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**QUASI-FROZEN SPIN CONCEPT AND ITS POSSIBLE APPLICATION
INTO COSY RING**

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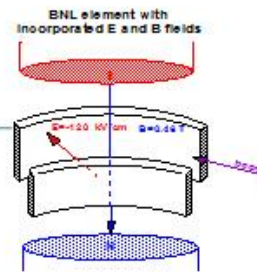
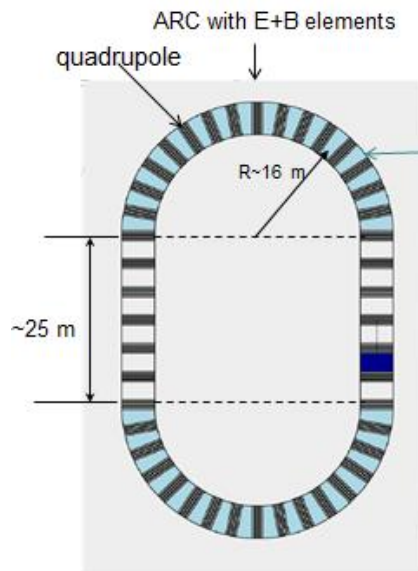
Frozen Spin (FS) lattice

The condition of the zero MDM spin precession frequency in FS lattice [1,2]

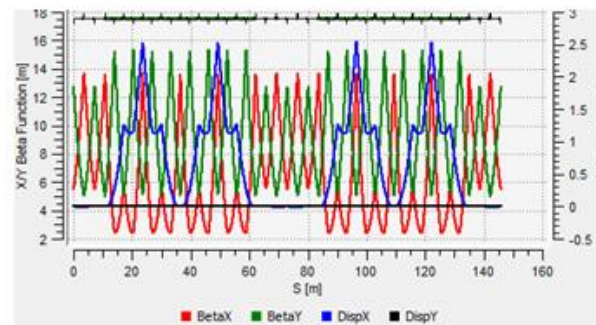
$$G\vec{B}_y + \left(\frac{1}{\gamma^2 - 1} - G \right) \left(\frac{\vec{\beta}_z}{c} \times \vec{E}_x \right) = 0$$

creates the relation between E and B fields in incorporated bending elements:

$$E_r \approx GBc\beta\gamma^2$$



Frozen Spin lattice based on E+B elements and TWISS functions



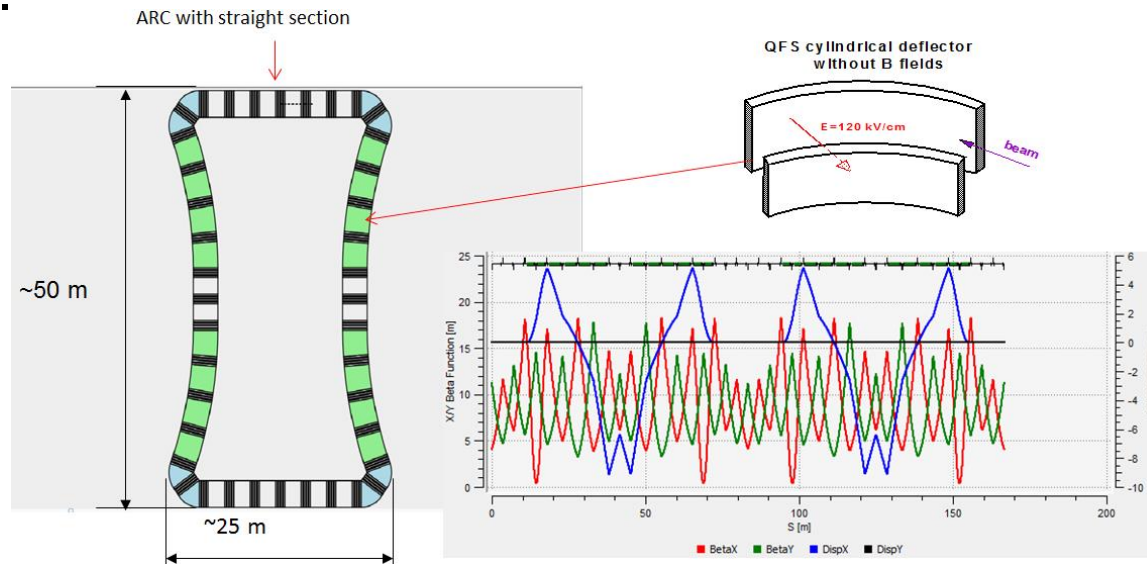
First message to Quasi-Frozen Spin (QFS) lattice

From T-BMT equations follows that the growth of the EDM signal is directly dependent on the angle between the spin and the spin precession axis (or momentum direction):

Exact fulfillment of the frozen spin condition is not required. We need an equal deviation of spin in magnetic and electric fields totally on the ring

First option of QFS lattice

In the first option the electrical and magnetic fields are fully spatially separated.



