Study of Metastable Vacuum Decay with Gravity

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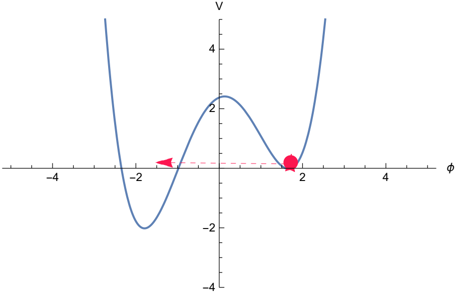
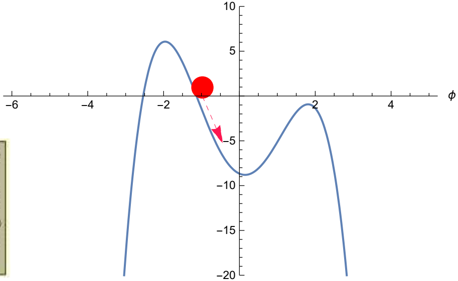
Introduction

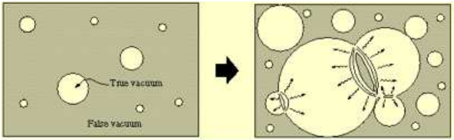
$$E_f = E_0 - \gamma \qquad (1)$$

$$\gamma = N \int D\phi e^{iS(\phi)/\hbar}, \qquad \hbar \to 0 \qquad (2)$$

$$\gamma = Ae^{-B} \qquad (3)$$







$$A = \frac{\int D\delta\phi e^{-S^{(2)}(\delta\phi)}|_{\phi=\phi^b}}{\int D\delta\phi e^{-S^{(2)}(\delta\phi)}|_{\phi=\phi^f}} \quad (4)$$

$B = S_E(\phi) - S_E(\phi_f)$. $S_E = \int d^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$ is Euclidean Action $S_E(\phi_f)$ is Euclidean action calculated at false vacuum and $S^{(2)}$ is quadratic action[1], [2],[3].

$$S_E^{(2)} = \pi^2 \int d\eta\delta\phi\left(-\frac{d}{d\eta}\eta^3\frac{d}{d\eta} + \eta^3 V''(\phi)\right)\delta\phi \quad (5)$$

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[1]Coleman:1977py S. R. Coleman, "The Fate of the False Vacuum. 1. Semiclassical Theory," Phys. Rev. D **15**, 2929 (1977); Erratum: [Phys. Rev. D **16**, 1248 (1977)]

[2]Curtis G. Callan, Jr. and S. Coleman, " Fate of the false vacuum. II. First quantim corrections" Phys. Rev. D **16**, no 6, (1977)

[3]S. R. Coleman and F. De Luccia, "Gravitational Effects on and of Vacuum Decay," Phys. Rev. D **21**, 3305 (1980)

$$S_E^{(2)} = \pi^2 \int d\eta \Phi \Bigg(- \frac{d}{d\eta} \left(\frac{\rho^3(\eta)}{Q(\eta)} \frac{d}{d\eta} \right) + \rho^3(\eta) V[\phi(\eta), \rho(\eta)] \Bigg) \Phi \quad (6)$$

where Φ is small perturbation $V[\phi(\eta), \rho(\eta)]$ is potential, expressed in terms of field equation solutions. For Q coefficient we have: [4] [5], [6],[7], [8].

$$Q_{KLT} \equiv Q = 1 - \frac{\kappa \rho^2 \phi'^2}{6} \quad (7)$$

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[4] A. Khvedelidze, G. V. Lavrelashvili and T. Tanaka, "On cosmological perturbations in closed FRW model with scalar field and false vacuum decay," Phys. Rev. D **62**, 083501 (2000)

[5] M. Koehn, G. Lavrelashvili and J. L. Lehners, "Towards a Solution of the Negative Mode Problem in Quantum Tunnelling with Gravity," Phys. Rev. D **92**, no. 2, 023506 (2015)

