Hadron physics in the era of large computers

Akaki Rusetsky, University of Bonn

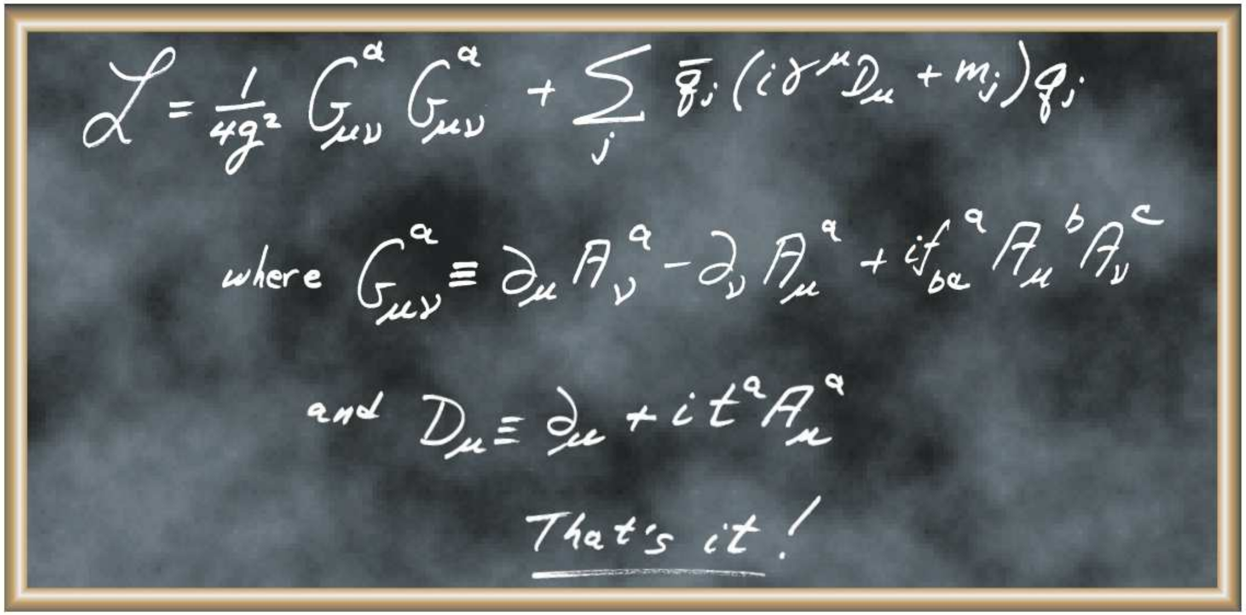
8th Georgian-German School and Workshop in Basic Science (GGSWBS'18)
Tbilisi, August 20th, 2018

Plan

- Introduction
- QCD input in the BSM physics searches:
  - Neurtinoless double β-decay
  - Direct dark matter searches
  - Muon $g-2$
  - Electric dipole moment of hadrons
- QCD on the lattice:
  - Getting started: Lagrangian, hadron masses, decay constants
  - Hadron scattering, resonances, multihadron reactions
  - Lattice QCD and effective field theories
- Conclusions, outlook

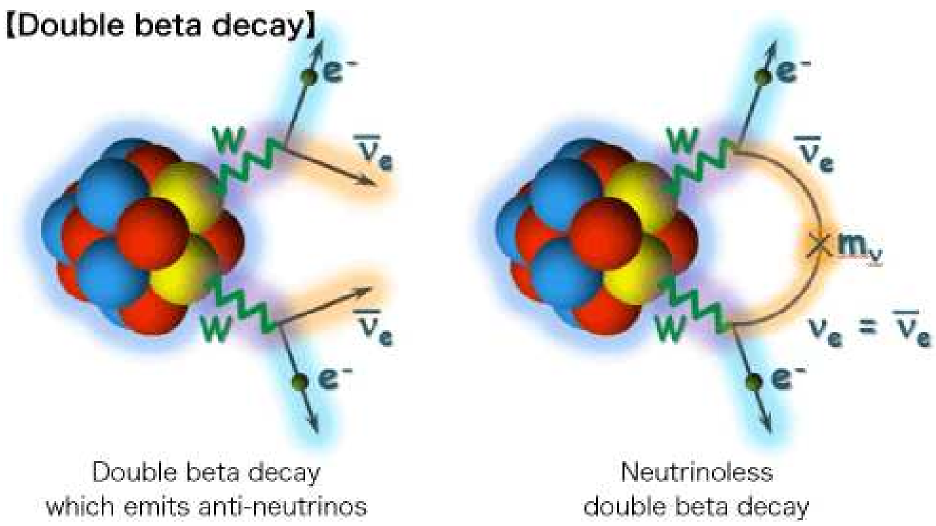
Why QCD?

- Because it is a beautiful and challenging theory on its own!
- Because the QCD input is needed in the searches of physics beyond the Standard Model!



© http://frankwilczek.com/Wilczek_Easy_Pieces/298_QCD_Made_Simple.pdf

Ex. 1: Neutrinoless double β-decay



© http://www1cm.phys.sci.osaka-u.ac.jp/en/research/r01.html

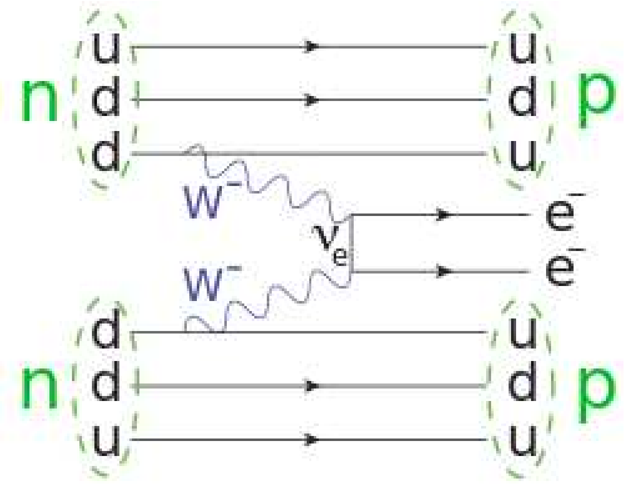
- What is the nature of the neutrino mass?
- Is the lepton number violated?
- What is the new physics beyond all this?

