Effective field theory and the derivation of the Salam-Weinberg model

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Outline

- Briefly about the Salam-Weinberg model;
- Derivation of the Salam-Weinber model from the principles of EFT;
- Summary;

The standard model (SM) is widely accepted as a consistent theory of the strong, electromagnetic and weak interactions.

Invariance under Lorentz and local gauge $SU(3)_C \times SU(2)_L \times U(1)$ transformations is taken as the underlying symmetry of the SM.

Salam-Weinberg (SW) model is the theory of electroweak interactions based on $SU(2)_L \times U(1)$ group.

Particle content of the model, i.e. quarks, leptons, Higgs particle, and also *vector bosons* is an input.

It is very often claimed that the existence and the interactions of the vector bosons is a consequence, i.e. an output of the non-Abelian gauge group ...

Non-Abelian Gauge Theories

The quantum field theories that have proved successful in describing the real world are all non-Abelian gauge theories, theories based on principles of gauge invariance more general than the simple $U(1)$ gauge invariance of quantum electrodynamics. These theories share with electrodynamics the attractive feature, outlined at the end of Section 8.1, that the existence and some of the properties of the gauge fields follow from a principle of invariance under local gauge transformations. In electrodynamics, fields $\psi_n(x)$ of charge e_n undergo the gauge transformation $\psi_n(x) \to \exp(ie_n\Lambda(x))\psi_n(x)$ with arbitrary $\Lambda(x)$. Since $\partial_\mu\psi_n(x)$ does not transform like $\psi_n(x)$, we must introduce a field $A_\mu(x)$ with the gauge transformation property $A_\mu(x) \to A_\mu(x)+\partial_\mu\Lambda(x)$, and use it to construct a gauge-covariant derivative $\partial_\mu\psi_n(x)-ie_nA_\mu(x)\psi_n(x)$, which transforms just like $\psi_n(x)$ and can therefore be used with $\psi_n(x)$ to construct a gauge-invariant Lagrangian. In a similar way, the existence and some of the properties of the gravitational field $g_{\mu\nu}(x)$ in general relativity follow from a symmetry principle, under general coordinate transformations.* Given these distinguished precedents, it was natural that local gauge invariance should be extended to invariance under local non-Abelian gauge transformations.

In the original 1954 work of Yang and Mills,[1] the non-Abelian gauge group was taken to be the $SU(2)$ group of isotopic spin rotations, and the vector fields analogous to the photon field were interpreted as the fields of strongly-interacting vector mesons of isotopic spin unity. This proposal immediately encountered the obstacle that these vector mesons would have to have zero mass, like photons, and it seemed that any such particles would already have been detected. Another problem was that, like all strong-interaction theories at that time, there was nothing that

* Of course, both local gauge invariance and general covariance can be realized in a trivial way, by taking $A_\mu(x)$ and $g_{\mu\nu}(x)$ to be non-dynamical c-number functions that simply characterize a choice of phase or coordinate system, respectively. These symmetries become physically significant when we treat $A_\mu(x)$ and $g_{\mu\nu}(x)$ as dynamical fields, over which we integrate in calculating S-matrix elements.

In connection with the SW model one often hears/reads that Higgs mechanism is the origin of the masses ... of quarks and leptons ...
which make only a tiny part of the baryon masses!

Where do the masses come from?

Equally valid question:
Where do the all kind of charges come from? ...
From the same place where the particles come from ...
that is: I do not know!!!

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Right question:

If $SU(2)_L \times U(1)$ is an underlying fundamental symmetry of the EW interaction then where do the quark and lepton masses come from?

Answer: Higgs mechanism is responsible for that!

However ... where does this gauge symmetry come from?

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However ... where does this gauge symmetry come from?

The electromagnetic and gravitational forces are long-ranged and therefore if they are indeed mediated by massless photons and gravitons, then the corresponding local Lorentz-invariant quantum field theories must be gauge theories
S. Weinberg, *The Quantum Theory Of Fields. Vol. 1: Foundations* (Cambridge University Press, Cambridge, England, 1995).

The weak interaction is mediated by massive particles,
Why should it be described by a gauge theory?

