Challenges of the Standard Model: studying the quark mass dependence

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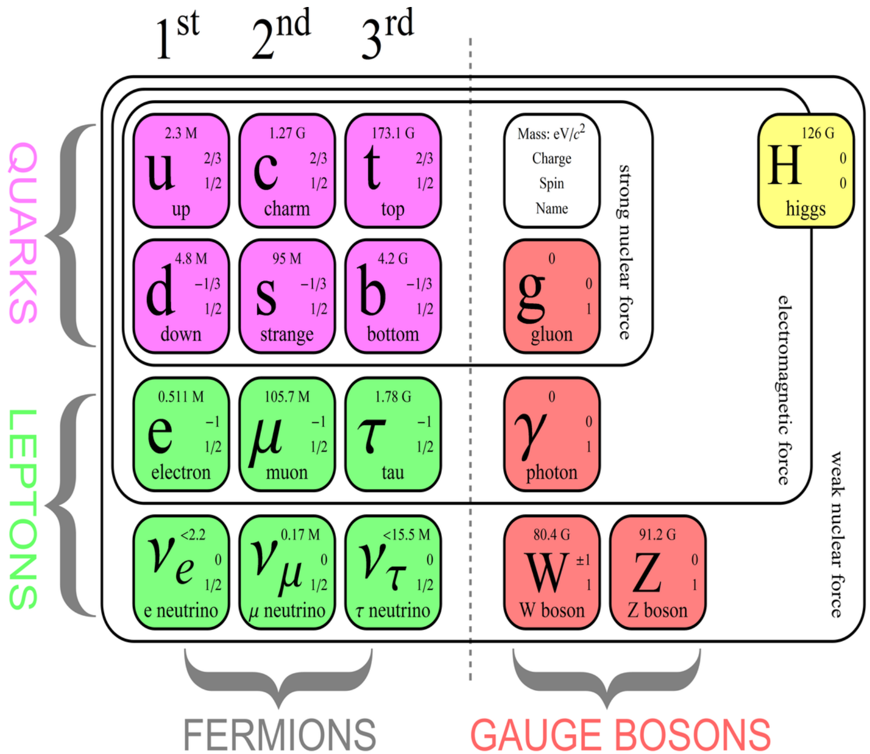
Standard Model



The masses of the quarks and leptons emerge through the spontaneous symmetry breaking. . .

Hadron/nuclear physics frontier of the Standard Model

Only quarks and gluons:

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \bar{\Psi}(i \not{D} - \mathcal{M})\Psi + \theta\text{-term}$$

$$\mathcal{M} = \text{diag}(\underbrace{m_u, m_d, m_s,}_{\text{light}} \underbrace{m_c, m_b, m_t}_{\text{heavy } \to \infty})$$

- Describes *all* phenomena of hadron / nuclear physics
- Confinement: only colorless states are observed
- Inherently non-perturbative: Lattice QCD, EFT methods...

How does the world looks like, if the values of m_u, m_d, m_s are different?

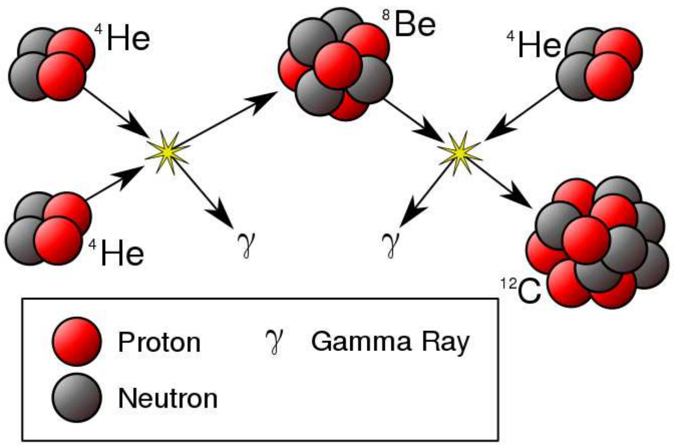
Why this can be useful?

