{m_p} + \dots$$

**CPT**

$$\mu^+ \leftrightarrow \mu^-$$

$$\Delta m_{\mu} / m_{\mu} = 0.05\text{ppm}$$

D. Groom et al. (PDG)

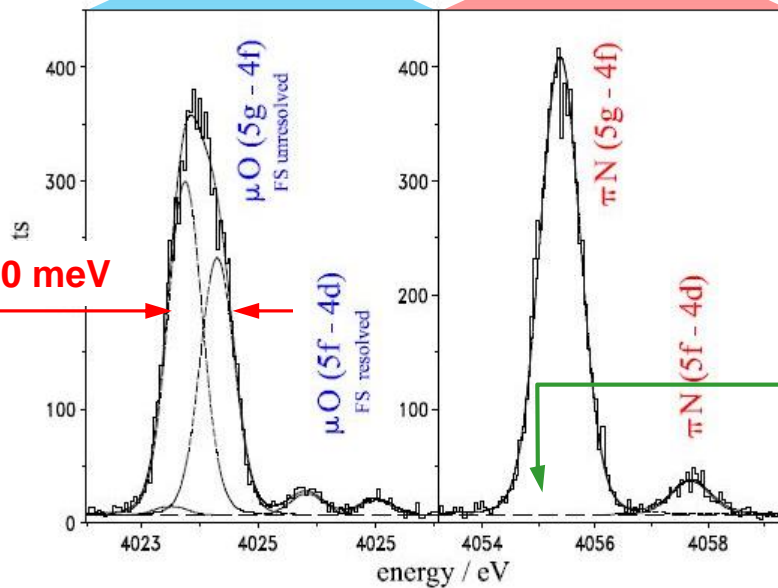
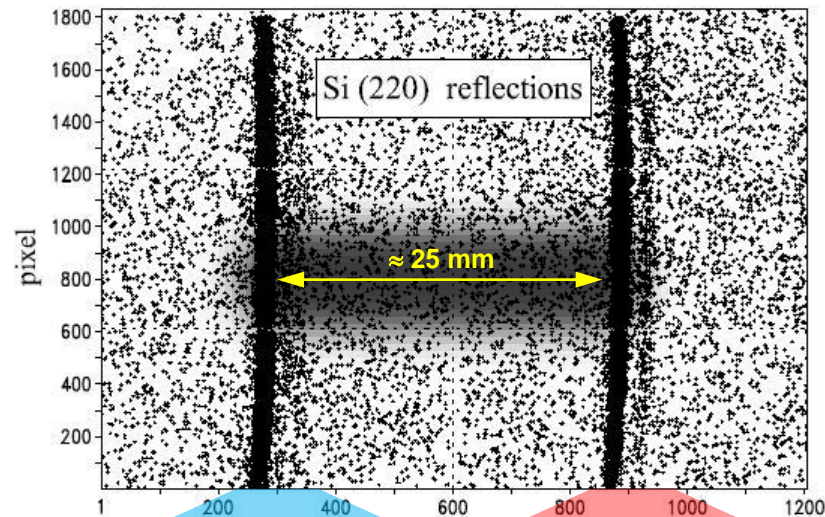