[6] G. Lavrelashvili, V. A. Rubakov and P. G. Tinyakov, "Tunneling Transitions With Gravitation: Breaking Of The Quasiclassical Approximation," Phys. Lett. **161B**, 280 (1985)

[7] T. Tanaka, M. Sasaki "False Vacuum Decay With Gravity: Negative Mode Problem" Vistas in Astronomy, Vol. 37, pp. 641-644, 1993

[8] A. Khvedelidze, G. V. Lavrelashvili and T. Tanaka, "On cosmological perturbations in closed FRW model with scalar field and false vacuum decay," Phys. Rev. D **62**, 083501 (2000)

Is the negative mode problem related to physics at Planck scale?

$$S_E = \int d^4x\sqrt{g}\Big(-\frac{1}{2\kappa}R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V(\phi)\Big) \tag{8}$$

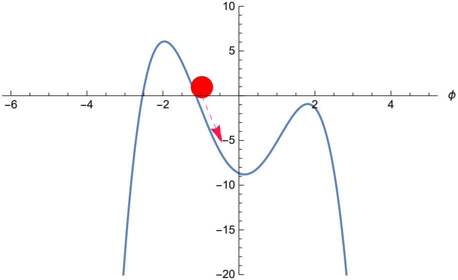
$$ds^2 = N^2(\eta)d\eta^2 + \rho^2(\eta)d\Omega_3^2 \tag{9}$$

where $\rho(\eta)$ is scale factor. $N(\eta)$ is Lapse function and $d\Omega_3^2$ is metric of the unit three-sphere

$$d\Omega_3^2 = d\chi^2 + sin^2\chi(d\theta^2 + sin^2(\theta)d\phi^2) \tag{10}$$

$$\phi'' + \frac{3\rho'}{\rho}\phi' = \frac{\partial V(\phi)}{\partial \phi} \quad (11)$$

$$\rho'' = -\frac{\kappa\rho}{3}(\phi'^2 + V) \quad (12)$$



$$\phi(0) = \phi_0, \qquad \phi'(0) = 0, \qquad \rho(0) = 0, \qquad \rho'(0) = 1 \quad (13)$$ [9]

$$V = V_0 + \frac{\epsilon}{2\mu}(\phi + \mu) + V_t \quad (14)$$

where $V_0 = \frac{\lambda}{8}(\phi^2 - \frac{\mu^2}{\lambda})^2$

[9] M. Koehn, G. Lavrelashvili and J. L. Lehners, "Towards a Solution of the Negative Mode Problem in Quantum Tunnelling with Gravity," Phys. Rev. D **92**, no. 2, 023506 (2015)

parameters: $\lambda, \mu, V_t, \epsilon$. $V_{top} << m_p^4$

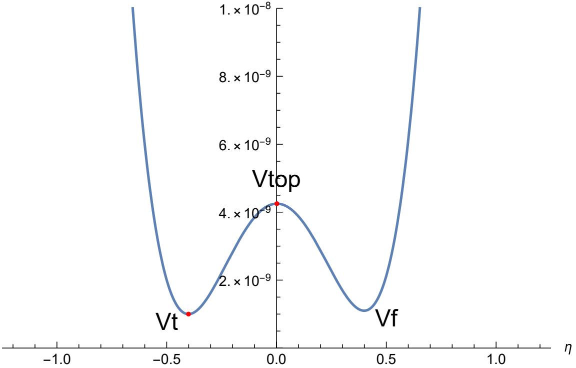


Figure: $V_t = 10^{-9}$, $\mu = 0.4$, $\lambda = 0.000001$,$\epsilon = 10^{-10}$.

