The width of the Roper resonance in baryon chiral perturbation theory

Jambul Gegelia

in collaboration with Ulf.-G. Meißner and De-Liang Yao

Forschungszentrum Jülich, D-52425 Jülich, Germany

7th Georgian-German School and Workshop in Basic Science
GGSWBS'16
August 28 - September 2, 2016, Tbilisi, Georgia

Outline

- Low-energy chiral EFT;
- Roper resonance in chiral EFT;
- The width of the Roper resonance;
- The width of the Roper resonance obtained from the decay amplitudes;
- Numerical results;
- Summary;

Low energy chiral EFT

- A very well motivated assumption:
  QCD is a correct theory of the strong interaction.
- Perturbation theory is not applicable at low energies.

  Chiral EFT provides with a solution to this problem.
- Started with:
  S. Weinberg, Physica A **96**, 327 (1979).

- Based on the symmetries of QCD, chiral EFT aims at reproducing the S-matrix of QCD in low-energy region.
- Hadronic one-particle states are represented by dynamical fields in EFT.
  Effective degrees of freedom: pions, nucleons, $\Delta(1232)$, ...
- Chiral EFT provides with a systematic expansion of physical quantities in powers of (small scale(s)/ large scale)
- Bound states require resummation of infinitely many diagrams.

- Most general Lagrangian of the EFT of Hadrons with symmetries of QCD gives the most general S-matrix with these symmetries.
- To obtain S-matrix of QCD one needs to fix properly the parameters of EFT $\cdots$
- $\cdots$ a finite number of them to achieve a finite accuracy!
- $\cdots$ EFT $\neq$ QCD.

  QCD calculates physical quantities in terms of fundamental parameters, EFT only relates physical quantities to each other at low-energies, like

$$\sigma_k(E) = F(E, \sigma_1(\mu_i), \sigma_2(\mu_i), \cdots, \mu).$$

What to do?

- Write down the most general effective Lagrangian.
- Consider *all* Feynman diagrams contributing to the process in question.
- Renormalize/subtract loop diagrams.
- Apply power counting to renormalized diagrams.
- Sum up all *renormalized* diagrams' contributions up to the given order.
- Only a finite number of diagrams contribute at any given order.

Roper resonance in chiral EFT

Roper resonance is the first nucleon resonance that decays into a nucleon and two pions, besides decaying into a nucleon and a pion.

Despite the fact that the Roper resonance was found a long time ago

L. D. Roper, Phys. Rev. Lett. **12**, 340 (1964).

a satisfactory theory of this state is still missing.

First steps in this direction within chiral EFT have been made in

B. Borasoy, P. C. Bruns, U.-G. Meißner and R. Lewis, Phys. Lett. B **641**, 294 (2006).
D. Djukanovic, J. Gegelia, S. Scherer, Phys. Lett. B **690**, 123 (2010).
B. Long and U. van Kolck, Nucl. Phys. A **870-871**, 72 (2011).
T. Bauer, J. Gegelia, S. Scherer, Phys. Lett. B **715**, 234 (2012).
E. Epelbaum, J. Gegelia, U.-G. Meißner and D. L. Yao, Eur. Phys. J. C **75**, no. 10, 499 (2015).

We present the calculation of the width of the Roper resonance at leading two-loop order in baryon chiral perturbation theory(BChPT) of pions, nucleons, the delta and Roper resonances.