Best experimental limits on the lifetime:

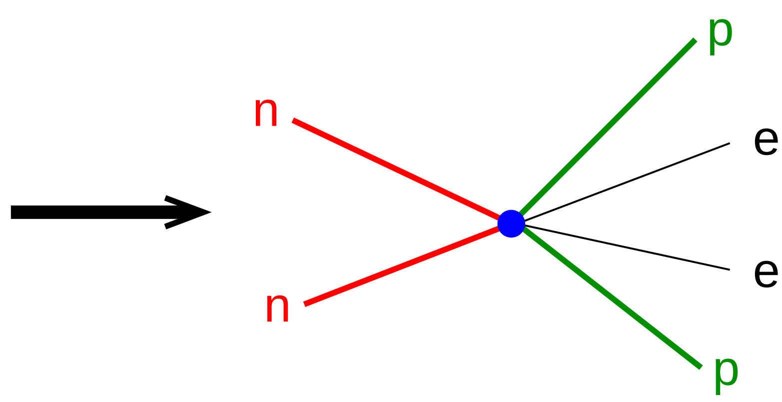
$$\tau(^{76}\text{Ge}) > 5.2 \cdot 10^{25}\ \text{yr}\,, \qquad \tau(^{136}\text{Xe}) > 10.7 \cdot 10^{25}\ \text{yr}$$

Double β-decay: hadronic input

Scaling down from high energies to hadronic scale $\sim\ 1$ GeV ...



© https://en.wikipedia.org/wiki/Double_beta_decay

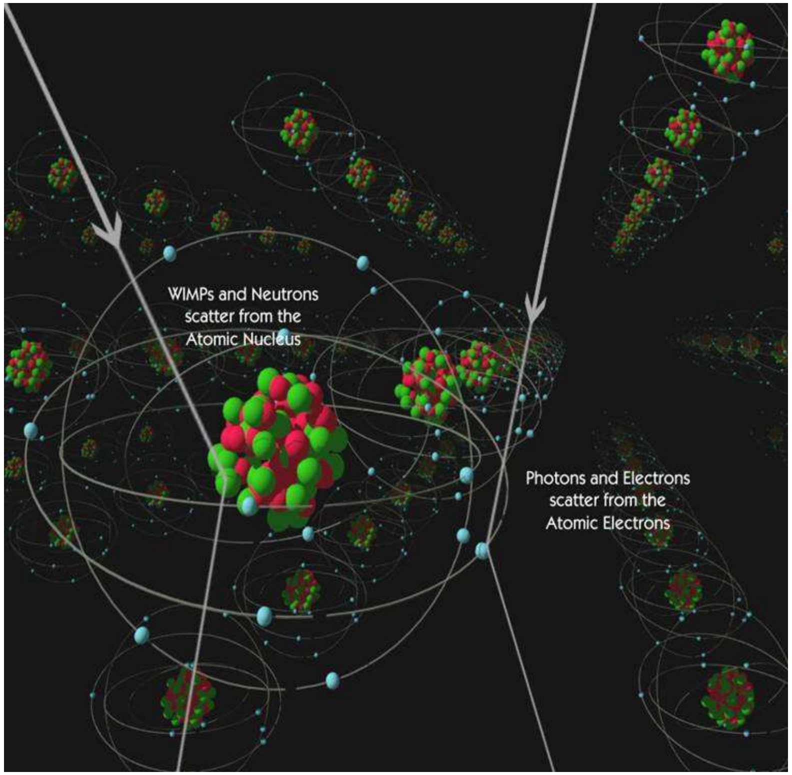


+ nuclear many-body calculations

Input: $\langle pp|T\mathcal{J}_L^-(x)\mathcal{J}_L^-(y)|nn\rangle$ with $\mathcal{J}_L^-(x)=\bar{u}(x)\frac{1}{2}(1-\gamma^5)d(x)$

$\hookrightarrow$ Strong interactions non-perturbative, calculations should be carried out from the first principles

Ex. 2: Direct DM searches: WIMPS



© http://cdms.berkeley.edu/Education/DMpages/essays/science/science/images/

- Looking for the nuclear recoil due to interaction with WIMPs
- Estimate for the scattering cross section?

Scattering cross section

$$\mathcal{L} = \sum_q \alpha_{3q} \bar\chi \chi \bar q q$$ (spin-independent)

T. Falk, A. Ferstl and K.A. Olive, PRD 59(1999) 055009

$$\sigma_{SI} = \frac{4m_r^2}{\pi}(Zf_p + (A-Z)f_n)^2$$

$$\frac{f_N}{m_N} = \sum_{q=u,d,s} f_{T_q}^{(N)} \frac{\alpha_{3q}}{m_q} + \frac{2}{27} F_{TG}^{(N)} \sum_{q=c,b,t} \frac{\alpha_{3q}}{m_q}$$

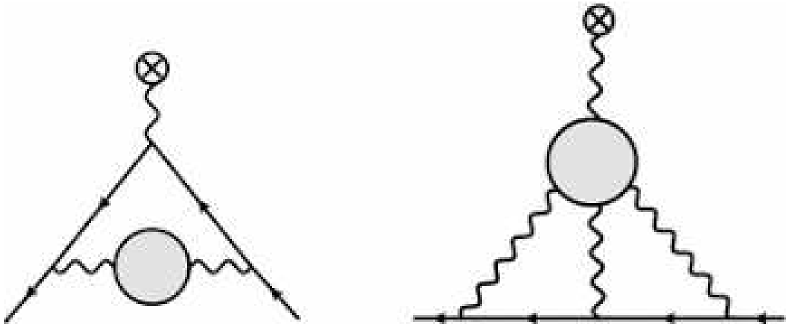
$$F_{TG}^{(N)} = 1 - \sum_{q=u,d,s} f_{T_q}^{(N)}$$

Input: the σ-terms $m_N f_{T_q}^{(N)} = \langle N | m_q \bar q q | N \rangle$

M. Hoferichter *et al.*, Phys.Rept. 625 (2016) 88 (Roy equations)

ETM coll., PRL 116 (2016) 252001 (lattice)

