EFFECTS OF FIELDS GRADIENTS ON THE SPIN PRECESSION

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BMT EQUATION

V. Bargamnn, L. Michel, V.L. Telegdi, *Phys. Rev. Lett. 2 (1959) 435*

- **important assumption – homogeneous electromagnetic field**
- **in storage ring this assumption is not strictly fulfilled**
- **fields gradients are present – dipole fringe field, quadrupole field**

commonly used for description of spin behaviour in storage ring

$\vec{s}$ – polarization vector, $I = {}^{1}/_{2}, 1, \cdots$ – particle spin

equation of motion – only simple Lorentz force

$$mc\frac{d(\gamma\vec{\beta})}{dt} = e\left(\vec{E} + c\vec{\beta} \times \vec{B}\right)$$

spin precession equation

$$\frac{d\vec{s}}{dt} = \frac{e}{mc}\vec{s} \times \left[\left(\frac{g}{2} - 1 + \frac{1}{\gamma}\right)c\vec{B} - \left(\frac{g}{2} - 1\right)\frac{\gamma c}{\gamma + 1}(\vec{\beta} \cdot \vec{B})\vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma + 1}\right)\vec{\beta} \times \vec{E}\right]$$

SPIN PRECESSION DUE TO EDM INTERACTION WITH ELECTROMAGNETIC FIELD

for particles with permanent electric dipole moment – *D*
BMT equation may be generalized
T. Fukuyama, A.J. Silenko, *Int. J. Mod. Phys. A28 (2013) 1350147*

it is done under the same assumptions under which BMT equation was derived

equation of motion remains unchanged

$$mc\frac{d(\gamma\vec{\beta})}{dt} = e\left(\vec{E} + c\vec{\beta}\times\vec{B}\right)$$

spin precession equation is modified by additional term

$$\frac{d\vec{s}}{dt} = \frac{e}{mc}\vec{s}\times\left[\left(\frac{g}{2}-1+\frac{1}{\gamma}\right)c\vec{B} - \left(\frac{g}{2}-1\right)\frac{\gamma c}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} - \left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right]$$

$$+\frac{D}{\hbar}\vec{s}\times\left[\vec{E} - \frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{E}\right)\vec{\beta} + c\vec{\beta}\times\vec{B}\right]$$

PARTICLES INTERACTION WITH FIELD GRADIENT

expected precision for EDM is of order $10^{-29}\ e \cdot cm \sim 10^{-50}\ C \cdot m$

EDM term in BMT equation is very small
→ it is obligatory to check the influence
of second order effects due to interaction with electromagnetic field gradients

in a particle rest frame:

magnetic dipole moment μ interacts with magnetic field gradient
additional force and torque → changes in equation of motion and spin precession

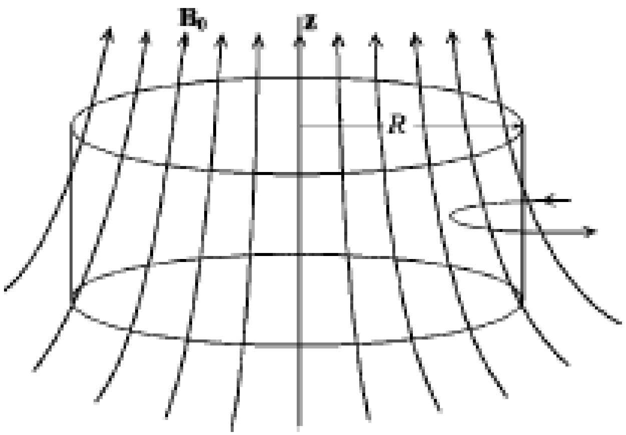
electric quadrupole moment Q interact with electric field gradient
additional torque → changes in spin precession

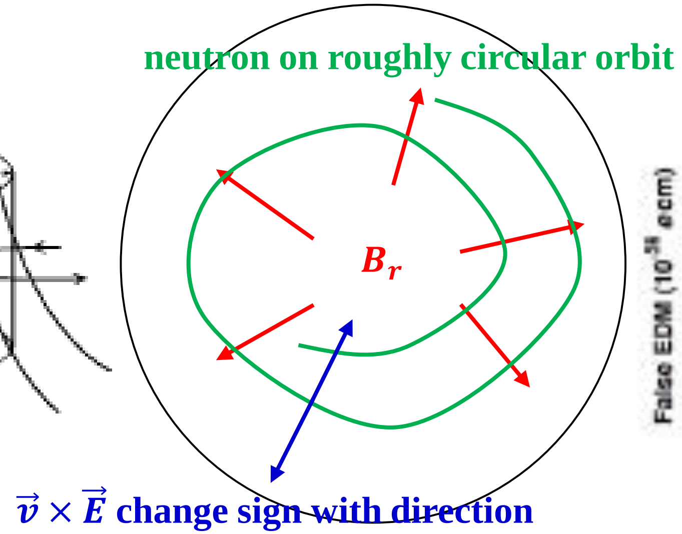
in LAB frame μ and Q interacts with electric and magnetic fields gradients