$N_2/O_2$  mixture (10% / 90%)  
@ 1.4 bar

9000 events per element  
5 weeks measuring time  
15 events / hour

$\mu O(5g-4f)$

$\pi N(5g-4f)$



*resolution should be better!*

*electronic satellite?  
Bayesian analysis  
 $< 3 \cdot 10^{-6}$*

# How to measure the spectrometer response?

**measurement**

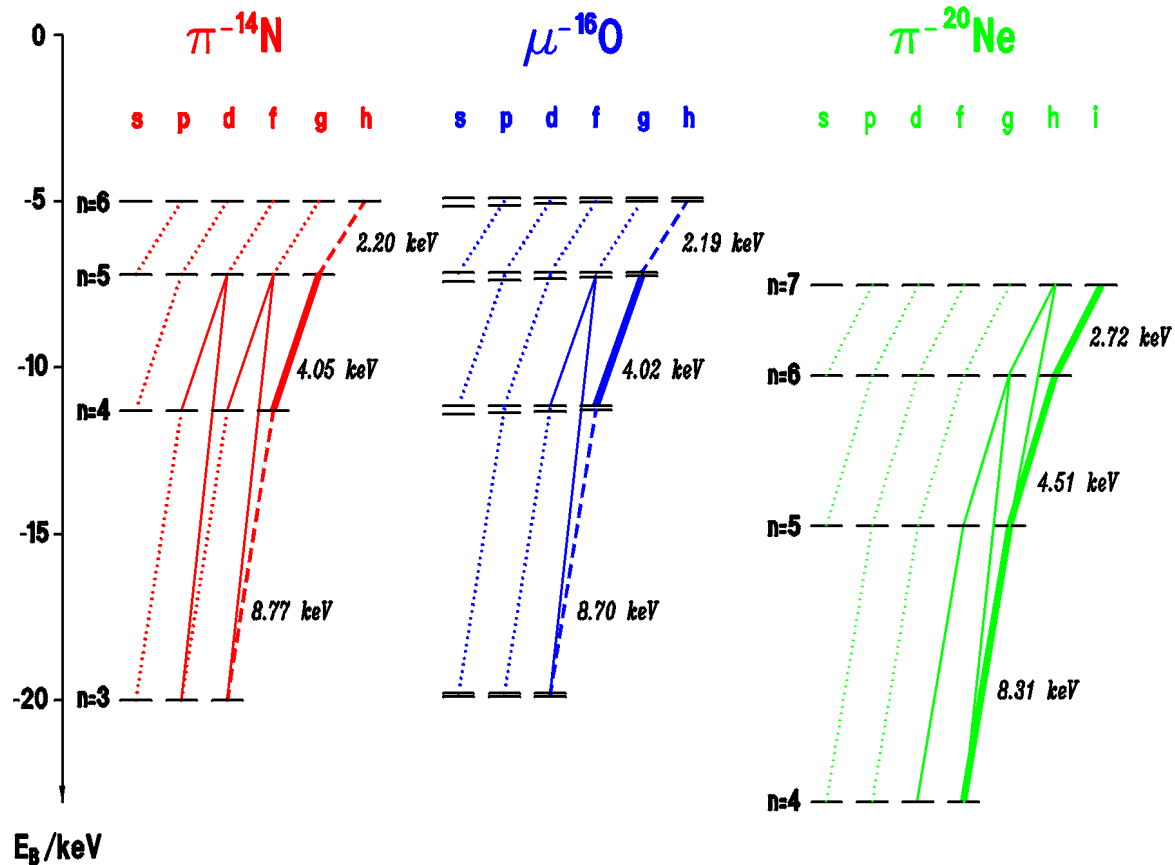
$\pi\text{N}(5g-4f)$

**calibration**

$\mu\text{O}(5g-4f)$

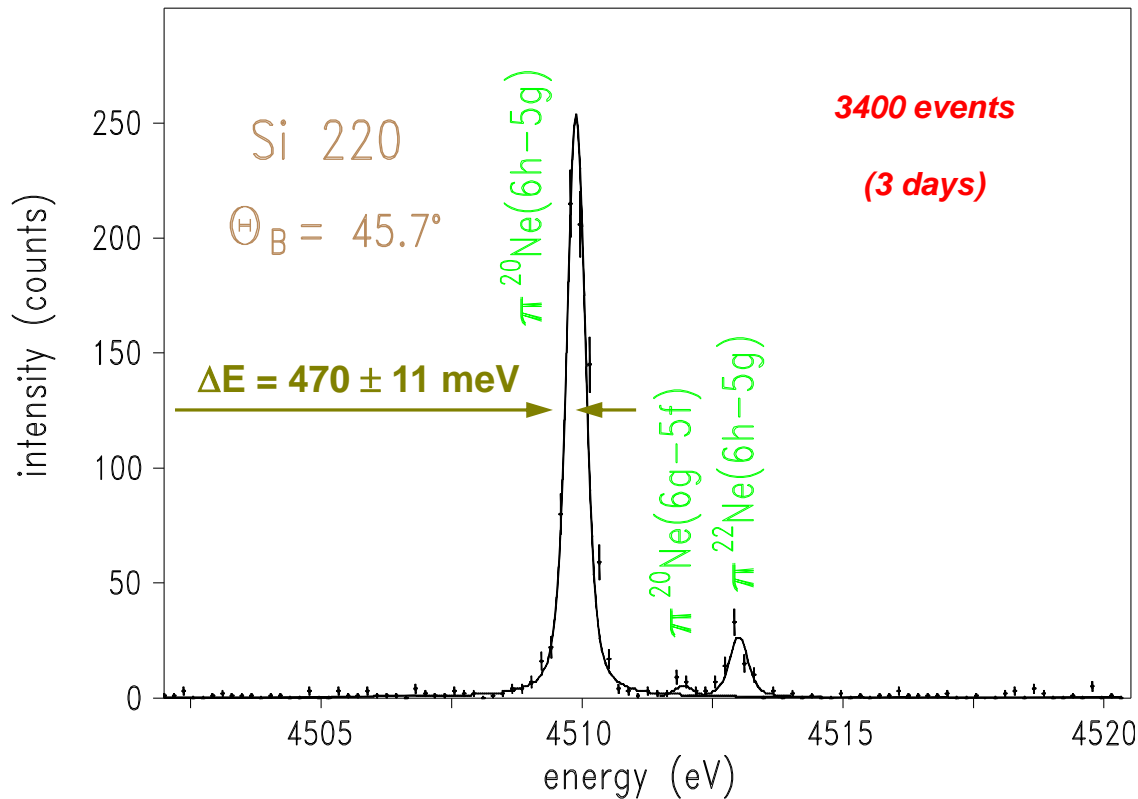
**response function**

$\pi\text{Ne}(6h-5g)$



# RESPONSE FUNCTION *from exotic atoms*

*no narrow  $\gamma$ - rays available for these energies*