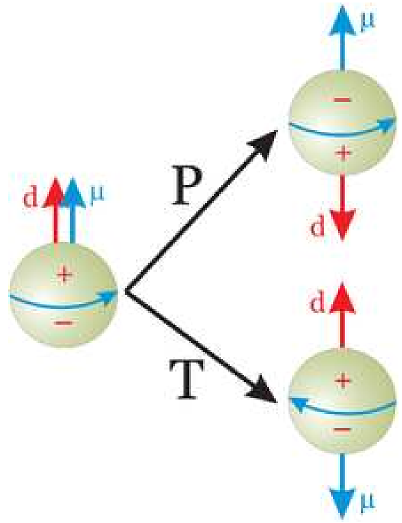
Ex. 3: Muon $g-2$



	$a_\mu \times 10^{10}$	
QED (5 loops)	11658471.8951 ± 0.0080	Aoyama et al '12
HVP LO	692.6 ± 3.3	Davier et al '16
HVP NLO	-9.84 ± 0.07	Hagiwara et al '11
HVP NNLO	1.24 ± 0.01	Kurz et al '11
HLbL	10.5 ± 2.6	Prades et al '09
Weak (2 loops)	15.36 ± 0.10	Gnendiger et al '13
SM total	11659180.2 ± 4.9	Davier et al '11
Experiment	11659208.9 ± 6.3	Bennett et al '06
Experiment-SM	28.7 ± 8.0	Davier et al '11

$a_\mu(\text{exp}) \neq a_\mu(\text{theory})$?

Ex. 4: Electric dipole moment



$$t \to -t \qquad \mathbf{S} \to -\mathbf{S} \qquad \mathbf{d} \to \mathbf{d}$$

$$\mathbf{d} = d\,\frac{\mathbf{S}}{S} \quad \hookrightarrow \quad d = 0$$

$\not{T},\ \not{C}P \quad \to$ baryogenesis

	Exp.	SM (CKM phases, θ-term)
n	$< 3 \cdot 10^{-26}$	$< 10^{-31}$
$^{199}Hg \hookrightarrow p$	$< 7.9 \cdot 10^{-25}$	$< 10^{-31}$

JEDI coll. at FZ Jülich:
First measurement of the deuteron EDM planned using storage ring method (2019-2020), . . .

The EDM of hadrons

$$\mathcal{L}_6 = \underbrace{-\bar\theta\frac{g_s^2}{64\pi^2}G\tilde G}_{\theta\text{-term}} - \underbrace{\frac{i}{2}\sum_q d_q\bar q\gamma_5\sigma_{\mu\nu}F^{\mu\nu}q}_{\text{quark EDM}} - \underbrace{\frac{i}{2}\sum_q \tilde d_q\bar q\gamma_5\frac{1}{2}\lambda^a\sigma_{\mu\nu}G^{a\,\mu\nu}q}_{\text{quark chromo-EDM}}$$

$$+ \underbrace{\frac{d_W}{6}f_{abc}\varepsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}G^b_{\mu\rho}G_\nu^{c\,\rho}}_{\text{gluon chromo-EDM}} + \underbrace{\sum_{ijkl}C^{4q}{}_{ijkl}\bar q_i\Gamma q_j\bar q_k\Gamma' q_l}_{\text{4-quark EDM}}$$

$$\hookrightarrow \quad \mathcal{L}_{\text{eff}} = -d_n\bar N(1-\tau_3)S^\mu N v^\nu F_{\mu\nu} - d_p\bar N(1+\tau_3)S^\mu N v^\nu F_{\mu\nu} + m_M\Delta\pi_3\boldsymbol{\pi}^2 + \cdots$$

Hadronic input:

$$\langle p+q|J^\mu|p\rangle = \bar u(p+q)\left(F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma_{\mu\nu}q_\nu}{2m_N}\right)u(p)$$

Why lattice QCD?

- At low energies the coupling α_s becomes large
- Emergence of bound states: mesons, baryons, glueballs, . . .
  The elementary constituents – quarks and gluons – are not observed in the experiment (confinement)
- Chiral symmetry is spontaneously broken

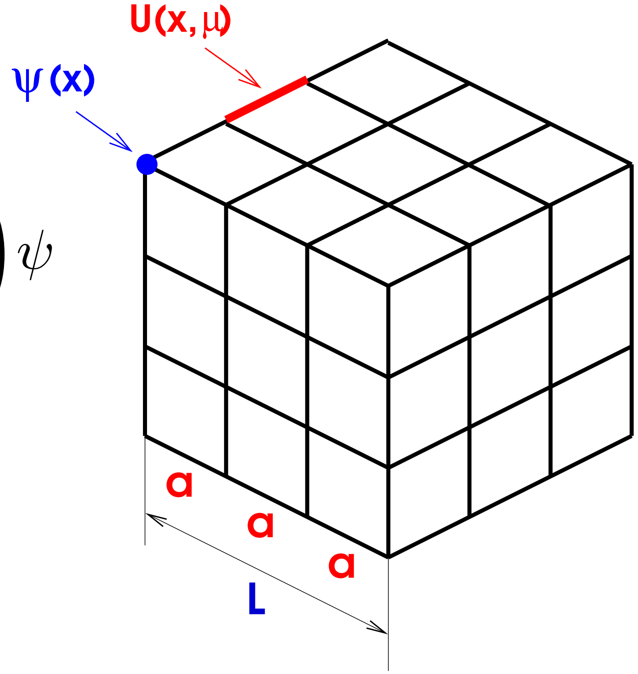
$\hookrightarrow$ In the low-energy region, using perturbation theory becomes impossible. Non-perturbative methods should be developed and applied!