First option of ring lattice based on QFS concept: ring view with main elements and TWISS functions

Basic relations of first option of QFS lattice

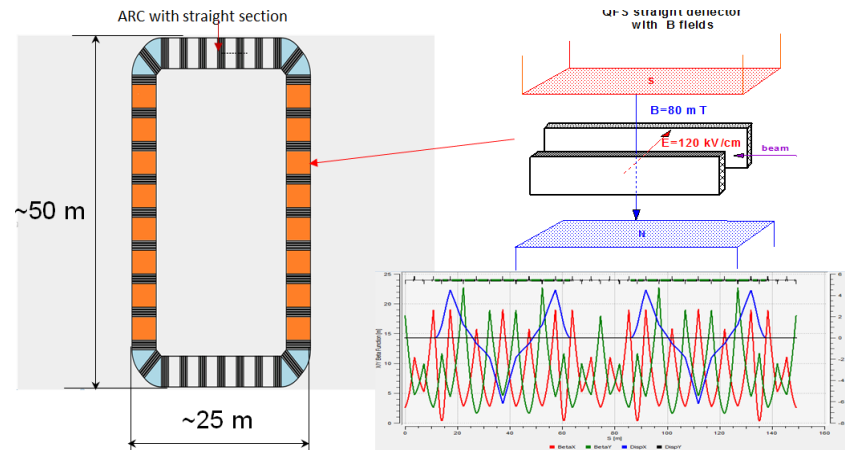
Basic relations in first option of quasi-frozen spin lattice [3]:

$$\gamma G \cdot \Phi_B = \left[\frac{1}{\gamma} (1 - G) + \gamma G \right] \cdot \Phi_E$$

However, this concept inherits one drawback of cylindricalelectrodes, namely the whole set of high-order nonlinearities.

Second option of QFS lattice

In second option of QFS lattice we introduced a magnetic field of small value ~ 80 mT, compensating the Lorentz force of the electric field in electrostatic deflector located on the straight sections.



Second option of ring lattice based on QFS concept: ring view with main elements and TWISS functions

Basic relations of second option of QFS lattice

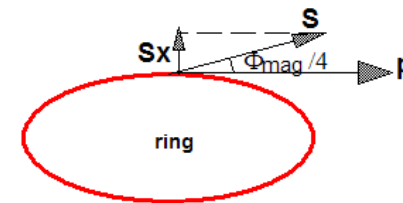
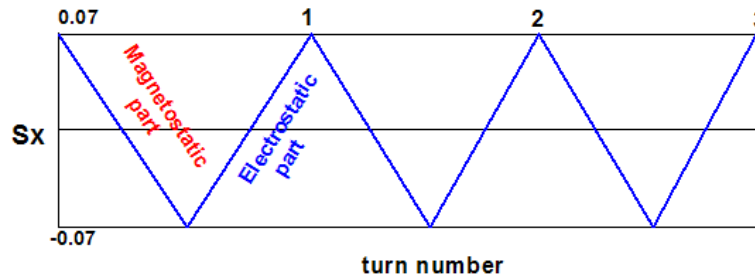
Spin direction is recovered in E+B incorporated elements located on the straight sections.