**QED**

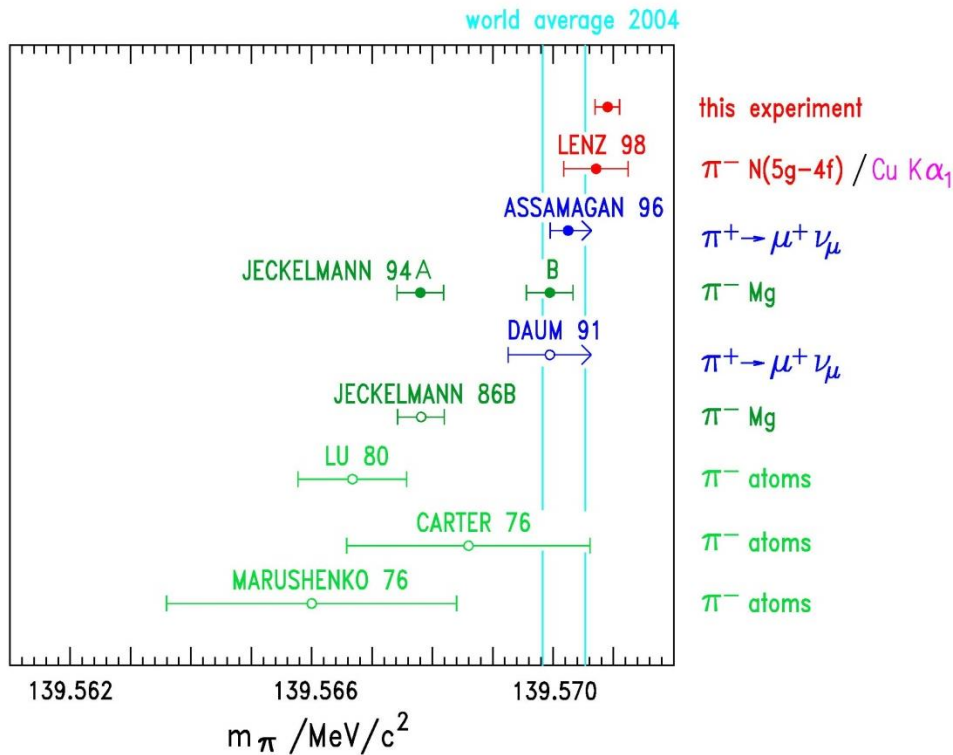
$\pi\text{Ne}(6h-5g)$   
 $(4509.894 \pm 0.001) \text{ eV}$

*closest  
to energy of  
 $\pi\text{N}/\mu\text{O}(5g-4f)$*

- MOTIVATION
- LABORATORY
- EXPERIMENT
- **ASSESSMENT of RESULTS**

# RESULTS *relative to world average PDG 2004 ( $\pm 2.5$ ppm)*

$\pi\text{N}/\text{Cu } K\alpha$	1998	$+ 3.8 \pm 3.8$ ppm
$\pi\text{Mg}$	1994 B	$- 1.7 \pm 2.5$ ppm
$\pi\text{N}/\mu\text{O}$	2016	$+ 4.2 \pm 0.8_{\text{stat}} \pm 1.0_{\text{sys}} (\pm 1.3)$ ppm