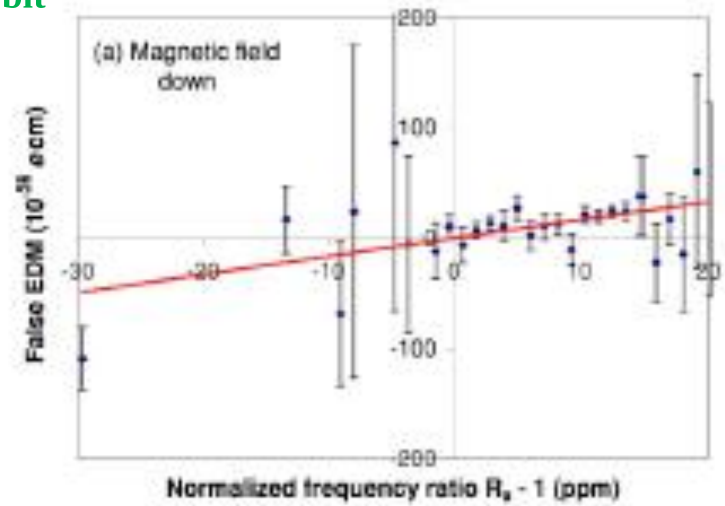
EDM searches are planned for proton, deuteron and ^{3}He
for these particles $\mu = (+2.8;\ +0.9; -2.1)\mu_N \sim (5 \div 15) \cdot 10^{-27}\ J/T$
proton and ^{3}He are spin $^1/_2$ particles ⇒ $Q = 0$
for deuteron $Q = 0.2859\ e \cdot b \sim 5 \cdot 10^{-50}\ C \cdot m^2$

FALSE NEUTRON EDM

J. M. Pendlebury et al., *Phys. Rev. A70 (2004) 032102*





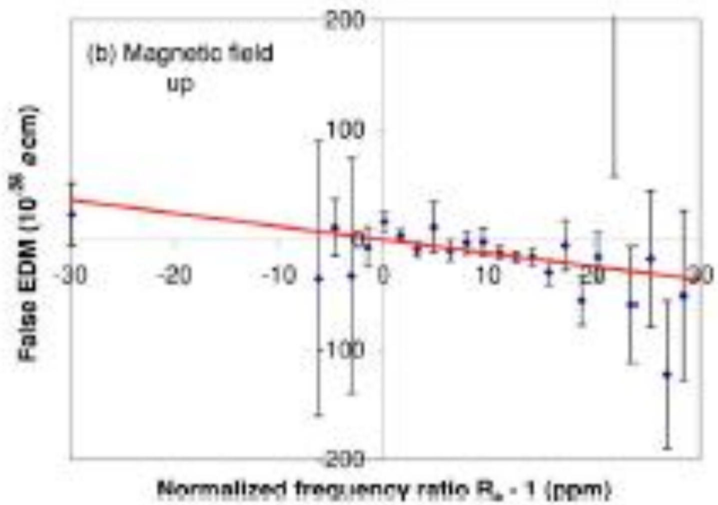


tiny effect arising due to combination of gravitation and magnetic field gradient

$$B_r = \frac{r}{2}\frac{\partial B_z}{\partial z}$$

for neutron EDM limit ~ 10^{-29} $e \cdot cm$

$$\frac{1}{B_z}\left\langle\frac{\partial B_z}{\partial z}\right\rangle \approx 10^{-4}/m$$



EQUATION OF MOTION IN PRESENCE OF FIELDS GRADIENTS

R.H. Good, *Phys. Rev. 15 (1962) 2112*

A.I. Solomon, *Nuovo Cim. 26 (1962) 1320*; P. Nyborg, *Nuovo Cim. 31 (1964) 1209*

$$mc\frac{d(\gamma\vec{\beta})}{dt} = e\left(\vec{E} + c\vec{\beta}\times\vec{B}\right) + \mu\gamma\left[\vec{\nabla} + \vec{\beta}\times\left(\vec{\beta}\times\vec{\nabla}\right) + \vec{\beta}\frac{\partial}{c\partial t}\right]\vec{s}\cdot\left(\vec{M} + \gamma\vec{N}\right)$$

$$\vec{M} = c\vec{B} + \gamma(\gamma+1)^{-1}\vec{E}\times\vec{\beta} \qquad \vec{N} = \gamma(\gamma+1)^{-1}\left(\vec{E} + c\vec{\beta}\times\vec{B}\right)\times\vec{\beta}$$

neglected EDM interaction with fields gradients (expected to be very small)