J. Gegelia, U. G. Meißner and D. L. Yao,
"The width of the Roper resonance in baryon chiral perturbation theory,"
arXiv:1606.04873 [hep-ph]. To appear in Phys. Lett. B

Effective Lagrangian of pions, nucleons, the delta and Roper resonances as dynamical degrees of freedom:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{\pi\Delta} + \mathcal{L}_{\pi R} + \mathcal{L}_{\pi N\Delta} + \mathcal{L}_{\pi NR} + \mathcal{L}_{\pi\Delta R}.$$

From the purely mesonic sector we need the following structures

$$\begin{aligned}
\mathcal{L}_{\pi\pi}^{(2)} &= \frac{F^2}{4}\langle\partial_\mu U\partial^\mu U^\dagger\rangle + \frac{F^2M^2}{4}\langle U^\dagger + U\rangle, \\
\mathcal{L}_{\pi\pi}^{(4)} &= \frac{1}{8}l_4\langle u^\mu u_\mu\rangle\langle\chi_+\rangle + \frac{1}{16}(l_3+l_4)\langle\chi_+\rangle^2,
\end{aligned}$$

where $\langle\ \ \rangle$ denotes the trace in flavor space, F is the pion decay constant in the chiral limit and M is the pion mass at leading order. The pion fields are contained in the 2×2 matrix U, with $u=\sqrt{U}$ and

$$\begin{aligned}
u_\mu &= i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right], \\
\chi^+ &= u^\dagger\chi u^\dagger + u\chi^\dagger u\,, \quad \chi = \begin{bmatrix} M^2 & 0 \\ 0 & M^2 \end{bmatrix}.
\end{aligned}$$

Terms of the Lagrangian with pions and baryons:

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} &= \bar{\Psi}_N \left\{ i\not{D} - m + \frac{1}{2} g \not{u} \gamma^5 \right\} \Psi_N \,, \\
\mathcal{L}_{\pi R}^{(1)} &= \bar{\Psi}_R \left\{ i\not{D} - m_R + \frac{1}{2} g_R \not{u} \gamma^5 \right\} \Psi_R \,, \\
\mathcal{L}_{\pi R}^{(2)} &= \bar{\Psi}_R \left\{ c_1^R \langle \chi^+ \rangle \right\} \Psi_R \,, \\
\mathcal{L}_{\pi NR}^{(1)} &= \bar{\Psi}_R \left\{ \frac{g_{\pi NR}}{2} \gamma^\mu \gamma_5 u_\mu \right\} \Psi_N + \text{h.c.} \,, \\
\mathcal{L}_{\pi \Delta}^{(1)} &= -\bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \left\{ \left(i\not{D}^{jk} - m_\Delta \delta^{jk} \right) g^{\mu\nu} - i \left(\gamma^\mu D^{\nu,jk} + \gamma^\nu D^{\mu,jk} \right) \right. \\
&+ i\gamma^\mu \not{D}^{jk} \gamma^\nu + m_\Delta \delta^{jk} \gamma^\mu \gamma^\nu + \frac{g_1}{2} \not{u}^{jk} \gamma_5 g^{\mu\nu} \\
&\left. + \frac{g_2}{2} (\gamma^\mu u^{\nu,jk} + u^{\nu,jk} \gamma^\mu) \gamma_5 + \frac{g_3}{2} \gamma^\mu \not{u}^{jk} \gamma_5 \gamma^\nu \right\} \xi_{kl}^{\frac{3}{2}} \Psi_\nu^l \,, \\
\mathcal{L}_{\pi N\Delta}^{(1)} &= h \bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\alpha}(z_1)\, \omega_\alpha^j \Psi_N + \text{h.c.} \,, \\
\mathcal{L}_{\pi \Delta R}^{(1)} &= h_R \bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\alpha}(\tilde{z})\, \omega_\alpha^j \Psi_R + \text{h.c.} \,.
\end{aligned}
$$

Here Ψ_N and Ψ_R are the fields of the nucleon and the Roper resonance, respectively.

The Rarita-Schwinger field Ψ_ν represents the Δ resonance. $\xi^{\frac{3}{2}}$ is the isospin-3/2 projector, $\omega_\alpha^i = \frac{1}{2}\langle \tau^i u_\alpha \rangle$ and $\Theta^{\mu\alpha}(z) = g^{\mu\alpha} + z\gamma^\mu\gamma^\nu$, where z is a so-called off-shell parameter.