Lattice QCD:

- Calculates properties of hadrons in QCD from first principles
- Path integrals in field theory are evaluated numerically, using Monte-Carlo technique

The lattice QCD action

$$
\begin{aligned}
S_W &= \frac{1}{g^2}\sum_{x\mu\nu} \text{Re}\,\text{tr}(1 - P_{\mu\nu}(x)) \\
&+ \sum_{x\mu} \bar\psi\left(\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \frac{a}{2}\nabla_\mu\nabla^*_\mu\right)\psi \\
&+ \sum_x \bar\psi\, m\, \psi
\end{aligned}
$$



The covariant derivative:

$$\nabla_\mu\psi(x) = \frac{1}{a}\left(U(x,\mu)\psi(x+a\hat\mu) - \psi(x)\right) \to \Lambda(x)\,\nabla_\mu\psi(x)$$

The plaquette:

$$
\begin{aligned}
P_{\mu\nu}(x) &= U(x,\mu)U(x+a\hat\mu,\nu)U(x+a\hat\nu,\mu)^{-1}U(x,\nu)^{-1} \\
\text{tr}(P_{\mu\nu}(x)) &= N_c - \frac{1}{2}a^4\text{tr}(G_{\mu\nu}(x)G_{\mu\nu}(x)) + O(a^5)
\end{aligned}
$$

How does one calculate the pion mass on the lattice?

The choice of the pion field:

$$O_\pi(x) = \bar{d}(x)\gamma_5 u(x)\,, \qquad x = (\mathbf{x}, t)$$

The two-point function:

$$D_\pi(x) = \langle 0|O_\pi(x)O_\pi^\dagger(0)|0\rangle = \frac{\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\, O_\pi(x)O_\pi^\dagger(0)\, e^{-S_W}}{\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{-S_W}}$$

Wick contractions:

$$\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\, O_\pi(x)O_\pi^\dagger(0)\, e^{-S_W} = \int \mathcal{D}U \,\text{tr}\,(\gamma_5 S_u(x)\gamma_5 S_d(-x))\, e^{-S_G}\,\text{det}$$

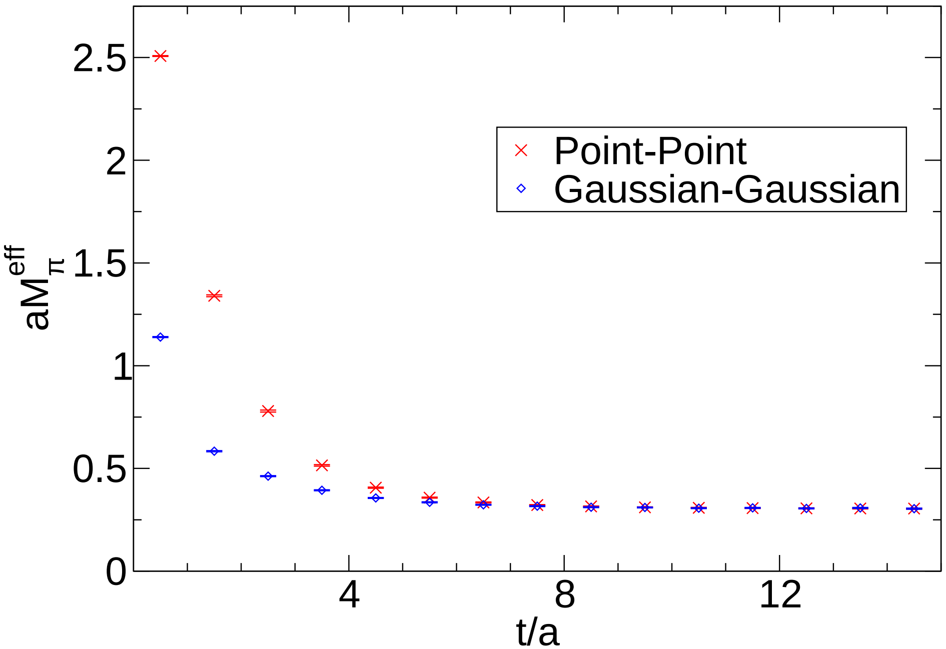
Projection of the zero momentum:

$$D_\pi(t) = \sum_{\mathbf{x}} D_\pi(\mathbf{x}, t)$$

The effective mass

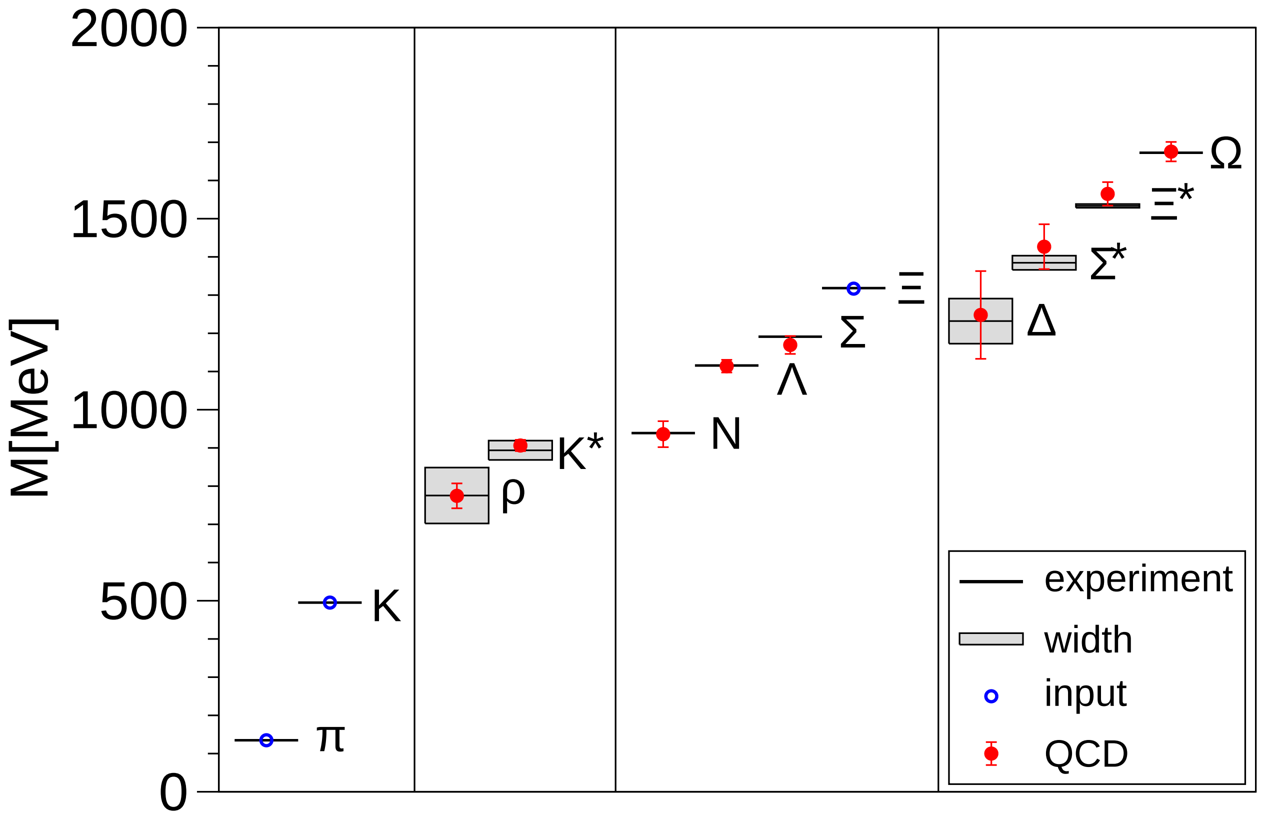
As $t \to \infty$, $\quad D_\pi(t) \to \text{const} \times \exp(-M_\pi t)$

