

Theory of Three-Nucleon Forces

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Motivation

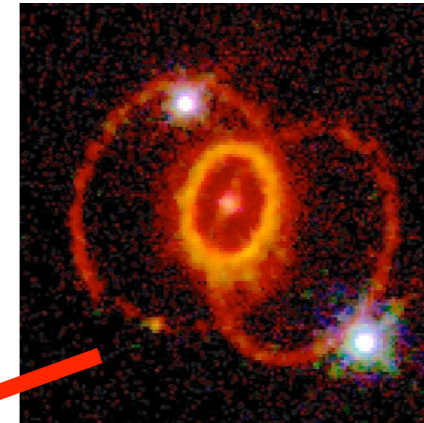
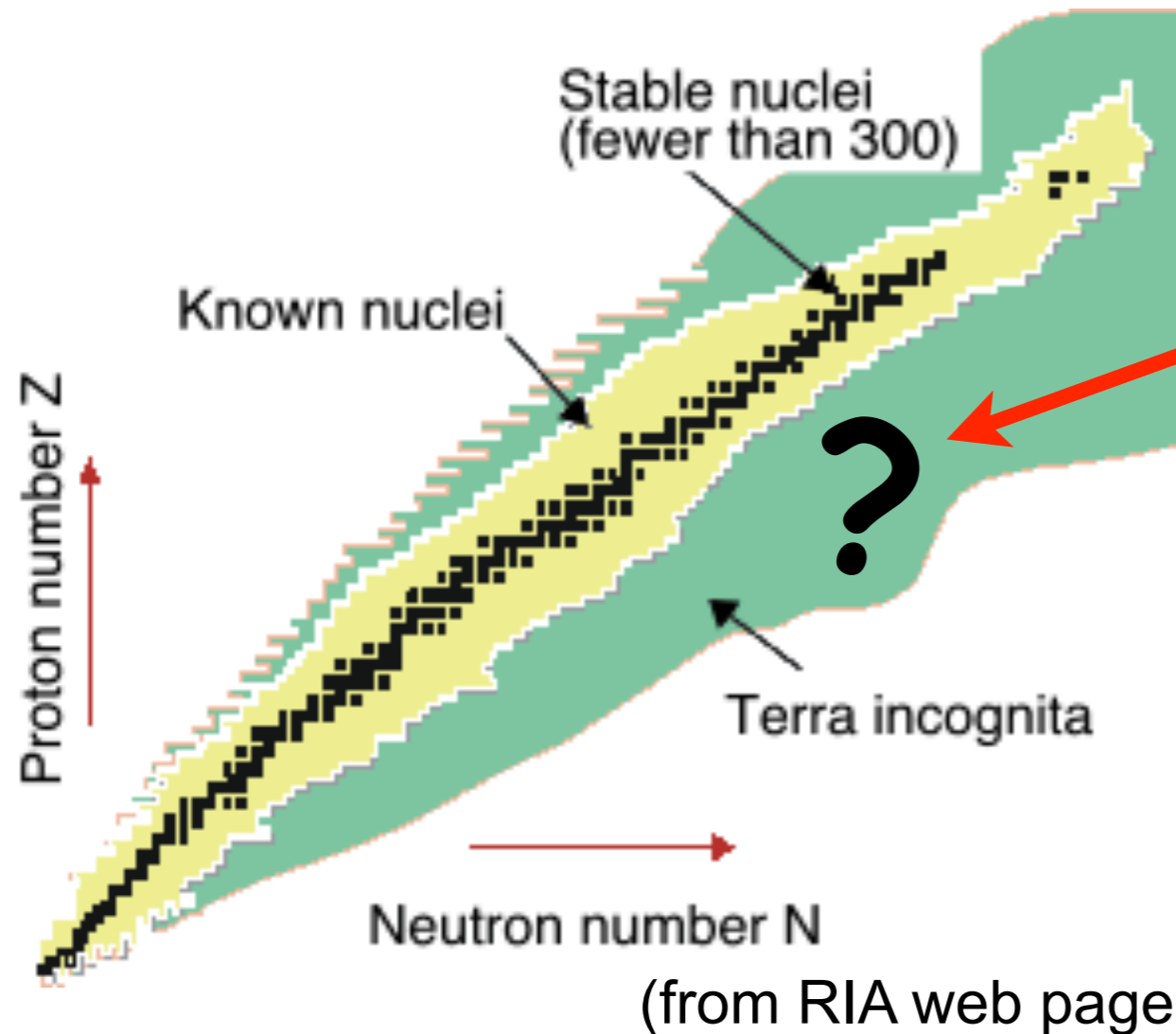
Signatures of 3NFs in light nuclei and neutron-deuteron scattering

Chiral nuclear interactions

Effects of chiral 3NFs on low energy observables

Summary & Outlook

Why theoretical nuclear physics?



(SN1987a)

- stable nuclei ✓
- nuclear drip line for light nuclei ✓
- complex, unstable nuclei ?
- nuclear dripline ?

experimental programs: Isolde, ISAC, FAIR, FRIB, ...
more reliable theoretical predictions are required

but: predictions are difficult

Nuclei from QCD?

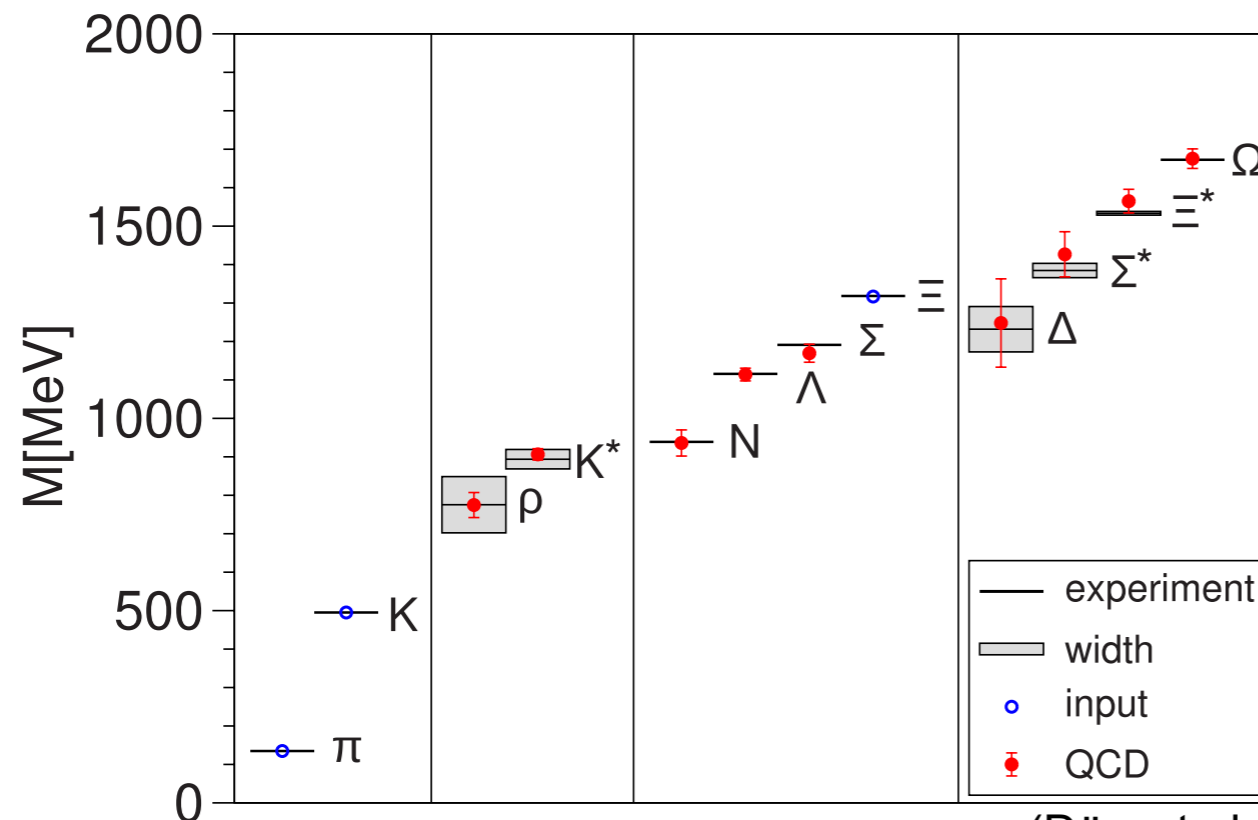
Quantum Chromo Dynamics \longrightarrow binding of nuclei!

but: QCD is non-perturbative at energies relevant for nuclei

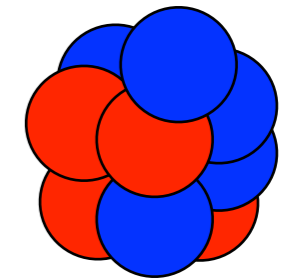
confinement: observed degrees of freedom are hadrons

Lattice QCD \longrightarrow masses of hadrons \checkmark

selected few-(two-)nucleon observables (?)



(Dürr et al., Science 2008)



nucleons (p,n) are more suitable for the description on nuclei

“microscopic” description of nuclei:

nuclear potential + non-relativistic Schrödinger equation

Signatures of 3NFs in light nuclei and neutron-deuteron scattering

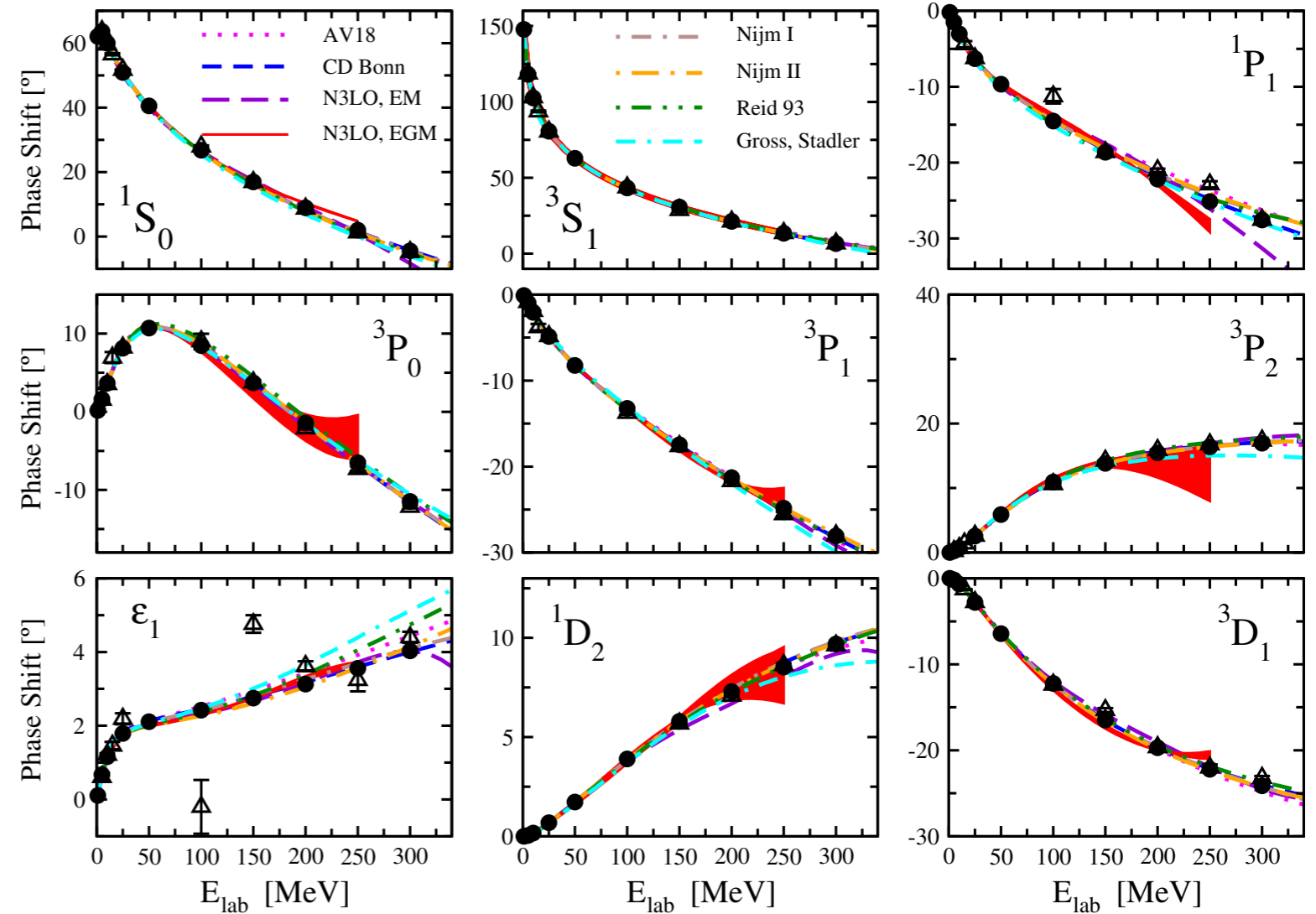
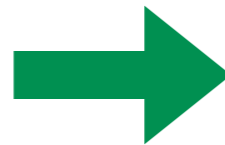
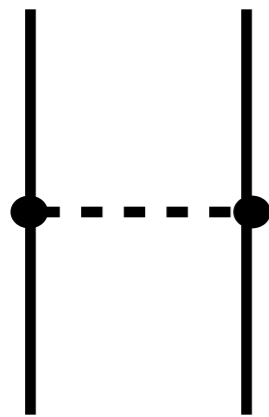
- neutron-deuteron scattering is the most important tools to study 3NFs
- phenomenological models of the nuclear force require significant 3NFs
- models fail to describe data

Models to describe NN data

NN force models (AV18, CD-Bonn, Nijmegen, ...)

describe the data (~ 4000) up to $T_{\text{lab}} \sim 300$ MeV ($p_{\text{lab}} \sim 700$ MeV) **perfectly** ($\chi^2/\text{datum} \sim 1$)

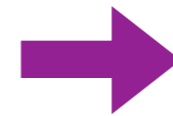
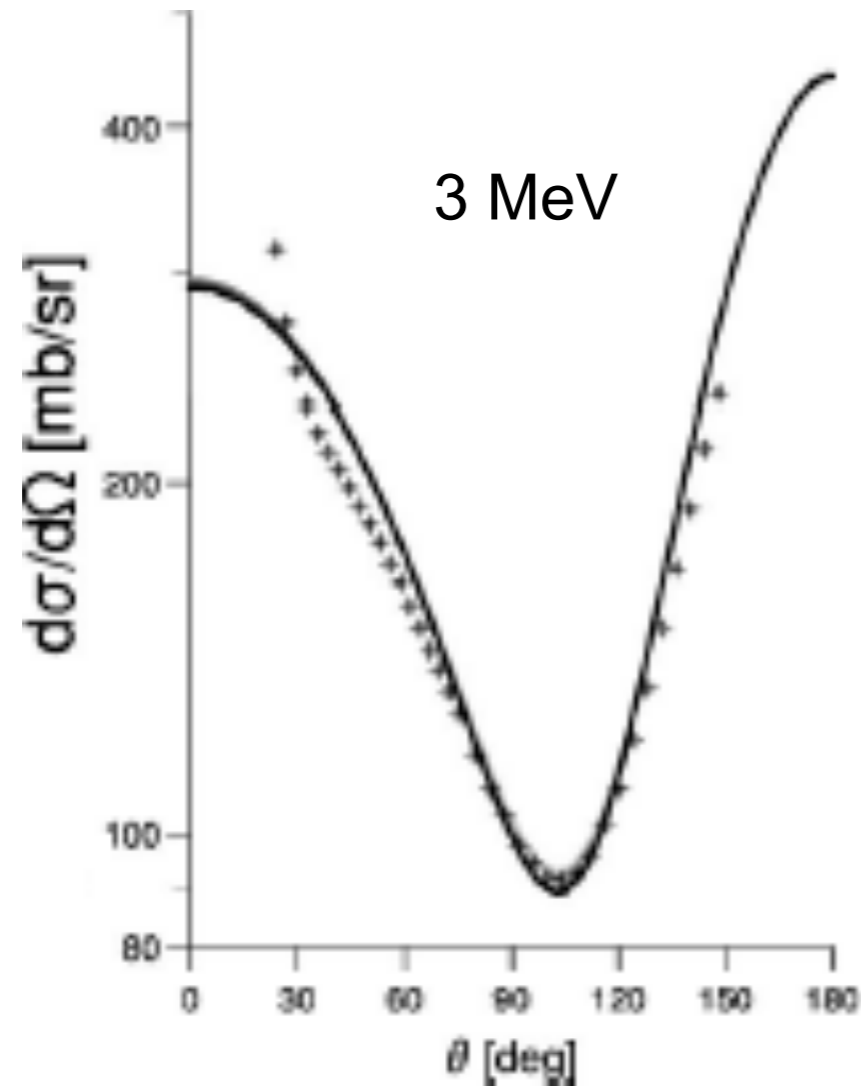
1 π -exchange
+ short-range



(from Kalantar-Nayestanaki et al., 2012)

3N system based on NN forces

Many low energy few-nucleon observables are well & model independently described !



Approximation to the nuclear Hamiltonian does not seem to be too bad, but

(see e.g. Wiatała et al., 2001)

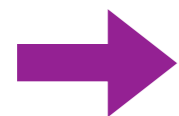
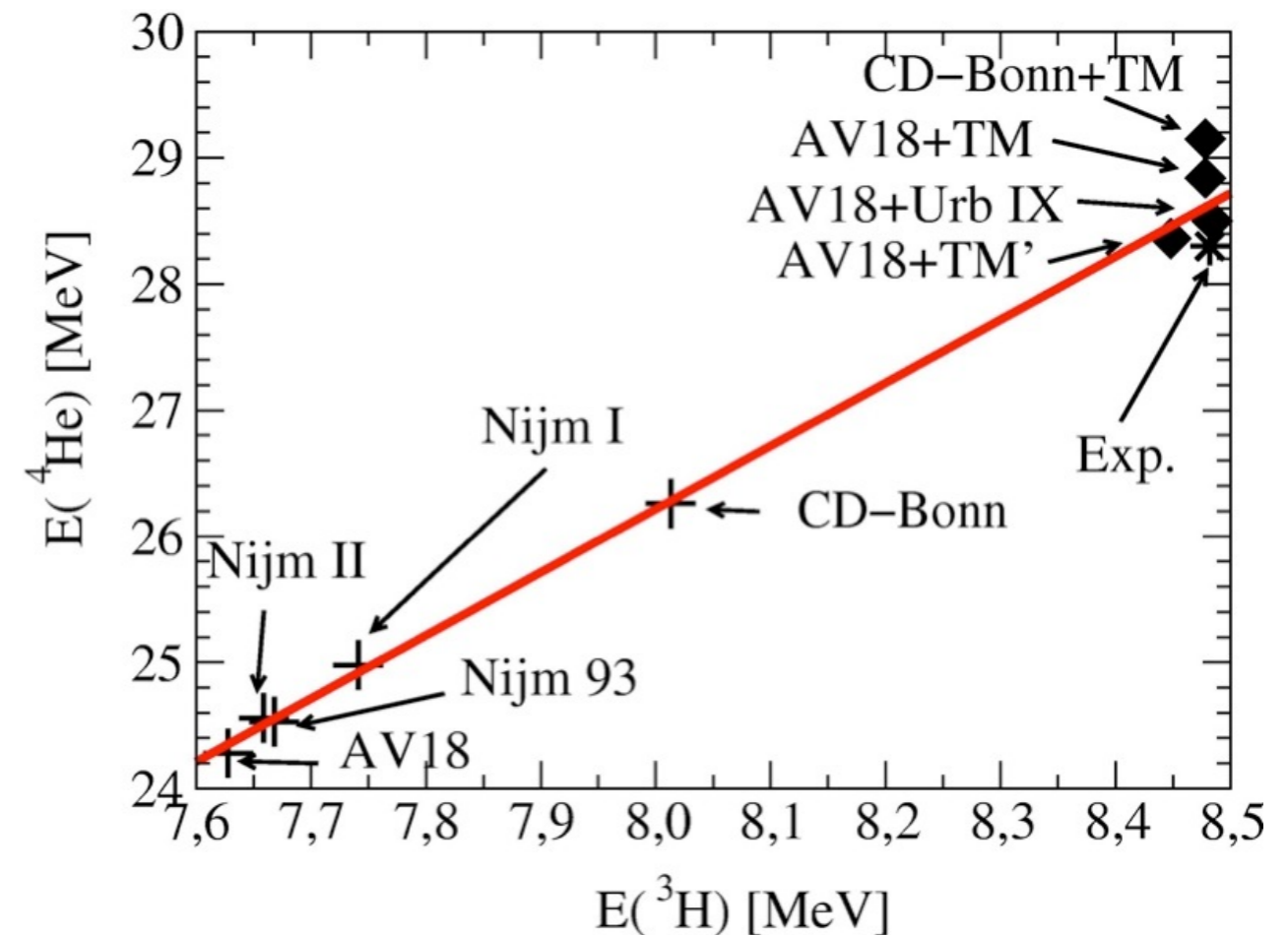
3N system based on NN forces

Binding energies are not model-independent

& the results do not agree with experiment

	3	4
CD-Bonn	-8.013	-26.23
AV18	-7.628	-24.25
Nijm I	-7.741	-24.99
Nijm II	-7.659	-24.55
Expt	-8.482	-28.30

(see e.g. A.N. et al., 2002)



3NF's are quantitatively important for binding energies.

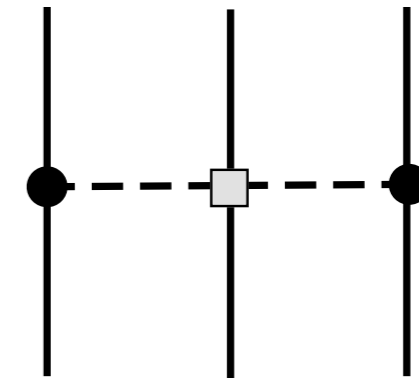
Cancelation of kinetic and potential energy!