**$139,57077 \pm 0.00018 \text{ MeV}/c^2$  2016**

***M. Trassinelli et al., PL B 759 (2016) 583***

***S. Lenz et al., PL B 416 (1998) 50***

**cosmological limit (CMB) 2014**

$$\sum_j m_{i=} = 1.3 \text{ eV}/c^2$$

***K.A. Olive et al. (PDG),  
Chin. Phys. C 38,09001 (1998)***

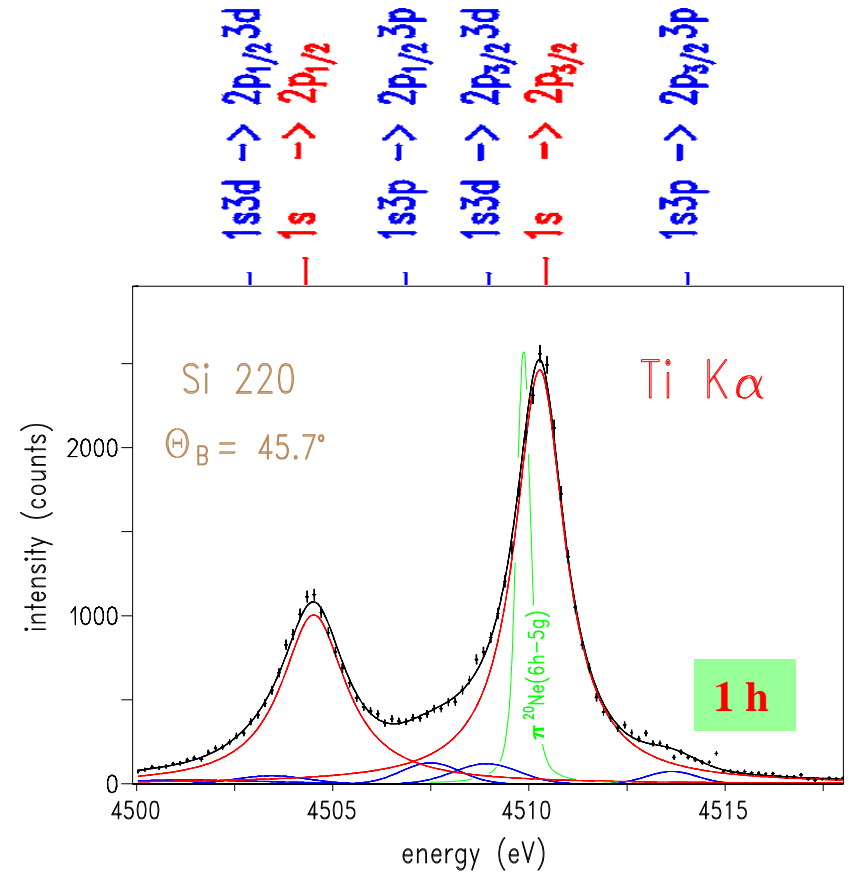
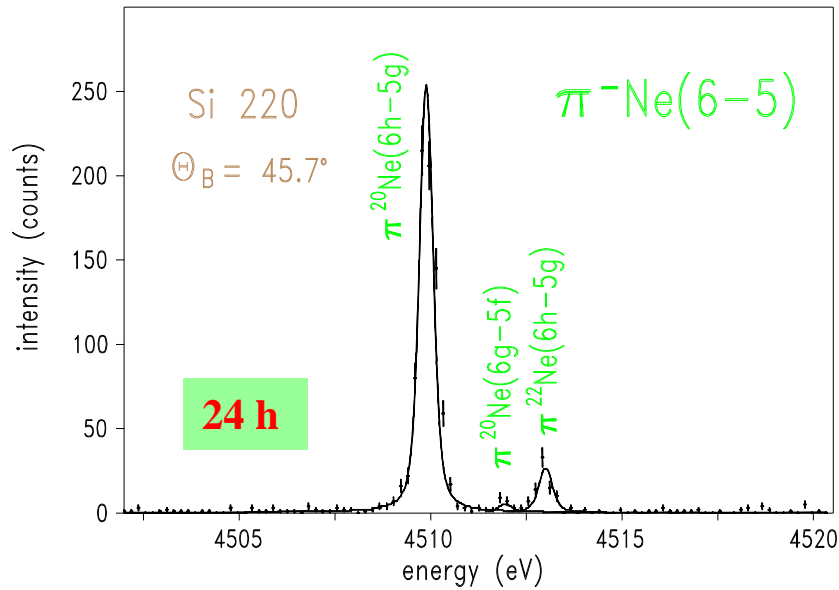
# Side result: new X-ray standards