We fix the off-shell structure of the interactions involving the delta by adopting $g_2 = -g_3 = 0$ and $z_1 = \tilde{z} = 0$.

The covariant derivatives are defined as follows:

$$\begin{aligned}
D_\mu \Psi_{N/R} &= (\partial_\mu + \Gamma_\mu)\,\Psi_{N/R}\,, \\
(D_\mu \Psi)_{\nu,i} &= \partial_\mu \Psi_{\nu,i} - 2\,i\,\epsilon_{ijk}\Gamma_{\mu,k}\Psi_{\nu,j} + \Gamma_\mu \Psi_{\nu,i}\,, \\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right] = \tau_k \Gamma_{\mu,k}\ .
\end{aligned}$$

The width of the Roper resonance

The dressed propagator of the Roper resonance can be written as

$$i\, S_R(p) = \frac{i}{\not{p} - m_{R0} - \Sigma_R(\not{p}\,)}\,,$$

where $-i\,\Sigma_R(\not{p}\,)$ is the self-energy.

The pole of the dressed propagator S_R is obtained by solving

$$S_R^{-1}(z) \equiv z - m_{R0} - \Sigma_R(z) = 0\,.$$

We define the physical mass and the width of the Roper resonance by parameterizing the pole as

$$z = m_R - i\,\frac{\Gamma_R}{2}\,.$$

Topologies of the one- and two-loop Roper self-energy diagrams

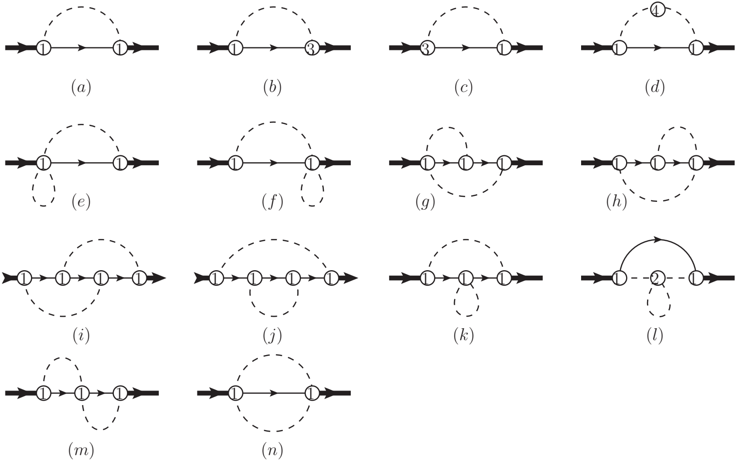

Figure: One and two-loop self-energy diagrams of the Roper resonance. The dashed and thick solid lines represent the pions and the Roper resonances, respectively. The thin solid lines in the loops stand for either nucleons, Roper or delta resonances.

We parameterize the pole as

$$z = m_2 + \hbar\delta z_1 + \hbar^2\delta z_2 + \mathcal{O}(\hbar^3),$$

where $m_2 = m_R^0 + 4c_1^R M^2$, with m_R^0 the Roper mass in the chiral limit and write the self-energy as an expansion in the number of loops

$$\Sigma_R = \hbar\Sigma_1 + \hbar^2\Sigma_2 + \mathcal{O}(\hbar^3)\,.$$

By expanding the equation for z in powers of $\hbar$, we get

$$\hbar\delta z_1 + \hbar^2\delta z_2 - \hbar\Sigma_1(m_2) - \hbar^2\delta z_1\Sigma_1'(m_2) - \hbar^2\Sigma_2(m_2) + \mathcal{O}(\hbar^3) = 0\,.$$

Solving order by order we obtain

$$\begin{aligned}
\delta z_1 &= \Sigma_1(m_2), \\
\delta z_2 &= \Sigma_1(m_2)\,\Sigma_1'(m_2) + \Sigma_2(m_2).
\end{aligned}$$

