# THEORY OF THE MASS GAP IN QCD AND ITS VACUUM

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# Lagrangian of QCD

$$L_{QCD} = L_{YM} + L_{qg}$$

$$L_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + L_{g.f.} + L_{gh.}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \quad a = N_c^2 - 1, \quad N_c = 3$$

$$L_{qg} = i\bar{q}^j_\alpha D_{\alpha\beta} q^j_\beta + \bar{q}^j_\alpha m^j_0 q^j_\beta, \quad \alpha, \beta = 1, 2, 3, \quad j = 1, 2, 3, ...N_f$$

$$D_{\alpha\beta} q^j_\beta = (\delta_{\alpha\beta}\partial_\mu - ig(1/2)\lambda^a_{\alpha\beta} A^a_\mu)\gamma_\mu q^j_\beta \qquad (cov.\ der.)$$

# Properties of the QCD Lagrangian

**QCD or, equivalently Quantum ChromoDynamics**

1). Unit coupling constant $g$

2). $g \sim 1$ while in QED (Quantum ElecroDynmics) it is $g \ll 1$.

3). No mass scale parameter to which can be assigned a physical meaning.

Current quark mass $m_0$ cannot be used, since the quark is a colored object

**QCD** without quarks is called **Yang-Mills, YM**

# The Jaffe-Witten (JW) theorem:

**Yang-Mills Existence And Mass Gap: Prove that for any compact simple gauge group $G$, quantum Yang-Mills theory on $R^4$ exists and has a mass gap $\Delta > 0$.**

(i). It must have a "mass gap". Every excitation of the vacuum has energy at least $\Delta$ (to explain why the nuclear force is strong but short-range).

(ii). It must have "quark confinement" (why the physical particles are $SU(3)$-invariant).

(iii). It must have "chiral symmetry breaking" (to account for the "current algebra" theory of soft pions).

We need Mass Gap responsible for the NP dynamics

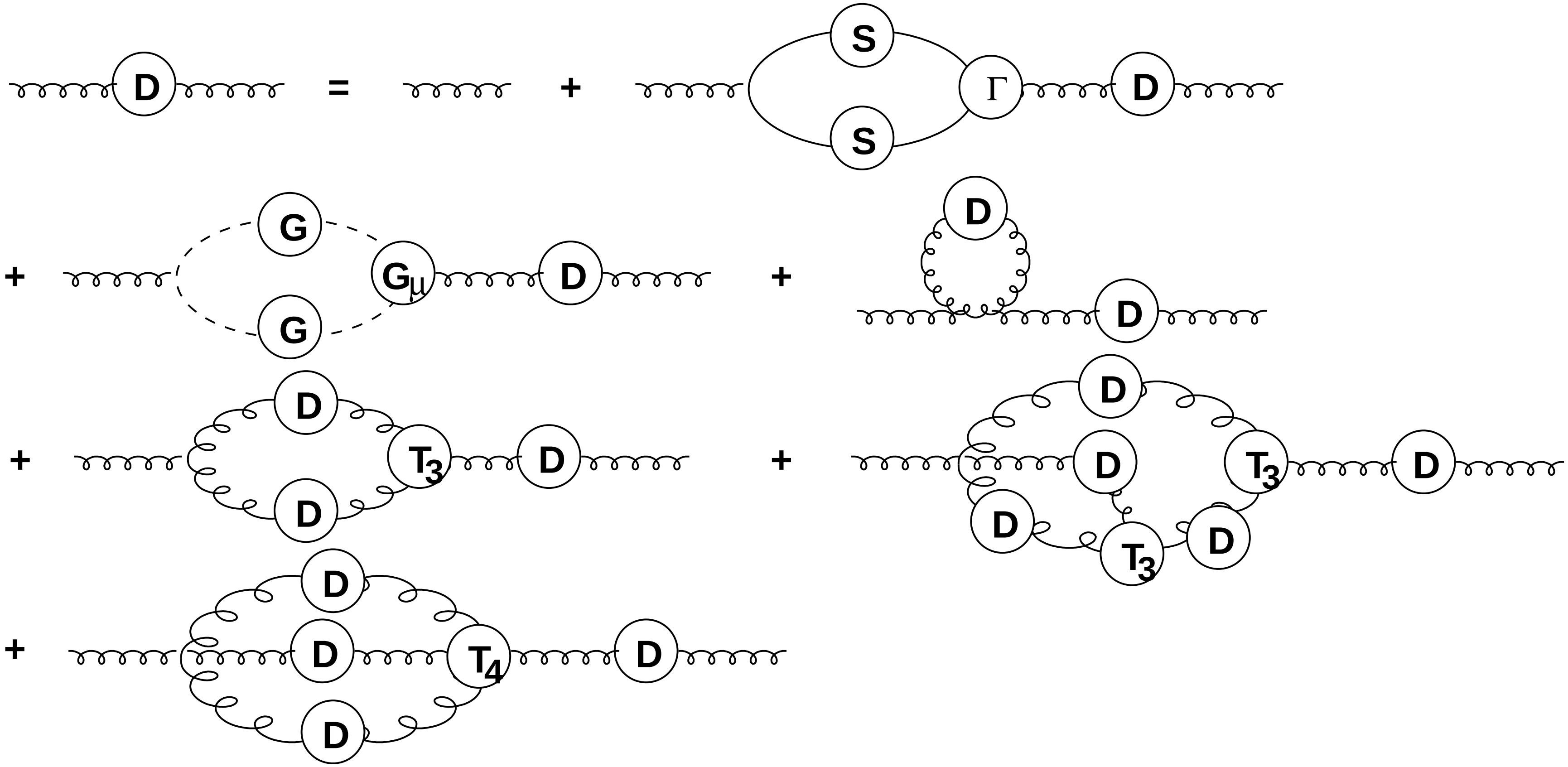
# Gluon SD equation

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i\Pi_{\rho\sigma}(q; D) D_{\sigma\nu}(q)$$