connect *exotic-* and *electronic-atom* X-rays

**QED**

$\pi$ Ne (6h-5g)

$(4509.894 \pm 0.001) \text{ eV}$



D.F. Anagnostopoulos et al., Phys. Rev. Lett. 91 (1999) 2018

# *Publication*

## A new determination of the mass of the charged pion

Physics Letters B 416 (1998) 50–55

**1. STEP**

## Measurement of the charged pion mass using X-ray spectroscopy of

Physics Letters B 759 (2016) 583–588



**2. STEP**

VOLUME 91, NUMBER 24

PHYSICAL REVIEW LETTERS

week ending  
12 DECEMBER 2003

## Low-Energy X-Ray Standards from Hydrogenlike Pionic Atoms

**Side result**

VOLUME 84, NUMBER 20

PHYSICAL REVIEW LETTERS

15 MAY 2000

## First Direct Observation of Coulomb Explosion during the Formation of Exotic Atoms

**Side result**

# Limits

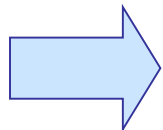
exotic atoms and molecules



$\pi N / \mu O$

Coulomb explosion

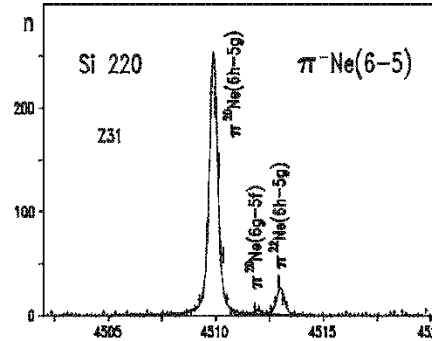
molecule fragmentation



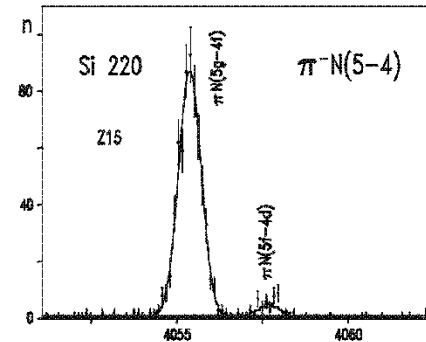
$\mu O$

count rate ( $\approx 1/10$  of  $\pi N$ )

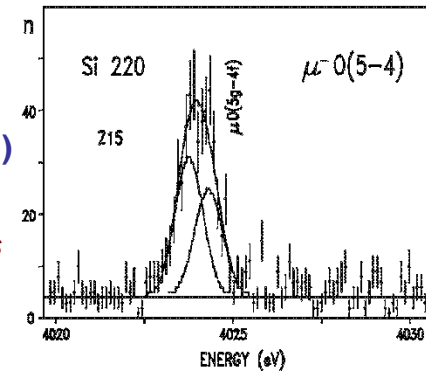
$\approx 10000$  per line / 6 weeks



response function  
 $\Delta E = 520$  meV



$\Delta E_{\text{Doppler}} \approx 800 \pm 100$  meV  
 $\Rightarrow q_1 \cdot q_2 \approx 9 \pm 2$