Small parts of the nuclear Hamiltonian are relevant

Phenomenological 3NF's

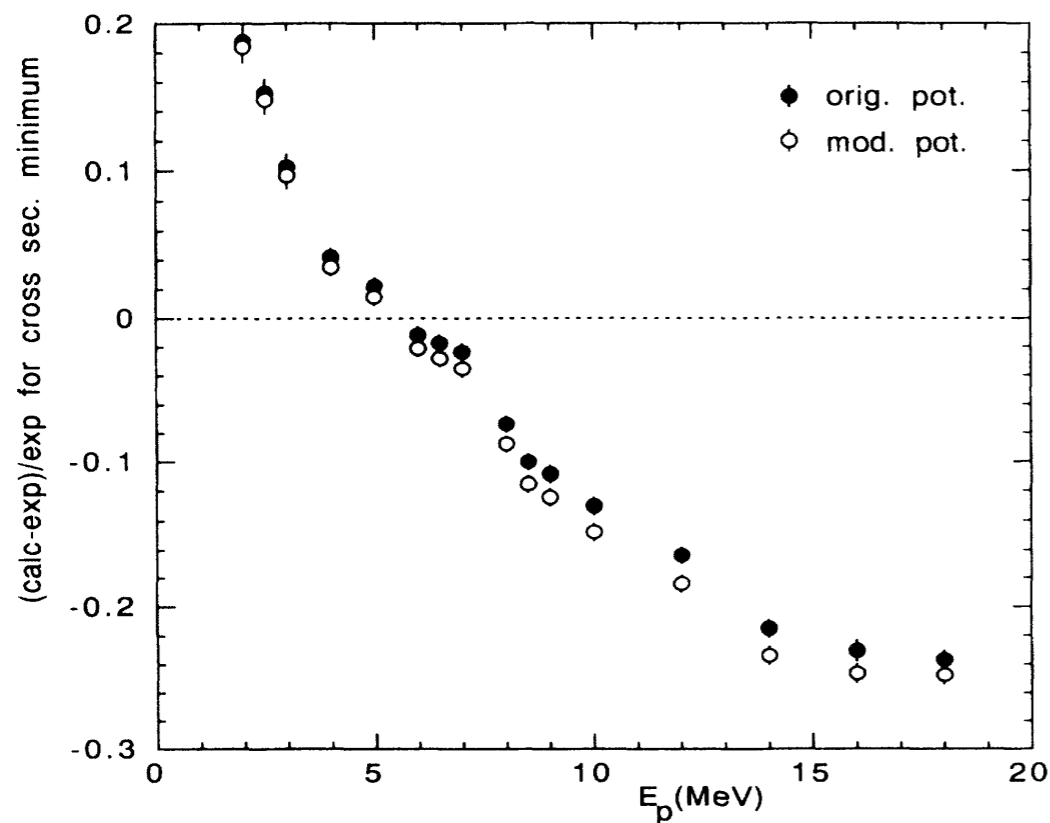
NN interactions can be augmented by phenomenological 3N interactions based on 2π -exchange

(Fujita-Miyazawa, Tuscon-Melbourne, Urbana, Illinois, ...)



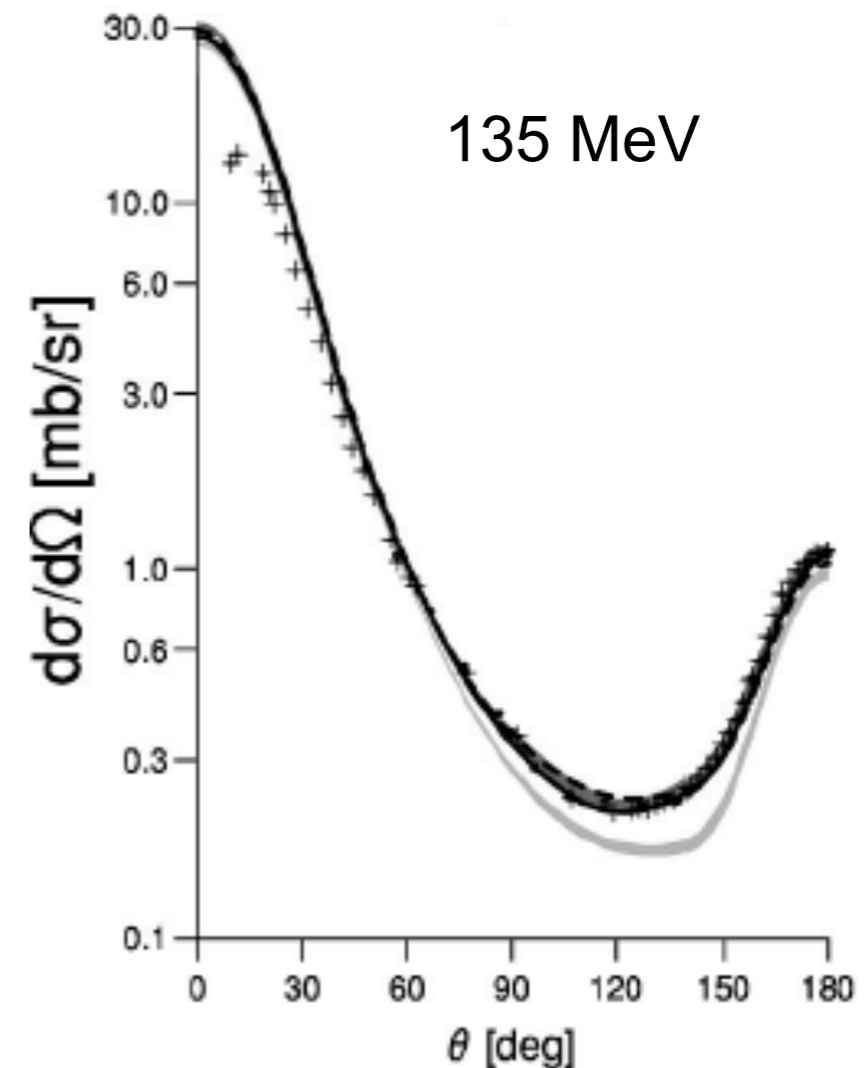
Adjust so that the ${}^3\text{H}$ / ${}^4\text{He}$ binding energy is described correctly (remember Tjon-line correlation)

→ binding energy problem and “Sagara discrepancy” was resolved



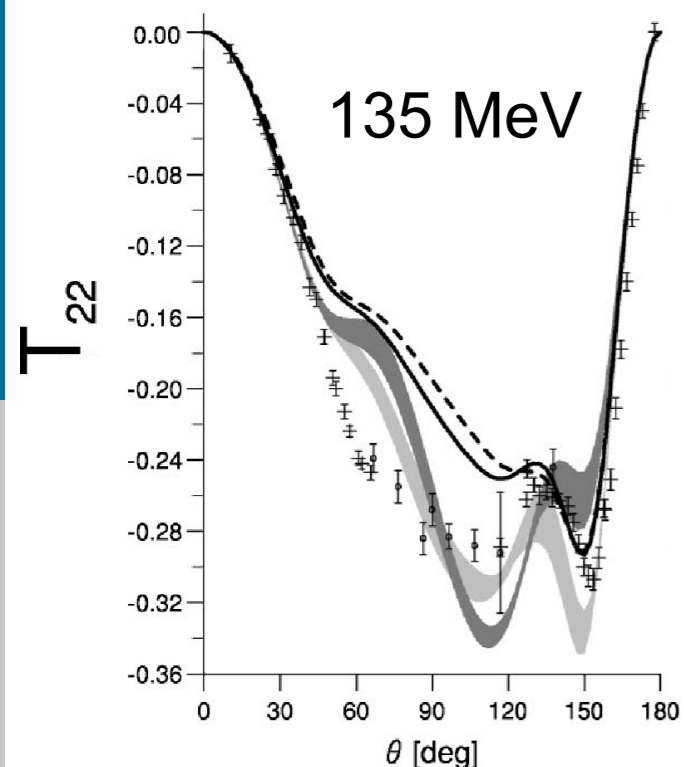
Sagara et al. (1994)

with 3NF



Phenomenological 3NF's

But: none of the phenomenological models describes all the data!

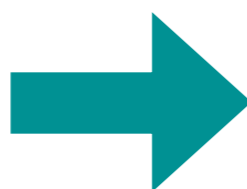


relativistic effects are small at these energies

(see e.g. Sekiguchi et al., 2005)

Phenomenological combinations are very useful to **identify signatures of 3NF's**

triggered a lot of experiments for pd scattering
(RIKEN, KVI, IUCF, ...)



