TWO FACES OF QCD

V. Gogokhia

gogohia.vahtang@wigner.mta.hu and gogokhia@rmi.ge

Wigner RCP, Inst. for Part. Nucl. Phys., HAS, Budapest, Hungary

A. Razmadze Math. Inst. of TSU, Tbilisi, Georgia

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Lagrangian of QCD

H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B **47** (1973) 365

$$L_{QCD} = L_{YM} + L_{qg}$$

$$L_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + L_{g.f.} + L_{gh.}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \quad a = N_c^2 - 1, \quad N_c = 3$$

$$L_{qg} = i\bar{q}^j_\alpha D_{\alpha\beta} q^j_\beta + \bar{q}^j_\alpha m^j_0 q^j_\beta, \quad \alpha, \beta = 1, 2, 3, \quad j = 1, 2, 3, ...N_f$$

$$D_{\alpha\beta} q^j_\beta = (\delta_{\alpha\beta}\partial_\mu - ig(1/2)\lambda^a_{\alpha\beta} A^a_\mu)\gamma_\mu q^j_\beta \qquad (cov.\ der.)$$

The λ^as are generators of $SU(3)$ color gauge group

$$[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c$$

$$q^A \;\to\; U(x)q^A(x), \quad U(x) = \exp(i\Theta^a(x)\lambda^a/2)$$

$$A_\mu(x) = A^a_\mu(x)\lambda^a/2 \;\to\; U(x)A_\mu(x)U^{-1}(x) + \frac{i}{g}U(x)\partial_\mu U^{-1}(x)$$

$$SU(N_f) \times SU(N_f) \times U_B(1) \times U_A(1), \quad q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$$

$$L_q = \bar{q}^A_\alpha \delta^{AB} m_0 q^B_\alpha$$

$$SU(N_f) \times SU(N_f) \to SU(N_f)$$

Properties of the QCD Lagrangian

QCD without quarks is Yang-Mills (YM)

1). Universal coupling constant $g \sim 1$.

2). $g \ll 1$ only in the AF regime, where the PT works.

3). Current quark mass m_0 after the renormalization program is performed remains un-physical since the quark is a colored object.

4). The massive gluon term $m_g^2 A_\mu A_\mu$ explicitly violates $SU(3)$ color gauge invariance of QCD.

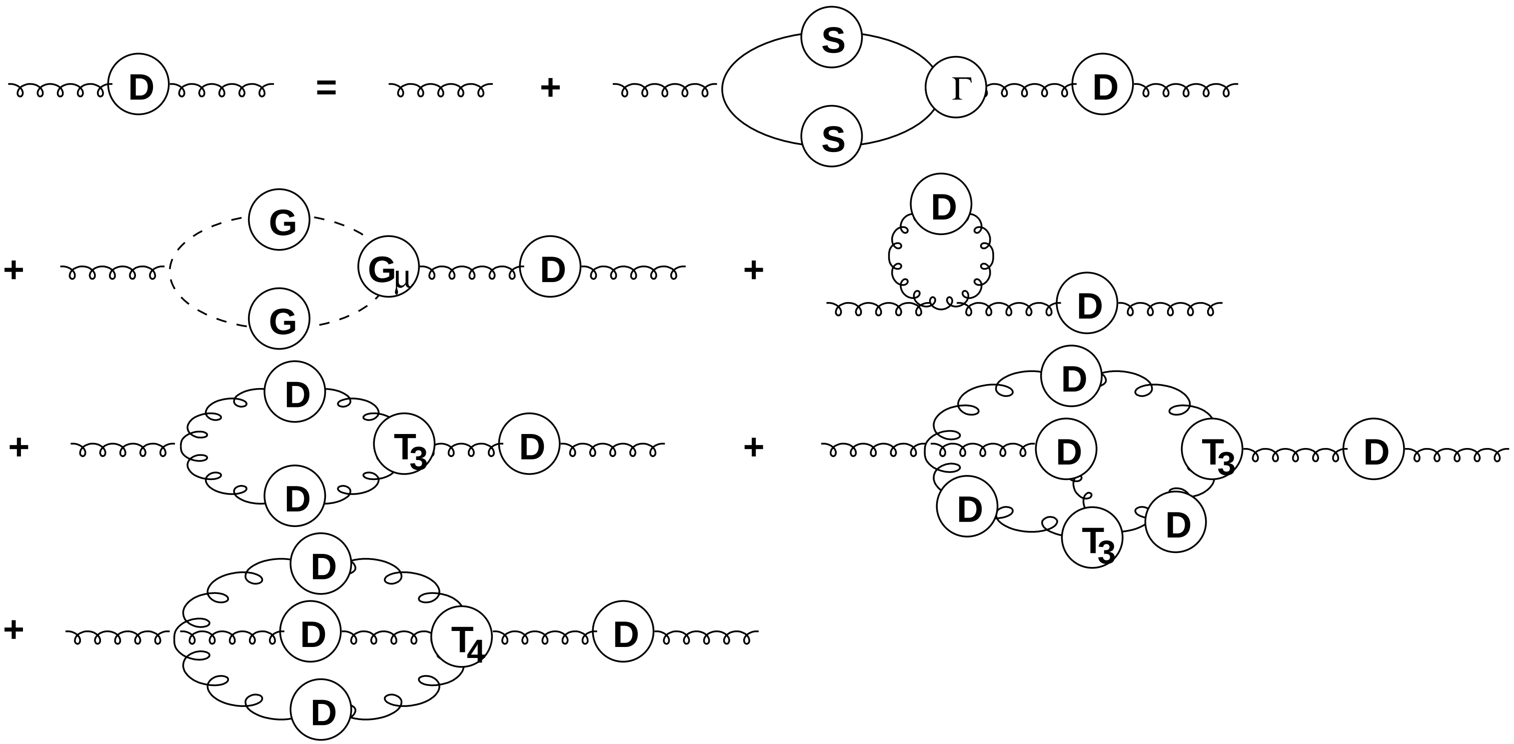
No mass scale parameter to which can be assigned a physical meaning even after the renormalization program is performed.
For some reasons I will call it as a mass gap.

H has no spectrum in the interval $(0, \Delta)$, $\quad 0 < \Delta < \infty$

QED. $m_e - (-m_e) = 2m_e > 0, \quad q_{ph}^2 \geq (2m_e)^2, \quad \Delta = m_e$

QCD. $0 - (-\Delta) = \Delta > 0, \qquad q^2 \geq \Delta^2, \qquad 0 < \frac{\Delta^2}{q^2} \leq 1$

Gluon SD equation



$$q^2 \to 0, \quad q_i \to 0 \, and \, vice \, versa \quad (Euclidean \, signature)$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i\Pi_{\rho\sigma}(q;D) D_{\sigma\nu}(q)$$

$$\Pi_{\rho\sigma}(q;D) = \Pi^q_{\rho\sigma}(q) + \Pi^{gh}_{\rho\sigma}(q) + \Pi^t_{\rho\sigma}(D) + \Pi^{(1)}_{\rho\sigma}(q;D^2)$$
$$+\Pi^{(2)}_{\rho\sigma}(q;D^4) + \Pi^{(2')}_{\rho\sigma}(q;D^3)$$

$$D^0_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$T_{\mu\nu}(q) = \delta_{\mu\nu} - (q_\mu q_\nu / q^2) = \delta_{\mu\nu} - L_{\mu\nu}(q)$$

$$\Pi_{\rho\sigma}(q;D) \equiv \Pi_{\rho\sigma}(q;\lambda,\alpha,D)$$

The transversality of the full gluon self-energy

$$\Pi_{\rho\sigma}(q;D) = \Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q;D) + \Pi^t_{\rho\sigma}(D)$$

$$\Pi^g_{\rho\sigma}(q;D) = \Pi^{gh}_{\rho\sigma}(q) + \Pi^{(1)}_{\rho\sigma}(q;D^2) + \Pi^{(2)}_{\rho\sigma}(q;D^4) + \Pi^{(2')}_{\rho\sigma}(q;D^3)$$

$$\Pi^t_{\rho\sigma}(D) \sim \int d^4k D_{\alpha\beta}(k) T^0_{\rho\sigma\alpha\beta} = \delta_{\rho\sigma}\Delta^2_t(D)$$

$$\begin{aligned} q_\rho\Pi_{\rho\sigma}(q;D) &= q_\rho\Pi^q_{\rho\sigma}(q) + q_\rho\Pi^g_{\rho\sigma}(q;D) + q_\rho\Pi^t_{\rho\sigma}(D) \\ &= q_\rho\Pi^q_{\rho\sigma}(q) + q_\rho\Pi^g_{\rho\sigma}(q;D) + q_\sigma\Delta^2_t(D) \end{aligned}$$