Plan

- Introduction: emergence of the Carbon-based life
- Hadronic input in BSM physics searches
- σ-terms
- The Feynman-Hellmann theorem and the quest for exotic states
- Resonance states
- Conclusions, outlook

Ex. 1: The Hoyle state and the fate of the Carbon-based life

How are the life-essential elements ^{12}C and ^{16}O generated in the stars?



Triple-α process (Bethe 1938, Öpik 1952, Salpeter 1952, Hoyle 1954):

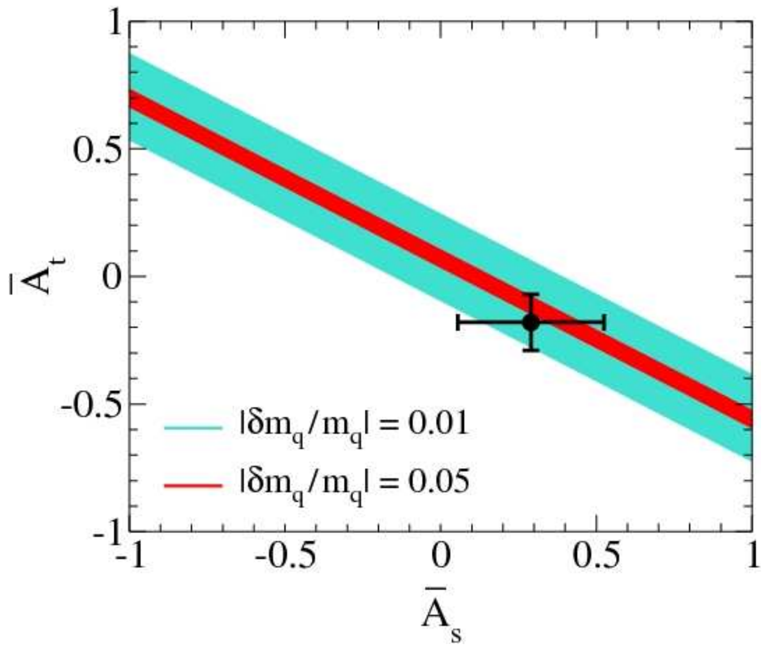
$$
\begin{aligned}
{}^4He + {}^4He &\leftrightharpoons {}^8Be \\
{}^8Be + {}^4He &\leftrightharpoons {}^{12}C^* \rightarrow {}^{12}C + \gamma \\
{}^{12}C + {}^4He &\leftrightharpoons {}^{16}O + \gamma
\end{aligned}
$$

Need an excited 0^+ state ~ 7.7 MeV above $^8Be + {}^4He$ threshold!

cogito ergo mundus talis est

Weak anthropic principle (Barrow and Tipler):

"The observed values of all physical and cosmological quantities are not equally probable but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirements that the universe be old enough for it to have already done so."



Creating Hoyle state at a right place:

Light quark masses fine-tuned at 2-3% precision

α_{EM} fine-tuned at 2.5% precision

Nuclear EFT on the lattice:

E. Epelbaum, H. Krebs, T.A. Lähde, D. Lee and U.-G. Meißner, PRL 110 (2013) 112502

Ex. 2: Hadronic input in BSM searches: EDM

$$\mathcal{L}_{CPV} = \mathcal{L}_{CKM} + \mathcal{L}_\theta + \frac{1}{M^2}\sum_i c_i O_i^{(6)}$$

quark EDM
quark CEDM
3-gluon term
4-quark operators

- Multiple experimental probes are needed to disentangle the origin of the CPV effects

↪ Measuring the EDM's of nucleons and light nuclei

↪ Using ChPT and/or lattice QCD to relate hadronic observables to the CPV parameters

The θ-term

$$\mathcal{L} = \mathcal{L}_{QCD} - \frac{i\theta g^2 N_f}{16\pi^2} \varepsilon_{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \quad \text{(Euclidean)}$$

The θ-term can be eliminated via chiral transformations

$$\Psi_R \to \exp(i\theta/2N_f)\Psi_R\,, \quad \Psi_L \to \exp(-i\theta/2N_f)\Psi_L$$