so that the intermediate energy
3N data base became quite extensive

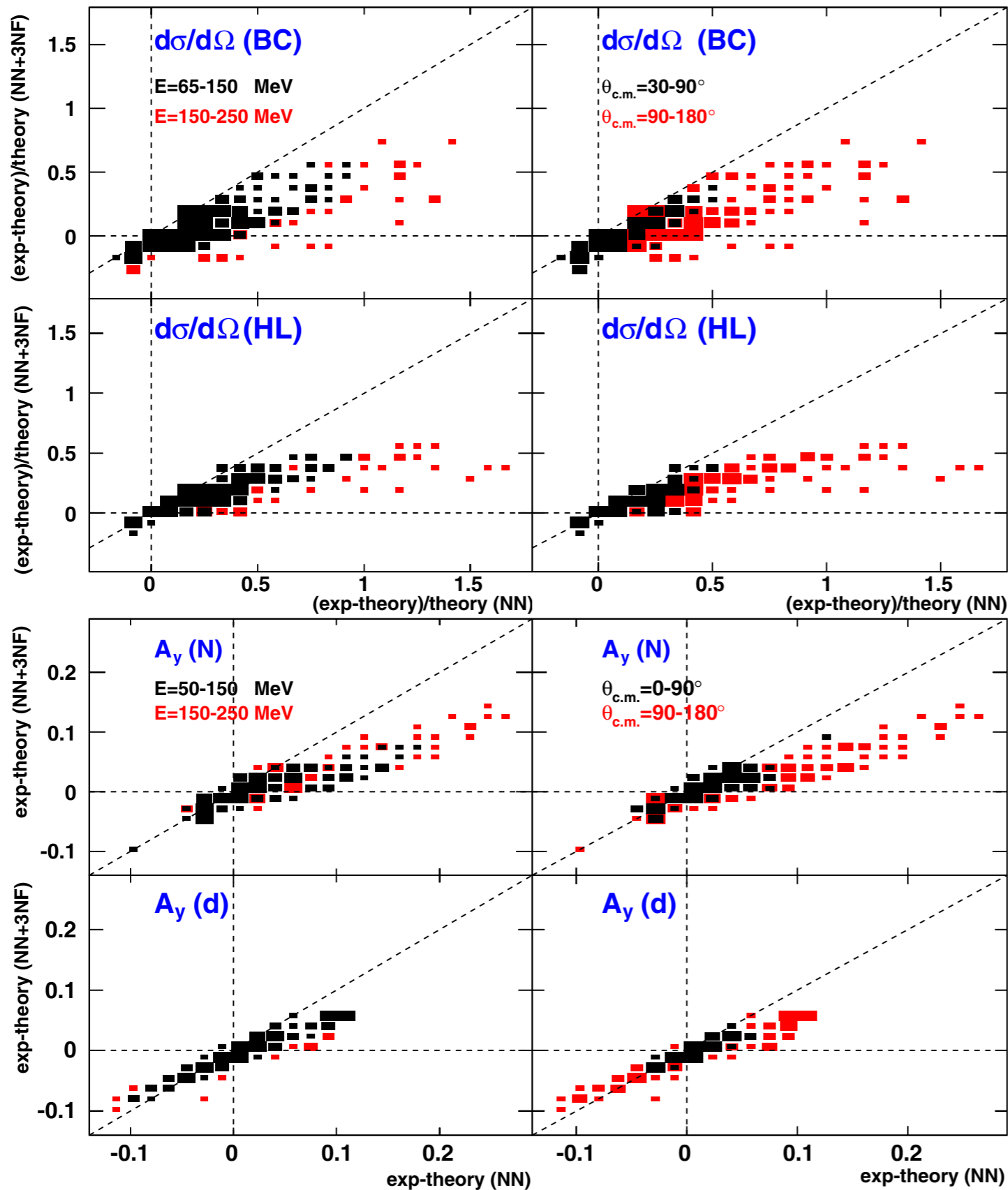
Nd elastic scattering

	100	200
$\frac{d\sigma}{d\Omega}$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$ $\blacksquare \blacksquare \blacksquare$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$ $\blacksquare \blacksquare$
$\vec{p} \rightarrow \vec{n}$ $A_y(N)$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$ $\blacksquare \blacksquare$
$\vec{d} \rightarrow \vec{p}$ $A_y(d)$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$
A_{yy}	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$
A_{xx}	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$
A_{xz}	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$	$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$
$\vec{p} \rightarrow \vec{p}$ K_i^j		\bullet
$\vec{d} \rightarrow \vec{p}$ $K_y^{y'}$	$\bullet \bullet$	
$K_{ij}^{y'}$	$\bullet \bullet$	
$\vec{p} + \vec{d}$ C_{ij}		$\bullet \bullet$

Nd break-up

	100	200
$\frac{d\sigma}{d\Omega}$	$\circ \bullet$	$\bullet \bullet$
$\vec{p} \rightarrow \vec{p}$ A_y	\bullet	$\bullet \bullet$
A_z		\bullet
$\vec{d} \rightarrow \vec{d}$ $A_y(d)$	$\circ \bullet$	\bullet
A_{yy}	$\circ \bullet$	\bullet
A_{xx}	$\circ \bullet$	\bullet
A_{xz}		\bullet
$\vec{p} \rightarrow \vec{p}$ K_i^j		
$\vec{d} \rightarrow \vec{p}$ $K_y^{y'}$		\bullet
K_{yy}		\bullet
$\vec{p} + \vec{d}$ C_{ij}		\bullet

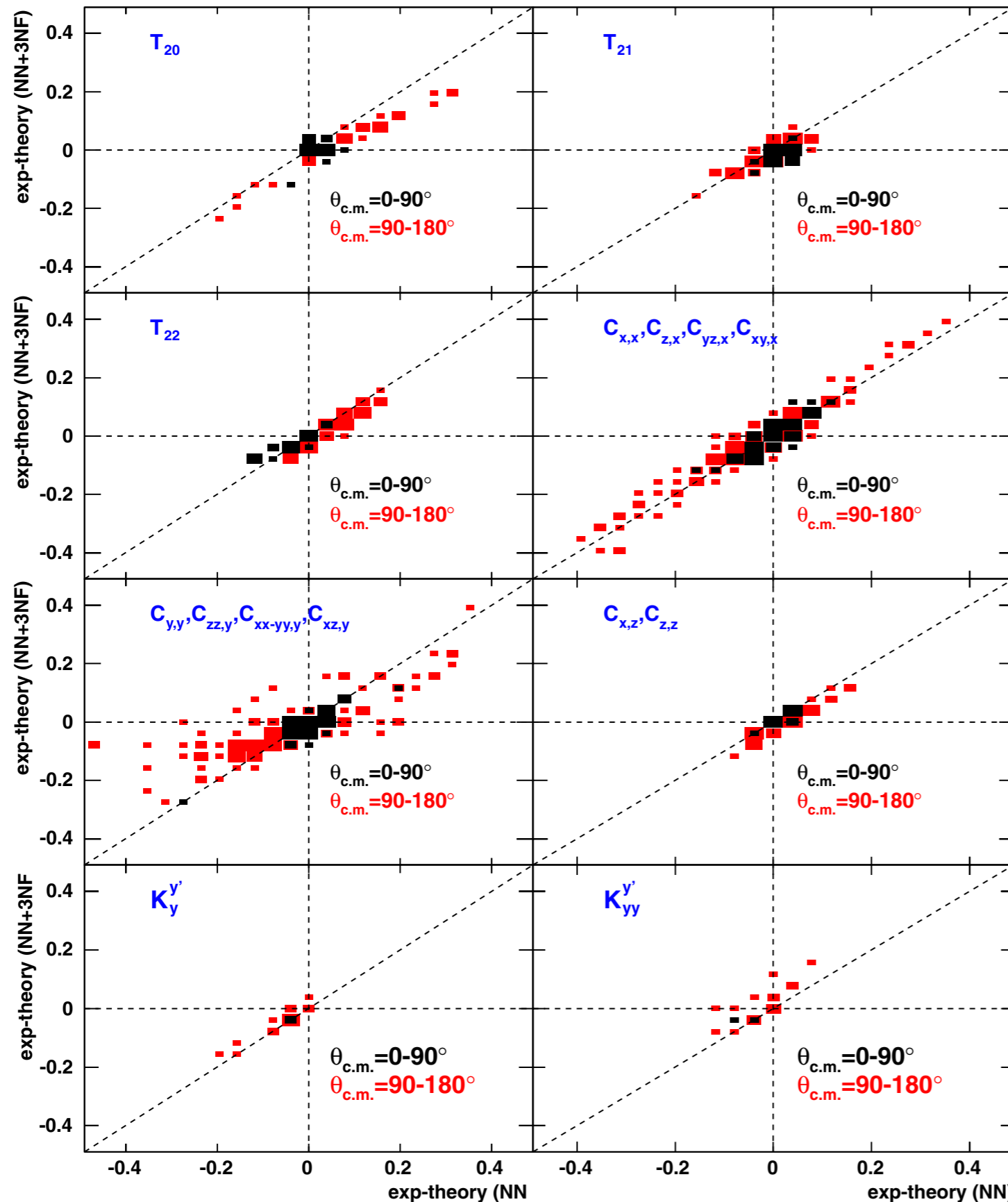
Overview data & 3NF (elastic)



3NF models improve description of elastic cross sections and A_y

(Kalantar-Nayestanaki et al. RPP (2012))

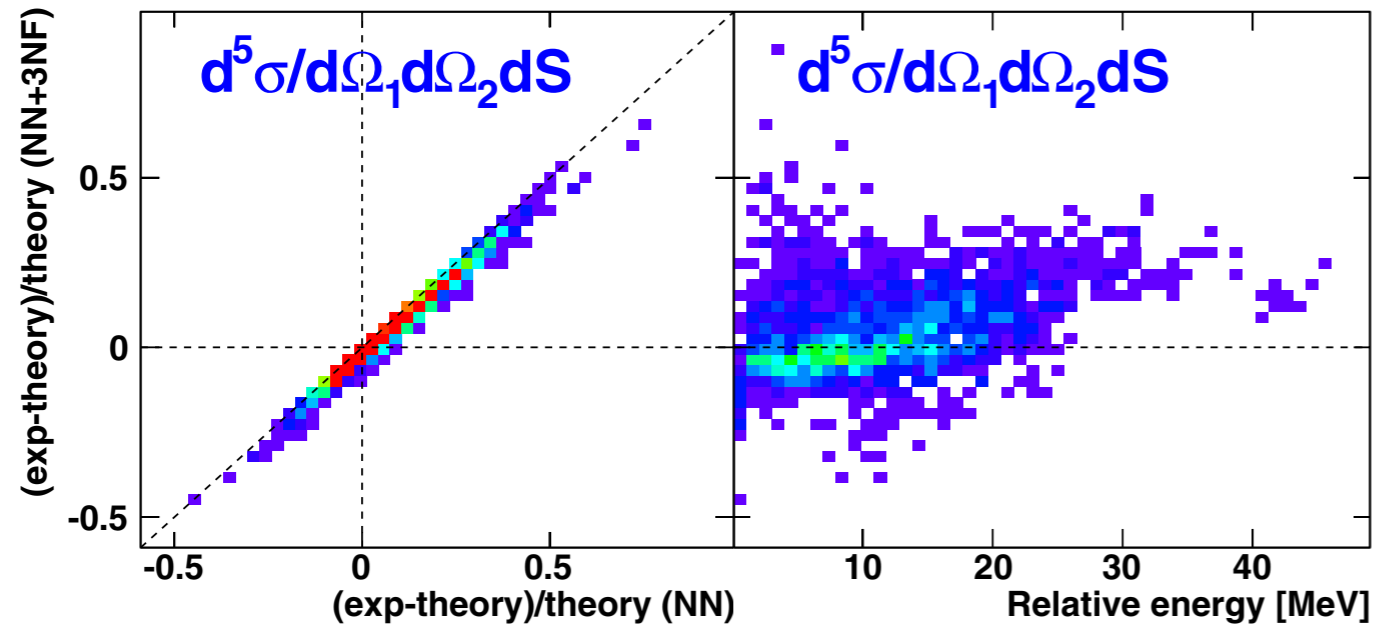
Overview data & 3NF (elastic)