$$D_{\mu\nu}(q) = i \left\{ T_{\mu\nu}(q) d(q^2) + \xi L_{\mu\nu}(q) \right\} \frac{1}{q^2}$$

$$D^0_{\mu\nu}(q) = i \left\{ T_{\mu\nu}(q) + \xi L_{\mu\nu}(q) \right\} \frac{1}{q^2}$$

$$T_{\mu\nu}(q) = \delta_{\mu\nu} - (q_\mu q_\nu / q^2) = \delta_{\mu\nu} - L_{\mu\nu}(q)$$




$$\Pi_{\rho\sigma}(q;D) = \Pi^{q}_{\rho\sigma}(q) + \Pi^{gh}_{\rho\sigma}(q) + \Pi^{t}_{\rho\sigma}(D) + \Pi^{(1)}_{\rho\sigma}(q;D^2)$$
$$+\Pi^{(2)}_{\rho\sigma}(q;D^4) + \Pi^{(2')}_{\rho\sigma}(q;D^3).$$

# The Mass Gap

$$\Pi^s_{\rho\sigma}(q;D) = \Pi_{\rho\sigma}(q;D) - \Pi_{\rho\sigma}(0;D) = \Pi_{\rho\sigma}(q;D) - \delta_{\rho\sigma}\Delta^2(D)$$

$$\Delta^2(D) \equiv \Delta^2(\lambda, \alpha; D)$$

**Problems**

**A**. The first problem is how to satisfy the ST identity $q_\mu q_\nu D_{\mu\nu}(q) = i\xi$ but without going to $\Delta^2(D) = 0$ limit.

**B**. The second problem is how to make the relevant gluon propagator purely transversal, since the ghosts will fail to do this when $\Delta^2(D)$ will be explicitly present.

# Methods

**Theory of matrices**
Subtractions at the gluon propagator level

**Theory of generalized functions (distributions)**

**Dimensional regularization method**

**Theory of functions of complex variable**

**The Weierstrass-Sokhotsky-Casorati theorem**

which describes the behavior of the Laurent expansions near their essential (zero) singularities

in order to get finite results (**Renormalization program)**

$$\lambda \to \infty, \quad \alpha \to 0$$

# Solution

$$D_{\mu\nu}(q;\Delta^2) = D_{\mu\nu}^{INP}(q;\Delta_R^2) + D_{\mu\nu}^{PT}(q)$$

$$D_{\mu\nu}^{INP}(q;\Delta_R^2) = iT_{\mu\nu}(q)d^{INP}(q^2;\Delta_R^2)\frac{1}{q^2}, \qquad d^{INP}(q^2;\Delta_R^2) = \frac{\Delta_R^2}{q^2}$$

$$D_{\mu\nu}^{PT}(q) = i\left\{T_{\mu\nu}(q)d^{PT}(q^2) + \xi L_{\mu\nu}(q)\right\}(1/q^2)$$

$$d^{PT}(q^2) = \frac{\alpha_s}{1+\alpha_s b\ln(q^2/\Lambda_{QCD}^2)} = \frac{1}{b\ln(q^2/\Lambda_{QCD}^2)},\ q^2 \to \infty,\ AF$$

$$\alpha_s(m_Z) = 0.1184,\ \Lambda_{QCD}^2 = 0.09\ GeV^2,\ b = (11/4\pi)\ for\ YM$$

# Some remarks

$$\Lambda^2_{INP} \xleftarrow[\infty \leftarrow \lambda]{\infty \leftarrow \alpha_s(\lambda)} \Delta^2(\lambda, \alpha_s(\lambda)) \xrightarrow[\lambda \to \infty]{\alpha_s(\lambda) \to 0} \Lambda^2_{PT},$$

$$\Lambda^2_{INP} \equiv \Delta^2_R, \quad \Lambda^2_{PT} \equiv \Lambda^2_{QCD}$$

$$INP\ QCD \Longleftarrow QCD \Longrightarrow PT\ QCD,$$

**Intrinsically non-perturbative QCD (INP QCD) confines gluons (no free gluons, dressed gluons are suppressed in incoming and outgoing states)**

**Perturbative QCD (PT QCD) is asymptotically free**

# VACUUM ENERGY DENSITY (VED) IN THE QUANTUM YM THEORY

**The vacuum of QCD is a very complicated confining medium. Its dynamical and topological complexity means that its structure can be organized at various levels: classical and quantum, dynamical and topological. It is mainly NP by origin, character and magnitude, since the corresponding coupling constant is large. However, the virtual gluon field configurations and excitations of the PT origin, character and magnitude, due to AF, are also present there.**

# The bag constant I

$$B = VED^{PT} - VED$$

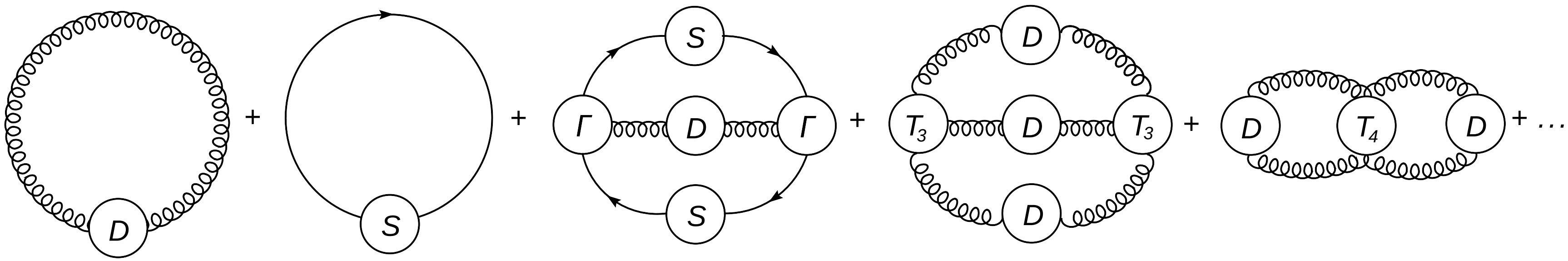
$$B = VED^{PT} - VED = VED^{PT} - [VED - VED^{PT} + VED^{PT}] =$$