The quark contribution

The color currents conservation condition implies

$$q_\rho \Pi^q_{\rho\sigma}(q) = q_\sigma \Pi^q_{\rho\sigma}(q) = 0$$

$$\Pi^{q(s)}_{\rho\sigma}(q) = \Pi^q_{\rho\sigma}(q) - \Pi^q_{\rho\sigma}(0) = \Pi^q_{\rho\sigma}(q) - \delta_{\rho\sigma}\Delta^2_q, \quad \Pi^{q(s)}_{\rho\sigma}(0) = 0$$

$$q^2 = -\mu^2 \to 0$$

$$\Pi^q_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^q_t(q^2) + q_\rho q_\sigma \Pi^q_l(q^2)$$

$$\Pi^{q(s)}_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^{q(s)}_t(q^2) + q_\rho q_\sigma \Pi^{q(s)}_l(q^2)$$

$$\Pi_l^q(q^2) = \Pi_l^{q(s)}(q^2) + \frac{\Delta_q^2}{q^2}, \quad \Pi_t^q(q^2) = \Pi_t^{q(s)}(q^2) + \frac{\Delta_q^2}{q^2}$$

$$\Pi_l^q(q^2) = \Pi_l^{q(s)}(q^2) + \frac{\Delta_q^2}{q^2} = 0$$

$$\Pi_l^{q(s)}(q^2) = -\frac{\Delta_q^2}{q^2}$$

$$\Delta_q^2 = 0, \quad \Pi_l^q(q^2) = \Pi_l^{q(s)}(q^2) = 0, \quad \Pi_t^q(q^2) = \Pi_t^{q(s)}(q^2)$$

$$\Pi_{\rho\sigma}^q(q) = \Pi_{\rho\sigma}^{q(s)}(q) = T_{\rho\sigma}(q) q^2 \Pi_t^{q(s)}(q^2)$$

The gluon contribution

$$q_\rho \Pi^g_{\rho\sigma}(q;D) =$$

$$q_\rho \Big[\Pi^{gh}_{\rho\sigma}(q) + \Pi^{(1)}_{\rho\sigma}(q;D^2) + \Pi^{(2)}_{\rho\sigma}(q;D^4) + \Pi^{(2')}_{\rho\sigma}(q;D^3) \Big] = 0$$

There is no transversality without Faddeev-Popov ghost term $\Pi^{gh}_{\rho\sigma}(q)$

$$\Pi^{g(s)}_{\rho\sigma}(q;D) = \Pi^g_{\rho\sigma}(q;D) - \Pi^g_{\rho\sigma}(0;D) = \Pi^g_{\rho\sigma}(q;D) - \delta_{\rho\sigma}\Delta^2_g(D)$$

$$\Delta^2_g(D) = \Pi^g(0;D) = \sum_a \Pi_a(0;D) = \sum_a \Delta^2_a(D), \quad a = gh,\ (1),\ (2)\ (2')$$

$$\Pi^{g(s)}_{\rho\sigma}(0;D) = 0$$

$$\Pi^{g}_{\rho\sigma}(q;D) = T_{\rho\sigma}(q)q^2\Pi^{g}_{t}(q^2;D) + q_\rho q_\sigma \Pi^{g}_{l}(q^2;D)$$

$$\Pi^{g(s)}_{\rho\sigma}(q;D) = T_{\rho\sigma}(q)q^2\Pi^{g(s)}_{t}(q^2;D) + q_\rho q_\sigma \Pi^{g(s)}_{l}(q^2;D)$$

$$\Pi^{g}_{t}(q^2;D) = \Pi^{g(s)}_{t}(q^2;D) + \frac{\Delta^2_g(D)}{q^2}$$

$$\Pi^{g}_{l}(q^2;D) = \Pi^{g(s)}_{l}(q^2;D) + \frac{\Delta^2_g(D)}{q^2}$$

$$\Pi_l^g(q^2; D) = \Pi_l^{g(s)}(q^2; D) + \frac{\Delta_g^2(D)}{q^2} = 0$$

$$\Pi_l^{g(s)}(q^2; D) = -\frac{\Delta_g^2(D)}{q^2}$$

$$\Delta_g^2(D) = 0, \quad \Pi_l^g(q^2; D) = \Pi_l^{g(s)}(q^2; D) = 0$$

$$\Pi_t^g(q^2; D) = \Pi_t^{g(s)}(q^2 : D)$$

$$\Pi_{\rho\sigma}^g(q; D) = \Pi_{\rho\sigma}^{g(s)}(q; D) = T_{\rho\sigma}(q) q^2 \Pi_t^{g(s)}(q^2; D)$$

The tadpole term contribution

$$\Pi_{\rho\sigma}(q;D) = \Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q;D) + \delta_{\rho\sigma}\Delta^2_t(D)$$

$$= T_{\rho\sigma}(q)\Big[q^2\Pi^s(q^2;D) + \Delta^2_t(D)\Big] + L_{\rho\sigma}(q)\Delta^2_t(D)$$

$$\Pi^s(q^2;D) = \Pi^{q(s)}_t(q^2) + \Pi^{g(s)}_t(q^2;D)$$

$$q_\rho\Pi_{\rho\sigma}(q;D) = q_\sigma\Delta^2_t(D) \neq 0$$

Hence the ghosts cannot make the full gluon self-energy or, equivalently, the full propagator transversal, unless the constant skeleton tadpole term $\Pi^t_{\rho\sigma}(D) = \delta_{\rho\sigma}\Delta^2_t(D)$ is discarded from the very beginning. Also, $\Delta^2_t(D) \equiv \Delta^2(D)$.

ST identity

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi$$

$$D_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d(q^2) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q)iT_{\rho\sigma}(q)[q^2\Pi^s(q^2;D) + \Delta^2(D)]D_{\sigma\nu}(q)$$

$$+D^0_{\mu\rho}(q)iL_{\rho\sigma}(q)\Delta^2(D)D_{\sigma\nu}(q)$$

$$d(q^2) = \frac{1}{1 + \Pi^s(q^2;D) + (\Delta^2(D)/q^2)}$$

$$q_\mu q_\nu D_{\mu\nu}(q;\Delta^2(D)) = i\xi\left(1+\xi\frac{\Delta^2(D)}{q^2}\right)$$

$$D_{\mu\nu}(q;\Delta^2(D)) = D^{PT}_{\mu\nu}(q), \quad at \quad \Delta^2(D)=0$$

$$D^{PT}_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d^{PT}(q^2)+\xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$D^{PT}_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) q^2 \Pi^s(q^2;D^{PT}) D^{PT}_{\sigma\nu}(q)$$

$$q_\mu q_\nu D^{PT}_{\mu\nu}(q) = i\xi, \quad d^{PT}(q^2) = \frac{1}{1+\Pi^s(q^2;D^{PT})}, \quad q_\rho \Pi_{\rho\sigma}(q;D^{PT}) = 0$$

Preliminary discussion

To identically put $\Delta^2(D) = 0$ everywhere is a way (but not unique, see discussion below) how to preserve the color gauge invaraince/symmetry in QCD. **Then why does $\Delta^2(D)$ (which is nothing but the tadpole term, explicitly present in the gluon SD eqution) exist in this theory at all?** There is no doubt that this symmetry should be maintained at non-zero $\Delta^2(D)$ as well.

A. The first problem is how to satisfy the ST identity but without going to $\Delta^2(D) = 0$ limit everywhere.

B. The second problem is how to make the relevant gluon propagator purely transversal, since the ghosts will fail to do this when $\Delta^2(D)$ is not disregarded.

A. Gauge invariance for the equation of motion. The spurious mechanism

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2 \Pi^s(q^2; D) + \Delta^2(D)] D_{\sigma\nu}(q)$$
$$+ D^0_{\mu\rho}(q) i L_{\rho\sigma}(q) \Delta^2(D) D_{\sigma\nu}(q)$$

$$D^0_{\mu\nu}(q) \to D^0_{\mu\nu}(q) + i\xi L_{\mu\nu}(q) d_0(q^2) \frac{1}{q^2}$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2 \Pi^s(q^2; D) + \Delta^2(D)] D_{\sigma\nu}(q)$$
$$+ i\xi L_{\mu\nu}(q) I(q^2; \Delta^2(D)) \frac{1}{q^2}$$

$$I(q^2; \Delta^2(D)) = \left[d_0(q^2) - \xi[1 + d_0(q^2)] \frac{\Delta^2(D)}{q^2} \right]$$

$$d_0(q^2) = \xi[1 + d_0(q^2)] \frac{\Delta^2(D)}{q^2}$$

$$d_0(q^2) = \xi \frac{\Delta^2(D)}{q^2 - \xi \Delta^2(D)}$$

$$I(q^2; \Delta^2(D)) = 0$$

QCD

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) iT_{\rho\sigma}(q)[q^2\Pi^s(q^2;D) + \Delta^2(D)]D_{\sigma\nu}(q)$$

$$d(q^2) = \frac{1}{1 + \Pi^s(q^2;D) + (\Delta^2(D)/q^2)}$$

PT QCD

$$D^{PT}_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) iT_{\rho\sigma}(q) q^2\Pi^s(q^2;D^{PT}) D^{PT}_{\sigma\nu}(q)$$

$$d^{PT}(q^2) = \frac{1}{1 + \Pi^s(q^2;D^{PT})}$$

Formal PT limit: $\Delta^2(D) = 0$ **or, equivalently,** $q^2 \to \infty$

The Mass Gap

$$d(q^2) = 1 - \left[\Pi^s(q^2; d) + \frac{\Delta^2(d)}{q^2}\right] d(q^2), \quad D \to d$$

$$\Delta^2(d) \equiv \Delta_t^2(d) = \Delta^2 \times c(d)$$

$$\Delta^2 > 0, \quad c(d) = c(\lambda, \alpha, \xi, g^2)$$

$$\Delta_R^2 = Z \times \Delta^2$$

The mass gap is generated by the point-like four-gluon vertex

B. Intrinsically non-perturbative (INP) gluon propagator

$$D^{INP}_{\mu\nu}(q;\Delta^2) = D_{\mu\nu}(q;\Delta^2) - D_{\mu\nu}(q;\Delta^2 = 0) = D_{\mu\nu}(q;\Delta^2) - D^{PT}_{\mu\nu}(q)$$

$$D_{\mu\nu}(q;\Delta^2) = i\left\{T_{\mu\nu}(q)d(q^2;\Delta^2) + \xi L_{\mu\nu}(q)\right\}(1/q^2)$$