$$\hookrightarrow \quad C_\pi(t) = \ln \frac{D_\pi(t)}{D_\pi(t+a)} \to aM_\pi$$



Pion effective mass plot for point and smeared sources,
S. Dürr *et al.*, Science 322 (2008) 1224

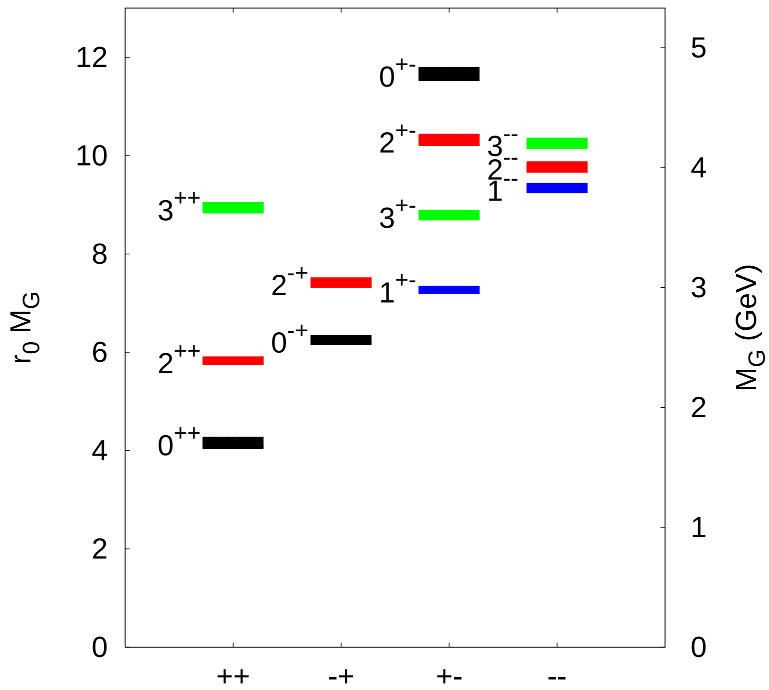
The low-energy spectrum of QCD



Recent results on the meson and baryon spectrum in QCD

S. Dürr *et al.*, Science 322 (2008) 1224

Exotic bound states

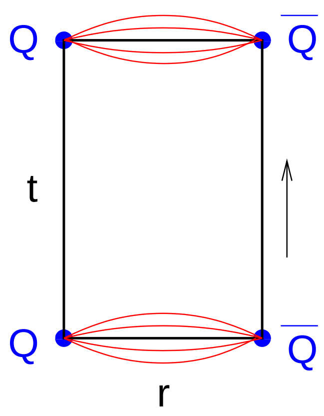


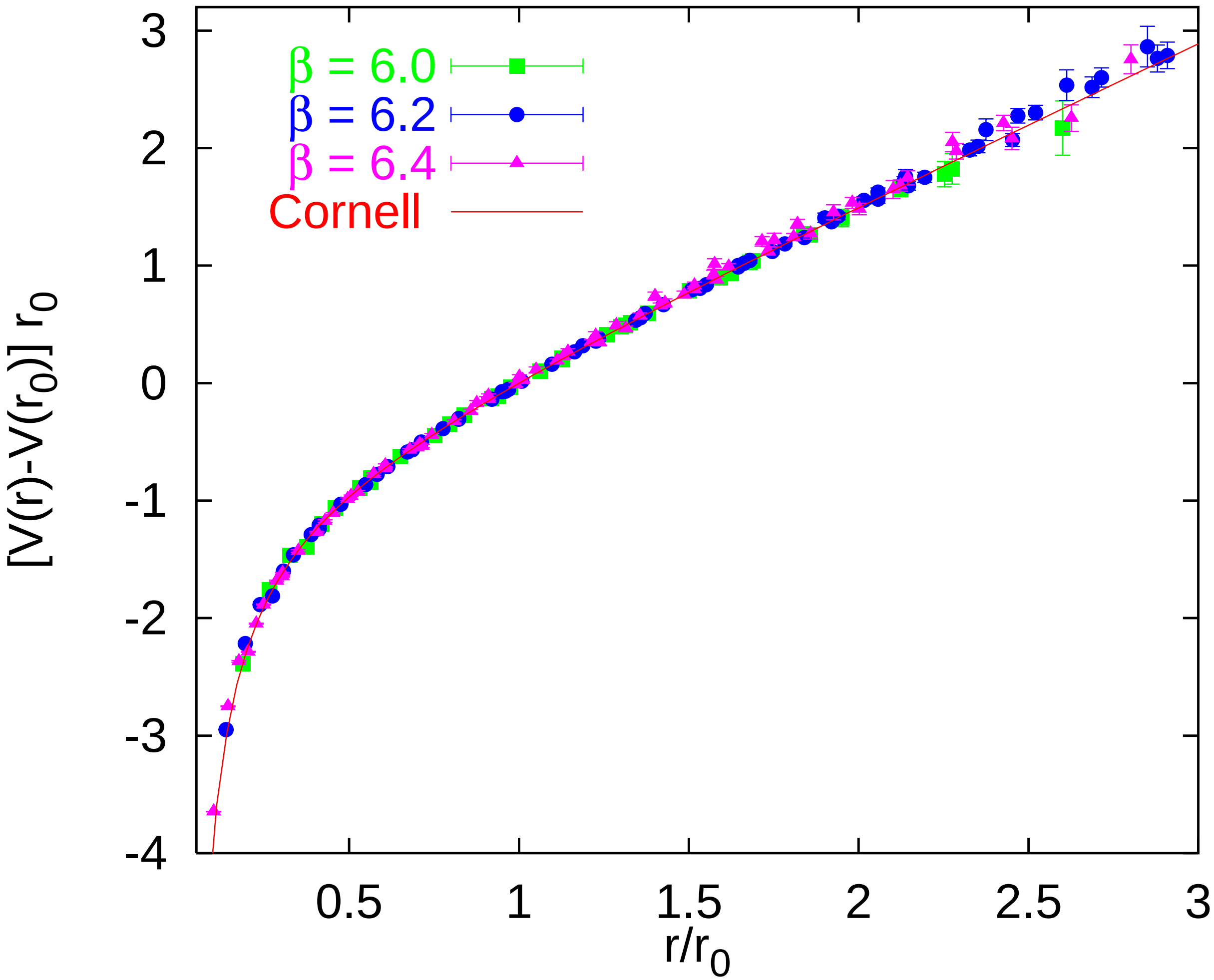
The mass spectrum of glueballs in the pure gauge theory