A gauge-invariant theory with the spontaneous symmetry breaking has been derived by demanding tree-order unitarity
C. H. Llewellyn Smith, Phys. Lett. B **46**, 233 (1973).
J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. Lett. **30**, 1268 (1973) [Erratum-ibid. **31**, 572 (1973)].
J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974) [Erratum-ibid. D **11**, 972 (1975)].
S. D. Joglekar, Annals Phys. **83**, 427 (1974).

This result could be considered as an answer to the above raised question, however, the modern point of view considers the SM as an EFT which inevitably violates the tree-order unitarity.

Original requirement of perturbative renormalizability which lead to a gauge theory of the EW is no longer a valid requirement either.

This motivated us to revisit the problem.

Our final aim is to construct a consistent most general Lorentz-invariant EFT of interacting three massive vector bosons, fermions and a scalar.

We start with vector bosons and a scalar.

Starting assumptions and required constraints

Free massive vector bosons are described by Proca Lagrangian which incorporates the second class constraints.

To have a consistent theory of interacting massive vector bosons interaction terms have to be consistent with these constraints.

This generates some relations between coupling constants.

Next we impose the renormalizability in the sense of an EFT.

The condition of perturbative renormalizability cannot be satisfied unless the coupling constants satisfy further restricting conditions.

SM is LO approximation to an EFT in which higher order operators are suppressed by powers of some large scale.
The value of this large scale is determined by new physics.

This leads to the next condition on our EFT - separation of scales.

That is, divergences of loop diagrams contributing in *physical* scattering amplitudes generated by the LO Lagrangian should be removable by renormalizing the parameters of the LO Lagrangian.

This condition is equivalent to demanding renormalizability of the LO EFT Lagrangian in the traditional sense, however not for off-shell Green's functions but for on-shell S-matrix.

To illustrate the problem with the scale separation consider an EFT specified by the following Lagrangian

$$\mathcal{L} = -\frac{1}{4}\, F^a_{\mu\nu} F^{a\mu\nu} + \frac{M^2}{2}\, W^a_\mu W^{a\mu} + \mathcal{L}_{\rm ho},$$

where $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu$, W^a_μ is a triplet of vector bosons and $\mathcal{L}_{\rm ho}$ contains all possible local terms with coupling constants of inverse mass dimensions which are invariant under local $SU(2)$ gauge transformations.

Massive Yang-Mills theory is perturbatively non-renormalizable.

Therefore to get rid of divergences generated by interactions with dimensionless couplings one needs to renormalize couplings of $\mathcal{L}_{\rm ho}$.

Consider the vector boson self-energy.

Calculating divergent parts of one-loop diagrams we obtain

$$\Sigma^{ab,\mu\nu}_{\rm div}(p) = \frac{g^2\delta_{ab}}{96\pi^2 M^4 (n-4)} \left[84M^4 - 14M^2p^2 - p^4\right] \left(p^\mu p^\nu - p^2 g^{\mu\nu}\right) .$$

The M^4 term is canceled by the vector field renormalization.

The second (magenta) term in the square brackets is removed by renormalizing the coupling constant of the following term:

$$\mathcal{L}_{HD} = \frac{g_{HD}}{4}\, D^{ab}_\mu F^b_{\nu\lambda} D^{ac,\mu} F^{c\nu\lambda} ,$$

where $D^{ab}_\mu = \delta^{ab}\partial_\mu - g\epsilon^{abc} W^c_\mu$.

Renormalization of the coupling g_{HD} leads to the following renormalized coupling:

$$g_{HD}(\mu) = g_{HD}(\mu_0) - \frac{7g^2 \ln \frac{\mu}{\mu_0}}{96\pi^2 M^2}.$$

Even if the renormalized coupling $g_{HD}(\mu_0)$, corresponding to $\mu = \mu_0$ is suppressed by some large scale $\Lambda \gg M$, for $\mu \sim e \times \mu_0$ the renormalized coupling will become $g_{HD}(\mu) \sim \frac{g^2}{16\pi^2}\frac{1}{M^2}$ - not suppressed by Λ.

Analogously, for all other couplings with inverse mass dimensions the scale of the renormalized couplings is set by M^2.

EFT Lagrangian and constraint analysis

We start with the most general Lorentz-invariant Lagrangian of charged vector fields $V^{\pm}_{\mu} = (V^1_{\mu} \mp iV^2_{\mu})/\sqrt{2}$, and charge-neutral vector boson V^3_{μ} and scalar Φ fields.