The mass term is replaced by

$$\bar\Psi\mathcal{M}\Psi \to \bar\Psi_L \mathcal{M} \exp(i\theta/N_f)\Psi_R + \bar\Psi_R \mathcal{M}^\dagger \exp(-i\theta/N_f)\Psi_L$$

This is equivalent to the replacement of the mass matrix

$$\mathcal{M} \to \mathcal{M} \exp(i\theta/N_f)$$

Chiral Perturbation theory

- Hadronic degrees of freedom (pions, nucleons. . .) instead of quarks & gluons
- Effective theory of QCD at low energy
- Gives a systematic expansion of the observables in powers of (small) momenta and light quark masses

$$\mathcal{L} = \frac{F_\pi^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle + \frac{F_\pi^2 B}{2}\langle \mathcal{M}U^\dagger + U\mathcal{M}^\dagger\rangle + \ldots$$

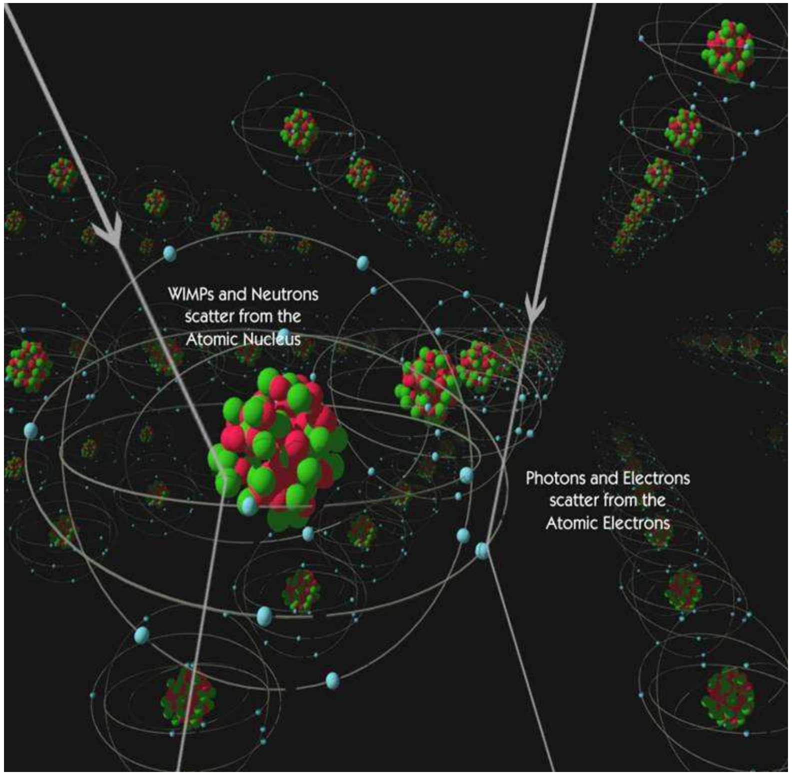
$$B \sim \langle 0|\Psi\Psi|0\rangle \quad \text{quark condensate}$$

- θ-dependence of the pion mass (R. Brower *et al.,* PLB 560 (2003) 64)

$$\mathcal{M} \to \mathcal{M}\exp(i\theta/N_f) \to \quad M_\pi^2(\theta) = M_\pi^2(0)\cos\theta/N_f$$

- The θ-dependence of the lightest meson resonance masses
  N. Acharyia *et al.,* PRD 92 (2015) 054023
- The method can be extended to the sectors with non-zero baryon number

Ex. 3: Hadronic input in BSM searches: WIMPS



- Looking for the nuclear recoil due to interaction with WIMPs
- Estimate for the scattering cross section?