The basic relations are

$$L_{\Sigma} E_{ss} = \frac{G}{G+1} \cdot \frac{mc^2}{e} \cdot \pi \beta^2 \gamma^3 \quad B_{ss} = -\frac{E_{ss}}{c\beta}$$

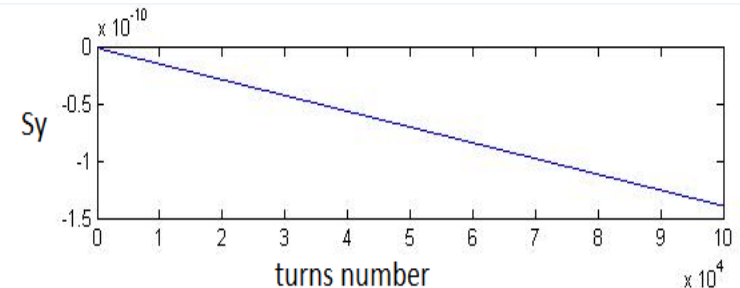
where L_{Σ} is the total length of straight elements in one straight section and B_{ss} , L_{ss} are the magnetic field and length of the straight element.

EDM growth: 3D spin orbital simulation by MODE and COSY Infinity codes



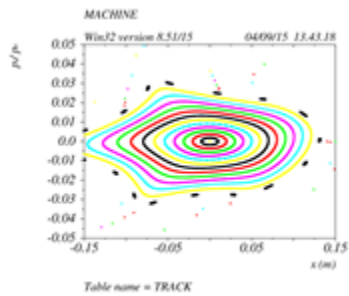
Results of 3D spin-orbital simulation:

- Due to S_x oscillation (QFS) the EDM signal decreases by 1%
- In each magnet EDM signal grows by $-2.14133779995135 \cdot 10^{-16}$ and in each deflector by $3.20268895179507 \cdot 10^{-17}$
- Total EDM signal grows by $-1.39074513140842 \cdot 10^{-15}$ per turn
- In order to get total EDM signal $\sim 10^{-6}$ we have to keep the beam in ring during $N_{\text{turn}} \sim 10^9$ or **~ 800 sec**

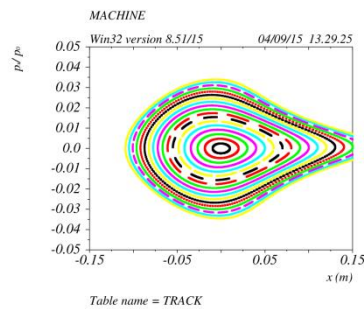


DA for different lattices

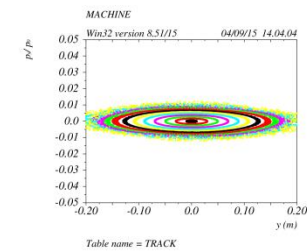
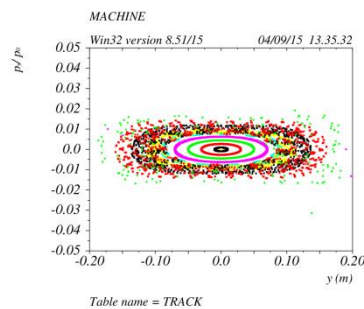
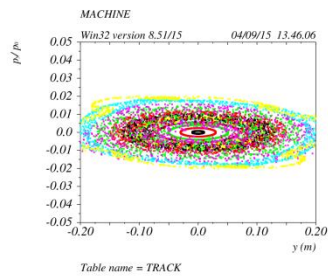
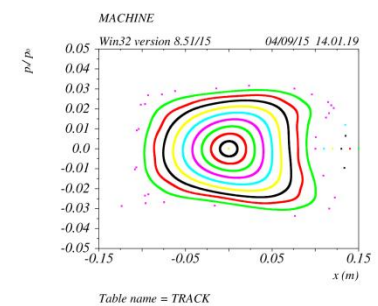
Arc with BNL elements



B arc+ E recovery



B arc+(E+B) recovery



Spin tune Decoherence Time vs RF field and sextupole families

The spin tune spread in the magnetic field relative to momentum:

$$v_s^B = \frac{\Omega_S^B - \Omega_p^B}{\Omega_p^B} = -\gamma|G| \longrightarrow \Delta v_s^B = -\Delta\gamma \cdot |G|$$