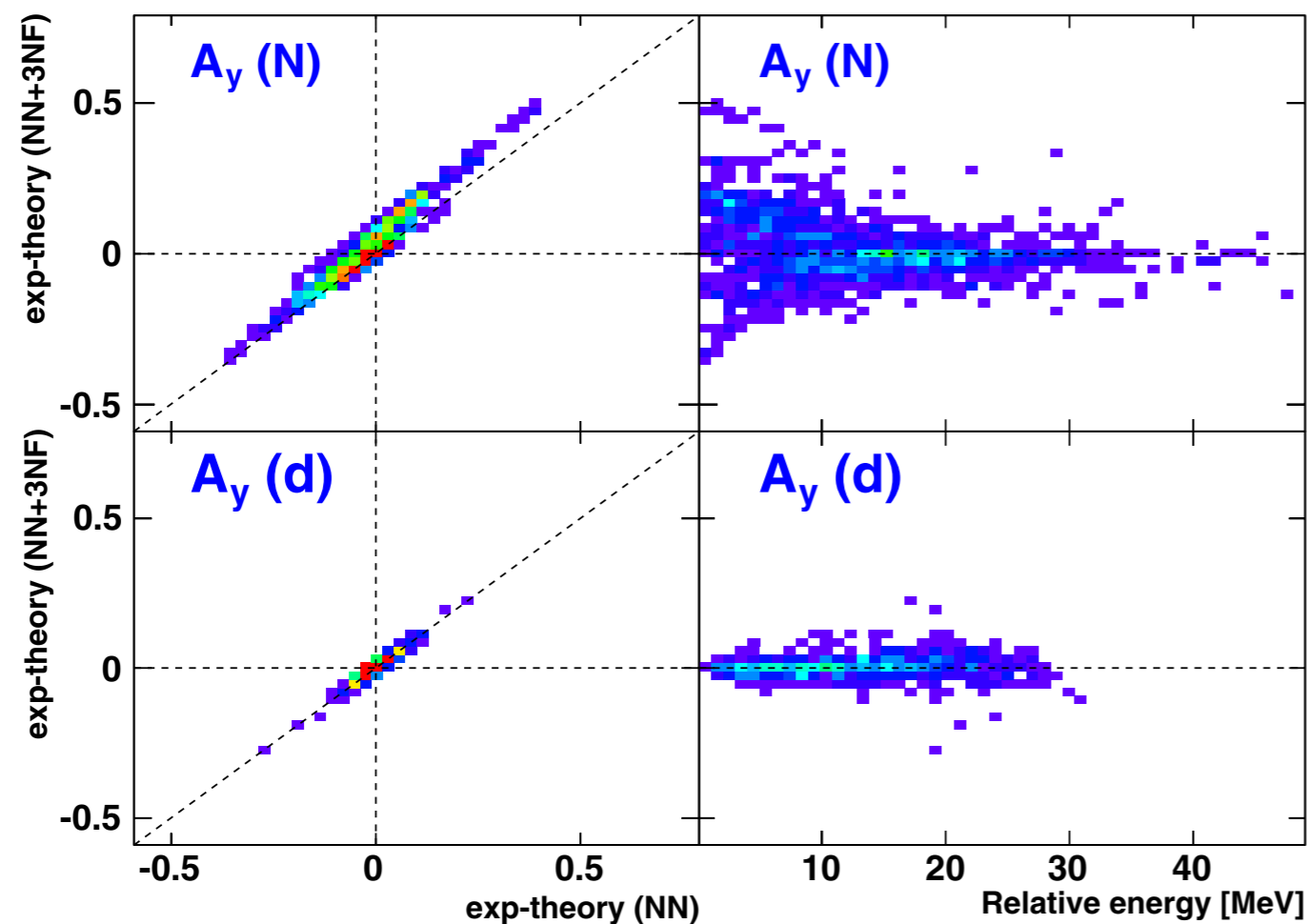
But no improvement for many other spin observables !

(Kalantar-Nayestanaki et al. RPP (2012))

Overview data & 3NF (breakup)



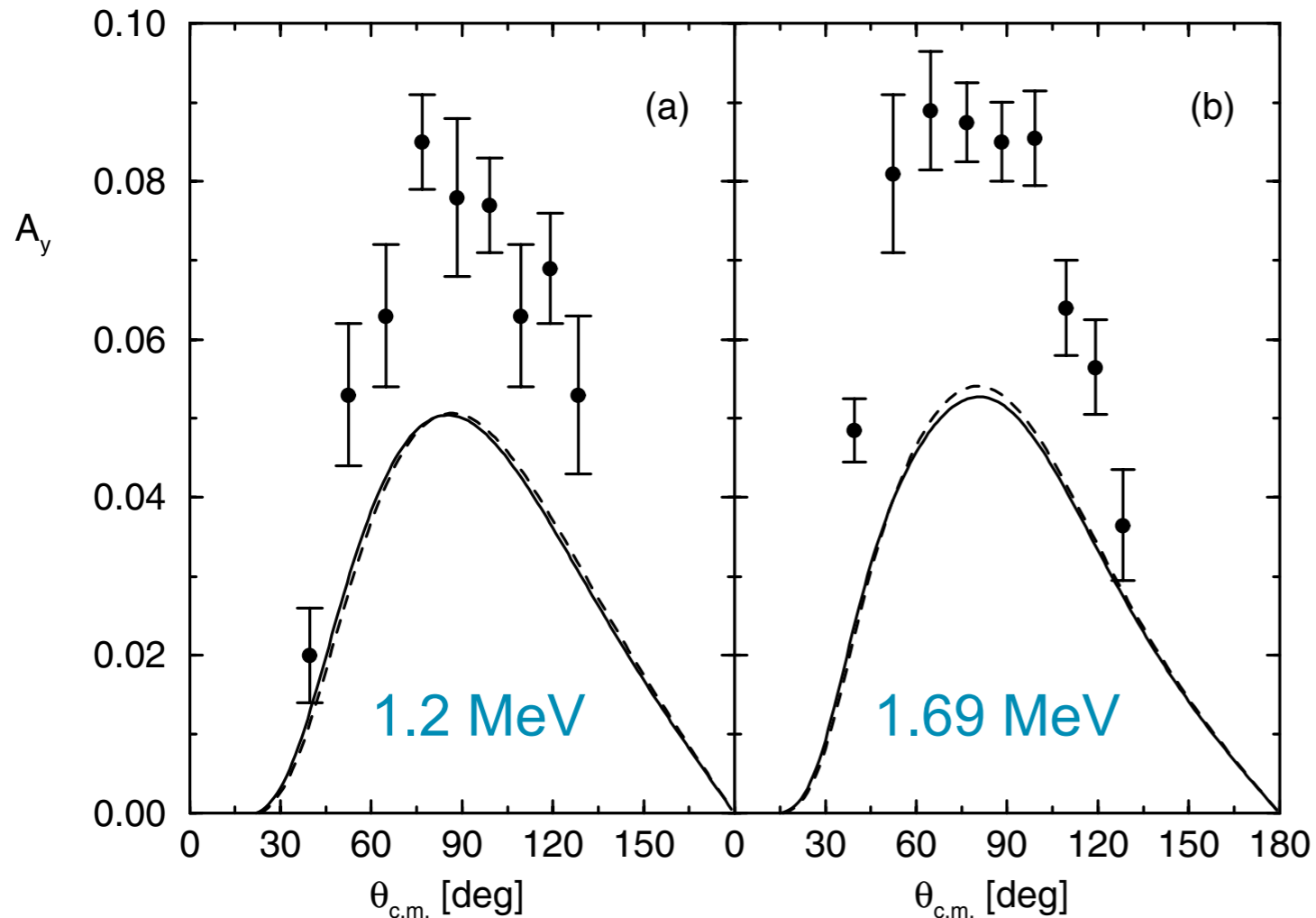
But no improvement for breakup!



(Kalantar-Nayestanaki et al. RPP (2012))

Deviation from data without effects of 3NF at low energies

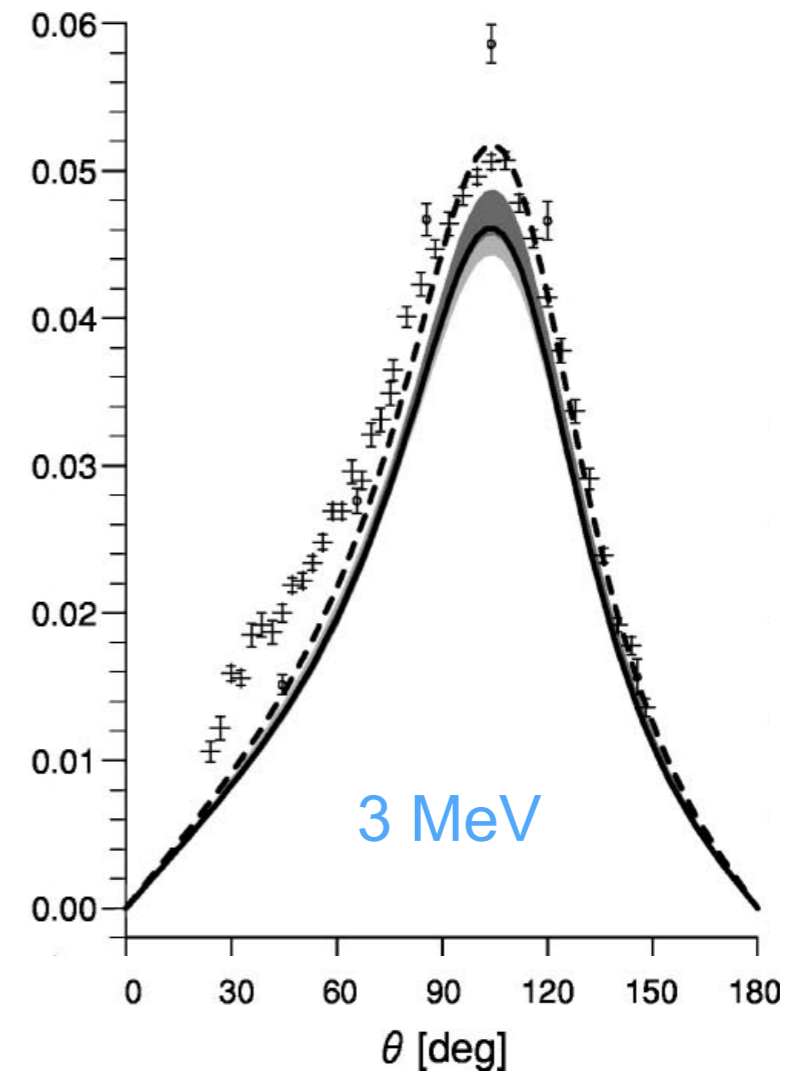
e.g. $p - {}^3\text{He}$ A_y



(Viviani et al., 2001)

**Deviation 5 % level
of cross section**

e.g. nd A_y



(see e.g. Witała et al., 2001)

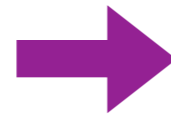
**Deviation 1 % level
of cross section**

Chiral nuclear interactions

systematic way to derive consistent NN and 3N forces

QCD & approximate chiral symmetry

symmetries



Effective Field Theory of QCD:
relevant degrees of freedom
nucleons & pions

$$\mathcal{L}_{QCD} = \bar{q} i \not{D} q - \frac{1}{2} \text{Tr} G_{\mu\nu} G^{\mu\nu} - \bar{q} m q$$

expansion in $\frac{Q}{\Lambda_\chi}$

$Q \approx m_\pi$, typical momentum

$$\Lambda_\chi \propto m_\Delta - m_N, m_\rho, \sqrt{m_\pi m_N}, 4\pi F_\pi, \dots$$

$$\approx 300 \text{ MeV} \dots 1200 \text{ MeV}$$



spontaneously & explicitly broken chiral symmetry

Goldstone bosons: pions



Chiral Perturbation Theory (ChPT)

„power counting“

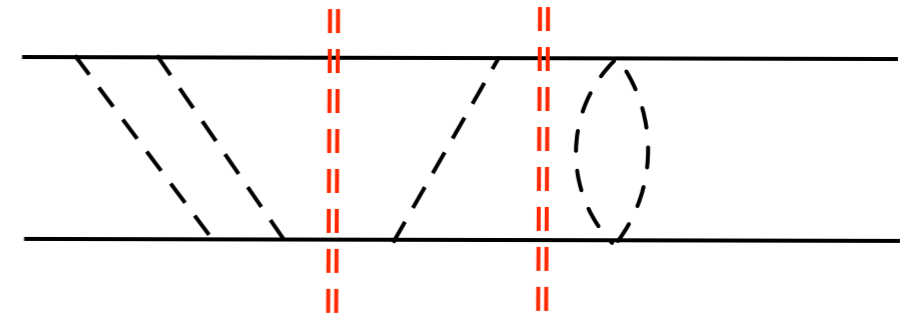
a systematic scheme to identify a finite numbers of diagrams contributing at a given order

ChPT for $A \geq 2$ is non-trivial:

the problem is non-perturbative

Weinberg's observation:

purely nucleonic intermediate states enhance diagrams at low energies
("reducible diagrams")