$$D^{PT}_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d^{PT}(q^2) + \xi L_{\mu\nu}(q)\right\}(1/q^2)$$

$$D^{INP}_{\mu\nu}(q;\Delta^2) = iT_{\mu\nu}(q)d^{INP}(q^2;\Delta^2)\frac{1}{q^2}$$

$$D_{\mu\nu}(q;\Delta^2) = D^{INP}_{\mu\nu}(q;\Delta^2) + D^{PT}_{\mu\nu}(q)$$

The exact separation between the INP and PT dynamics

$$D_{\mu\nu}(q;\Delta^2) = i\left\{T_{\mu\nu}(q)d(q^2;\Delta^2) + \xi L_{\mu\nu}(q)\right\}(1/q^2)$$

$$-iT_{\mu\nu}(q)d^{PT}(q^2)(1/q^2) + iT_{\mu\nu}(q)d^{PT}(q^2)(1/q^2)$$

$$= D^{INP}_{\mu\nu}(q;\Delta^2) + D^{PT}_{\mu\nu}(q)$$

$$d^{INP}(q^2;\Delta^2) = d(q^2;\Delta^2) - d^{PT}(q^2)$$

The SDE for the INP gluon propagator

$$D^{INP}_{\mu\nu}(q;\Delta^2) = D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2\Pi^s(q^2;D) - q^2\Pi^s(q^2;D^{PT}) + \Delta^2] D^{PT}_{\sigma\nu}(q)$$

$$+ D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)\left[q^2\Pi^s(q^2;D) + \Delta^2\right] D^{INP}_{\sigma\nu}(q;\Delta^2)$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2\Pi^s(q^2;D) + \Delta^2(D)] D_{\sigma\nu}(q)$$

$$D^{PT}_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) q^2\Pi^s(q^2;D^{PT}) D^{PT}_{\sigma\nu}(q)$$

No free gluons in INP QCD

Singular solution

$$d(q^2) \equiv d(q^2; z) = \frac{1}{1 + \Pi^s(q^2; d) + zc(d)}, \quad 0 < z = \frac{\Delta^2}{q^2} < 1$$

$$d(q^2; z = 0) = d^{PT}(q^2) = \frac{1}{1 + \Pi^s(q^2; d^{PT})}$$

$$d(q^2; z) = \sum_{k=0}^{\infty} z^k f_k(q^2)$$

$$f_k(q^2) = (-1)^k d^{PT}(q^2)[d^{PT}(q^2)c(d^{PT})]^k$$

$$d(q^2;z) = \sum_{k=0}^{\infty} z^k f_k(q^2) = d^{PT}(q^2) + d^{INP}(q^2;z)$$

$$d^{INP}(q^2;z) = \sum_{k=1}^{\infty} z^k f_k(q^2) = z \sum_{k=0}^{\infty} z^k f_{k+1}(q^2)$$

$$f_{k+1}(q^2) = \sum_{n=0}^{\infty} z^{-n} f_{k+1}^{(n)}(0), \quad 0 < z^{-1} = \frac{q^2}{\Delta^2} < 1$$

$$d^{INP}(q^2;z) = z \sum_{k=0}^{\infty} z^k f_{k+1}(q^2) = z \sum_{k=0}^{\infty} z^k \sum_{n=0}^{\infty} z^{-n} f_{k+1}^{(n)}(0)$$

$$d^{INP}(q^2; z) = z \sum_{k=0}^{\infty} z^k c_k(0) + z \sum_{k=1}^{\infty} z^{-k} c_{-k}(0)$$

$$c_k(0) = \sum_{m=0}^{\infty} f_{k+m+1}^{(m)}(0),$$

$$c_{-k}(0) = \sum_{m=0}^{\infty} f_{m+1}^{(k+m)}(0)$$

$$d^{INP}(q^2; z) = z \sum_{k=-\infty}^{\infty} z^k c_k(0), \quad 0 < z < 1$$

$$D_{\mu\nu}(q;\Delta^2) = D_{\mu\nu}^{INP}(q;\Delta^2) + D_{\mu\nu}^{PT}(q)$$

$$D_{\mu\nu}^{INP}(q;\Delta^2) = iT_{\mu\nu}(q)d^{INP}(q^2;\Delta^2)\frac{1}{q^2} = iT_{\mu\nu}(q)\frac{\Delta^2}{(q^2)^2}L(q^2)$$

$$L(q^2) = \sum_{k=-\infty}^{\infty}\Big(\frac{\Delta^2}{q^2}\Big)^k \Phi_k(\lambda,\alpha,\xi,g^2), \quad 0 < \frac{\Delta^2}{q^2} < 1$$

$$D_{\mu\nu}^{PT}(q) = i\left\{T_{\mu\nu}(q)d^{PT}(q^2) + \xi L_{\mu\nu}(q)\right\}(1/q^2)$$

$$d^{PT}(q^2) = \frac{1}{1 + \Pi^s(q^2; d^{PT})}$$

Massive solution

$$\frac{1}{q^2}d(q^2) = \frac{1}{q^2 + q^2\Pi^s(q^2;\xi) + \Delta^2 c(\xi)}$$

If the denominator has a zero at point $q^2 = -m_g^2$
(Eucl. sign.), where $m_g^2 \equiv m_g^2(\xi)$ is an effective gluon mass

$$-m_g^2 - m_g^2\Pi^s(-m_g^2;\xi) + \Delta^2 c(\xi) = 0$$

$$q^2 + q^2\Pi^s(q^2;\xi) + \Delta^2 c(\xi) = q^2 + m_g^2 + q^2\Pi^s(q^2;\xi) + m_g^2\Pi^s(-m_g^2;\xi)$$

$$\Pi^s(q^2;\xi) = \Pi^s(-m_g^2;\xi) + (q^2 + m_g^2)\Pi'^s(-m_g^2;\xi) + O\Big((q^2 + m_g^2)^2\Big)$$

$$q^2 + m_g^2 + q^2\Pi^s(q^2;\xi) + m_g^2\Pi^s(-m_g^2;\xi)$$

$$= (q^2 + m_g^2)[1 + \Pi^s(-m_g^2;\xi) - m_g^2\Pi'^s(-m_g^2;\xi)][1 + \Pi^{s,R}(q^2;\xi)]$$

$\Pi^{s,R}(q^2;\xi) = 0$ at $q^2 = -m_g^2$ and regular at small q^2.

$$D_{\mu\nu}(q;m_g^2) = iT_{\mu\nu}(q)\frac{Z_3(m_g^2)}{(q^2 + m_g^2)[1 + \Pi^{s,R}(q^2;m_g^2)]} + i\xi L_{\mu\nu}(q)\frac{1}{q^2}$$

$$Z_3(m_g^2) = \frac{1}{1 + \Pi^s(-m_g^2;\xi) - m_g^2\Pi'^s(-m_g^2;\xi)}$$

An effective gluon mass is the NP effect. It cannot be interpreted as the "physical" gluon mass, since it remains explicitly gauge-dependent quantity. We were unable to renormalize it along with the gluon propagator. In the formal PT $\Delta^2 = m_g^2(\xi) = 0$ limit the gluon renormalization constant becomes the standard one

$$Z_3(0) = \frac{1}{1 + \Pi^s(0; \xi)}$$

The existence of the massive solution shows the general possibility for a massless vector particles to acquire masses dynamically, i.e., without so-called Higgs mechanism.

$$1 + \Pi^s(-m_g^2; \xi) = \frac{\Delta^2 c(\xi)}{m_g^2}, \quad 1 + \Pi^s(-m_g^2; \xi) = 0 \quad ?$$

$$D_{\mu\nu}(q;m_g^2) = D_{\mu\nu}^{TNP}(q;m_g^2) + D_{\mu\nu}^{PT}(q)$$

$$D_{\mu\nu}^{TNP}(q) = iT_{\mu\nu}(q)\left[\frac{Z_3(m_g^2)}{(q^2+m_g^2)[1+\Pi^{s,R}(q^2;m_g^2)]} - \frac{Z_3(0)}{q^2[1+\Pi^{s,R}(q^2;0)]}\right]$$

$$D_{\mu\nu}^{PT}(q) = i\left[T_{\mu\nu}(q)\frac{Z_3(0)}{[1+\Pi^{s,R}(q^2;0)]} + \xi L_{\mu\nu}(q)\right]\frac{1}{q^2}$$

Preliminary conclusions

The mass gap is generated in the gluon sector of QCD
due to the self-interaction of massless gluon modes.
The role of the point-like four-gluon vertex is to be emphasized.

It is explicitly present in $\Pi(q; D)$ in the form of of the
tadpole term $\Delta^2(D) \equiv \Delta_t^2(D)$, so it is not introduced by hand.

**No any truncations/approximations/asumptions, no
special gauge choice, only algebraic (i.e., exact) derivations**.

The common belief (of PT) that mass gap contradicts
the color gauge invariance/symmetry of QCD is false.

This fundamental symmetry is maintained/preserved
at non-zero mass gap as well at the level of equation of motion.

The correct SDE for the full gluon propagator in the presence of the mass gap is derived.

We distinguish between INP and PT QCD by the explicit presence of $\Delta^2(D)$, and not by the strength of g.

The INP gluon propagator exactly reproduces the NP context of the full gluon propagator, and it is purely transversal in a gauge-invariant way.

The PT gluon propagator exactly reproduces the non-trivial PT context of the full gluon propagator. It is of the arbitrary gauge, but becomes transversal by means of ghosts.