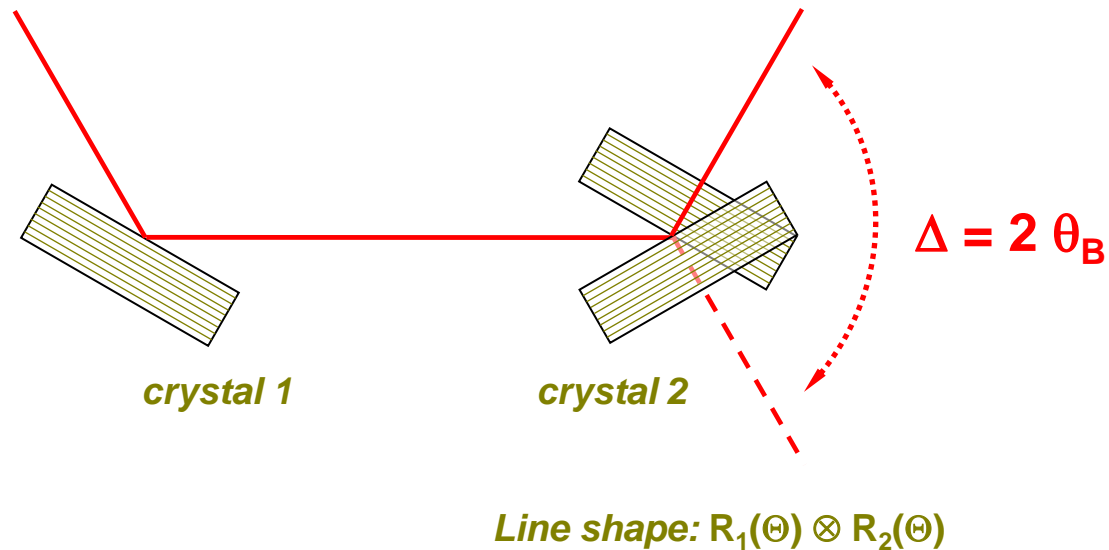


$\Delta E_{\text{Doppler}} \approx 900 \pm 300$  meV  
 $\Rightarrow q_1 \cdot q_2 \approx 19 \pm 10$