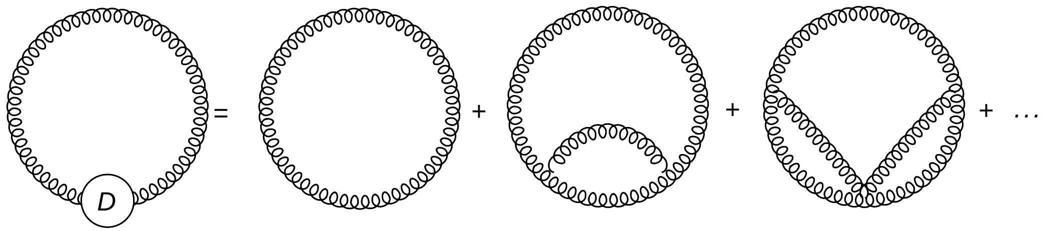
$$VED^{PT} - [VED^{INP} + VED^{PT}] = -VED^{INP} > 0$$

**The formalism which makes it possible to calculate the VED from first principles is the effective potential approach for composite operators**

**J.M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D, 10 (1974) 2428.**

# The VED




$$V(D) = \frac{i}{2}\int \frac{d^4q}{(2\pi)^4} Tr\left\{\ln(D_0^{-1}D) - (D_0^{-1}D) + 1\right\}, \quad V(D_0) = 0$$

# The bag constant II

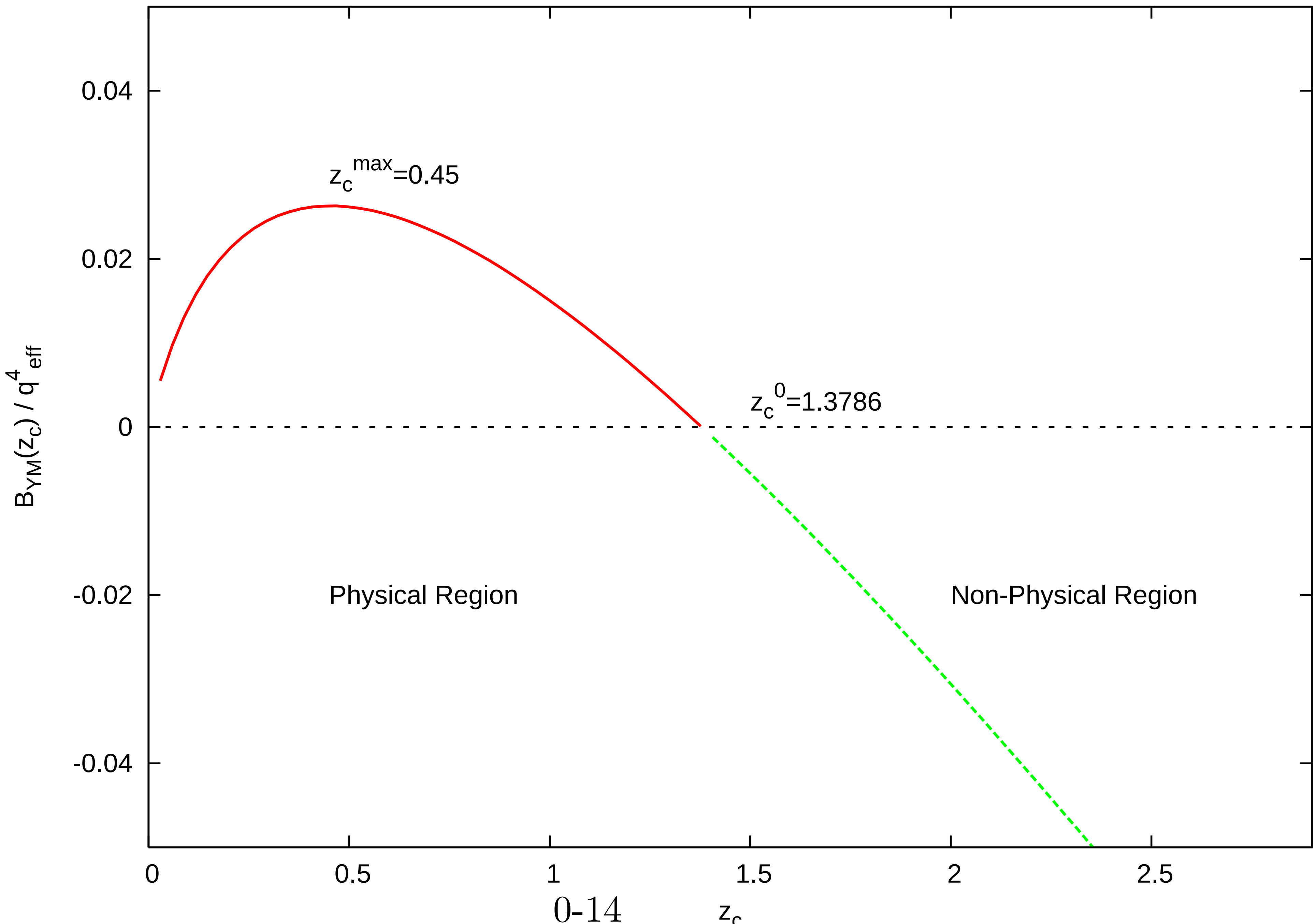
$$B_{YM} = -\epsilon_{YM}$$

$$= \frac{1}{\pi^2} \int_0^{q^2_{eff}} \; dq^2 \; q^2 \left[ \ln[1 + 3\alpha^{INP}(q^2)] - \frac{3}{4}\alpha^{INP}(q^2) \right],$$