Two formal exact solutions for the full gluon propagator in the explicit presence of the mass gap:

Singular solution and **Massive solution**

INP QCD

$$D_{\mu\nu}^{INP}(q;\Delta^2) = iT_{\mu\nu}(q)\frac{\Delta^2}{(q^2)^2}L(q^2), \quad q^2 \in (0,\infty)$$

$$L(q^2) = \sum_{k=-\infty}^{\infty}\Big(\frac{\Delta^2}{q^2}\Big)^k \Phi_k(\lambda,\alpha,\xi,g^2), \quad 0 < \frac{\Delta^2}{q^2} < 1$$

A. Transversal and depending on Δ^2

B. No PT infrared (IR) singularity $\sim (q^2)^{-1}$

C. NP (severe) IR singularities only $\sim (q^2)^{-2-k}, \ k = 0, 1, 2, 3, ...$

D. $q^2 \to \infty$ **is also essential singularity like** $q^2 \to 0$

Theory of functions of complex variable

Two theorems which describe the behavior of the meromorphic functions near their essential singularities

Picard theorem: *If* z_0 *is an essential singularity of the function* $f(z)$*, then for any complex/real number* $Z \neq \infty$*, excepting, may be, one value* $Z = Z_0$*, every neighborhood of* z_0 *contains infinite set of points* z*, such that*

$$f(z) = Z, \quad z \to z_0$$

or, equivalently,

Weierstrass-Sokhatsky-Casorati (WSC) theorem:

If z_0 is an essential singularity of the function $f(z)$, then for any complex/real number Z (including $Z = \infty$) there exists the sequence of points $z_k \to z_0$, such that

$$\lim_{k\to\infty} f(z_k) = Z, \quad Z = Z(z_k)$$

So both theorems (Picard/WSC) tell us that the function $f(z)$ in the close neighborhood of its essential singularity can be replaced by the constant Z, which value depends only on how precisely $z_k \to z_0$.

Classical example I (Picard)

$$f(z) = \sin(1/z) = \sum_{n=0}^{\infty} \frac{(-1)^{n+2}}{(2n+1)!} \left(\frac{1}{z}\right)^{2n+1}, \quad 0 < |z| < \infty, \quad (z \neq 0)$$

$$z_k = \frac{1}{(4k+1)(\pi/2) + a} \to 0, \quad k = 0, 1, 2, 3....$$

$$\lim_{k\to\infty} \sin(1/z_k) = \lim_{k\to\infty} \sin[(4k+1)(\pi/2) + a] =$$

$$\sin[(4k+1)(\pi/2)]\cos a + \cos[(4k+1)(\pi/2)]\sin a = \cos a = Z$$

So the last equation for any given real number $Z \neq \infty$ has solutions without exceptional point, for example

$$Z = 1, \frac{\sqrt{2}}{4}(\sqrt{3}+1), 0, ..., \quad a = 0, \frac{\pi}{12}, \frac{\pi}{2}, ...$$

Classical example II (WSC)

$$f(z) = e^{1/z} = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{z^n}, \qquad 0 < |z| < \infty, \quad (z \neq 0)$$

(i). $z_k = (1/k) \to 0, \quad k = 1, 2, 3, ...$

$$\lim_{k\to\infty} f(z_k) = \lim_{k\to\infty} e^k = \infty$$

(ii). $z_k = -(1/k) \to 0, \quad k = 1, 2, 3, ...$

$$\lim_{k\to\infty} f(z_k) = \lim_{k\to\infty} e^{-k} = 0$$

(iii). $z_k = (1/\ln A + 2k\pi i) \to 0, \quad k = 0, 1, 2, 3, ...$

$$\lim_{k\to\infty} f(z_k) = \lim_{k\to\infty} e^{lnA+2k\pi i} = A \neq 0$$

TWO PHASES IN QCD

$$D^{INP}_{\mu\nu}(q;\Delta^2) = iT_{\mu\nu}(q)\frac{\Delta^2}{(q^2)^2}L(q^2)$$

$$= iT_{\mu\nu}(q)\frac{\Delta^2}{(q^2)^2}\sum_{k=-\infty}^{\infty}\Big(\frac{\Delta^2}{q^2}\Big)^k\Phi_k(\lambda,\alpha,\xi,g^2),$$

$$q^2 \in (0,\infty)$$

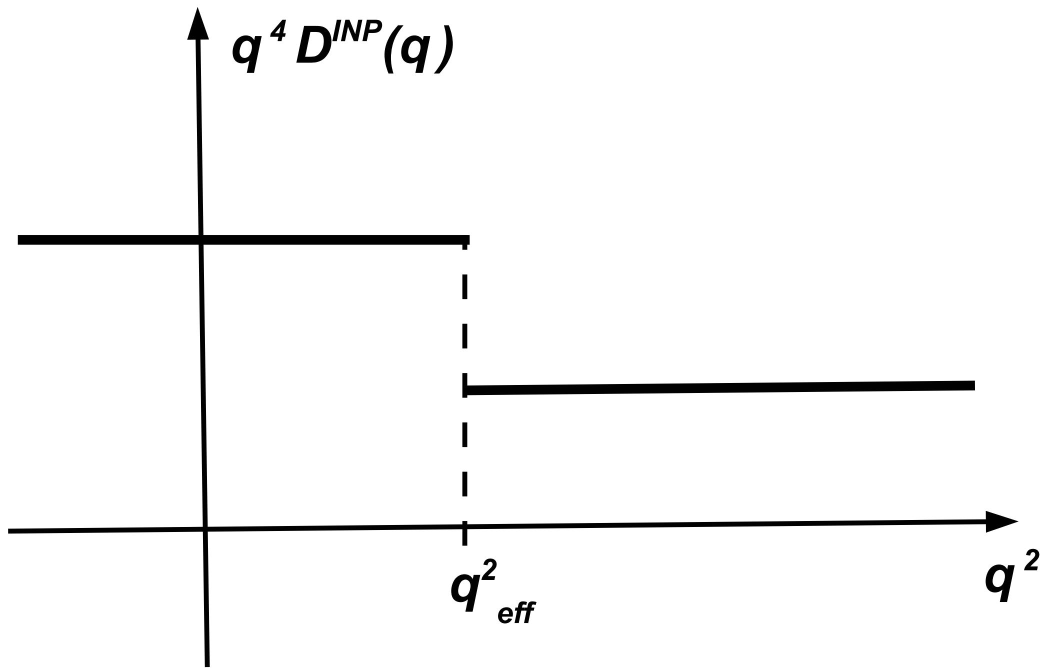
$$0 < \frac{\Delta^2}{q^2} < 1$$

$$D_{\mu\nu}^{INP}(q;\Delta^2) = iT_{\mu\nu}(q)\frac{\Delta^2}{(q^2)^2} \times Z, \quad q^2 \to 0$$

$$D_{\mu\nu}^{INP}(q;\Delta^2) = iT_{\mu\nu}(q)\frac{\Delta^2}{(q^2)^2} \times Z', \quad q^2 \to \infty$$

The two different renormalized constants
$\Delta^2 \times Z = \Delta_R^2$ **and** $\Delta^2 \times Z' = \Delta_R'^2 \equiv \Lambda_{YM}^2$
appear in the two different regimes $q^2 \to 0$ **and** $q^2 \to \infty$**,**
respectively. But the mass gap Δ^2 **itself comes from the IR.**

Suppression of all the severe IR singularities $(q^2)^{-2-k}, \ k = 1, 2, 3, ...$
apart from $(q^2)^{-2}$ **due to the Picard/WSC theorem.**



Existence of the effective scale separating the two different phases in QCD.

QCD: $D_{\mu\nu}(q;\Delta^2) = D^{INP}_{\mu\nu}(q;\Delta^2) + D^{PT}_{\mu\nu}(q)$

a) $q_\rho \Pi_{\rho\sigma}(q;D) = q_\sigma \Delta^2(D) \neq 0$

b) free gluons at large distances

INP QCD: $D^{INP}_{\mu\nu}(q;\Delta^2) = D_{\mu\nu}(q;\Delta^2) - D^{PT}_{\mu\nu}(q)$

PT QCD: $D^{PT}_{\mu\nu}(q) = D_{\mu\nu}(q;\Delta^2) - D^{INP}_{\mu\nu}(q;\Delta^2)$

General prescription: All the PT/INP contributions (”contaminations”) should be always omitted/subtracted in order to get to INP QCD/PT QCD, respectively, making thus the separation between them exact and unique.