Y. Chen *et al.*, PRD 73 (2006) 014516

Static quark-antiqiark potential

Wilson loop





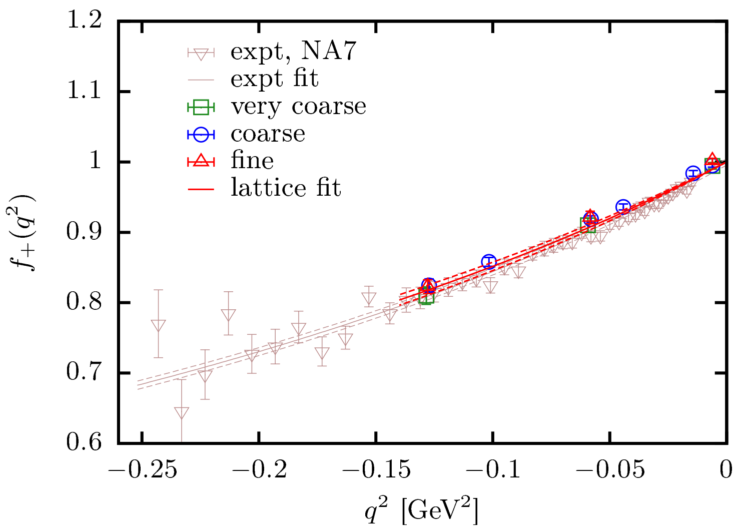
G. Bali, Phys. Rep. 343 (2001) 1

The form factors

$$G^\mu(t,\tau,\mathbf{p}',\mathbf{p}) = \sum_{\mathbf{x},\xi} e^{-\mathbf{p}'(\mathbf{x}-\xi)-i\mathbf{p}\boldsymbol{\xi}} \langle 0|O_\pi(\mathbf{x},t')J^\mu(\boldsymbol{\xi},\tau)O_\pi^\dagger(0)|0\rangle$$

$$G(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}}\langle 0|O_\pi(\mathbf{x},t)O_\pi^\dagger(0)|0\rangle$$

$$R^\mu(\tau,\mathbf{p}',\mathbf{p}) = G^\mu(t,\tau,\mathbf{p}',\mathbf{p})\sqrt{\frac{G(t-\tau,\mathbf{p})G(\tau,\mathbf{p}')}{G(\tau,\mathbf{p})G(t-\tau,\mathbf{p}')G(t,\mathbf{p})G(t,\mathbf{p}')}} \to F_\pi(Q^2)$$



Pion e.m. form factor

J. Koponen *et al.*, arXiv:1710.07467

Hadronic reactions

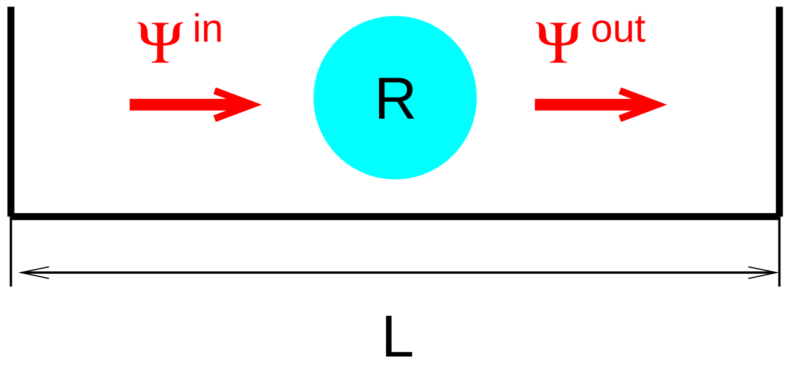
- Hadron-hadron scattering
- Resonances
- Multihadron reactions
- Final-state interactions in the matrix elements

↪ Lattice QCD simulations are carried out in the Euclidean space, in a finite box

↪ The measured spectrum is discrete, the observables are real

How does one relate the results of the measurements to the observables in the scattering sector (continuous spectrum, complex amplitudes, ...)?

The Lüscher method in one dimension



- $R \ll L$: the interaction range is much smaller than the box size
- The wave function: $\Psi^{\text{in}}(x) \propto e^{ipx}$ and $\Psi^{\text{out}}(x) \propto e^{2i\delta(p)+ipx}$
- Periodic boundary conditions: $\Psi(x+L) = \Psi(x)$

$$\hookrightarrow \quad e^{2i\delta(p)+ipL} = 1 \quad \hookrightarrow \quad 2i\delta(p) + ipL = 2\pi n$$

Measured energy levels $\leftrightarrow$ phase shift

How this method can be used to study hadronic observables?

Why effective field theories?