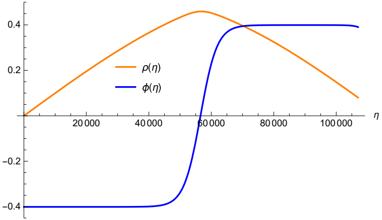
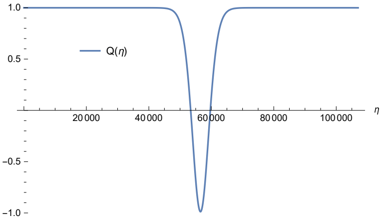


Figure: Bounce profile $V_t = 10^{-9}$, $\mu = 0.4$, $\lambda = 0.000001$
$\epsilon = 10^{-10}$ $\rho(\eta) \times 10^{-5}$ $\phi(\eta)$



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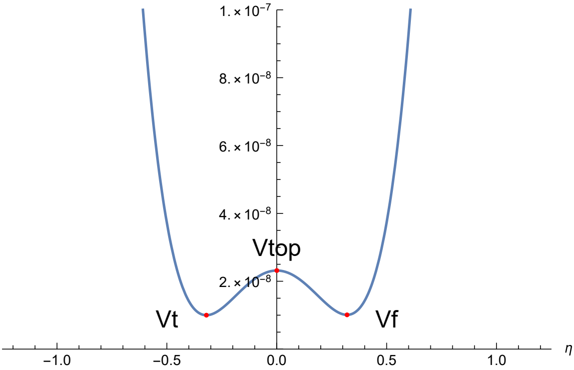

Figure: $V_t = 10^{-8}$, $\mu = 0.32$, $\lambda = 0.00001$,$\epsilon = 10^{-10}$.

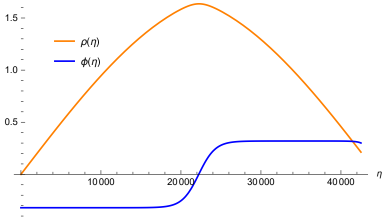

Figure: $V_t = 10^{-8}$, $\mu = 0.32$, $\lambda = 0.00001$ $\epsilon = 10^{-10}$ $\rho(\eta) \times 10^{-4}$ $\phi(\eta)$.

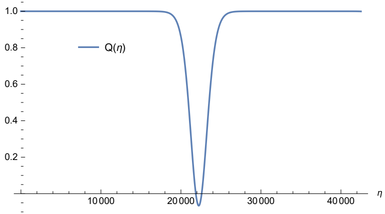

Figure: $V_t = 10^{-8}$, $\mu = 0.32$, $\lambda = 0.00001$, $\epsilon = 10^{-10}$.

Conclusion and future plans

- Study of pre-exponential factor A shows that in quadratic action before the kinetic term appears factor Q, which becomes negative along some instantons.
- It is shown that for some parameter combinations Q becomes negative far from Planck's scale.

Thank you for your attention!

$$\phi(\eta) = \phi_0 + \frac{V'(\phi_0)}{8}\eta^2 + \frac{V'(\phi_0)}{192}\left[V''(\phi_0) + \frac{2\kappa V(\phi_0)}{3}\right]\eta^4$$
$$+ \frac{V'(\phi_0)}{829440}\Big[135V'(\phi_0)V'''(\phi_0) + 90V''(\phi_0)^2 + 162\kappa V'(\phi_0)^2$$
$$+ 180\kappa V(\phi_0)V''(\phi_0) + 112\kappa^2 V(\phi_0)^2\Big]\eta^6 + \mathcal{O}(\eta^8) \quad (15)$$

$$\rho(\eta) = \eta - \frac{\kappa}{18}V(\phi_0)\eta^3 - \frac{\kappa}{120}\left[\frac{3}{8}V'(\phi_0)^2 - \frac{\kappa}{9}V(\phi_0)^2\right]\eta^5$$
$$- \frac{\kappa}{2177280}[405V'(\phi_0)^2V''(\phi_0) - 54\kappa V(\phi_0)V'(\phi_0)^2$$
$$+ 16\kappa^2 V(\phi_0)^3]\eta^7 + \mathcal{O}(\eta^9) \quad (16)$$