RENORMALIZATION OF INP QCD

$$D_{\mu\nu}^{INP}(q;\Delta_R^2) = iT_{\mu\nu}(q)\frac{\Delta_R^2}{(q^2)^2}, \quad q^2 \in [0,\infty)$$

I. Distribution Theory (DT)

$$P(q) = q_0^2 + q_1^2 + q_2^2 + ... + q_{n-1}^2 = q^2, \quad P(q) > 0$$

$$(P^\lambda, \varphi) = \int_{P>0} d^d q P^\lambda(q)\varphi(q)$$

$Re\lambda \geq 0,$ integral is convergent and analytic function of λ

$Re\lambda < 0$, integral has a simple poles at points

$$\lambda = -\frac{n}{2} - k, \quad k = 0, 1, 2, 3...$$

$$P^{\lambda}(q) = (q^2)^{\lambda} = \frac{C_{-1}^{(k)}}{\lambda + (d/2) + k} + finite\, terms$$

$$C_{-1}^{(k)} = \frac{\pi^{n/2}}{2^{2k} k! \Gamma((n/2) + k)} \cdot L^k \delta^n(q)$$

$$L = \frac{\partial^2}{\partial q_0^2} + \frac{\partial^2}{\partial q_1^2} + ... + \frac{\partial^2}{\partial q_{n-1}^2}$$

II. Dimensional Regularization Method (DRM)

$$d = n - 2\delta, \quad \delta \to 0^+, \quad UV$$

$$d = n + 2\epsilon, \quad \epsilon \to 0^+, \quad IR$$

$$(q^2)^{-(n/2)-k} = \frac{1}{\epsilon} C_{-1}^{(k)} + finite\, terms, \quad \epsilon \to 0^+$$

$$\int d^d q \frac{q_{\mu_1} \ldots q_{\mu_p}}{(q^2)^m}$$

$d + p - 2m \leq 0$ IRD; $\quad d + p - 2m \geq 0$ UVD

$$\mathbf{n = 4:} \quad q^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2$$

$$(q^2)^{-2-k} = \frac{1}{\epsilon}\left[a(k)[\delta^4(q)]^{(k)} + O_k(\epsilon)\right], \quad k = 0, 1, 2, 3..., \quad \epsilon \to 0^+$$

$$a(k) = \pi^2/2^{2k}k!\Gamma(2+k)$$

$$[\delta^4(q)]^{(k)} = \left[(\partial^2/\partial q_0^2) + (\partial^2/\partial q_1^2) + (\partial^2/\partial q_2^2) + (\partial^2/\partial q_3^2)\right]^k \delta^4(q)$$

$$\mathbf{k = 0:} \quad (q^2)^{-2} = \frac{1}{\epsilon}\left[\pi^2\delta^4(q) + O(\epsilon)\right], \quad \epsilon \to 0^+$$

III. IRMP renormalization

$$D_{\mu\nu}^{INP}(q;\Delta_R^2) = \frac{1}{\epsilon}\Big[iT_{\mu\nu}(q)\Delta_R^2\pi^2\delta^4(q) + O(\epsilon)\Big], \quad \epsilon \to 0^+$$

$$\Delta_R^2 = X(\epsilon)\bar{\Delta}_R^2 = \epsilon\bar{A}[1+O(\epsilon)]\bar{\Delta}_R^2 = \epsilon\bar{A}\cdot\bar{\Delta}_R^2 = \epsilon\Delta_{JW}^2, \quad \epsilon \to 0^+$$

$X(\epsilon)$ is IRMP renormalization constant for the mass gap Δ_R^2.

$$D_{\mu\nu}^{INP}(q;\Delta_{JW}^2) = iT_{\mu\nu}(q)\Delta_{JW}^2\pi^2\delta^4(q) + O(\epsilon), \quad \epsilon \to 0^+$$

However, this expression is valid only when q^2 is an independent loop variable. Also, in order to avoid the multiplications of at least two δ-functions at the same point (which is not defined in the DT) it is necessary to go to the general formula, which leads to the correct result for the multi-loop skeleton diagrams.

The general criterion of gluon confinement

$$D_{\mu\nu}^{INP}(q;\Delta_{JW}^2) = \epsilon \times iT_{\mu\nu}(q)\frac{\Delta_{JW}^2}{(q^2)^2}, \quad \epsilon \to 0^+$$

1). q^2 is independent skeleton loop variable

$$D_{\mu\nu}^{INP}(q;\Delta_{JW}^2) = iT_{\mu\nu}(q)\Delta_{JW}^2 \pi^2 \delta^4(q) + O(\epsilon), \quad \epsilon \to 0^+$$

2). q^2 is not a loop variable

$$D_{\mu\nu}^{INP}(q;\Delta_{JW}^2) = \epsilon \times iT_{\mu\nu}(q)\frac{\Delta_{JW}^2}{(q^2)^2} \sim \epsilon, \quad \epsilon \to 0^+$$

3). No massless free gluons in this theory ($q^2 \neq 0$)

INP QCD confines transversal gluons in the gauge-invariant way.

Color gluons can never be isolated. Infrared slavery (IRS).

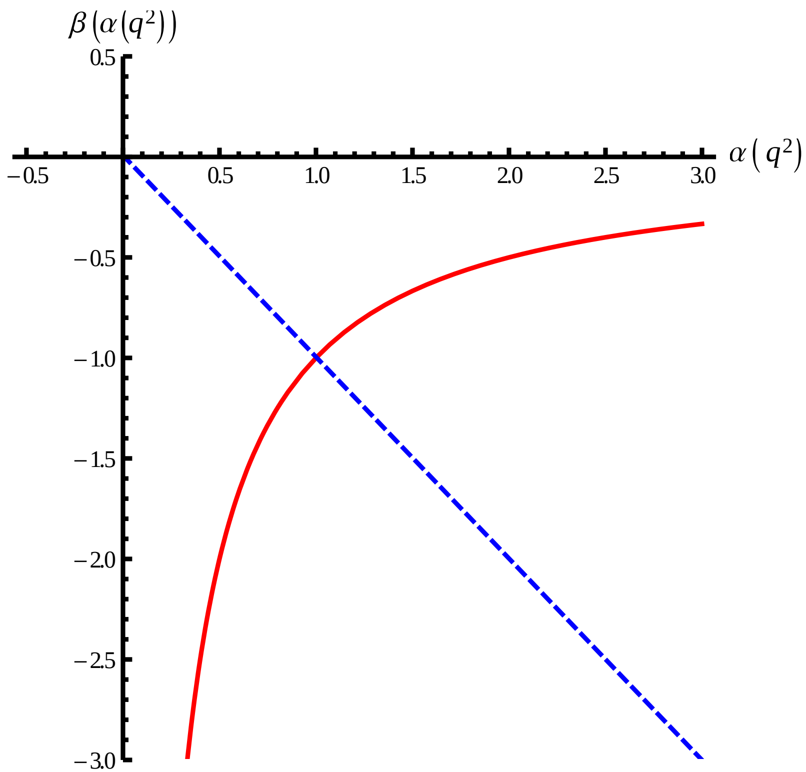
Renormalized running effective charge and β function

$$d(q^2; \Delta^2_{JW}) \equiv \alpha(q^2; \Delta^2_{JW}) = \frac{\Delta^2_{JW}}{q^2}$$

$$q^2 \frac{d\alpha(q^2; \Delta^2_{JW})}{dq^2} = \beta(\alpha(q^2; \Delta^2_{JW}))$$

$$\beta(\alpha(q^2; \Delta^2_{JW})) = -\alpha(q^2; \Delta^2_{JW}) = -\frac{\Delta^2_{JW}}{q^2}$$

$$\frac{\beta(\alpha(q^2; \Delta^2_{JW}))}{\alpha(q^2; \Delta^2_{JW})} = -1$$



$$\beta(\alpha(q^2;\Delta_{JW}^2)) = -\alpha(q^2;\Delta_{JW}^2) = -\frac{\Delta_{JW}^2}{q^2}$$

Relation between the string tension and the mass gap

$$V(r) = -4\pi C_2(R) \int \frac{d^3q}{(2\pi)^3} e^{i\bar{q}\bar{r}} \frac{\alpha(q^2; \Delta^2_{JW})}{q^2}$$

$$\alpha(q^2; \Delta^2_{JW}) = \frac{\Delta^2_{JW}}{q^2}$$

$$\int \frac{d^nq}{(2\pi)^n} e^{iqx} \frac{1}{(q^2)^\lambda} = \frac{\Gamma(n/2 - \lambda)}{4^\lambda \pi^{n/2} \Gamma(\lambda)} (x^2)^{\lambda - n/2}$$

$$\lambda = 2,\ n = 3,\ C_2(R) = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3},\ \Gamma(-1/2) = -2\sqrt{\pi}$$