Most general effective Lagrangian: all terms with assumed symmetry

$\hookrightarrow$ Most general S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and symmetries (Weinberg, 1979)

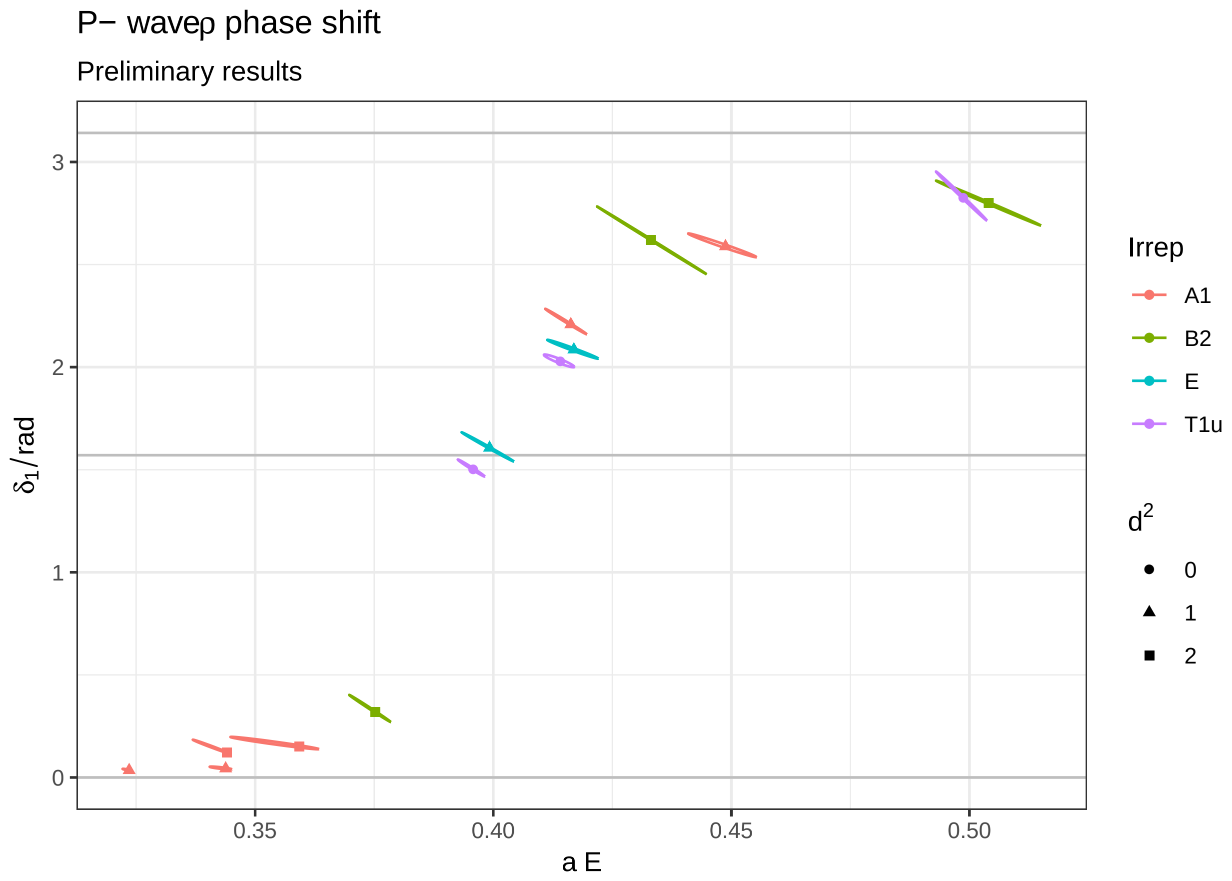
QCD $\longrightarrow$ ChPT $(p \sim M_\pi)$ $\longrightarrow$ non-rel. EFT $(p \ll M_\pi)$

- Lattice QCD $\longrightarrow$ EFT in a finite volume
- Short-distance behavior not changed: **the same Lagrangian**

$\hookrightarrow$ A bridge between the measured spectrum and scattering sector:

Multichannel resonances
Twisted boundary conditions
Lüscher-Lellouch formula, timelike pion formfactor
Resonance formfactors
Few-body systems

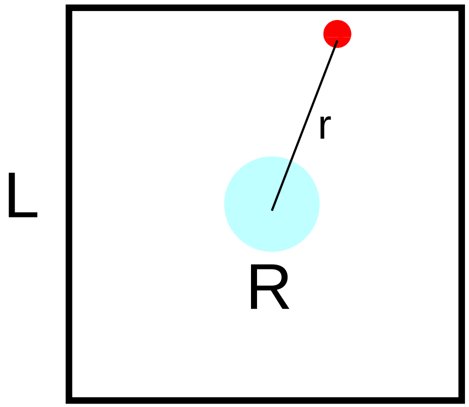
Example: the P-wave $\pi\pi$ phase shift



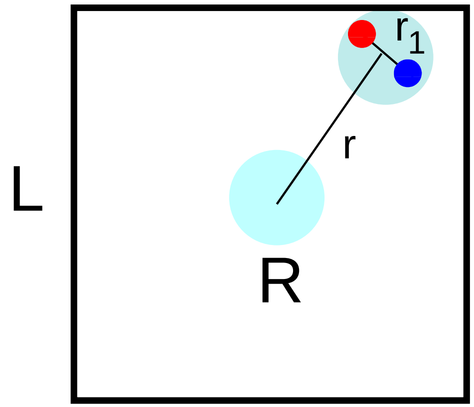
Preliminary result from ETM collaboration

Three particles in a finite volume: the problem

Two-particle scattering: The wave function always in the asymptotic form near the walls: no off-shell effects!



2 particles



3 particles

- The three-particle wave function near the box walls is not always described by the asymptotic wave function
- Is the three-particle spectrum determined solely in terms of the S-matrix?