Scattering cross section

$$\mathcal{L} = \sum_q \alpha_{3q} \bar{\chi} \chi \bar{\Psi}_q \Psi_q$$ (spin-independent)

T. Falk, A. Ferstl and K.A. Olive, PRD 59(1999) 055009

$$\sigma_{SI} = \frac{4m_r^2}{\pi}(Zf_p + (A-Z)f_n)^2$$

$$\frac{f_N}{m_N} = \sum_{q=u,d,s} f_{T_q}^{(N)} \frac{\alpha_{3q}}{m_q} + \frac{2}{27} F_{TG}^{(N)} \sum_{q=c,b,t} \frac{\alpha_{3q}}{m_q}$$

$$F_{TG}^{(N)} = 1 - \sum_{q=u,d,s} f_{T_q}^{(N)}$$

The σ-terms: $m_N f_{T_q}^{(N)} = \langle N | m_q \bar{\Psi}_q \Psi_q | N \rangle$

Feynman-Hellmann theorem

The Hamiltonian depends on the external parameter: $H = H(\lambda)$

$$
\begin{aligned}
H(\lambda)|\lambda\rangle &= E(\lambda)|\lambda\rangle \\
\frac{\partial E(\lambda)}{\partial\lambda} &= \frac{\partial}{\partial\lambda}\Big(\langle\lambda|H(\lambda)|\lambda\rangle\Big) = \left\langle\lambda\left|\frac{\partial H(\lambda)}{\partial\lambda}\right|\lambda\right\rangle
\end{aligned}
$$

If $\lambda =$ quark masses m_u, m_d, m_s ,

$$H(\lambda) = H(m_q = 0) + m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s$$

Light and strange σ-terms of the nucleon:

$$
\begin{aligned}
\sigma_\ell &= \frac{1}{2m_N}\langle N|\hat{m}(\bar{u}u + \bar{d}d)|N\rangle = \frac{\partial m_N}{\partial\hat{m}} \\
\sigma_s &= \frac{1}{2m_N}\langle N|m_s\bar{s}s|N\rangle = \frac{\partial m_N}{\partial m_s}
\end{aligned}
$$

Various determinations of the σ_ℓ: a puzzle

1) Lattice QCD (both direct calculation and Feynman-Hellmann theorem, artifacts should be studied further):

BMW	$38(3)(3)$ MeV	arXiv:1510.08013
χQCD	$44.4(3.2)(5.5)$ MeV	arXiv:1511.09089
ETM	$37.22(2.57)^{+0.99}_{-0.63}$ MeV	arXiv:1601.01624
RQCD	$35(6)$ MeV	arXiv:1603.00827
Average	$38.2(2.0)$	(H. Leutwyler)

2) Chiral Perturbation Theory + Roy-Steiner equation + data

M. Hoferichter *et al.,* PRL 115 (2015) 092301

$$\sigma_\ell = 59.1(3.5) \text{ MeV}$$

- Significant violation of the OZI rule
- Large strangeness content of the nucleon?

Ex. 4: Quest for exotica

Gell-Mann-Okubo relations for the σ-terms:

$$\begin{aligned}\langle H|\bar{u}u+\bar{d}d|H\rangle &= A_l+B_lY+C_l(I(I+1)-\tfrac{1}{4}Y^2)\\ \langle H|\bar{s}s|H\rangle &= A_s+B_sY+C_s(I(I+1)-\tfrac{1}{4}Y^2)\end{aligned}$$

Mesons : $$b_f=\frac{B_f}{A_f}=0\,,\quad c_f=\frac{C_f}{A_f}=\frac{2(\sigma_\pi^f-\sigma_K^f)}{4\sigma_K^f-\sigma_\pi^f}$$

Baryons : $$b_f=\frac{B_f}{A_f}=\frac{\sigma_N^f-\sigma_\Xi^f}{2(\sigma_N^f+\sigma_\Xi^f)-\sigma_\Sigma^f}$$

$$c_f=\frac{C_f}{A_f}=\frac{2\sigma_\Sigma^f-\sigma_N^f-\sigma_\Xi^f}{2(\sigma_N^f+\sigma_\Xi^f)-\sigma_\Sigma^f}$$

Counting valence quarks in hadrons

$$\langle H|\bar{\Psi}_q\Psi_q|H\rangle = n_q\langle H|H\rangle$$

V. Bernard, M. Lage, U.-G. Meißner and AR, JHEP 1101 (2011) 019

Strictly valid in: Quark model

$N_c \to \infty$

Quark Model values:

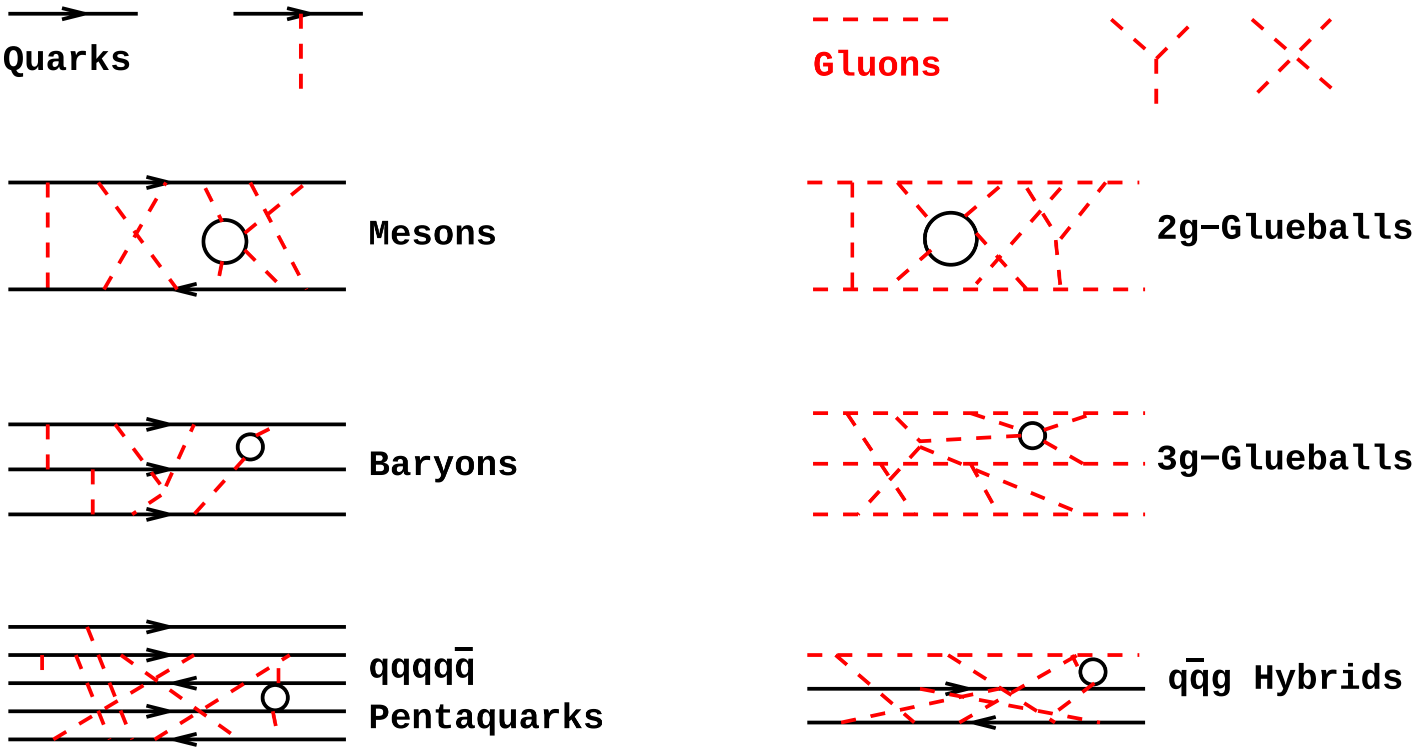
Mesons, $\bar{q}q$: $c_s = -\frac{1}{2}\,,\quad c_l = 1$

Baryons, qqq : $c_s = c_l = 0\,,\quad b_s = -\frac{2}{3}\,,\quad b_l = \frac{1}{3}$

For b_l, c_l, in addition, $SU(3)$ invariance assumed: $m_u = m_d = m_s$

- Exotic states in QCD are <u>defined</u> as being not contained in the Quark Model spectrum
- A criterion for the exotic particles: If b_f, c_f significantly differ from the quark model values, this is interpreted as a signal for exotic states

Exotica



e.g., Tetraquarks, $\bar{q}\bar{q}qq$: $c_s = -1\,, \quad c_l = -\dfrac{1}{5}$

Can be tested in the EFT and on the lattice…

Feynman-Hellmann theorem for resonances (in progress)