$$V(r) = \frac{2}{3}\Delta^2_{JW}\, r = \sigma\, r$$

$$\Delta^2_{JW} = \frac{3}{2}\sigma$$

$$\sqrt{\sigma} = 0.420\, GeV, \quad \sigma = 0.1764\, GeV^2$$

$$\Delta^2_{JW} = 0.2646\, GeV^2, \quad \Delta_{JW} = 0.5144\, GeV$$

$$Gerber,\ Leutwyler \longrightarrow F^0_\pi = 88.30\ MeV$$

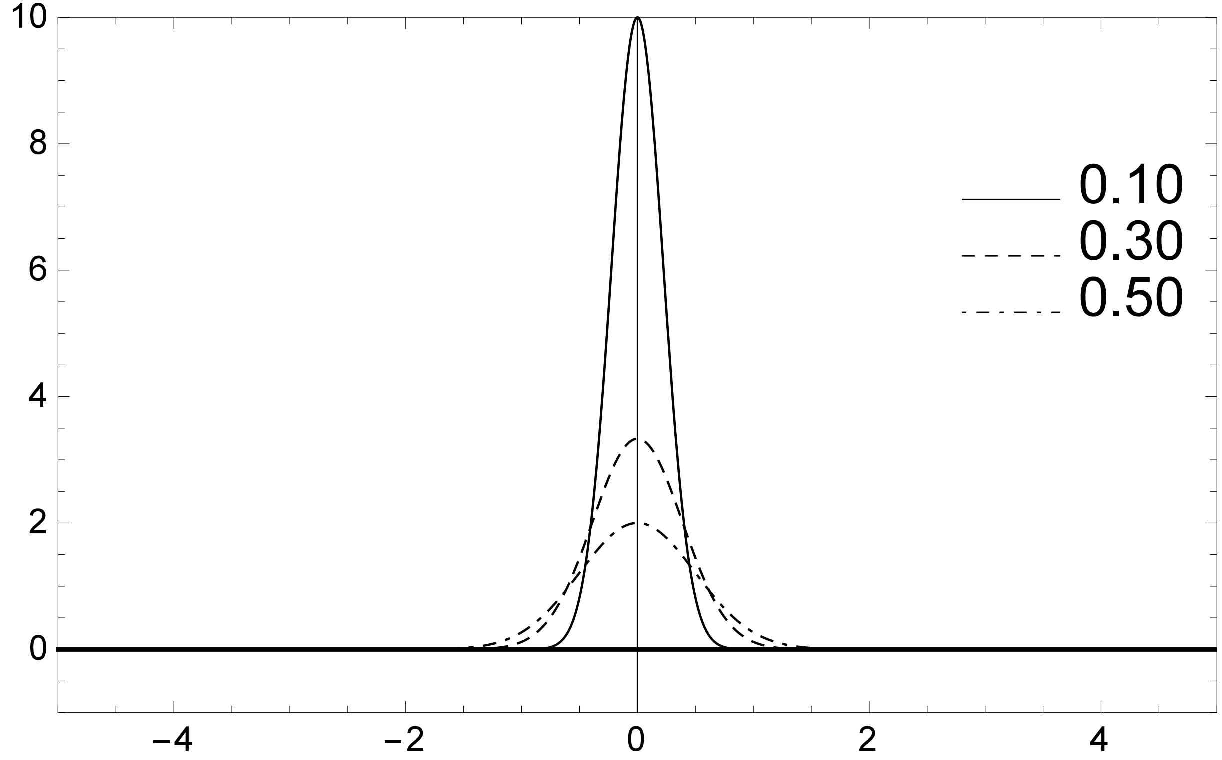
$$\Delta^2_{JW} = 0.2996\, GeV^2, \quad \Delta_{JW} = 0.5474\, GeV$$

The <u>strong</u> character of the delta-function potential with the mass gap

$$D(q;\Delta^2_{JW}) = \Delta^2_{JW}\pi^2\delta^4(q)$$

$$\delta(q_i) = \lim_{\Delta_{JW}\to 0}\frac{1}{\Delta_{JW}\sqrt{\pi}}e^{-\frac{q_i^2}{\Delta^2_{JW}}}, \quad i = 0, 1, 2, 3.$$

$$D(q;\Delta^2_{JW}) = \lim_{\Delta_{JW}\to 0}\frac{1}{\Delta^2_{JW}}e^{-\frac{q^2}{\Delta^2_{JW}}}$$



The delta-function potential with the mass gap squared
as the sequences of zero-centered normal distributions

The short-range behavior of the delta-function potential with the mass gap

$$D(q;\Delta^2_{JW}) \sim \Delta^2_{JW}\delta^4(q) \sim \Delta^2_{JW}\delta^4(\Delta_{JW}x) \sim \frac{1}{\Delta^2_{JW}}\delta^4(x), \quad q = \Delta_{JW}x$$

$$D(r;\Delta^2_{JW}) \sim \int d^3q e^{i\bar{q}\bar{r}} D(q;\Delta^2_{JW})$$

$$R^{-1}(r;\Delta^2_{JW}) \sim \int dx_0 e^{iq_0x_0} D(r;\Delta^2_{JW})$$

$$R^{-1}(r;\Delta^2_{JW}) \sim \Delta_{JW}\int d^4x e^{ixr}\delta^4(x) \sim \Delta_{JW} \sim 0.5\, GeV$$

$$R(r;\Delta^2_{JW}) \sim 0.4fm \sim 0.4\times 10^{-13}cm$$

$$L_l \sim 0.7fm \sim 0.7\times 10^{-13}cm$$

$$nucl.\,forse \sim a\;few\;fm \sim c\times 10^{-13}cm, \quad c>1$$

$$a_0 = \frac{1}{m_e\alpha} \sim 10^{-9}cm$$

$$L_p \sim 10^{-33}cm$$

Asymptotic freedom and the mass gap

$$d(q^2) = \frac{1}{1 + \Pi^s(q^2) + (\Delta^2 c(d)/q^2)}$$

$$\Delta^2 \neq 0, \quad q^2 \to \infty$$

$$d^{PT}(q^2) = \frac{1}{1 + b_0 \alpha_s(\lambda) \ln(q^2/\Lambda^2)}$$

$$d^{PT}(q^2) = \alpha_s(q^2;\Lambda^2)/\alpha_s(\lambda)$$

$$\alpha_s(q^2;\Lambda^2) = \frac{\alpha_s(\lambda)}{1 + b_0 \alpha_s(\lambda) \ln(q^2/\Lambda^2)}$$

$$\Lambda^2 = f(\lambda)\Delta^2 = f(\lambda) Z'^{-1} \Lambda_{YM}^2$$

$$\alpha_s(q^2) = \frac{\alpha_s}{1 + b_0 \alpha_s \ln(q^2/\Lambda_{YM}^2)}, \quad \frac{q^2}{\Lambda_{YM}^2} \geq 1$$

$$\alpha_s = \frac{\alpha_s(\lambda)}{1 + b_0 \alpha_s(\lambda) \ln(f/Z')}, \quad \lambda \to \infty, \quad \alpha(\lambda) \to 0$$

$$\alpha_s \equiv \alpha_s(M_Z) = 0.1184, \quad b_0 = \frac{1}{4\pi}(11 - \frac{2}{3} N_f)$$

Asymptotic Freedom (AF): Khriplovich, t' Hooft, Parisi,
Gross, Wilczek, Politzer.

$$\ln(f/Z') = \frac{\alpha_s - \alpha_s(\lambda)}{\alpha_s b_0 \alpha_s(\lambda)} \to \frac{1}{b_0 \alpha_s(\lambda)}, \quad \lambda \to \infty, \quad \alpha(\lambda) \to 0$$

$$\lim_{(\Lambda^2,\lambda)\to\infty} \Lambda^2 exp\left[-\frac{1}{b_0 \alpha_s(\lambda)}\right] = \Lambda^2_{YM} \equiv \Delta'^2_R, \quad \alpha(\lambda) \to 0$$

$$\alpha_s(q^2) = \frac{1}{b_0 \ln(q^2/\Lambda^2_{YM})} \to 0, \quad q^2 \to \infty$$

Mystery of AF (scale violation at high energies) is resolved.

$\Lambda^2_{YM} = \Lambda^2_{QCD}(N_f = 0) \equiv \Lambda^2_{PT}$ survives at $q^2 \to \infty$, since it is nothing else but the renormalized mass gap squared in the weak coupling regime.

Preliminary conclusions

1. It cannot be disregarded on general grounds (beyond PT).
2. It is compatible with the $SU(3)$ color gauge invariance/symmetry of QCD.
3. It accumulates/summarizes all the severe IR singularities of QCD into the full gluon propagator.
4. All other vertices thus become regular at zero gluon momenta involved.
5. It survives the renormalization program due to P/WSC.
6. The existence of the two phase transitions in the strong and weak coupling limits due to P/WSC:
7. The mass gap exhibits confinement of gluons (finite result+zero (no free gluons) at $q^2 \to 0$).
8. The mass gap explains AF (zero result at $q^2 \to \infty$).

QCD

$$D_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)\alpha(q^2) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2} = D^{INP}_{\mu\nu}(q) + D^{PT}_{\mu\nu}(q)$$

INP QCD

$$D^{INP}_{\mu\nu}(q) = \epsilon \times i T_{\mu\nu}(q)\frac{\alpha^{INP}(q^2)}{q^2}$$

$$\alpha^{INP}(q) = \Delta^2_{JW}/q^2$$

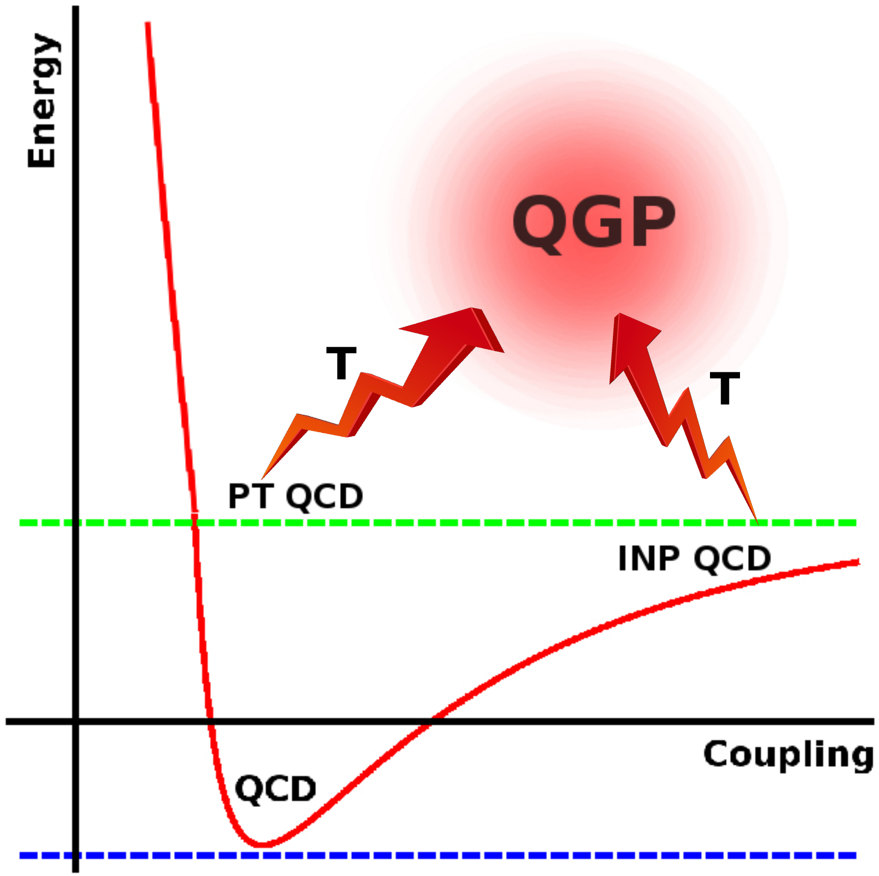
GC, $\Delta^2_{JW} \sim 0.3\, GeV^2$

PT QCD

$$D^{PT}_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)\alpha^{PT}(q^2) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$\alpha^{PT}(q^2) = \alpha_s/[1 + \Pi^s(q^2; \Lambda^2_{PT})]$$