K. Polejaeva and AR, EPJA 48 (2012) 67: Yes!

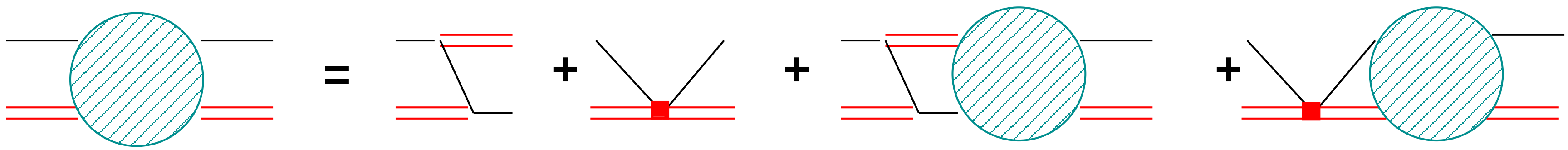
The strategy

H.-W. Hammer, J.-Y. Pang and AR, JHEP 1709 (2017) 109, JHEP 1710 (2017) 115

- If $R \ll L$ (large boxes $\to$ small momenta), the energy spectrum can be calculated, using non-relativistic EFT in a finite volume
- Effective couplings matched to the observables in the infinite volume *on the mass shell*
- Analysis of the lattice data: determine these couplings from the fit to the spectrum, calculate the S-matrix from the dynamical equations

$\hookrightarrow$ Effective couplings form a convenient set of the parameters to be determined on the lattice $\to$ contain only exponentially suppressed effects at large L.

Skornyakov-Ter-Martirosian equation in a finite volume



$$\mathcal{M}_L(\mathbf{p},\mathbf{q};E) = Z(\mathbf{p},\mathbf{q};E) + \frac{8\pi}{L^3}\sum_{\mathbf{k}}^{\Lambda} Z(\mathbf{p},\mathbf{q};E)\tau_L(\mathbf{k};E)\mathcal{M}_L(\mathbf{k},\mathbf{q};E)$$

$$Z(\mathbf{p},\mathbf{q};E) = \frac{1}{\mathbf{p}^2+\mathbf{q}^2+\mathbf{pq}-mE} + H_0 + H_2(\mathbf{p}^2+\mathbf{q}^2) + \cdots$$

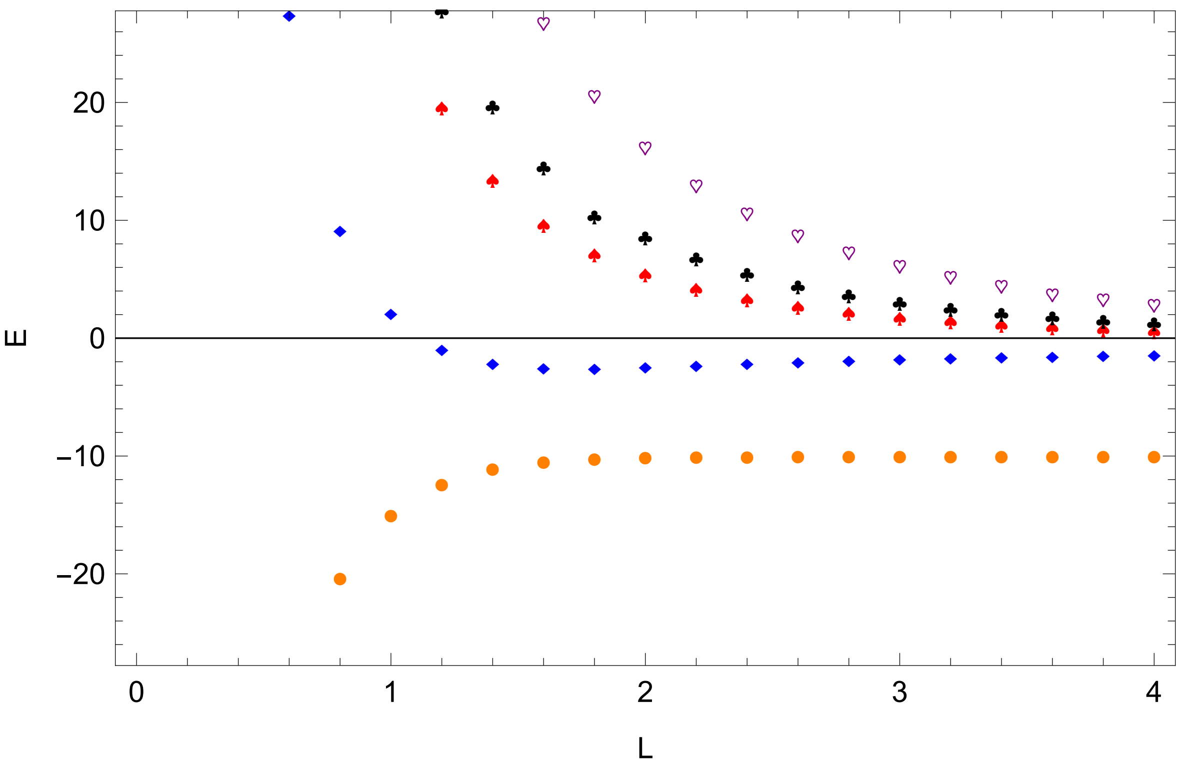
$$\tau_L^{-1}(\mathbf{k};E) = k^*\cot\delta(k^*) - \frac{4\pi}{L^3}\sum_{\mathbf{l}}\frac{1}{\mathbf{k}^2+\mathbf{l}^2+\mathbf{kl}-mE}$$

$\hookrightarrow$ Poles in the amplitude $\rightarrow$ finite-volume energy spectrum

$\hookrightarrow$ $H_0, H_2, \ldots$ should be fitted to the three-particle energies

The finite-volume spectrum

M. Döring, H.-W. Hammer, M. Mai, J.-Y. Pang, AR and J. Wu, PRD 97 (2018) 114508



The spectrum both below and above the three-particle threshold

Conclusions/outlook

- The study of QCD on the lattice enables one to calculate hadronic observables *in the Standard Model* from the first principles
- It also provides the hadronic input, needed in the searches of the physics *beyond the Standard Model*
- Calculating the hadronic reaction amplitudes on a finite lattice constitutes a big challenge. The use of the effective field theory enables one to systematically relate the hadronic observables in the infinite volume to the results of lattice measurements
- Two particle (coupled channel) scattering: standard by now
- Three and more particles: rapidly advancing
  - Three-particle decays
  - Inelastic resonances (Roper, etc)
  - Applications in nuclear physics and so on …