Re-deriving known result for the pion:

$$D(p^2) = i\int d^4x e^{ipx}\langle 0|T\phi_\pi(x)\phi_\pi^\dagger(0)|0\rangle \to \frac{Z_\pi}{(M_\pi^2-p^2)} + \text{regular}$$

$$\frac{\partial D(p^2)}{\partial m_q} \to -\frac{Z_\pi}{(M_\pi^2-p^2)^2}\frac{\partial M_\pi^2}{\partial m_q} + \text{less singular terms}$$

since $\mathcal{L} = \mathcal{L}_0 - \sum_q m_q Z_F^{-1} Z_m \bar\Psi_q^0 \Psi_q^0 = \mathcal{L}_0 - \sum_q m_q \bar\Psi_q \Psi_q\,,$

$$\hookrightarrow \frac{\partial D(p^2)}{\partial m_q} = \int d^4x d^4y e^{ip(x-y)}\langle 0|T\phi_\pi(x)\phi_\pi^\dagger(y)\bar\Psi_q(0)\Psi_q(0)|0\rangle$$

$$\to -\frac{Z_\pi\langle\pi|\bar\Psi_q\Psi_q|\pi\rangle}{(M_\pi^2-p^2)^2}\frac{\partial M_\pi^2}{\partial m_q} + \text{less singular terms}$$

$$\hookrightarrow \frac{\partial M_\pi^2}{\partial m_q} = \langle\pi|\bar\Psi_q\Psi_q|\pi\rangle$$

What changes in case of a resonance?

- Resonances emerge as poles on the unphysical Riemann sheets, $s \to z_R$, in the Green functions of the appropriately chosen operators
- The matrix elements between the resonance states are determined from the residues of the pertinent Green functions at the double pole
- The Feynman-Hellmann theorem for the resonances has the same form as for the stable states

$$\frac{\partial z_R}{\partial m_q} = \langle \text{res}|\bar{\Psi}_q \Psi_q|\text{res}\rangle$$

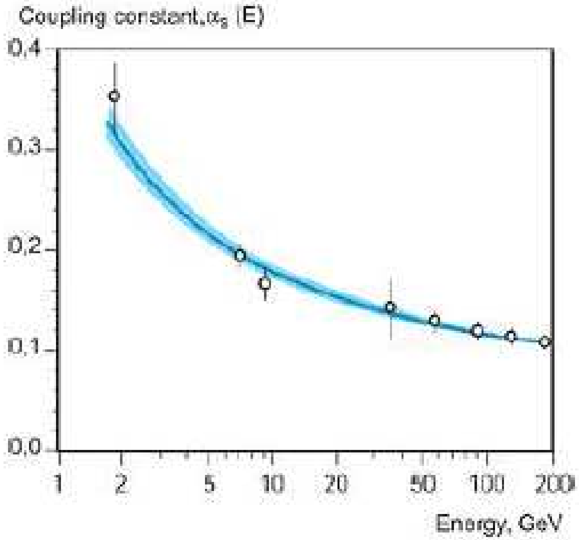
- Can be used to test the exotic nature of the unstable states

Is the 0^{++} octet a good candidate for exotica?

QCD on the lattice

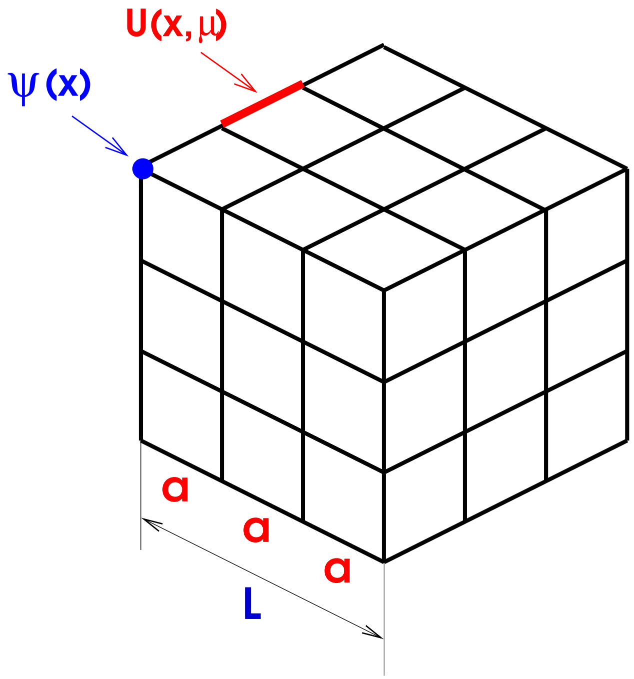
$$\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \bar{\Psi}(\gamma_\mu(\partial_\mu - igT^aG^a_\mu) + \mathcal{M})\Psi$$