The width takes the form

$$
\begin{aligned}
\Gamma_R &= \hbar\, 2i\, \mathrm{Im}\left[\Sigma_1(m_2)\right] \\
&+ \hbar^2\, 2i \left\{ \mathrm{Im}\left[\Sigma_1(m_2)\right] \mathrm{Re}\left[\Sigma_1'(m_2)\right] + \mathrm{Re}\left[\Sigma_1(m_2)\right] \mathrm{Im}\left[\Sigma_1'(m_2)\right] \right\} \\
&+ \hbar^2\, 2i\, \mathrm{Im}\left[\Sigma_2(m_2)\right] + \mathcal{O}(\hbar^3).
\end{aligned}
$$

It turns out that the contribution of the second term is of an order higher than the accuracy of our calculation, which is δ^5.

To calculate the contributions to the width of the Roper resonance, we use the Cutkosky cutting rules.

Only contributions obtained by cutting the lines, corresponding to stable particles, are needed.

The width of the Roper resonance obtained from the decay amplitudes

By applying the cutting rules to the self-energy diagrams we obtain the Feynman graphs contributing in the decay amplitudes of the Roper resonance into πN and $\pi\pi N$ systems.

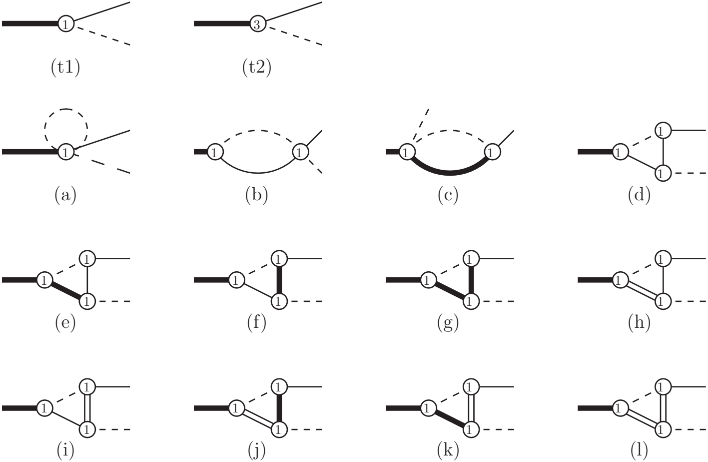

Figure: Diagrams contribution to the decay $R \to N\pi$ up to leading one-loop order. Dashed, solid, double and thick solid lines correspond to pions, nucleons, deltas and Roper resonances, respectively. The numbers in the circles give the chiral orders of the vertices.

The decay amplitude of $R(p) \to N(p')\pi^a(q)$ can be written as

$$\mathcal{A}^a = \bar{u}_N(p')\left\{A\,\not{q}\gamma_5\tau^a\right\}u_R(p)\,,$$

where a is an isospin index of the pion, and the $\bar{u}, u$ are spinors. The corresponding decay width reads

$$\Gamma_{R\to\pi N} = \frac{\lambda^{1/2}(m_R^2, m_N^2, M^2)}{16\pi\, m_R^3}\,|\mathcal{M}_1|^2\,,$$

with $\lambda(x,y,z) = (x-y-z)^2 - 4yz$ and

$$|\mathcal{M}_1|^2 \;=\; 3(m_N+m_R)^2\left[(m_N-m_R)^2 - M_\pi^2\right]A^*A\,.$$

The leading order tree diagrams contributing to the $R \to \pi\pi N$ decay are shown below.

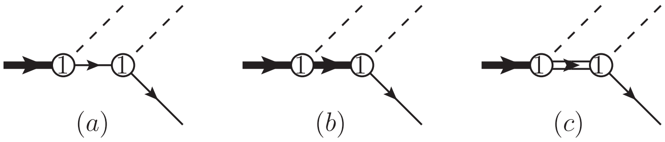

Figure: Tree diagrams contributing to the $R \to \pi\pi N$ decay. Crossed diagrams are not shown. Dashed, solid, double and thick solid lines correspond to pions, nucleons, deltas and Roper resonances, respectively. The numbers in the circles give the chiral orders of the vertices.