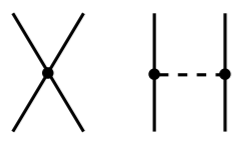
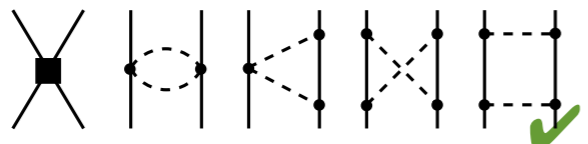

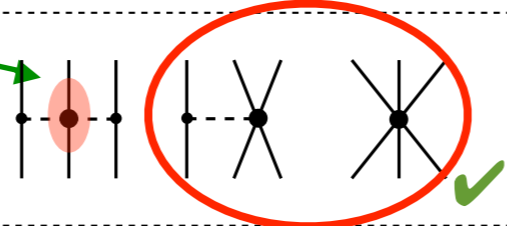
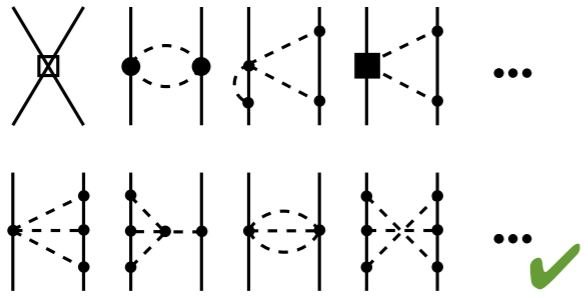
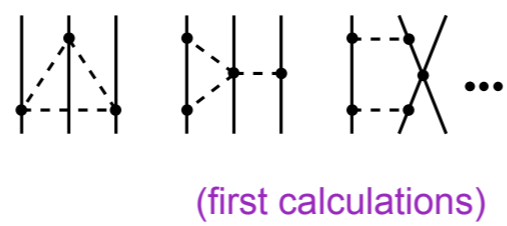
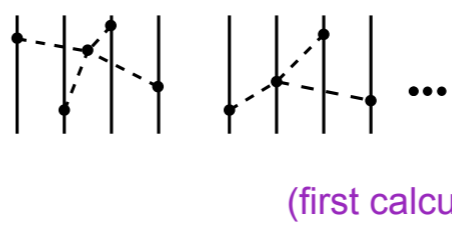
Weinberg's suggestion:

apply ChPT to irreducible diagrams and sum these to all orders using a LS equation
(Weinberg, 1991)

- the expected hierarchy of forces, $2N \gg 3N \gg 4N \dots$, follows naturally
- NN , $3N$, ... interactions can be consistently derived
& are connected to, e.g., πN scattering amplitudes
- ChPT results in a potential
- regularization required \rightarrow introduces cutoff parameter

NN, 3N and 4N sector

adjust to 2 few-body data

	2N force	3N force	4N force	
LO	 ✓	—	—	2 NN short range parameters (poor)
NLO	 ✓	—	—	
N ² LO	 ✓	 ✓	—	9 NN short range parameters (fair)
N ³ LO	 ✓	 (first calculations)	 (first calculations)	24 NN short range parameters (nice)

(from Epelbaum, 2008)

- systematically improvable NN, 3N, 4N, ... interactions
- qualitatively: NN >> 3N >> 4N ...
- quantitatively successful
- **cutoff dependence** can be used to estimate higher order effects

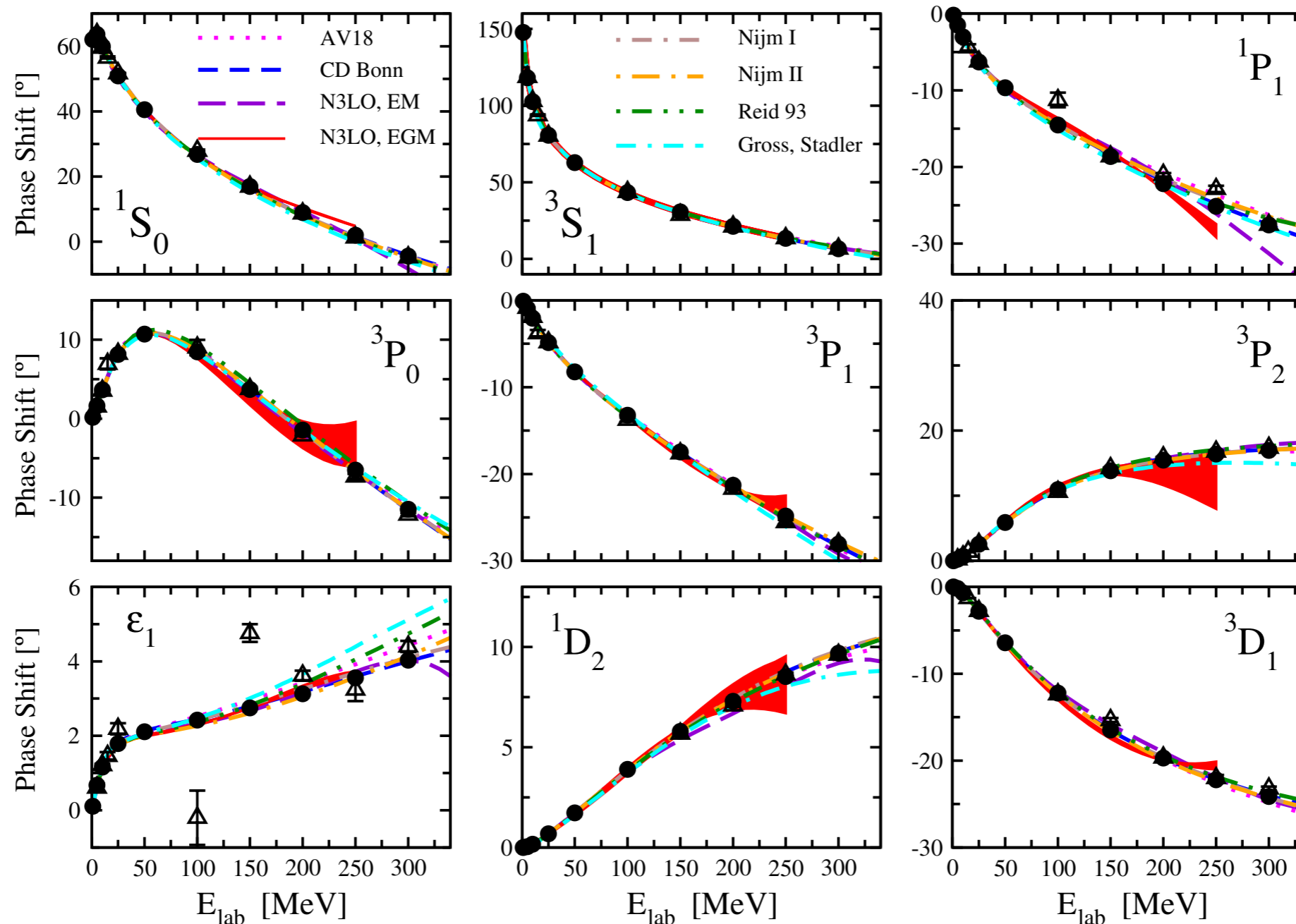
Effects of chiral 3NFs on low energy observables

- accurate predictions are possible
- predictions for 3N observables are currently restricted to low energy
- significant effects of leading 3NFs

Description of NN data

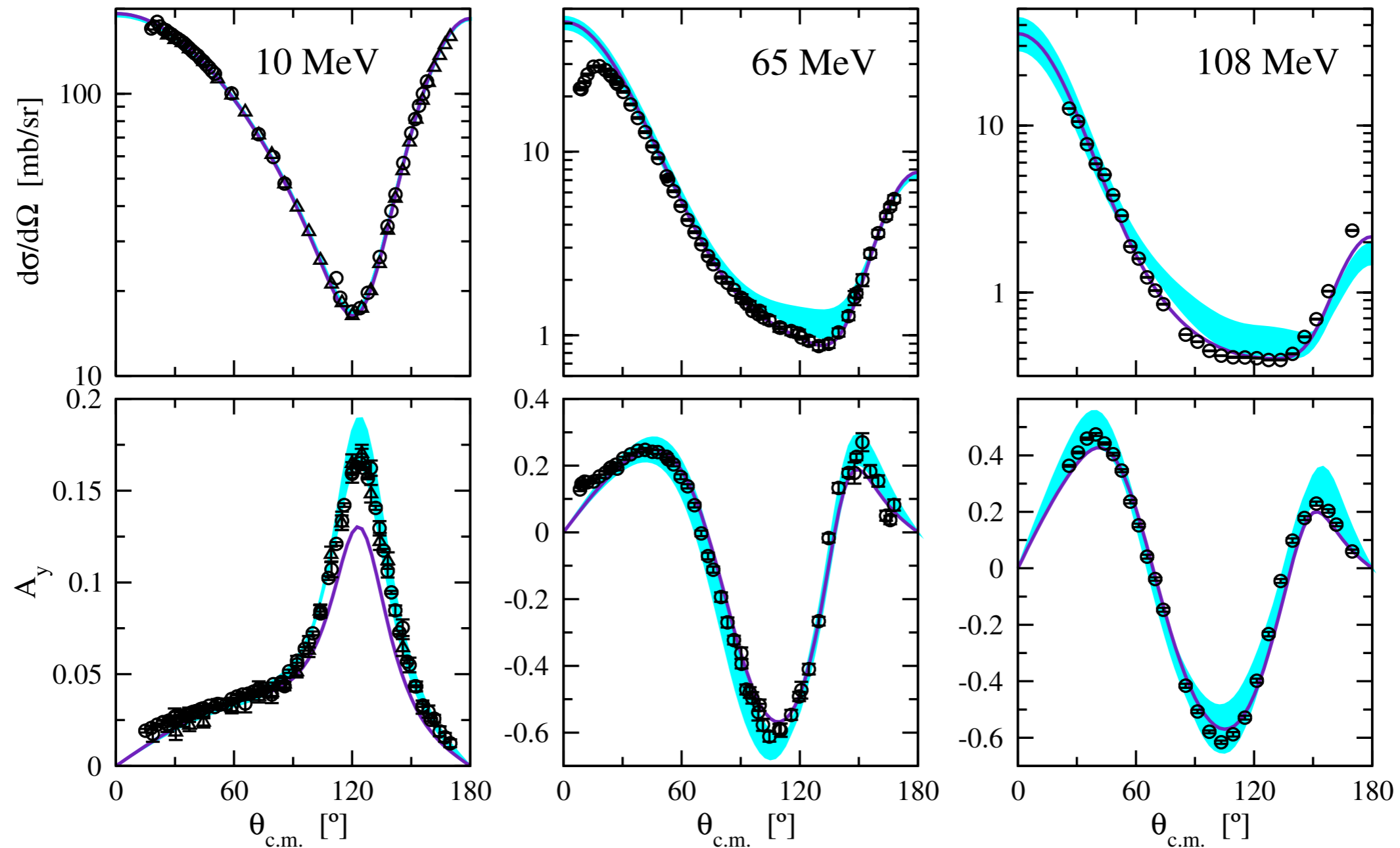
NN force models (AV18, CD-Bonn, Nijmegen, ...) and chiral forces are **phase equivalent**