$$G^a_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - igf^{abc}G^b_\mu G^c_\nu\,, \qquad \mathcal{M} = \text{diag}\,(m_u, m_d, \cdots)$$



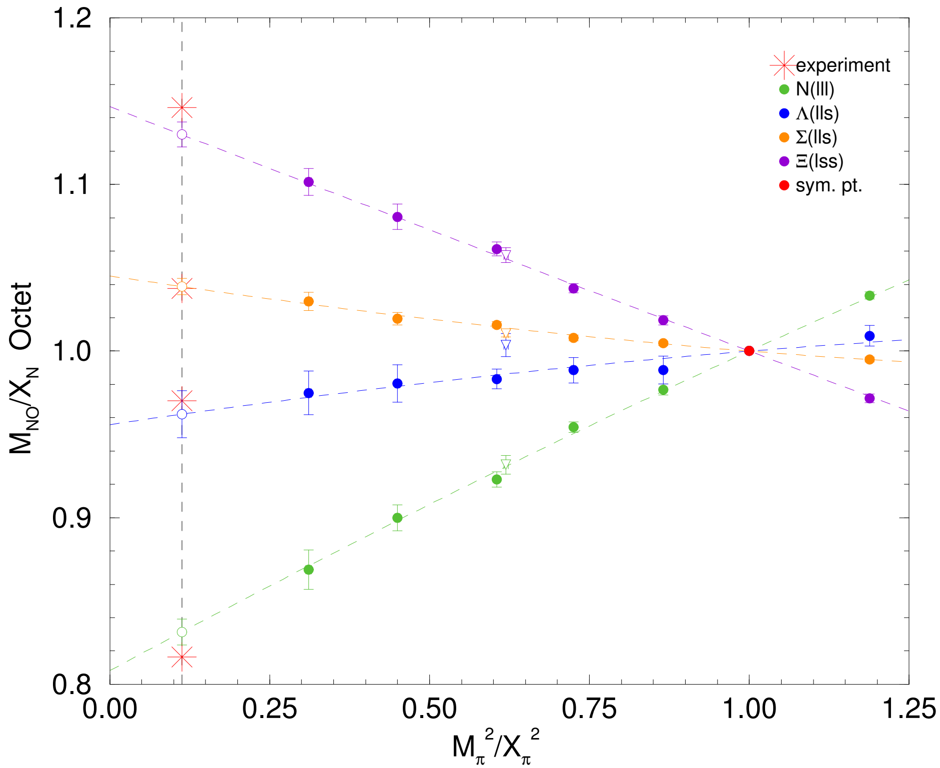
Non-perturbative at low energies:

- Confinement
- Spontaneous chiral symmetry breaking
- Quark masses are free parameters!



$\hookrightarrow$ QCD on the lattice

Lattice tests



W. Bietenholz *et al.,* PRD 84 (2011) 054509

- The approach tested for the pseudoscalar, vector meson octets, for the baryon octet
- Preliminary: as expected, predominately quark-model states

Conclusions

- The study of the quark mass dependence allows one to extract important information about the real world, where the quark masses are fixed. The particular examples are provided by:
  - Synthesis of ^{12}C in stars and the emergence of the Carbon-based life
  - CP violation and the EDM of hadrons and nuclei
  - Interaction of the dark matter with ordinary matter. . .
- The goal can be achieved by using theoretical tools only:
  - Lattice QCD
  - Chiral effecitive field theories. . .
- The quark mass dependence of the masses of the QCD bound states and resonances provides a criterion to judge about the exotic nature of these states