T. Siems et al,  
Phys. Rev. Lett. 84 (2000) 4573



## Possible solution: double flat crystal spectrometer



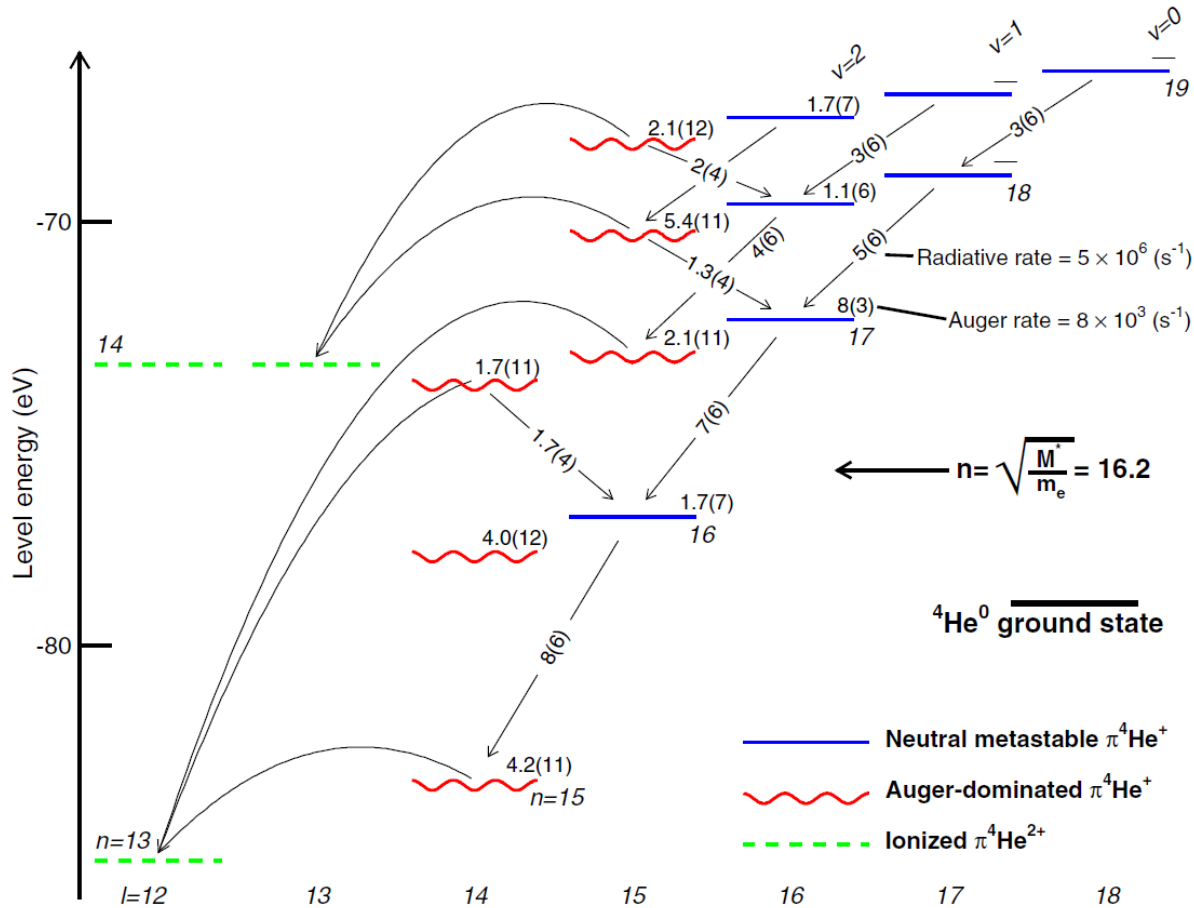
### advantage

- absolute angle calibration
- no Coulomb explosion (noble gas)

### disadvantage

- accurate knowledge of lattice constant  $d$   $\text{Si } \Delta d/d \approx 10^{-8}$
- “ “ of  $\Delta\Theta_{\text{ind}}$
- measuring time (one measurement per bin)

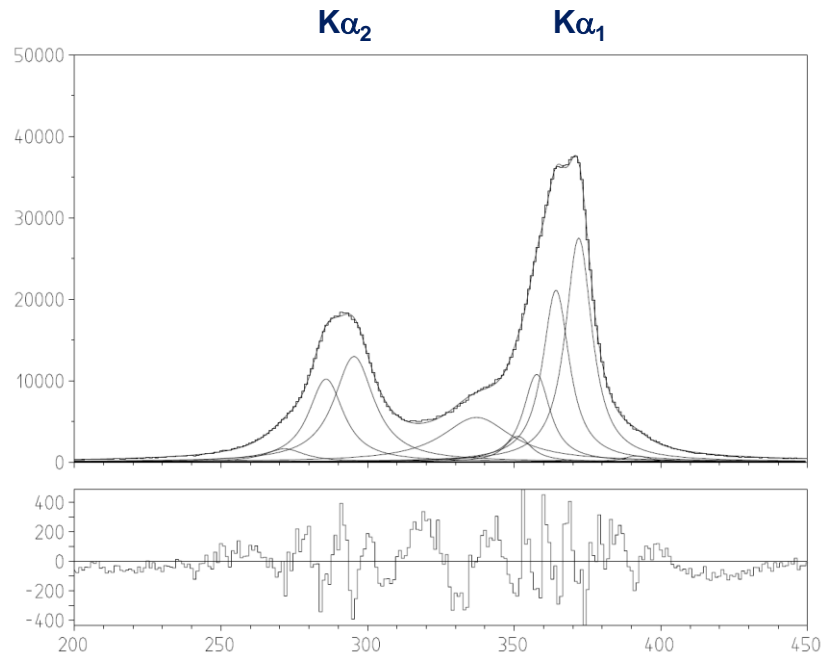
# Outlook: Laser-induced excitation of metastable $\pi^- \text{He}^+$ states



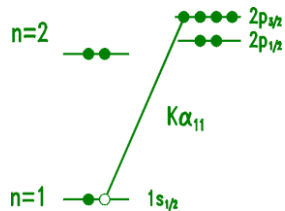
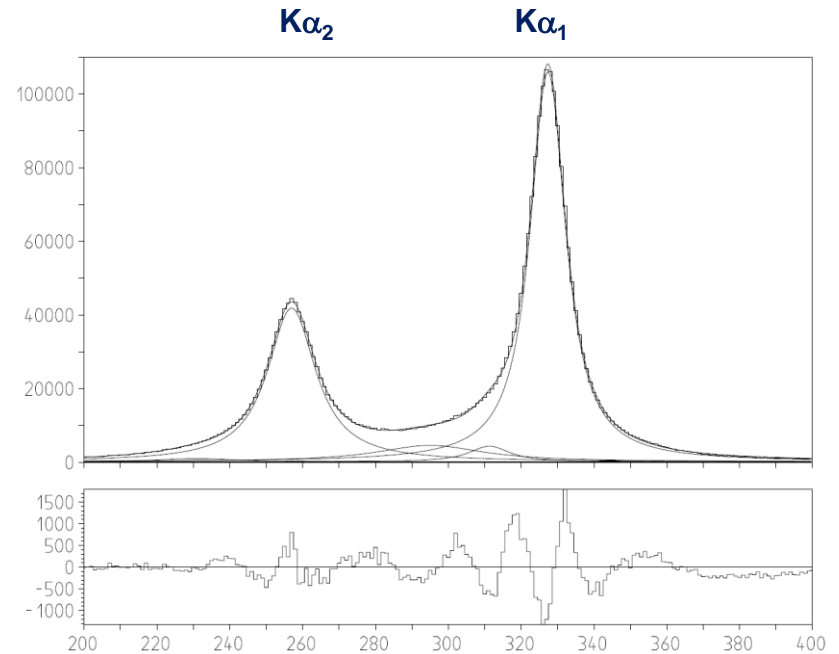
M. Hori, A. Sótér, V. I. Korobov, PR A 89, 042515 (2014)