Describe the data (~ 4000) up to $T_{\text{lab}} \sim 300$ MeV ($p_{\text{lab}} \sim 700$ MeV) **perfectly** ($\chi^2/\text{datum} \sim 1$)



(from Kalantar-Nayestanaki et al., 2012)

Current status: complete calculations up to N²LO

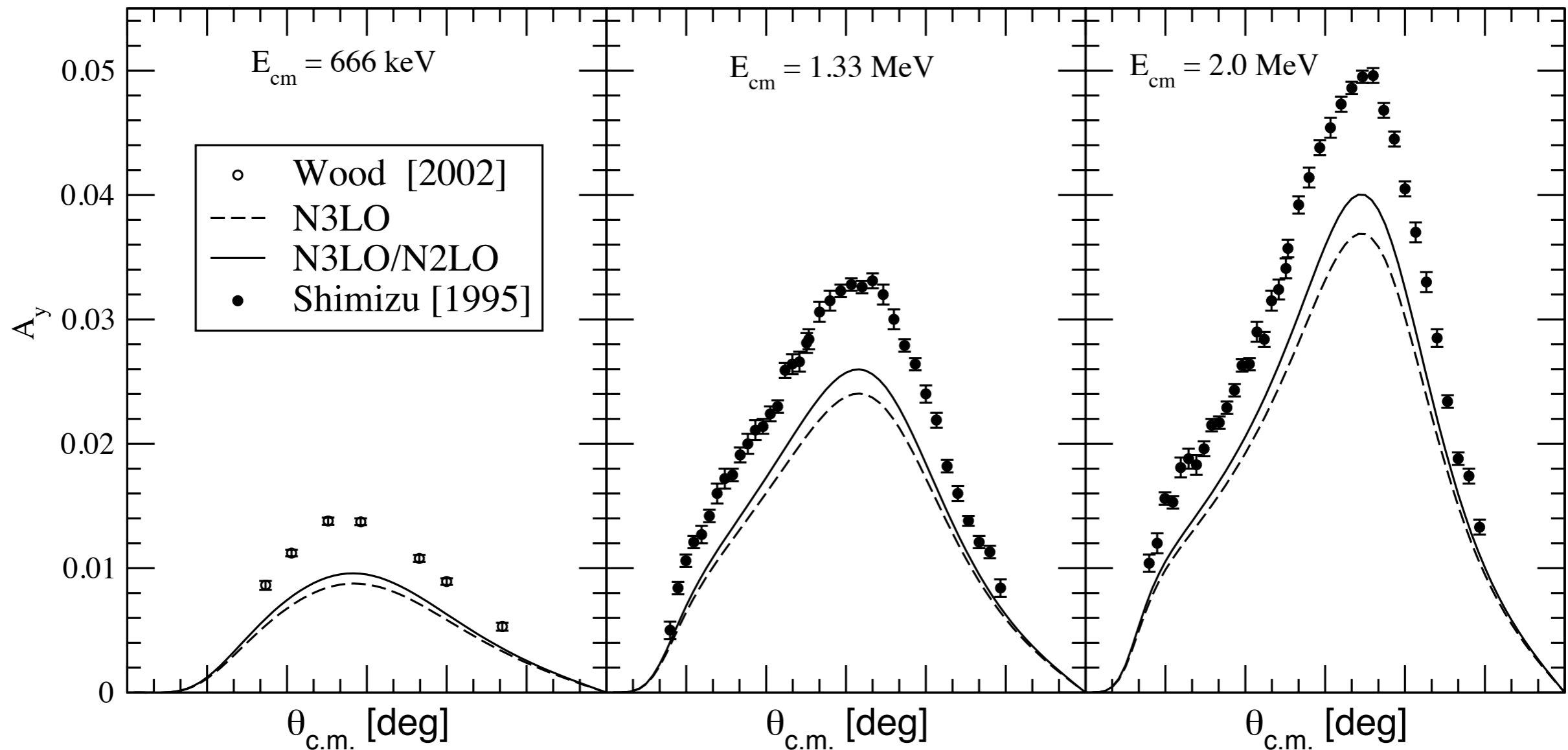


(from Kalantar-Nayestanaki et al., 2012)

cutoff dependence indicates that uncertainties are significant at energies above 60 MeV
N³LO calculations & other regularizations are currently studied

Back to low energy puzzles

e.g. A_y of pd and nd elastic scattering

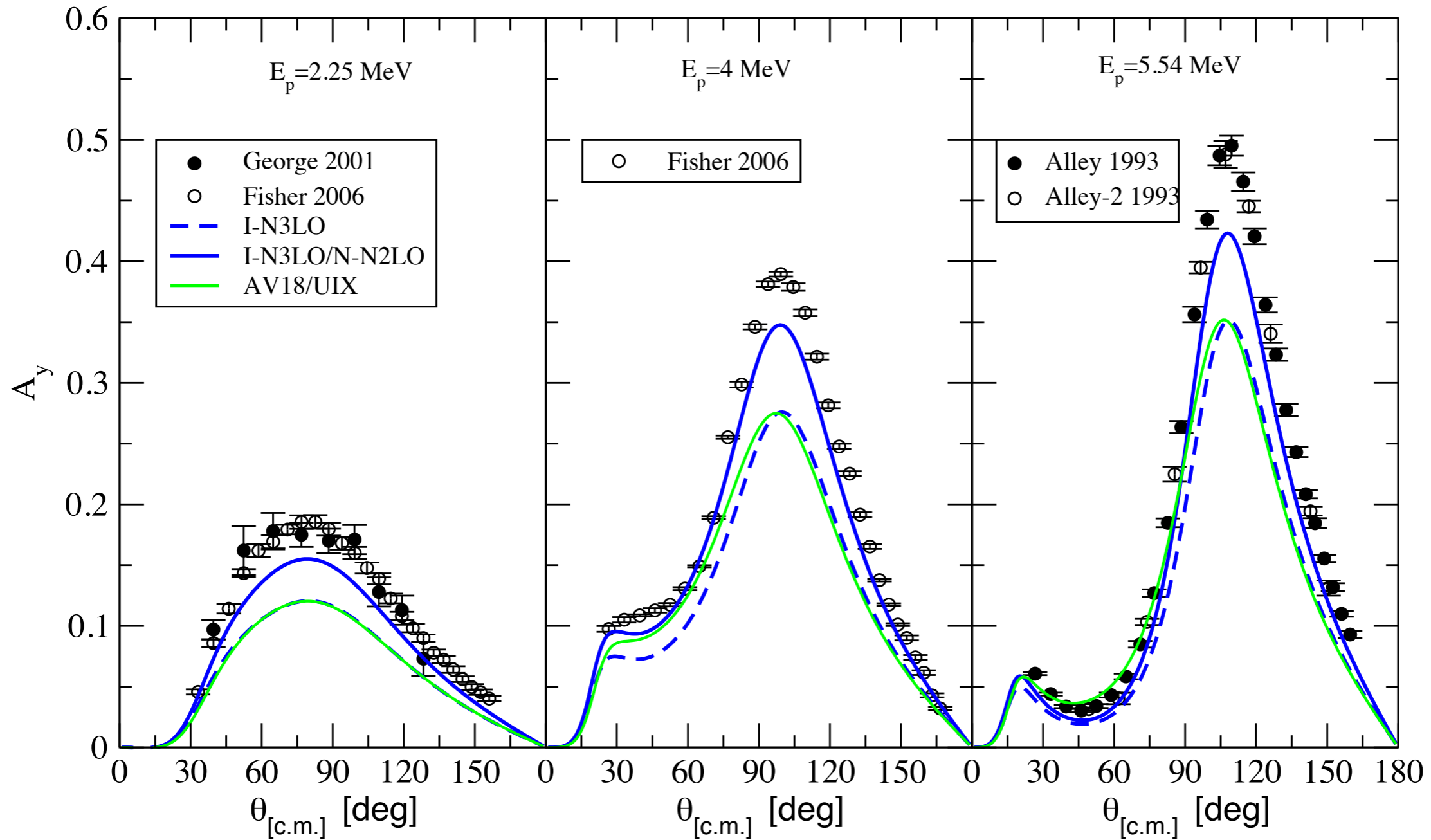


**A_y deviation remains on the 1 % level !
Important N^3LO contribution? No indication yet.**

(L.E. Marcucci et al., 2009)