Below we analyse the Lagrangian containing only interaction terms with coupling constants of non-negative mass dimensions:

$$
\begin{aligned}
\mathcal{L} \;=\;& -\frac{1}{4}\, V^a_{\mu\nu} V^{a\mu\nu} + \frac{M_a^2}{2} V^a_{\mu} V^{a\mu} - g_V^{abc}\, V^a_{\mu} V^b_{\nu} \partial^{\mu} V^{c\nu} \\
&- g_A^{abc}\, \epsilon^{\mu\nu\alpha\beta} V^a_{\mu} V^b_{\nu} \partial_{\alpha} V^c_{\beta} - h^{abcd}\, V^a_{\mu} V^b_{\nu} V^{c\mu} V^{d\nu} \\
&+ \frac{1}{2}\, \partial_{\mu}\Phi\, \partial^{\mu}\Phi - \frac{m^2}{2}\, \Phi^2 - a\,\Phi - \frac{b}{3!}\, \Phi^3 - \frac{\lambda}{4!}\, \Phi^4 \\
&- g_{vss}\, \partial_{\mu} V^{3\mu} \Phi^2 - g^{ab}_{vvs}\, V^{a\mu}\, V^b_{\mu} \Phi - g^{ab}_{vvss}\, V^{a\mu}\, V^b_{\mu} \Phi^2\,,
\end{aligned}
$$

where $V^a_{\mu\nu} = \partial_{\mu} V^a_{\nu} - \partial_{\nu} V^a_{\mu}$, M_a ($M_1 = M_2 = M$) and m are masses and the summations run from 1 to 3.

Coupling of the linear term a vanishes at tree order. and further corrections can be fixed by demanding that the VEV of Φ vanishes.

The interaction terms of Φ with two vector fields can be written as

$$g_{1,s} = g^{11}_{vvs} = g^{22}_{vvs}, \quad g_{2,s} = g^{33}_{vvs}, \quad g_{1,ss} = g^{11}_{vvss} = g^{22}_{vvss}, \quad g_{2,ss} = g^{33}_{vvss},$$

and all other g^{ab}_{vvs} and g^{ab}_{vvss} couplings do not contribute.

The three-vector boson interaction term depends on ten parameters,

$$\begin{aligned}
g_V^{333} &= g_1, \quad g_V^{113} = g_2, \quad g_V^{123} = -g_3, \quad g_V^{213} = g_3, \\
g_V^{223} &= g_2, \quad g_V^{311} = g_4, \quad g_V^{321} = -g_5, \quad g_V^{312} = g_5, \\
g_V^{322} &= g_4, \quad g_V^{131} = g_6, \quad g_V^{231} = -g_7, \quad g_V^{132} = g_7, \quad g_V^{232} = g_6\,, \\
g_A^{213} &= -g_A^{123} = g_{A1}, \quad g_A^{311} = g_A^{322} = -g_A^{131} = -g_A^{232} = g_{A2}, \\
g_A^{312} &= -g_A^{321} = -g_A^{132} = g_A^{231} = g_{A3}\,.
\end{aligned}$$

All other constants vanish.

Charge conservation relates the couplings h^{abcd} to each other.

Four-vector interaction term can be parameterized in terms of five parameter $d_1, \cdots, d_5$.

The canonical momenta corresponding to Φ, V_0^a and V_i^a:

$$\begin{aligned}
p &= \frac{\partial \mathcal{L}}{\partial \dot\Phi} = \dot\Phi \,, \\
\pi_0^a &= \frac{\partial \mathcal{L}}{\partial \dot V_0^a} = -g_V^{bca} V_0^b V_0^c - g_{vss} \delta_{a3}\, \Phi^2, \\
\pi_i^a &= \frac{\partial \mathcal{L}}{\partial \dot V_i^a} = V_{0i}^a + g_V^{bca} V_0^b V_i^c + g_A^{bca} \epsilon^{ijk0} V_j^b V_k^c \,.
\end{aligned}$$

Second equation leads to the primary constraints

$$\phi_1^a = \pi_0^a + g_V^{bca} V_0^b V_0^c + g_{vss}\, \delta_{a3}\, \Phi^2 \,.$$