# „Spin-off“: Chemical effects – Mn $K\alpha$

**MnF<sub>2</sub> - core Mn<sup>2+</sup>**



**Mn(V)-complex - core Mn<sup>5+</sup>**



**$K\alpha_2$      $\approx 5888$  eV**  
 **$K\alpha_1$      $\approx 5899$  eV**

**M. Jabua / GTU 2016 PhD 2016**

# PION MASS collaboration

*experiments R-94.01 & R-97.02*

*Paul-Scherrer-Institut (PSI), Villigen, Switzerland*

*Ioannina<sup>1</sup> – Jülich<sup>2</sup> – Leicester<sup>3</sup> – Paris<sup>4</sup> – PSI<sup>5</sup>*

D. F. Anagnostopoulos<sup>1</sup>, G. Borchert<sup>2</sup>, A. Dax<sup>5</sup>, D Gotta<sup>2</sup>, M. Hennebach<sup>2</sup>, P. Indelicato<sup>4</sup>,  
Y.-W. Liu<sup>5</sup>, B. Manil<sup>4</sup>, N. Nelms<sup>3</sup>, L. M. Simons<sup>5</sup>, M. Trassinelli<sup>4</sup>, A. Wells<sup>3</sup>

*CCDs*

*Crystal spectrometer*

*Cyclotron trap*

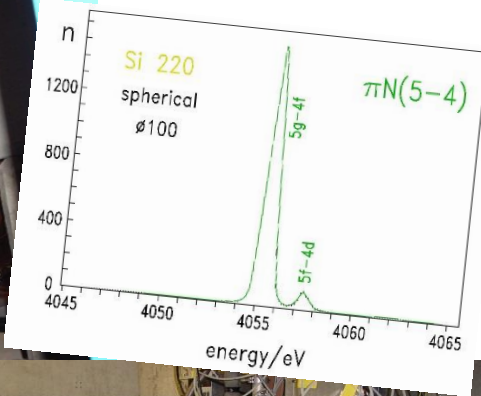
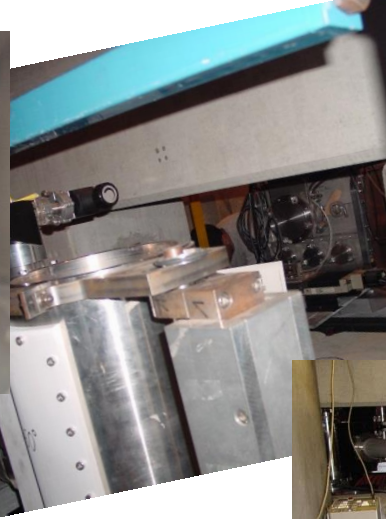
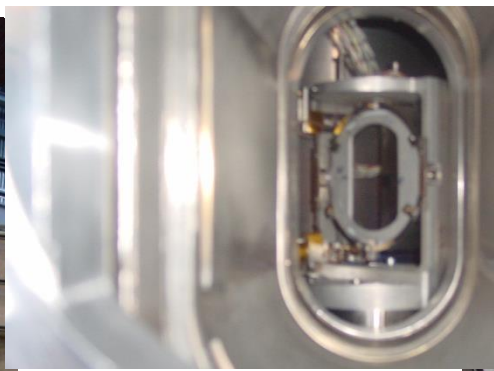
*Data analysis*

*Leicester, PSI*

*Jülich*

*PSI*

*Ioannina, Jülich, Paris*



THANK YOU