AF, $\Lambda^2_{PT} = \Lambda^2_{YM} = 0.09\, GeV^2$


Energy
QGP
T
T
PT QCD
INP QCD
Coupling
QCD

QCD

$$\alpha(q^2) = \alpha^{INP}(q^2) + \alpha^{PT}(q^2)$$

INP QCD

PT QCD

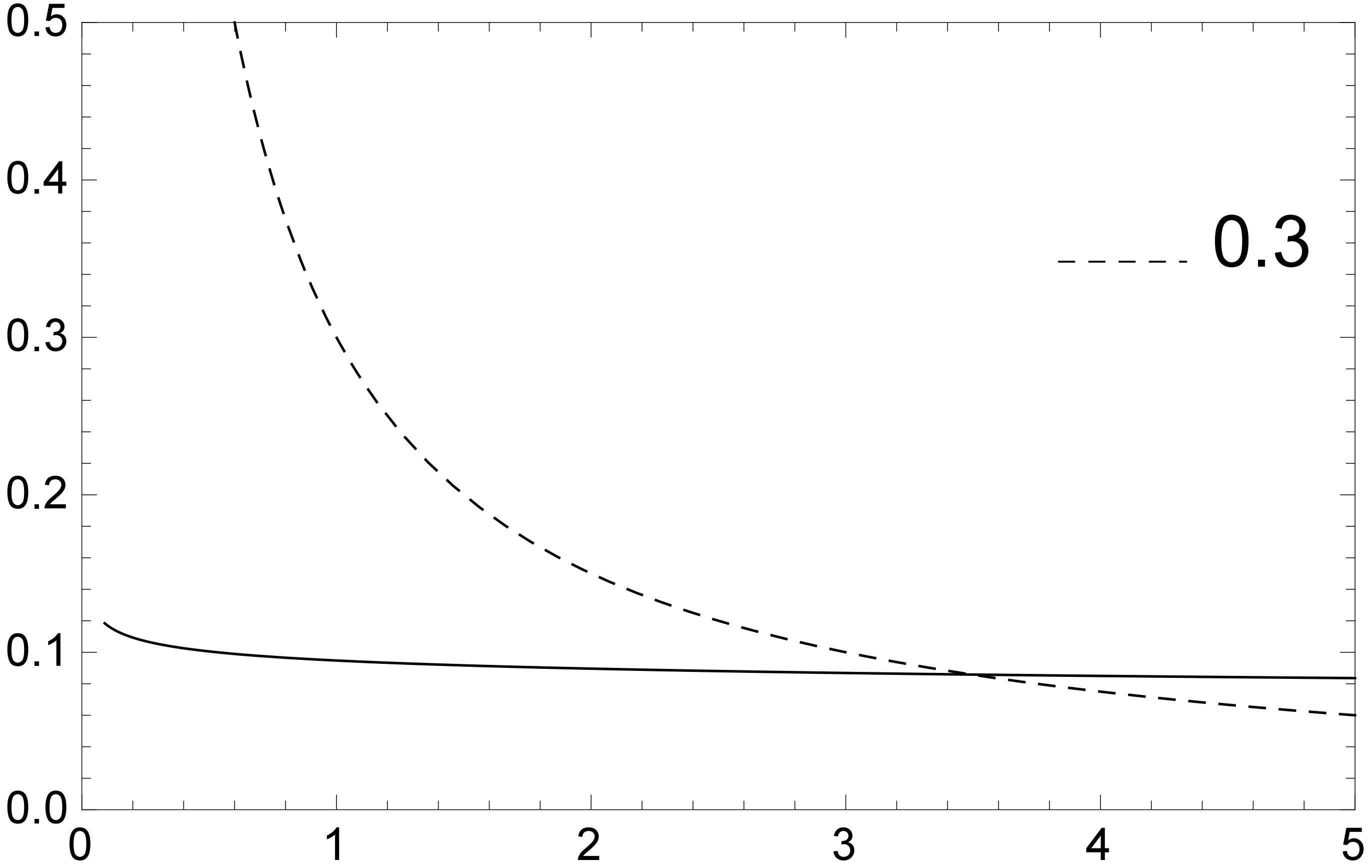
$$\alpha^{INP}(q) = \Delta^2_{JW}/q^2$$

$$\alpha^{PT}(q^2) = \frac{\alpha_s}{1 + b_0\alpha_s \ln(q^2/\Lambda^2_{YM})}$$

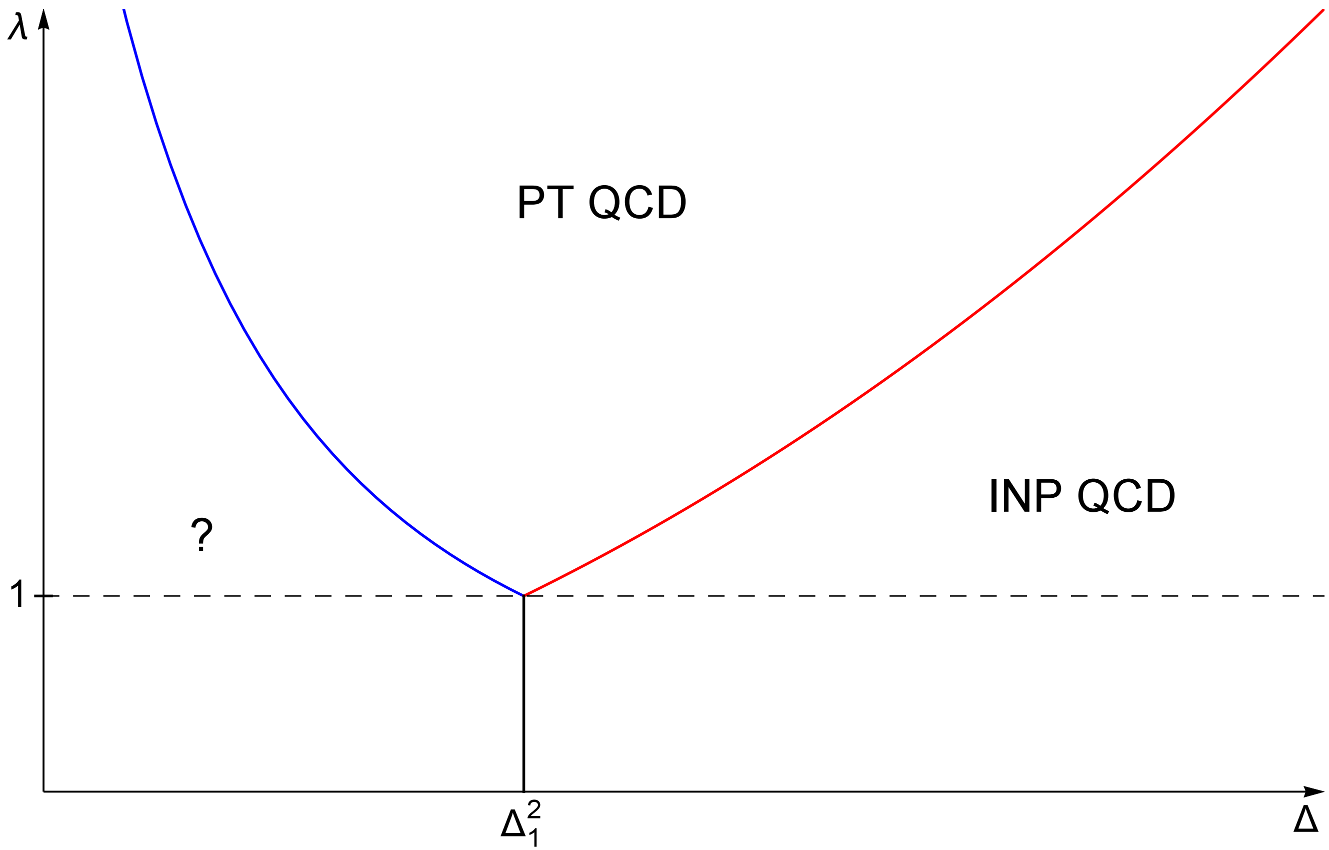
$$\alpha^{INP}(\Delta) = \Delta^2_{JW}/q^2_{eff} = \Delta^2$$

$$\alpha_s = 0.1184, \quad b_0 = 11/4\pi$$

$$\alpha^{PT}(\lambda) = \frac{\alpha_s}{1 + b_0\alpha_s \ln \lambda^2}, \qquad \lambda^2 = q^2_{eff}/\Lambda^2_{YM} \geq 1$$



$\alpha^{PT}(q^2)$ and $\alpha^{INP}(q^2)$ as functions of q^2 (GeV^2 units); solid and dashed lines, respectively, with $\Delta^2_{JW} = 0.3\, GeV^2$ and $\Lambda^2_{YM} = 0.09\, GeV^2$.



$$\Delta_1^2 = \Delta_{JW}^2/\Lambda_{YM}^2$$

The Jaffe-Witten (JW) theorem (2000)

Yang-Mills Existence and Mass Gap: Prove that for any compact simple gauge group G, quantum Yang-Mills theory on R^4 exists and has a mass gap $\Delta > 0$.