The delta propagators in these diagrams are dressed. The non-pole parts are of higher orders and therefore can be dropped.

The contributions of the loop diagrams are suppressed by additional powers of δ so that they do not contribute at order δ^5.

Kinematical variables for the decay $R(p) \to N(p')\pi^a(q_1)\pi^b(q_2)$ via

$$s_1 = (q_1 + q_2)^2 \,, \qquad s_2 = (p' + q_1)^2 \,, \qquad s_3 = (p' + q_2)^2 \,,$$

subject to the contraint

$$s_1 + s_2 + s_3 = m_R^2 + m_N^2 + 2M_\pi^2 \ .$$

The isospin and the Lorentz decomposition of the decay amplitude:

$$\begin{aligned} \mathcal{A}^{ab} &= \chi_N^\dagger \left\{ \delta^{ab} F_+ + i\epsilon^{abc}\tau^c F_- \right\} \chi_R \ , \\ F_\pm &= \bar{u}_N(p') \left\{ F_\pm^{(1)} - \frac{1}{2(m_N + m_R)} \left[\not{q}_1 , \not{q}_2 \right] F_\pm^{(2)} \right\} u_R(p) \,, \end{aligned}$$

with the χ being isospinors, a and b are isospin indices of the pions.

The unpolarized squared invariant amplitude is given by

$$
\begin{aligned}
|\mathcal{M}|^2 &= \sum_{i,j=1}^{2} \mathcal{Y}_{ij} \left[\frac{3}{2} F_+^{(i)*} F_+^{(j)} + 3\, F_-^{(i)*} F_-^{(j)} \right], \\
\mathcal{Y}_{11} &= 2 \left[(m_N + m_R)^2 - s_1 \right], \\
\mathcal{Y}_{12} &= \mathcal{Y}_{21} = -s_1 \nu \,, \\
\mathcal{Y}_{22} &= \frac{1}{2} \left[(4M_\pi^2 - s_1)(s_1 - (m_R - m_N)^2) - s_1 \nu^2 \right],
\end{aligned}
$$

with ν given by

$$\nu = \frac{s_2 - s_3}{m_N + m_R} \,.$$

The decay width corresponding to the $\pi\pi N$ final state is obtained by substituting $|\mathcal{M}|^2$ in the following formula

$$\Gamma_{R\to\pi\pi N} = \frac{1}{32 m_R^3 (2\pi)^3} \int_{4M_\pi^2}^{(m_R-m_N)^2} \mathrm{d}s_1 \int_{s_{2-}}^{s_{2+}} \mathrm{d}s_2 \, |\mathcal{M}|^2 \,,$$

where the integration limits over s_2 are given by

$$s_{2\pm} = \frac{m_R^2 + m_N^2 + 2M_\pi^2 - s_1}{2} \pm \frac{1}{2s_1} \lambda^{1/2}(s_1, m_R^2, m_N^2) \lambda^{1/2}(s_1, M_\pi^2, M_\pi^2) \,.$$

We consider $m_R - m_N \sim 400$ MeV as a small parameter of the order δ^1 and count $M_\pi \sim \delta^2$.

The kinematical variable ν for the $R \to \pi\pi N$ decay varies from $m_N - m_R$ to $m_R - m_N$ (for $M_\pi = 0$) within the range of integration and therefore we count $\nu \sim \delta$.

As s_1 varies from $4M_\pi^2$ to $(m_R - m_N)^2$, we assign the order δ^2 to it. We also count $m_R - m_\Delta \sim \delta^2$.

The $R \to \pi N$ width is of order $\delta^3 \times \text{order of } A^* A$.

The $R \to \pi\pi N$ width is of order $\delta^3 \times \text{order of } |\mathcal{M}|^2$.