On the other hand, from the first and third equations we solve

$$\begin{aligned}
\dot V_i^a &= \pi_i^a + \partial_i V_0^a - g_V^{bca} V_0^b V_i^c - g_A^{bca} \epsilon^{ijk0} V_j^b V_k^c \,, \\
\dot\Phi &= p \,.
\end{aligned}$$

For the total Hamiltonian we have:

$$H_1 = \int d^3\mathbf{x}\ (\phi_1^a z^a + \mathcal{H})$$

with

$$\mathcal{H} = \frac{\pi_i^a \pi_i^a}{2} + \pi_i^a \partial_i V_0^a + \frac{1}{4} V_{ij}^a V_{ij}^a - \frac{M_a^2}{2} V_\mu^a V^{a\mu} + \ldots ,$$

and the z^a are arbitrary functions which must be determined.

Condition of conserving primary constraints ϕ_1^a leads to

$$\{\phi_1^a, H_1\} = A^{ab} z^b + \chi^a = 0.$$

The 3×3 matrix A is given by

$$A = \begin{pmatrix} 0 & -2\gamma_1 V_0^3 & \gamma_2 V_0^1 - \gamma_1 V_0^2 \\ 2\gamma_1 V_0^3 & 0 & \gamma_1 V_0^1 + \gamma_2 V_0^2 \\ -(\gamma_2 V_0^1 - \gamma_1 V_0^2) & -(\gamma_1 V_0^1 + \gamma_2 V_0^2) & 0 \end{pmatrix},$$

where $\gamma_1 = g_5 + g_7$ and $\gamma_2 = g_4 + g_6 - 2g_2$. The above system of equations can be satisfied only if (secondary constraint):

$$\phi_2 = \chi^1 (\gamma_1 V_0^1 + \gamma_2 V_0^2) + \chi^2 (\gamma_1 V_0^2 - \gamma_2 V_0^1) - \chi^3 2\gamma_1 V_0^3 = 0 .$$

If at least one of γ_1 or γ_2 is non-zero then we obtain that

$$\begin{aligned} z^1 &= \frac{\chi_3 + \gamma_1 z^2 V_0^1 + \gamma_2 z^2 V_0^2}{\gamma_1 V_0^2 - \gamma_2 V_0^1}, \\ z^3 &= \frac{\chi_1 + 2\gamma_1 z^2 V_0^3}{\gamma_2 V_0^1 - \gamma_1 V_0^2} \end{aligned}$$

and z^2 can be solved from conservation of ϕ_2, $\{\phi_2, H_1\} = 0$.

However, in this case we obtain four constraints of the second class instead of six.
Therefore, for a self-consistent theory we must require

$$\gamma_1 = \gamma_2 = 0 \Rightarrow \; g_7 = -g_5 \,, \;\; 2g_2 = g_4 + g_6 \,.$$

Thus we are left with secondary constraints:

$$\{\phi_1^a, H_1\} \;\; = \;\; \partial_i \pi_i^a + g_V^{abc} V_i^b \pi_i^c + \ldots \equiv \phi_2^a, \quad a = 1, 2, 3.$$

If no more constraints appear then our Lagrangian describes a system with the right number of constraints.

If this is the case, then all z^a have to be solvable from the condition of the constraints ϕ_2^a being conserved in time.

From the condition of conservation of ϕ_2^a in time we obtain

$$\{\phi_2^a, H_1\} = \mathcal{M}^{ab} z^b + Y^a = 0, \quad a = 1, 2, 3,$$

where

$$\mathcal{M}^{ab} = M_a^2 \delta^{ab} - \left(g_V^{bca} + g_V^{cba} \right) \partial_i V_i^c + \dots,$$

and the particular form of Y^a is not important for our purposes. To obtain a self-consistent field theory we demand that $\det \mathcal{M}$ does not vanish.

For small fluctuations this is indeed the case and we proceed by quantizing these small fluctuations and deriving further constraints on the couplings by investigating the conditions of perturbative renormalizability and scale separation.

Perturbative renormalizability

We analyze one-loop diagrams using dimensional regularization.

We impose the on-mass-shell renormalization condition, i.e. require that all divergences in physical quantities should be removable by redefining the parameters of the effective Lagrangian.