The spin tune spread in the electrical field relative to momentum

$$v_s^E = \frac{\Omega_S^E - \Omega_p^E}{\Omega_p^E} = \frac{1}{\gamma}(1 - |G|) + \gamma|G| \longrightarrow \Delta v_s^E = \Delta\gamma \cdot |G| - \frac{1}{\gamma_0^2}(1 - |G|)\Delta\gamma + \frac{1}{\gamma_0^3}(1 - |G|)\Delta\gamma^2 - \dots$$

nonlinear term of spin tune

Longitudinal motion:

$$\frac{d\varphi}{dt} = -\omega_{rf} \left[\left(\alpha_0 - \frac{1}{\gamma^2} \right) \cdot \delta + \left(\alpha_1 - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4} \right) \cdot \delta^2 + \left(\frac{\Delta L}{L} \right)_\beta \right]$$

x and y orbit lengthening

$$\frac{d\delta}{dt} = \frac{eV_{rf}\omega_{rf}}{2\pi h\beta^2 E} \sin \varphi$$

nonlinear term of energy oscillation

Spin Coherence Time vs RF field and sextupole families

The energy oscillation:

$$\Delta\gamma = \gamma_0 \beta_0^2 \left\{ \delta_0 \cos 2\pi\nu_z n + \left(\frac{\alpha_1}{\eta} - \frac{1}{\gamma_0^2} \right) \delta_0^2 \cos 4\pi\nu_z n + \left(\frac{\alpha_1}{\eta} - \frac{1}{\gamma_0^2} \right) \delta_0^2 + \frac{1}{\eta} \left(\frac{\Delta L}{L} \right)_\beta \right\}$$

Substituting $\Delta\gamma$ in the spin tune spread in the electrical

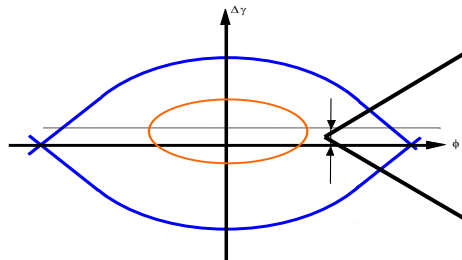
$$\Delta v_s^E = \Delta\gamma \cdot |G| - \frac{1}{\gamma_0^2} (1 - |G|) \Delta\gamma + \frac{1}{\gamma_0^3} (1 - |G|) \Delta\gamma^2 - \dots$$

$\rightarrow \langle \Delta v_s^E \rangle_{N \text{ turns}} \approx$

$\frac{\alpha_1}{\gamma_0 \eta} \delta_0^2$
Nonlinear Z motion

$\frac{\beta_0^2}{\eta \gamma_0} \left(\frac{\Delta L}{L} \right)_\beta$
Betatron motion

$\frac{\beta_0^4}{\gamma_0} \delta_0^2$
Nonlinearity of spin tune



and the magnetic fields:

$$\Delta v_s^B = -\Delta\gamma \cdot |G|$$

 $\rightarrow \langle \Delta v_s^B \rangle_{turns} \approx$

$\frac{\alpha_1}{\gamma_0 \eta} \delta_0^2$
Nonlinear Z motion

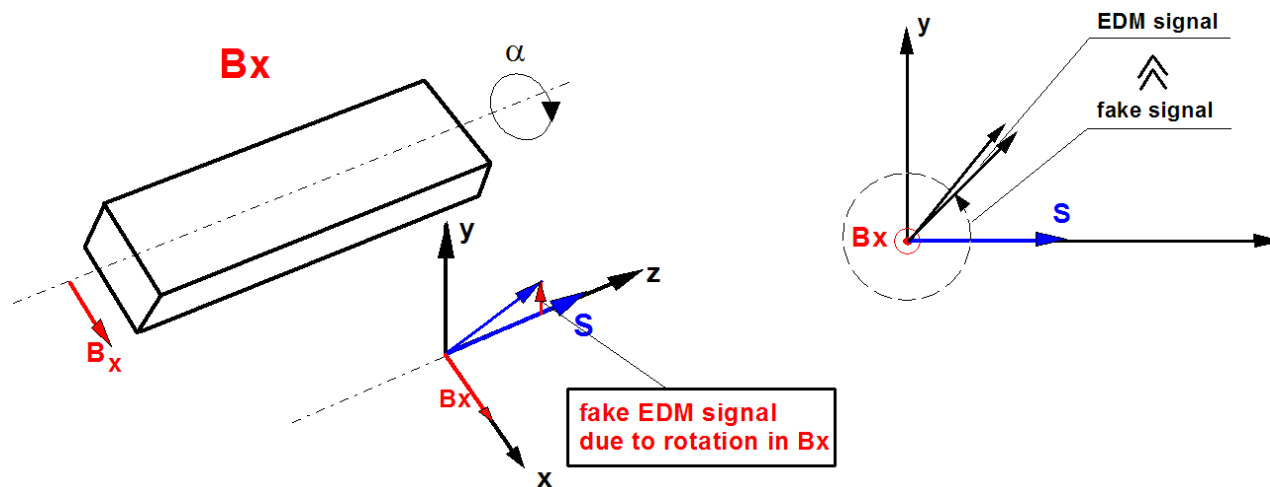
$\frac{\beta_0^2}{\eta \gamma_0} \left(\frac{\Delta L}{L} \right)_\beta$
Betatron motion