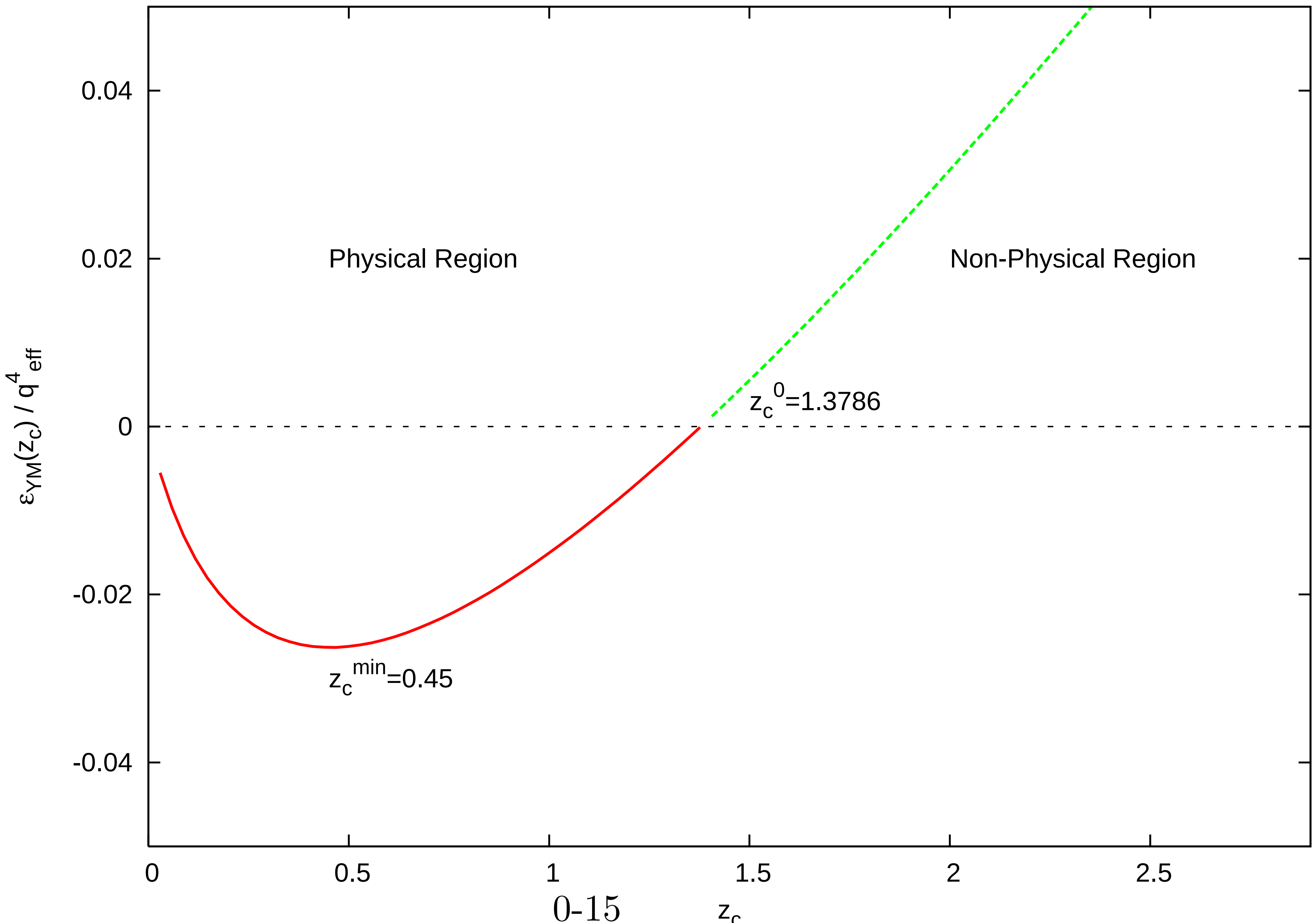
$$d^{INP}(q^2) \equiv \alpha^{INP}(z) = \frac{z_c}{z}, \quad z = \frac{q^2}{q^2_{eff}} \quad z_c = \frac{\Delta_R^2}{q^2_{eff}}$$

$$B_{YM} = q^4_{eff} \times \Omega_{YM},$$

$$\Omega_{YM} = \frac{1}{\pi^2} \int_0^1 \; dz \; z \left[ \ln[1 + 3\alpha_s^{INP}(z)] - \frac{3}{4}\alpha_s^{INP}(z) \right].$$



$z_c^{max}$=0.45
$z_c^0$=1.3786
Physical Region
Non-Physical Region
0.04
0.02
0
-0.02
-0.04
0
0.5
1
1.5
2
2.5
$B_{YM}(z_c) / q^4_{eff}$
$z_c$



0.04
0.02
0
-0.02
-0.04
0
0.5
1
1.5
2
2.5
Physical Region
Non-Physical Region
$z_c^0$=1.3786
$z_c^{min}$=0.45
$\varepsilon_{YM}(z_c) / q^4_{eff}$
$z_c$

$$\Omega_{YM}(z_c)/\partial z_c = 0$$

$$q^2_{eff} = (z_c^{max})^{-1}\Delta_R^2 = 2.2\Delta_R^4$$

$$B_{YM} = -\epsilon_{YM} = 0.1273 \times \Delta_R^4 = 0.0263\ GeV^4$$

**General properties of the bag constant being:**

- **colorless (color-singlet);**
- **electrically neutral;**
- **transversal, i.e., depending only on "physical" degrees of freedom of gauge bosons;**

- **the explicit gauge invariance;**
- **uniqueness, i.e., it is free of all the types of the PT contributions now;**
- **finiteness;**
- **positiveness;**
- **no imaginary part (stable vacuum);**
- **existence of the stationary state for the corresponding YM energy density (negative pressure);**
- **the final dependence on the mass gap only;**
- **a good numerical agreement with phenomenology (gluon condensate).**