We start by calculating the one-loop contribution to the scattering amplitude $V^3 V^3 \to V^3 V^3$, shown below

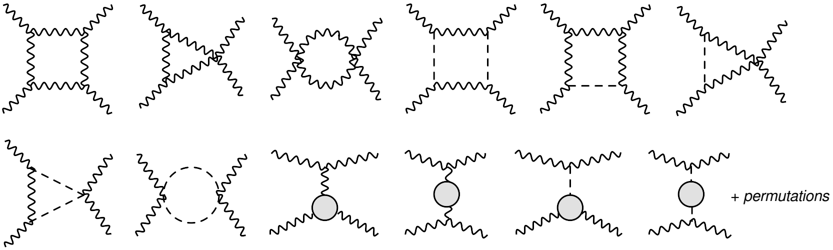

Figure: One-loop contributions to the four-vector vertex function. The dashed and the wiggly lines correspond to the scalar and the vector-boson, respectively. Blobs indicate the corresponding one-loop two- and

Non-pole parts of one-particle reducible diagrams have to be taken into account together with one-particle irreducible diagrams.

We write the sum of divergent parts of the loop diagrams in the form of a polynomial in terms of the Mandelstam variables:

$$V^{\mu\nu\lambda\sigma} = \sum_{i,j=0}^{4} u^i s^j C_{ij}^{\mu\nu\lambda\sigma} ,$$

where $C_{ij}^{\mu\nu\lambda\sigma}$ depend on the momenta, masses and couplings. Scale separation demands that terms with $i + j \neq 0$ must vanish.

Demanding that the term proportional to u^4 vanishes leads to

$$g_1^4 M_3^8 + 2g_2^4 M^8 = 0, \Rightarrow g_1 = 0,\ g_2 = 0 .$$

The next condition is obtained by demanding that the term proportional to u^2 also vanishes. This leads to:

$$\begin{aligned} d_5 &= 0, \quad g_4 = 0, \quad g_5 = -g_3, \\ g_{A2} &= 0, \quad g_{A3} = -g_{A1}, \quad d_3 = -g_3^2. \end{aligned}$$

Demanding the vanishing of the term proportional to s^2, we obtain

$$d_4 = g_3^2 \,.$$

Divergent part of the amplitude $V^3 V^3 \to V^3 V^3$ becomes

$$\sim \left[8M^8 \left(2M_3^2 g_{2,ss} + g_{2,s}^2\right)^2 + M_3^4 \left(g_3^2 M_3^4 - 4M^2 g_{1,s} g_{2,s}\right)^2\right]$$

For $d_5 = g_1 = 0$, the tree-order amplitude $V^3 V^3 \to V^3 V^3$ vanishes and therefore the one-loop divergent expression has also to vanish, leading to:

$$\begin{aligned} g_{2,ss} &= -\frac{g_3^4 M_3^6}{32 M^4 g_{1,s}^2}, \\ g_{2,s} &= \frac{g_3^2 M_3^4}{4M^2 g_{1,s}} \,. \end{aligned}$$

Next, as there is no tree order one-particle irreducible contribution in the amplitude $V^1 V^1 \to V^1 V^1$, we have to demand that the divergent part of the corresponding one-loop contribution vanishes.

Ba requiring that the terms with s^2 and $s\,t$ vanish, we obtain:

$$(d_1 + d_2)\left(d_2 + \frac{g_3^2}{2}\right) = 0\,,$$

$$\left(d_2 + \frac{g_3^2}{2}\right)^2 + (d_1 + d_2)^2 + \frac{1}{4}\,g_3^4\left(1 - \frac{M^4}{M_3^4}\right) = 0\,.$$