Back to low energy puzzles

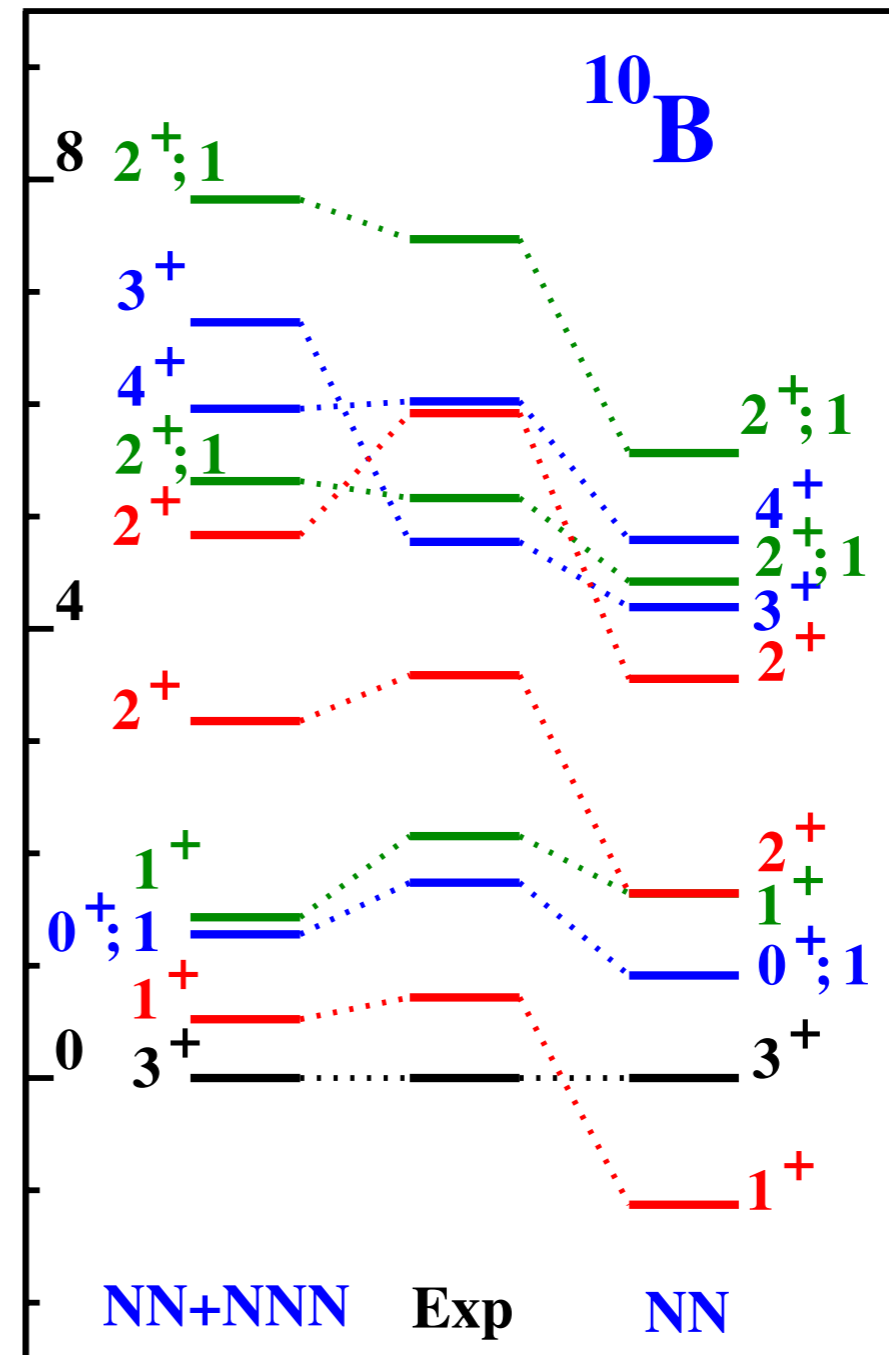
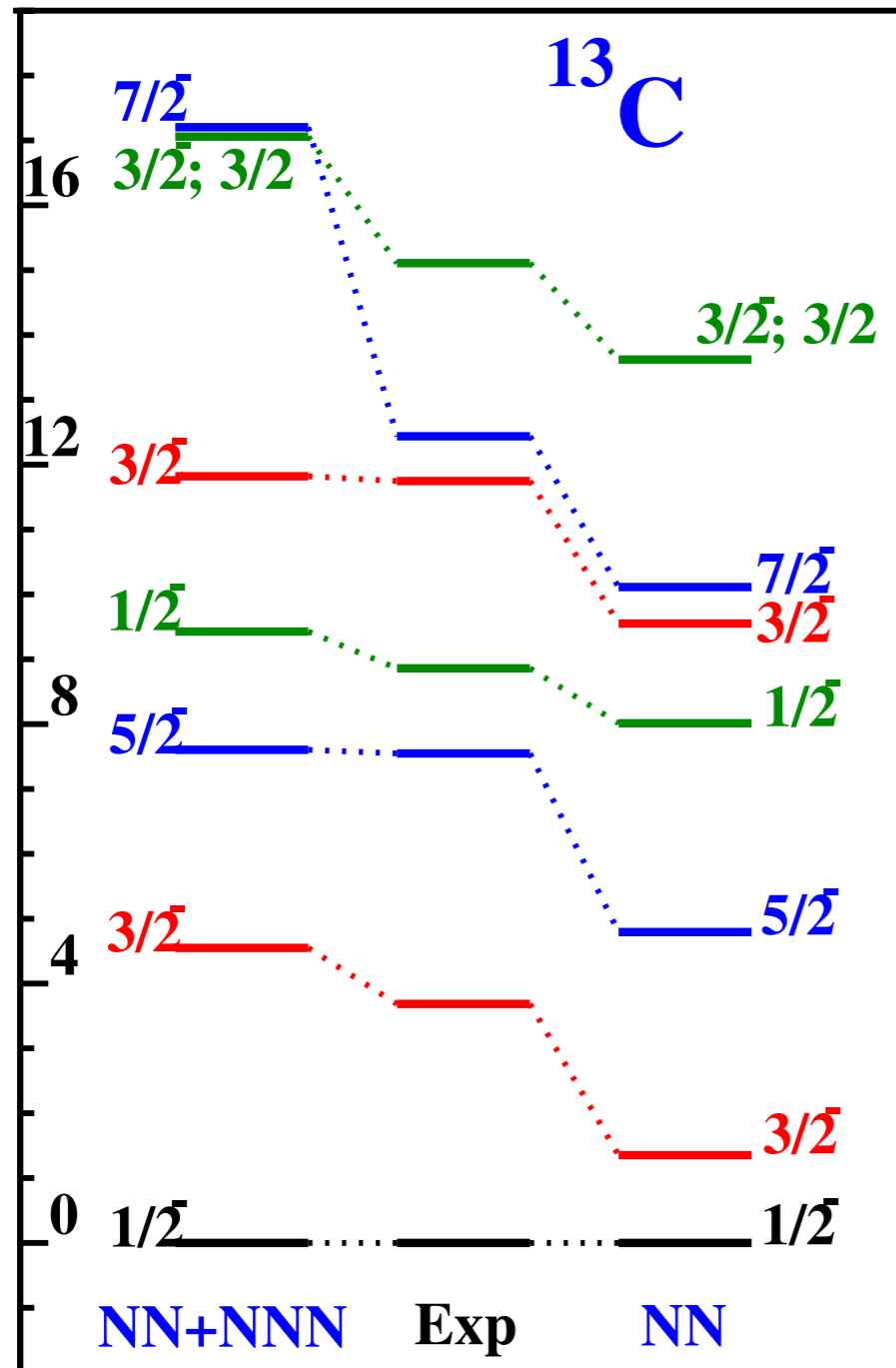
but p - ^3He A_y is affected !!!



Note that A_y deviation is finally on the 1 % level !

(Viviani et al., 2010,
Viviani et al., 2013)

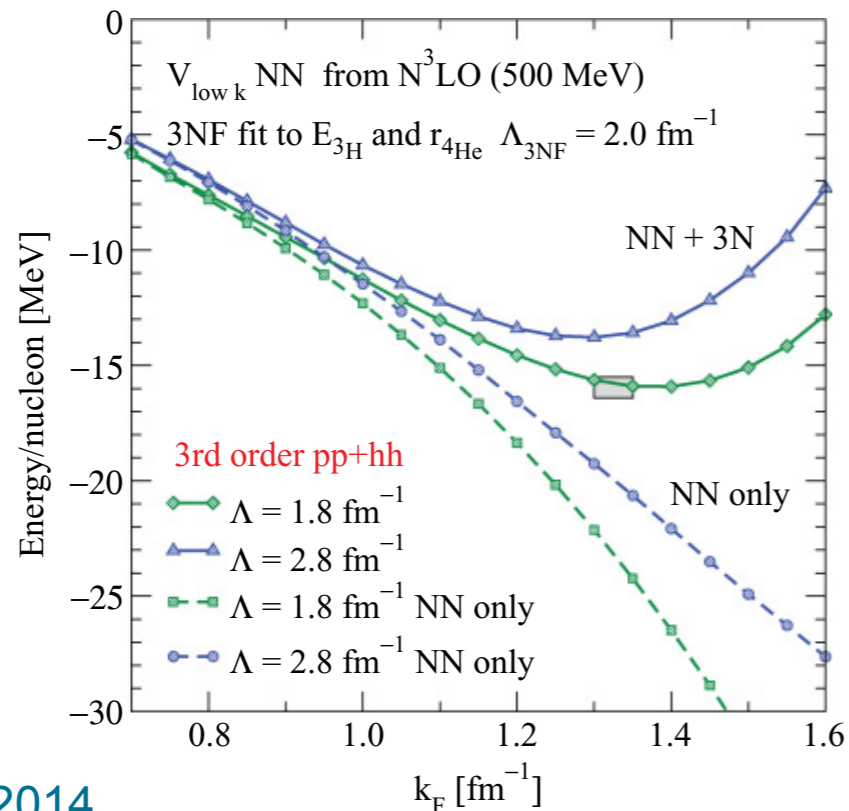
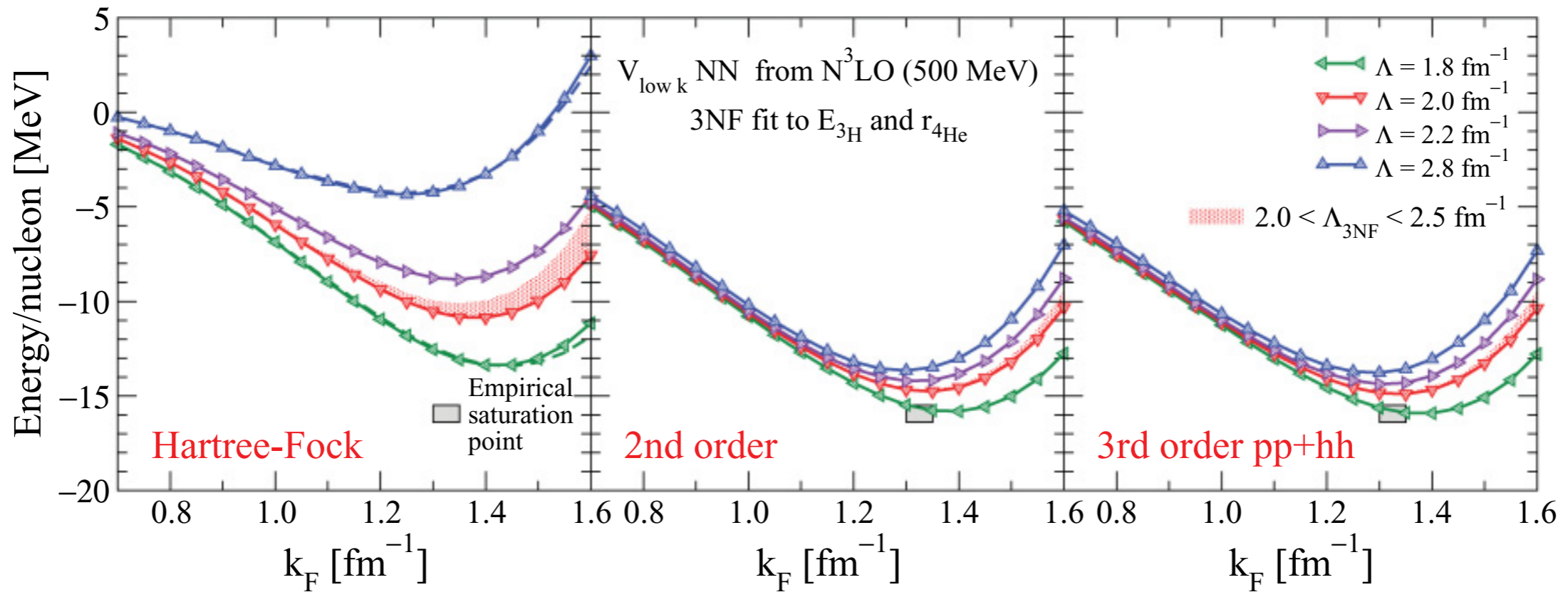
Spectra of ^{10}B and ^{13}C



(Navrátil et al., 2007)

Clear improvement of description compared to experiment.

Many-body calculations can be performed perturbatively if soft interactions are used



- 3NF are essential for predictions
- contribution of 3NF remains natural

(Hebeler et al. PRC (2011))

- **3NF significantly impact nuclei and nuclear reactions**
- 3NF are not well understood
 - *spin observables*
 - ***nd breakup** observables are a significant challenge*
- Chiral Perturbation Theory
 - *systematic framework to develop consistent NN and 3N forces*
 - *Predictions for low energy data improve using chiral interactions at N^2LO*
 - *Cutoff dependence at energies above 60 MeV is large*
- **Theory needs to be developed further**
 - *full N^3LO calculations (about to be achieved)*
 - *improved regularizations (implementation in progress)*
 - *ChEFT including explicit Δ (formulate)*
 - *fit of 3NF to 3N data*
- **Data at energies below 60 MeV is scarce**
 - *more accurate theory*
 - *onset of 3NF effects*