# Contribution of $B_{YM}$ to the dark energy problem

**Bag constant may also contribute to the so-called dark energy density. At least, from the qualitative point of view it satisfies almost all the criteria necessary for the dark energy/matter candidate.**

**From the quantitative numerical point of view it is also much better than the estimate from the Higgs field's contribution to the VED**

**by S. Weinberg, arXiv:astroph/9610044**

$$\varrho_H \sim 10^8 \ GeV^4.$$

$$\varrho_{our} \sim 10^{-2} \ GeV^4$$

$$\varrho_{vac}^{exp} \sim 10^{-46}\ GeV^4$$

**WMAP Collaboration, E. Komatsu et al., Astrophys. J. Suppl. 180 (2009) 330**

**So relatively to the value inferred from the cosm. const. (experimental value**

$$\varrho_H/\varrho_{vac}^{exp} \sim 10^{54},$$

$$\varrho_{our}/\varrho_{vac}^{exp} \sim 10^{44}$$

# Numerical value in different units

$$B_{YM} = 0.0263\ GeV^4$$

$$= 3.4\ GeV/fm^3$$

$$= 3.4 \times 10^{39}\ GeV/cm^3.$$

$$1\ GeV = 1.6 \times 10^{10} J = 4.45 \times 10^{-23} GWh,$$

$$1W = 10^{-3} kW = 10^{-6} MW = 10^{-9} GW$$

$$B_{YM} \sim 10^{17}\ GWh/cm^3$$

$$E_{YM} = B_{YM}\ cm^3 \sim 10^{17}\ GWh$$

**Total production of primary energy of the 25 EU countries in 2004 was**

$$E_t \sim 10^7\ GWh.$$

**The number is taken from "Energy FOR THE FUTURE", a position paper of the EPS, www.eps.org**

**Approximately** $1/3$ **of this energy was produced by nuclear power plants**

# Energy from the QCD vacuum

**The bag constant is the energy density of the purely transversal severely infrared singular virtual gluon field configurations which are not only stable (no imaginary part), but are being in the stationary state as well, i.e., in the state with the minimum of energy. That is why it makes sense to discuss the "releasing" of the bag energy $E_{YM}$ from the vacuum.**

$$E_{vac} = -B_{YM}\ V = -E_{YM}\frac{V}{cm^3} \sim -\lambda^3, \quad \lambda \to \infty$$

**Let us imagine now that we can release the finite portion $E_{YM}$ from the vacuum in $k$ different places (different "vacuum energy releasing facilities" (VERF)).**

**It can be done by $n_m$ times in each place, where $m = 1, 2, 3 ... k$. Then the releasing energy $E_r$ becomes**

$$E_r = E_{YM} \sum_{m=1}^{k} n_m = E_{YM}(n_1 + n_2 + n_3 + ... + n_k)$$

$$E_r = E_{YM} \times \lim_{(k,n_m)\to\infty} \sum_{m=1}^{k} n_m \sim \lambda^2, \quad \lambda \to \infty$$

$$E_R = E_{vac} - E_r = E_{vac}[1 + O(1/\lambda)], \quad \lambda \to \infty,$$

**i.e., the QCD vacuum is infinite and permanent reservoir of energy. The only problem is how to release the finite portion – the bag energy, and whether**

**it will be profitable or not by introducing some type of cyclic process.**

**"Perpetuum mobile" does not exist, but "perpetuum source" of energy does exist, and it is the QCD ground state.**

**V. Gogokhia, G.G. Barnafoldi, "The Mass Gap and its Application" (Word Scientific, 2012)**

**Scientific problem what amount of energy can be released is resolved**

**Technological problem how to release is not resolved?**

**Engineering roblem how to built VERF is not resolved ?**

# Transversality of the full gluon self-energy

$$\Pi_{\rho\sigma}(q;D) = \Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q;D) + \Pi^t_{\rho\sigma}(D)$$

$$\Pi^g_{\rho\sigma}(q;D) = \Pi^{gh}_{\rho\sigma}(q) + \Pi^{(1)}_{\rho\sigma}(q;D^2) + \Pi^{(2)}_{\rho\sigma}(q;D^4) + \Pi^{(2')}_{\rho\sigma}(q;D^3).$$

$$q_\rho\Pi_{\rho\sigma}(q;D) = q_\rho\Pi^q_{\rho\sigma}(q) + q_\rho\Pi^g_{\rho\sigma}(q;D) + q_\rho\Pi^t_{\rho\sigma}(D)$$

## The quark contribution

**The color currents conservation condition implies**

$$q_\rho \Pi^q_{\rho\sigma}(q) = q_\sigma \Pi^q_{\rho\sigma}(q) = 0$$

$$\Pi^{q(s)}_{\rho\sigma}(q) = \Pi^q_{\rho\sigma}(q) - \Pi^q_{\rho\sigma}(0) = \Pi^q_{\rho\sigma}(q) - \delta_{\rho\sigma}\Delta^2_q$$

$$\Pi^q_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^q_t(q^2) + q_\rho q_\sigma \Pi^q_l(q^2),$$

$$\Pi^{q(s)}_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^{q(s)}_t(q^2) + q_\rho q_\sigma \Pi^{q(s)}_l(q^2).$$

$$\Pi_l^q(q^2) = \Pi_l^{q(s)}(q^2) + \frac{\Delta_q^2}{q^2},$$

$$\Pi_t^q(q^2) = \Pi_t^{q(s)}(q^2) + \frac{\Delta_q^2}{q^2}$$

$$\Pi_l^q(q^2) = \Pi_l^{q(s)}(q^2) + \frac{\Delta_q^2}{q^2} = 0$$

$$\Pi_l^{q(s)}(q^2) = -\frac{\Delta_q^2}{q^2}$$

$$\Delta_q^2 = 0, \quad \Pi_l^q(q^2) = \Pi_l^{q(s)}(q^2) = 0, \quad \Pi_t^q(q^2) = \Pi_t^{q(s)}(q^2)$$

$$\Pi_{\rho\sigma}^q(q) = \Pi_{\rho\sigma}^{q(s)}(q) = T_{\rho\sigma}(q) q^2 \Pi_t^{q(s)}(q^2).$$