ORDER OF MAGNITUDE ESTIMATION FOR PROTON

$$mc\frac{d(\gamma\vec{\beta})}{dt} = 1.6\cdot 10^{-19}\left(\vec{E} + c\vec{\beta}\times\vec{B}\right) + 4.7\cdot 10^{-35}\gamma\left[\vec{\nabla} + \vec{\beta}\times\left(\vec{\beta}\times\vec{\nabla}\right) + \vec{\beta}\frac{\partial}{c\partial t}\right]\vec{s}\cdot\left(\vec{M} + \gamma\vec{N}\right)$$

in equation of motion effects of fields gradients are small

GENERALIZED BMT EQUATION INCLUDING FIELDS GRADIENTS

R.H. Good, *Phys. Rev. 15 (1962) 2112*

$$\frac{d\vec{s}}{dt} = \frac{e}{mc}\vec{s} \times \left[\left(\frac{g}{2} - 1 + \frac{1}{\gamma}\right)c\vec{B} - \left(\frac{g}{2} - 1\right)\frac{\gamma c}{\gamma + 1}(\vec{\beta} \cdot \vec{B})\vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma + 1}\right)\vec{\beta} \times \vec{E}\right]$$

$$+ \frac{D}{\hbar}\vec{s} \times \left[\vec{E} - \frac{\gamma}{\gamma + 1}(\overrightarrow{\beta \cdot} \vec{E})\vec{\beta} + c\vec{\beta} \times \vec{B}\right]$$

$$+ \frac{\mu}{mc}\frac{1}{\gamma + 1}\vec{s} \times (\vec{\beta} \times \vec{\nabla})\vec{s} \cdot (\vec{M} + \gamma\vec{N})$$

$$+ \frac{Q}{\hbar(2I - 1)}\vec{s} \cdot \left[\vec{\nabla} + \frac{\gamma}{\gamma + 1}\vec{\beta} \times (\vec{\beta} \times \vec{\nabla}) + \vec{\beta}\frac{\partial}{c\partial t}\right]\vec{s} \times (\vec{E} + \gamma\vec{\beta} \times \vec{M})$$

GENERALIZED SPIN PRECESSION EQUATION FOR SPECIFIC CASE

$$\vec{E} = E\hat{x} \qquad \vec{B} = B\hat{y} \qquad \vec{\beta} = \beta\hat{z} \qquad \vec{s} = s\hat{y}$$

$$\frac{ds_z}{dt} = -\frac{D}{\hbar}[E - c\beta B]s - \frac{\mu}{mc}\frac{\gamma\beta}{\gamma+1}\left[\beta\frac{dE}{dy} - c\frac{dB}{dy}\right]s - \frac{Q}{\hbar(2I-1)}\left[\frac{dE}{dy} - c\beta\frac{dB}{dy}\right]s$$

ORDER OF MAGNITUDE ESTIMATION $D = 10^{-29}\ e \cdot cm$

proton

$$\frac{ds_z}{dt} = -1.5 \cdot 10^{-16}[E - c\beta B]s$$

$$-9.4 \cdot 10^{-17}\ \frac{\gamma\beta}{\gamma+1}\left[\beta\frac{dE}{dy} - c\frac{dB}{dy}\right]s$$

deuteron

$$\frac{ds_z}{dt} = -1.5 \cdot 10^{-16}[E - c\beta B]s$$

$$-1.4 \cdot 10^{-17}\ \frac{\gamma\beta}{\gamma+1}\left[\beta\frac{dE}{dy} - c\frac{dB}{dy}\right]s - 4.3 \cdot 10^{-16}\ \left[\frac{dE}{dy} - c\beta\frac{dB}{dy}\right]s$$

$$\frac{\text{EDM effect}}{\text{field gradient effect}} = 100 \quad \rightarrow \quad \frac{1}{E}\left(\frac{\partial E}{\partial y}\right) \text{ and } \frac{1}{B}\left(\frac{\partial B}{\partial y}\right) \sim 10^{-2} \div 10^{-3}/m$$

SUMMARY

μ and Q interaction with fields gradients may induce the same effects as EDM with the magnitude comparable to EDM

it may have impact on design of dedicated storage ring

it is worth to apply generalized BMT equation (Good equation) for spin tracking simulations:

- **calculate fields gradients**
- **update generalized spin precession equation instead of BMT equation**

application of μ and Q interaction with fields gradients (inducing false EDM):

- **adding fields gradients in a controlled way**
- **detailed understanding of storage ring properties to the accuracy level necessary for expected EDM accuracy**
- **check polarization measurement methods at the level of EDM accuracy**