Considering $\Phi \to VV$ decay and requiring that the divergences of corresponding diagrams do not contribute in the renormalization of the couplings of the higher-order operators, we find:

$$g_{1,s}\left((d_1+d_2)+d_2+\frac{g_3^2}{2}\right)=0\,.$$

The coupling $g_{1,s}$ cannot be vanishing and therefore we obtain

$$d_1=-d_2=\frac{g_3^2}{2}\,,\quad M_3=M\,.$$

Analyzing the vertex function $V_1 V_2 V_3$ and demanding that the divergent part of the sum of loop diagrams has the same Lorentz structure as the tree one, we obtain

$$g_{1,s} = g_{2,s} = \pm\frac{g_3 M}{2} \, .$$

Going back to the $V_1 V_1 \to V_1 V_1$ amplitude the condition of the vanishing of its divergent part reduces to

$$\left(8g_{1,ss} + g_3^2\right)^2 = 0 \, ,$$

from which we obtain

$$g_{1,ss} = -\frac{g_3^2}{8} \, .$$

Next, we have calculated the divergent parts of one-loop diagrams contributing to the $\Phi V_3 \to \Phi V_3$ scattering amplitude.

As the coupling of the $V^3_\mu V^{3\mu}\Phi^2$ term is given by $g_{2,\mathrm{ss}} = -g_3^2/8$, i.e. in terms of the coupling of the three- and four-vector interactions, the divergent pieces of the corresponding amplitudes have to be correlated.

In a self-consistent theory the renormalized value for the coupling g_3 should be independent from the process that was used to fix it.

After a lengthy one-loop calculation we found that this consistency condition requires that the coupling g_{vss} has to vanish.

All obtained relations can be written as

$$
\begin{aligned}
M_1 &= M_2 = M_3 = M\,, \\
g_V^{abc} &= -g_3\,\epsilon^{abc}\,, \quad g_A^{abc} = g_{A1}\,\epsilon^{abc}\,, \\
h^{abcd} &= \frac{1}{4}\,g_V^{abe} g_V^{cde}\,, \quad g_{vss} = 0\,, \\
g_{1,s} &= g_{2,s} = \frac{g_3 M}{2}\,, \quad g_{1,ss} = g_{2,ss} = -\frac{g_3^2}{8}\,.
\end{aligned}
$$

Denoting $g_3 = g$ the effective Lagrangian takes the form

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}\,G^a_{\mu\nu} G^{a\mu\nu} + \frac{1}{2} V^a_\mu V^{a\mu} \left(M - \frac{g}{2}\Phi \right)^2 - g_{A1}\,\epsilon^{abc} \epsilon^{\mu\nu\alpha\beta}\, V^a_\mu V^b_\nu \partial_\alpha V^c_\beta \\
&+ \frac{1}{2}\,\partial_\mu \Phi\, \partial^\mu \Phi - \frac{m^2}{2}\,\Phi^2 - a\,\Phi - \frac{b}{3!}\,\Phi^3 - \frac{\lambda}{4!}\,\Phi^4\,,
\end{aligned}
$$

where

$$
G^a_{\mu\nu} = V^a_{\mu\nu} - g\,\epsilon^{abc}\, V^b_\mu V^c_\nu\,.
$$

This Lagrangian coincides with the SU(2) locally gauge invariant Lagrangian with spontaneous symmetry breaking in the unitary gauge except for the self-interaction terms of the scalars.

We checked that in all processes with three and four particles one-loop divergences are absorbed in coupling constants and masses and no further conditions on the couplings are obtained.

This leaves the two scalar self-interaction couplings unfixed.

We expect that the investigation of the one-loop diagrams contributing in five and six-point functions will fix the couplings of three and four scalar self-interactions.

Summary

- We revisited the problem of the uniqueness of a theory with spontaneously broken gauge symmetry.
- We analyzed the most general Lorentz-invariant LO EFT Lagrangian of massive vector bosons interacting with a massive scalar field.
- From the constraint structure of the effective Lagrangian we obtained consistency conditions.
- Further conditions were obtained by requiring perturbative renormalizability and scale separation for one-loop order amplitudes with three and four particles.

- All these conditions impose restrictions on the couplings such that the Lagrangian of spontaneously broken gauge symmetry in unitary gauge is obtained, except that the couplings of the self-interactions of the scalar field remain unfixed.
- These are not pinned down by the analysis of the UV divergences of all one-loop three- and four-point functions.
- We expect that condition of perturbative renormalizability for one-loop order amplitudes with five external legs will fix these two free couplings such that the Lagrangian with spontaneously broken SU(2) gauge symmetry taken in unitary gauge appears as an unique LO Lagrangian of a self-consistent EFT of a massive scalar interacting with massive vector bosons.