# QED

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi, \quad q_\rho \Pi_{\rho\sigma}(q) = q_\sigma \Pi_{\rho\sigma}(q) = 0$$

$$j^1_\mu(q) D_{\mu\nu}(q) j^2_\nu, \quad j^1_\mu(q) q_\mu = j^1_\nu(q) q_\nu = 0$$

$$D(q) = D^0(q) + D_0(q) i\Pi^s(q) D(q), \quad \Pi(q) \to \Pi^s(q) \sim O(q^2)$$

$$D(q) = \frac{D^0(q)}{1 - i\Pi^s(q) D_0(q)} =$$

$$D^0(q) + D_0(q) i\Pi^s(q) D_0(q) + D_0(q) i\Pi^s(q) D_0(q) i\Pi^s(q) D_0(q) + \ldots$$

## The gluon contribution

$$q_\rho \Pi^g_{\rho\sigma}(q;D) =$$

$$q_\rho \Big[ \Pi^{gh}_{\rho\sigma}(q) + \Pi^{(1)}_{\rho\sigma}(q;D^2) + \Pi^{(2)}_{\rho\sigma}(q;D^4) + \Pi^{(2')}_{\rho\sigma}(q;D^3) \Big] = 0,$$

$$\Pi^{g(s)}_{\rho\sigma}(q;D) = \Pi^g_{\rho\sigma}(q;D) - \Pi^g_{\rho\sigma}(0;D) = \Pi^g_{\rho\sigma}(q;D) - \delta_{\rho\sigma}\Delta^2_g(D)$$

$$\Delta^2_g(D) \equiv \Pi^g(0;D) = \sum_a \Pi_a(0;D) = \sum_a \Delta^2_a(D), \quad a = gh, (1), (2), (2')$$

$$\Delta^2_g(D) \equiv \Delta^2_g(\lambda, \alpha; D)$$

$$\Pi^g_{\rho\sigma}(q;D) = T_{\rho\sigma}(q)q^2\Pi^g_t(q^2;D) + q_\rho q_\sigma \Pi^g_l(q^2;D)$$

$$\Pi^{g(s)}_{\rho\sigma}(q;D) = T_{\rho\sigma}(q)q^2\Pi^{g(s)}_t(q^2;D) + q_\rho q_\sigma \Pi^{g(s)}_t(q^2;D)$$

$$\Pi^g_t(q^2;D) = \Pi^{g(s)}_t(q^2;D) + \frac{\Delta^2_g(D)}{q^2}.$$

$$\Pi^g_l(q^2;D) = \Pi^{g(s)}_l(q^2;D) + \frac{\Delta^2_g(D)}{q^2}.$$

$$\Pi_l^g(q^2;D) = \Pi_l^{g(s)}(q^2;D) + \frac{\Delta_g^2(D)}{q^2} = 0$$

$$\Pi_l^{g(s)}(q^2;D) = -\frac{\Delta_g^2(D)}{q^2}$$

$$\Delta_g^2(D) = 0, \quad \Pi_l^g(q^2;D) = \Pi_l^{g(s)}(q^2;D) = 0, \quad \Pi_t^g(q^2;D) = \Pi_t^{g(s)}(q^2;D)$$

$$\Pi_{\rho\sigma}^g(q;D) = \Pi_{\rho\sigma}^{g(s)}(q;D) = T_{\rho\sigma}(q)q^2\Pi_t^{g(s)}(q^2;D).$$

## The tadpole term contribution

$$q_\rho \Pi^{g}_{\rho\sigma}(q;D) = q_\rho \Pi^{t}_{\rho\sigma}(D) = q_\rho \delta_{\rho\sigma} \Delta_t^2(D) = q_\sigma \Delta_t^2(D) \neq 0$$

$$\Pi_{\rho\sigma}(q;D) = T_{\rho\sigma}(q) q^2 \Pi_t^f(q^2;D) + q_\rho q_\sigma \Pi_l^f(q^2;D)$$

$$\Pi_l^f(q^2;D) = \frac{\Delta_t^2(D)}{q^2}$$

$$\Pi_{\rho\sigma}(q;D) = T_{\rho\sigma}(q) q^2 \Pi_t^f(q^2;D) + L_{\rho\sigma} \Delta_t^2(D)$$

$$\Pi_{\rho\sigma}(q;D) = \Pi^{q}_{\rho\sigma}(q) + \Pi^{g}_{\rho\sigma}(q;D) + \delta_{\rho\sigma}\Delta^2_t(D)$$

$$q^2\Pi^f_t(q^2;D) = q^2[\Pi^{q(s)}_t(q^2) + \Pi^{g(s)}_t(q^2;D)] + \Delta^2_t(D)$$

$$\Pi_{\rho\sigma}(q;D) = T_{\rho\sigma}(q)[q^2\Pi(q^2;D) + \Delta^2_t(D)] + L_{\rho\sigma}\Delta^2_t(D)$$

$$\Pi(q^2;D) = [\Pi^{q(s)}_t(q^2) + \Pi^{g(s)}_t(q^2;D)].$$

$$\Delta^2_t(D) = \Delta^2(D)$$

# ST identity

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi$$

$$D_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d(q^2) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q)i\Pi_{\rho\sigma}(q;D)D_{\sigma\nu}(q) =$$

$$D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q)iT_{\rho\sigma}(q)[q^2\Pi(q^2;D) + \Delta^2(D)]D_{\sigma\nu}(q)$$

$$+D^0_{\mu\rho}(q)iL_{\rho\sigma}(q)\Delta^2(D)D_{\sigma\nu}(q)$$

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi - i\xi^2 \frac{\Delta^2(D)}{q^2}$$