(i). It must have a ”mass gap”. Every excitation of the vacuum has energy at least Δ (the nuclear force is strong but short-range).

(ii). It must have ”quark confinement” (the phys. part. are $SU(3)$-invariant).

(iii). It must have ”chiral symmetry breaking” (to account for the ”current algebra” theory of soft pions).

We understand the Mass Gap as the scale determining the dynamical structure of the QCD ground state (vacuum) at large distances (IR region).

General conclusions

Mass Gap Existence and Asymptotic Freedom: If quantum YM theory with compact simple gauge group $SU(3)$ exists on $\mathbf{R}^4$, then it has a mass gap and undergoing the phase transition in the weak coupling regime it becomes **PT QCD**, which has **a physical mass gap and asymptotic freedom**.

$$INP\,QCD \Longleftarrow QCD \Longrightarrow PT\,QCD$$

$$\Lambda^2_{INP} \longleftarrow^{\infty \leftarrow \alpha(\lambda)}_{\infty \leftarrow \lambda} \Delta^2(\lambda, \alpha(\lambda))^{\alpha(\lambda) \to 0}_{\lambda \to \infty} \longrightarrow \Lambda^2_{PT}$$

Mass Gap Existence and Gluon Confinement: If quantum YM theory with compact simple gauge group $SU(3)$ exists on $\mathbf{R}^4$, then it has a mass gap and undergoing the phase transition in the strong coupling regime it becomes **INP QCD**, which has **a physical mass gap and confines gluons**.

The Existence of YM theory includes establishing axiomatic properties of Euclidean Gauge Green's Function. Being the distribution (generalized function) with the singularity of the δ-function type, apparently it will violet none of the axioms for them (Streater, Strocchi, Osterwalder, Schrader) as well as properties (cluster one, for example) of the Wightman functions (observables)?

QED vs QCD

QED. The interaction between constituents in the **vacuum** and in the **real word** is the **same**. **No mass gap described here**.

QCD. The interaction between constituents in the **vacuum** and in the **real word** is **different**. **Mass gap exists**.
Without mass gap QCD behaviors like QED at zero gluon/photon momenta transfer.

The mass gap described here coincides with the JW mass gap by the properties and not by the definition (H **has no spectrum in the interval** $(0, \Delta)$**). It determines the large-scale structure of the QCD ground state. Being generated by the self-interaction of massless gluon modes, it explains the origin of mass in QCD. The mass gap is a unique mass scale parameter in QCD compatible with its** $SU(3)$ **color gauge symmetry** .

Structure of QCD ground state

INP QCD effectively becomes an Abelian gauge theory at the fundamental quark-gluon level by incorporating non-Abelian degrees of freedom into the full gluon propagator with the help of the mass gap. Being such a theory, it requires the dominance of flux tube configurations of gluon fields in the QCD ground-state (vacuum) not only between any massive (light or heavy) objects there (quarks, chromo-magnetic monopoles (if any), etc.). Such string-type configuration of gluon fields will dominate the Yang–Mills (YM) vacuum as well. It happens because of the survival of the simplest NP IR singularity $(q^2)^{-2}$ only due to the Picard/WSC theorem and the multiplicative renormalization program for the mass gap.

Such a dominance of purely transversal virtual gluon field configurations with low-frequency components (or, equivalently, large scale amplitudes) is a gauge invariant, i.e., it needs no special gauge construction.

The general criterion of quark confinement

(I). The first necessary condition is

$$S(p) \neq \frac{Z_2}{\hat{p} - m_{ph}}$$

The **quark** always remains an **off-mass-shell object**. This is also equivalent to the absence of the imaginary part in the quark propagator unlike to the electron propagator in QED.

(II). The second sufficient condition is the existence of the discrete spectrum only (no continuum) in bound-states. It comes apparently from the 't Hooft's model for two-dimensional QCD with large N_c limit.

This definition of quark confinement in the momentum space is gauge-invariant and flavor independent, and thus it is a general one. Color confinement is absolute and permanent.

No mass-shell for quarks $\hat{p} \neq m_{ph}$ **and gluons** $q^2 \neq 0$.

In QGP the most of the bound-states will be dissolved. However, by increasing temperature or density there is no way to put such liberated quarks and gluons on the mass-shell. The colored gluons can indeed propagate from one hadron to the next in the thermal state with many overlapping hadrons, but they are not free, i.e., $q^2 \neq 0$. So what is called as **de-confinement** phase transition in QGP is, in fact, **de-hadronization** phase transition.

De-confinement at the fundamental quark-gluon level is about the liberation of any colored objects (quarks, gluons, etc.) from the vacuum (which never happens) and not from the bound-states. So De-confinement phase transition does not exist, at all (temperature zero or finite).

The general criterion of dynamical/spontaneous breakdown of chiral symmetry

$$S^{-1}(p) = S_0^{-1}(p) + \int i d^4 q \Gamma_\mu(p,q) S(q) \gamma_\nu D_{\mu\nu}(p-q)$$

$$S^{-1}(p) = i[\hat{p} A(p^2) + B(p^2)], \quad S_0^{-1}(p) = i[\hat{p} + m_0]$$

$$S^{-1}(0) = iB(0) = i m_{eff}, \quad S_0^{-1}(0) = i m_0$$

$$m_{eff} = m_0 + m_d$$

(a). *chiral symmetry violating solution:* $m_{eff} = m_d \neq 0, \quad m_0 = 0$

(b). *chiral symmetry preserving solution:* $m_{eff} = m_d = 0, \quad m_0 = 0$

(I). The first necessary (dynamical) condition

$$m_{eff} = m_d \sim \int d^4q \Gamma_\mu(q) S(q) \gamma_\nu D_{\mu\nu}(q) \neq 0, \quad m_0 = 0$$

$$\{S^{-1}(p), \gamma_5\} = i\gamma_5 2B(p^2)_{p^2=0} = i\gamma_5 2m_d \neq 0, \quad m_0 = 0$$

(II). The second sufficient (phenomenological) condition

$$\langle 0 \,|\bar{q}q|\, 0\rangle_0 \sim \int d^4p \, TrS(p) \sim - \int d^4p \frac{B(p^2)}{p^2A^2(p^2) + B^2(p^2)} \neq 0$$

The dynamically generated mass m_d depends on the running quark mass $B(p^2)$ in the much more complicated way than the chiral quark condensate $\langle 0 \,|\bar{q}q|\, 0\rangle_0$. **So if the temperature-dependent chiral quark condensate drops it value at some T_c, this does not mean that dynamical quark mass will also automatically drop its value at T_c in the chiral limit $m_0 = 0$.**

Suppression of the NP IR singularities

$(q^2)^{-2-k}, \ k = 1, 2, 3, ...$ **apart from** $(q^2)^{-2}$ **in 4D QCD**

in agreement with the Picard/WSC theorem

$$d^{INP}(z) = zL_s(z) + zL_r(z), \quad z = \frac{\Delta^2}{q^2} = \epsilon \frac{\bar{\Delta}^2}{q^2} = \epsilon \bar{z}$$

$$L_s(z) = \sum_{k=0}^{\infty} z^k \sum_{m=0}^{\infty} f_{k+m+1}^{(m)}(0)$$

$$L_r(z) = \sum_{k=1}^{\infty} z^{-k} \sum_{m=0}^{\infty} f_{m+1}^{(k+m)}(0) = \sum_{k=0}^{\infty} z^{-k-1} \sum_{m=0}^{\infty} f_{m+1}^{(k+1+m)}(0)$$

Since the differentiation of $f_k(q^2) = (-1)^k d^{PT}(q^2)[d^{PT}(q^2)c(d^{PT})]^k$

$$z^{-1} = \frac{q^2}{\Delta^2} = \frac{1}{\epsilon}\frac{q^2}{\bar{\Delta}^2} = \frac{1}{\epsilon}\bar{z}^{-1}$$

$$f_{k+m+1}^{(m)}(0) = \epsilon^m \bar{f}_{k+m+1}^{(m)}(0), \quad \epsilon \to 0^+$$

$$f_{m+1}^{(k+1+m)}(0) = \epsilon^{k+1+m} \bar{f}_{m+1}^{(k+1+m)}(0), \quad \epsilon \to 0^+$$

$$zL_s(z) = \epsilon\bar{z}\sum_{k=0}^{\infty} \epsilon^k \bar{z}^k \sum_{m=0}^{\infty} \epsilon^m \bar{f}_{k+m+1}^{(m)}(0)$$

$$zL_r(z) = \epsilon\sum_{k=0}^{\infty} \bar{z}^{-k} \sum_{m=0}^{\infty} \epsilon^m \bar{f}_{m+1}^{(k+1+m)}(0)$$

$$zL_s(z) = \epsilon \times \frac{\bar{\Delta}^2}{q^2}[\bar{f}_1(0) + O_s(\epsilon)], \quad \epsilon \to 0^+$$

$$zL_r(z) = \epsilon \times O_r(q^2), \quad \epsilon \to 0^+$$

$$D^{INP}(q; \Delta^2_{JW}) = \epsilon \times \left[\frac{\Delta^2_{JW}}{(q^2)^2} + \frac{1}{q^2}O_r(q^2)\right], \quad \epsilon \to 0^+$$

$$D^{INP}_{\mu\nu}(q; \Delta^2_{JW}) = \epsilon \times iT_{\mu\nu}(q)\left[\frac{\Delta^2_{JW}}{(q^2)^2} + \frac{1}{q^2}O_r(q^2)\right], \quad \epsilon \to 0^+$$