$$\left(\frac{\Delta L}{L} \right)_\beta = \frac{\pi}{2L} \left[\frac{v_x}{\beta_x} x^2 + \frac{v_y}{\beta_y} y^2 \right]$$

Systematic errors with B_x

The systematic error is called the error component, which remains constant in repeated measurements and is caused by imperfections of the physical facility. The misalignments create conditions for systematic errors in EDM experiments and are associated with limited capabilities of the geodetic instruments:

$B_x \neq 0$



Systematic errors with B_x

Due to magnet slope (misalignments) relative to the longitudinal axis, the horizontal component of the magnetic field arises and causes the spin rotation with $\Omega_x = \Omega_{B_x}$ in the same plane where

we expect the EDM rotation. Then we can present components:

$$\Omega_x = \Omega_{B_x} + \Omega_{EDM} \text{ and } \Omega_y = 0 + \langle \delta\Omega_{decoh} \rangle .$$

At the magnets installation accuracy $\alpha_{max} = 10^{-5}$ rad Ω_{B_x} is 3 rad/sec.

In the same time, at presumable EDM value of 10^{-29} e·cm, the EDM rotation should be 10^{-9} rad/sec, that is $\Omega_{EDM} / \Omega_{B_x} \approx 10^{-9}$, and we can write:

$$\langle S_x(t) \rangle = \frac{\langle \delta\Omega_{decoh} \rangle}{\Omega_{B_x}} \sin \Omega_{B_x} t; \quad S_y(t) = -\sin(\Omega_{B_x} + \Omega_{EDM})t$$

Now the spin decoherence in the horizontal plane is stabilized at the level of $\langle \delta\Omega_{decoh} \rangle / \Omega_{B_x} \approx 10^{-3}$, and the spin decoherence is transformed in the vertical plane:

$$\langle \Omega_{B_x} \rangle = \frac{e}{m\gamma} (\langle \gamma \rangle G + 1) B_x$$

which one we can minimize by the same methods (sextupole, RF) as in horizontal plane.

Systematic errors with $B_z \neq 0$

At conditions $\Omega_z = \Omega_{Bz}$, $\Omega_y = \langle \delta\Omega_{\text{decoh}} \rangle$ and $\Omega_{Bz} \ll \langle \delta\Omega_{\text{decoh}} \rangle$ T-BMT equation has solution:

$$\langle S_x(t) \rangle = \sin \langle \Omega_{\text{decoh}} \rangle t; \quad \langle S_y(t) \rangle = \frac{\Omega_{Bz}}{\langle \Omega_{\text{decoh}} \rangle} \left[1 - \cos \langle \Omega_{\text{decoh}} \rangle t \right]$$

How we can see the fake signal depends on the ratio between $\langle \Omega_{\text{decoh}} \rangle$ and Ω_{Bz} .

Therefore, the only way is to minimize the longitudinal component of the magnetic field with $\Omega_{Bz} \sim 10^{-9}$ rad/sec, using additional trim coils with the longitudinal magnetic field.

Calibration B_x to get $\Omega_{B_x}^{CCW} - \Omega_{B_x}^{CW} \leq 10^{-10}$

We calibrate **By** component using measurement of spin precession in horizontal plane (where EDM contribution is absent) with relative accuracy $\sim 10^{-9}$ and then restore **Bx** with the same accuracy, since each magnet position remains to be unchanged.