$$\Delta^2(D) = 0$$

$$\Delta^2(D) = 0, \quad \longrightarrow D = D^{PT}$$

$$\Pi_{\rho\sigma}(q; D^{PT}) = T_{\rho\sigma}(q) q^2 \Pi(q^2; D^{PT})$$

$$q_\rho \Pi_{\rho\sigma}(q; D^{PT}) = 0$$

$$q_\mu q_\nu D^{PT}_{\mu\nu}(q) = i\xi,$$

$$D^{PT}_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d^{PT}(q^2) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$D^{PT}_{\mu\nu}(q) = D^{0}_{\mu\nu}(q) + D^{0}_{\mu\rho}(q)iT_{\rho\sigma}(q)q^2\Pi(q^2; D^{PT})D^{PT}_{\sigma\nu}(q)$$

$$d^{PT}(q^2) = \frac{1}{1+\Pi(q^2; D^{PT})}$$

# On the other hand

$$D_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d(q^2) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q)iT_{\rho\sigma}(q)[q^2\Pi^s(q^2;D) + \Delta^2(D)]D_{\sigma\nu}(q)$$

$$+D^0_{\mu\rho}(q)iL_{\rho\sigma}(q)q^2\tilde{\Pi}(q^2;D)D_{\sigma\nu}(q)$$

$$d(q^2) = \frac{1}{1 + \Pi^s(q^2;D) + (\Delta^2(D)/q^2)}$$

$$\tilde{\Pi}(q^2;D) = 0, \quad \rightarrow \quad \Delta^2(D) = 0$$

# Preliminary discussion

**The formal $\Delta^2(D) = 0$ limit is a real way how to preserve the color gauge invaraince/symmetry in QCD. Why does $\Delta^2(D)$ (which is nothing but the tadpole term) exist in this theory at all? There is no doubt that this symmetry should be maintained at non-zero $\Delta^2(D)$ as well.**

**A. The first problem is how to satisfy the ST identity but without going to $\Delta^2(D) = 0$ limit.**

**B. The second problem is how to make the relevant gluon propagator purely transversal, since the ghosts will fail to do this when $\Delta^2(D)$ will be explicitly present.**

# The spurious mechanism

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2\Pi(q^2;D) + \Delta^2(D)]D_{\sigma\nu}(q)$$
$$+ D^0_{\mu\rho}(q) i L_{\rho\sigma}(q)\Delta^2(D) D_{\sigma\nu}(q)$$

$$D^0_{\mu\nu}(q) \to D^0_{\mu\nu}(q;\Delta^2(D)) = D^0_{\mu\nu}(q) + i\xi L_{\mu\nu}(q) d_0(q^2;\Delta^2(D))\frac{1}{q^2}$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2\Pi(q^2;D) + \Delta^2(D)]D_{\sigma\nu}(q)$$
$$+ I_{\mu\nu}(q^2;\Delta^2(D))$$

$$I_{\mu\nu}(q^2;\Delta^2(D)) = I_{\mu\nu}(q^2)$$

$$I_{\mu\nu}(q^2) = i\xi L_{\mu\nu}(q)\left[d_0(q^2;\Delta^2(D)) - \xi[1 + d_0(q^2;\Delta^2(D)]\frac{\Delta^2(D)}{q^2}\right]\frac{1}{q^2}$$

$$d_0(q^2;\Delta^2(D)) = \xi[1 + d_0(q^2;\Delta^2(D))]\frac{\Delta^2(D)}{q^2}$$

$$I_{\mu\nu}(q;\Delta^2(D)) = 0$$

# NP QCD

$$D_{\mu\nu}(q) = D_{\mu\nu}(q; \Delta^2(D)) =$$

$$D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2 \Pi(q^2; D) + \Delta^2(D)] D_{\sigma\nu}(q; \Delta^2(D))$$

$$D_{\mu\nu}(q; \Delta^2(D)) = i\{T_{\mu\nu}(q) d(q^2; \Delta^2(D) + \xi L_{\mu\nu}(q)\} \frac{1}{q^2}$$

$$d(q^2; \Delta^2(D)) = \frac{1}{1 + \Pi(q^2; D) + (\Delta^2(D)/q^2)}.$$

# PT QCD

## formal PT $\Delta^2(D) = 0$ limit

$$D^{PT}_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) q^2 \Pi(q^2; D^{PT}) D^{PT}_{\sigma\nu}(q)$$

$$D^{PT}_{\mu\nu}(q) = i\{T_{\mu\nu}(q) d^{PT}(q^2) + \xi L_{\mu\nu}(q)\} \frac{1}{q^2}$$

$$d^{PT}(q^2) = \frac{1}{1 + \Pi(q^2; D^{PT})}.$$

# The Jaffe-Witten mass gap

$$d(q^2) = 1 - \left[\Pi(q^2; d) + \frac{\Delta^2(d)}{q^2}\right] d(q^2)$$

$$\Delta^2(d) = \Delta^2 c(d)$$

$$\Delta^2 \equiv \Delta^2(\lambda, \alpha, \xi, g^2)$$

$$\Delta_R^2 = Z(\lambda, \alpha, \xi, g^2)\Delta^2(\lambda, \alpha, \xi, g^2).$$