Thus, the contributions of the one- and two-loop self-energy diagrams in the width of the Roper resonance sum up to

$$\Gamma_R = \Gamma_{R\to\pi N} + \Gamma_{R\to\pi\pi N}.$$

Numerical results

To calculate the full decay width of the Roper resonance we use the following standard values of the parameters from PDG

$$M_\pi = 139\ \mathrm{MeV}, \quad m_N = 939\ \mathrm{MeV}, \quad m_\Delta = 1210 \pm 1\ \mathrm{MeV},$$
$$\Gamma_\Delta = 100 \pm 2\ \mathrm{MeV}, \quad m_R = 1365 \pm 15\ \mathrm{MeV}, F_\pi = 92.2\ \mathrm{MeV},$$

and obtain

$$\begin{aligned}
\Gamma_{R\to\pi N} &= 550(57.7)\, g_{\pi NR}^2\ \mathrm{MeV}, \\
\Gamma_{R\to\pi\pi N} &= \Big[1.49(0.58)\, g_A^2\, g_{\pi NR}^2 - 2.76(1.07)\, g_A\, g_{\pi NR}^2\, g_R \\
&+ 1.48(0.59)\, g_{\pi NR}^2\, g_R^2 + 2.96(0.94)\, g_A\, g_{\pi NR}\, h h_R \\
&- 3.79(1.37)\, g_{\pi NR}\, g_R\, h h_R + 9.93(5.45)\, h^2 h_R^2\Big]\ \mathrm{MeV}.
\end{aligned}$$

Further, we substitute $g_A = 1.27$ and $h = 1.42 \pm 0.02$.
The latter value is taken from

D. L. Yao, et.al. JHEP **1605**, 038 (2016).

We pin down $g_{\pi NR}$ by reproducing

$\Gamma_{R\to\pi N} = (123.5 \pm 19.0)$ MeV

from PDG, which yields

$g_{\pi NR} = \pm(0.47 \pm 0.11)$.

Following
S. R. Beane and U. van Kolck, J. Phys. G **31**, 921 (2005)
we assume $g_R = g_A$ and $h_R = h$ and obtain:

$$\begin{aligned}\Gamma_{R\to\pi\pi N} &= \big[0.53(32) - 0.98(60) + 0.53(32) \pm 3.57(1.41) \\ &\mp 4.57(1.97) + 40.4(22.2)\big]\ \mathrm{MeV} = 40.5(22.3)\ \mathrm{MeV}.\end{aligned}$$

The largest contribution comes from the decay diagram with intermediate Δ state.

Further, using the approach of
E. Epelbaum, H. Krebs and U.-G. Meißner, Eur. Phys. J. A **51**, no. 5, 53 (2015)
we estimate the theoretical error due to the omitting the higher order contributions and obtain

$$\Gamma_{R\to\pi\pi N} = (40.5 \pm 22.3 \pm 16.8)\ \mathrm{MeV},$$

which is consistent with

$$\Gamma_{\pi\pi N} = (66.5 \pm 9.5)\ \mathrm{MeV}$$

quoted by PDG.

Summary

- The width of the Roper resonance calculated up NLO of BChPT has been presented.
- The NLO calculation of the width requires obtaining the imaginary parts of one- and two-loop self-energy diagrams.
- We employed the Cutkosky cutting rules and obtained the width by squaring the decay amplitudes.
- One of the three unknown couplings we fix by reproducing the PDG value for $\Gamma_{R\to\pi N}$.
- Assuming that the remaining two couplings of the Roper interaction take values equal to those of the nucleon, we obtain the result for $\Gamma_{R\to\pi\pi N}$ consistent with the PDG value.
- To improve the accuracy of our calculation, three-loop self-energy diagrams need to be calculated.
  Moreover, an infinite number of diagrams, corresponding to the scalar-isoscalar pion-pion scattering need to be re-summed.