CW-CCW procedure

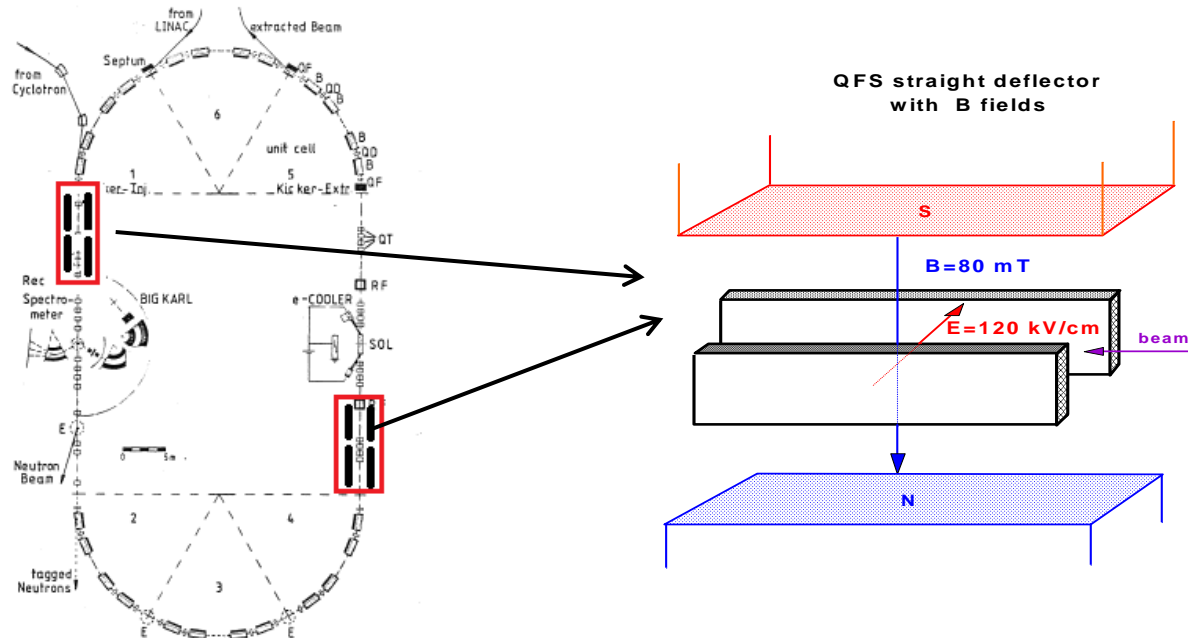
We measure $\Omega^{\text{CCW}} = -\Omega_{\text{BX}}^{\text{CCW}} + \Omega_{\text{EDM}}$ and compare with
 $\Omega^{\text{CW}} = -\Omega_{\text{BX}}^{\text{CW}} + \Omega_{\text{EDM}}$.

If to define: $\Omega_{\text{EDM}} = (\Omega^{\text{CW}} + \Omega^{\text{CCW}})/2 + (\Omega_{\text{BX}}^{\text{CCW}} - \Omega_{\text{BX}}^{\text{CW}})/2$
with accuracy $10^{-4} \div 10^{-5}$ rad/sec we can define

**the EDM in one measurement on the level of $10^{-24} \div 10^{-25}$ e·cm
and for one year measurement it can reach $10^{-29} \div 10^{-30}$ e·cm**

QFS-COSY ring

In precursor experiment we do not need a large statistics and we can start working on energy 75 MeV. This allows to use only 4 “E+B” straight elements, which is four times less than at 270 MeV. The total length is 2x7 m. Further, E+B elements can be used for a full scale experiment at 270 MeV. In result, it will provide Quasi Frozen Spin at energy of 75 MeV. Due to small B field value the E+B elements on the straight sections may be made using ordinary electrical coils with field 120-100 mT. The condition for spin recovery is fulfilled using E field (working regime 120 kV/cm).



Conclusion

- 3D simulation of both methods QFS and FS shows even a slight advantage of Quasi-Frozen Spin concept over Frozen Spin.
- Together with QFS the proposed field calibration method in the horizontal plane and then the transition to a EDM measurement in the vertical plane will allow measuring EDM with one year statistics to reach a level of **$10^{-29} \div 10^{-30} \text{e}\cdot\text{cm}$** .
- Testing of this method can be made on the COSY ring using a reduced number of E + B elements.
- The proposed concept of quasi-frozen spin significantly simplifies the design of the ring in comparison with the frozen spin concept and allows using already existing COSY ring.
- Minor changes on the straights sections of COSY ring will require investments at the level of <10 MEuro.