## B. Restoration of transversality of the gluon propagator

$$D_{\mu\nu}^{TNP}(q;\Delta^2) = D_{\mu\nu}(q;\Delta^2) - D_{\mu\nu}(q;\Delta^2=0) = D_{\mu\nu}(q;\Delta^2) - D_{\mu\nu}^{PT}(q)$$

$$D_{\mu\nu}^{TNP}(q;\Delta^2) = iT_{\mu\nu}(q)d^{TNP}(q^2;\Delta^2)\frac{1}{q^2}$$

$$D_{\mu\nu}^{PT}(q) = i\left\{T_{\mu\nu}(q)d^{PT}(q^2) + \xi L_{\mu\nu}(q)\right\}(1/q^2)$$

$$D_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d(q^2) + \xi L_{\mu\nu}(q)\right\}(1/q^2)$$

$$d(q^2;\Delta^2) = \frac{1}{1+\Pi^s(q^2;D) + (\Delta^2/q^2)}$$

$$d^{TNP}(q^2;\Delta^2) = \frac{[q^2\Pi^s(q^2;D^{PT}) - q^2\Pi^s(q^2;D) - \Delta^2]}{[q^2 + q^2\Pi^s(q^2;D) + \Delta^2][1+\Pi^s(q^2;D^{PT})]}$$

$$d^{PT}(q^2) = \frac{1}{1+\Pi^s(q^2;D^{PT})}$$

$$D_{\mu\nu}(q;\Delta^2) = i\left\{T_{\mu\nu}(q)d(q^2;\Delta^2) + \xi L_{\mu\nu}(q)\right\}(1/q^2)$$

$$= -iT_{\mu\nu}(q)d^{PT}(q^2)(1/q^2) + iT_{\mu\nu}(q)d^{PT}(q^2)(1/q^2)$$

$$= D^{TNP}_{\mu\nu}(q;\Delta^2) + D^{PT}_{\mu\nu}(q)$$

# Prescription

$$D_{\mu\nu}(q;\Delta^2) = D_{\mu\nu}^{TNP}(q;\Delta^2) + D_{\mu\nu}^{PT}(q)$$

$$D_{\mu\nu}(q;\Delta^2) \to D_{\mu\nu}^{TNP}(q;\Delta^2) = D_{\mu\nu}(q;\Delta^2) - D_{\mu\nu}^{PT}(q)$$

$$d(q^2;\Delta^2) \to d^{TNP}(q^2;\Delta^2) = d(q^2;\Delta^2) - d^{PT}(q^2).$$

$$D_{\mu\nu}(q;\Delta^2) \to D_{\mu\nu}^{PT}(q) = D_{\mu\nu}(q;\Delta^2) - D_{\mu\nu}^{TNP}(q;\Delta^2)$$

$$d(q^2;\Delta^2) \to d^{PT}(q^2) = d(q^2;\Delta^2) - d^{TNP}(q^2;\Delta^2).$$

# The Bag constant

**One of the important characteristics of the QCD ground state is the Bag constant.**

$$B = VED^{PT} - VED,$$

$VED$ **is the NP but "contaminated" by the PT contributions (i.e., it is a full** $VED$ **like the full gluon propagator).**

$$B = VED^{PT} - VED = VED^{PT} - [VED - VED^{PT} + VED^{PT}] =$$

$$VED^{PT} - [VED^{TNP} + VED^{PT}] = -VED^{TNP} > 0,$$

**Thus the Bag constant is completely free of the PT "contaminations".**

# The SDE for the TNP gluon propagator

$$D^{TNP}_{\mu\nu}(q;\Delta^2) = D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2\Pi^s(q^2;D) - q^2\Pi^s(q^2;D^{PT}) + \Delta^2] D^{PT}_{\sigma\nu}(q)$$

$$+ D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) \left[q^2\Pi^s(q^2;D) + \Delta^2\right] D^{TNP}_{\sigma\nu}(q;\Delta^2)$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)[q^2\Pi^s(q^2;D) + \Delta^2(D)] D_{\sigma\nu}(q)$$

$$D^{PT}_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) q^2\Pi^s(q^2;D^{PT}) D^{PT}_{\sigma\nu}(q),$$

**No free gluons in TNP QCD**

## The necessity of the proposed subtractions

**The first subtraction at the level of the full gluon self-energy makes it possible to introduce the mass gap**

**The second subtraction at the level of the full gluon propagator makes it possible:**

$$D_{\mu\nu}(q;\Delta^2) = D^{TNP}_{\mu\nu}(q;\Delta^2) + D^{PT}_{\mu\nu}(q)$$

**A. To achieve transversality in a gauge invariant way**

**B. Do not affect its true NP structure**
**Exact separation between the NP and PT dynamics**

## C. Exclude the free gluons from the theory

# Conclusions

**The mass gap is generated in the gluon sector of QCD mainly due to the self-interaction of massless gluon modes.**

**It is defined as the difference between the full gluon self-energy and its value at some point, so it is not introduced by hand.**

**No any truncations/approximations/asumptions, no special gauge choice, only algebraic (i.e., exact) derivations have been done.**

**The common belief (coming from PT) that mass gap contradicts the color gauge invariance/symmetry of QCD is false.**

**This fundamental symmetry is maintained/preserved at non-zero mass gap as well.**

**We distinguish between NP and PT QCD by the explicit presence of the mass gap, and not by the strength of the coupling constant. It plays no role when the mass gap is kept "alive".**

**Proposed subtraction makes it possible to make the relevant gluon propagator to become purely transversal in a gauge invariant way and to remove free gluons from the theory at the same time.**

**So unitarity of the $S$-matrix in TNP QCD is maintained.**

**No free gluons in TNP QCD.**

**Emission and absorbtion of "dressed" gluons will be**

**suppressed by the renormalization of the mass gap.**

# QED vs QCD

**QED. We cannot release the mass gap from the QED vacuum, while we can release the photons and the electron-positron pars from it.**

**QCD. We can release the mass gap from the QCD vacuum, as it has been described in this talk. But we cannot release the gluons and the quarks/antiquarks from its vacuum because of the color confinement phenomenon.**

**The next step is to find a formal solution(s) for the full gluon propagator as a function of the regularized mass gap and its renormalization.**

## Renormalization of the mass gap

**1. The two independent types of general solutions**

**(a). Smooth (Massive)**

**(b). Singular (NL iteration)**

**2. Renormalization of the relevant gluon propagator**

**3. Gluon and Quark confinement criteria**

**4. Asymptotic Freedom from the mass gap**

**5. The JW theorem is not completely correctly formulated**

**6. INP QCD $\Longleftarrow$ QCD $\Longrightarrow$ PT QCD**