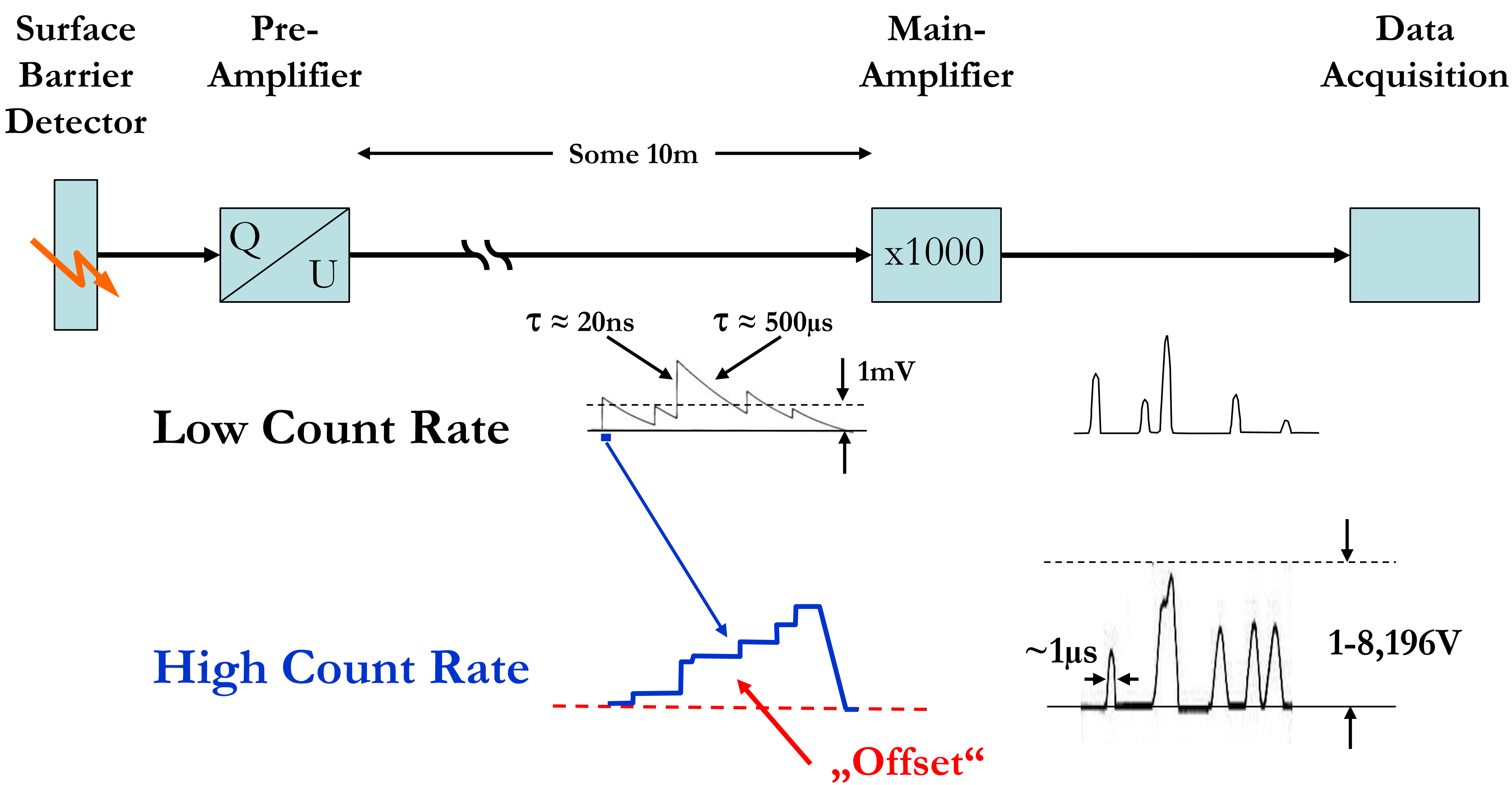
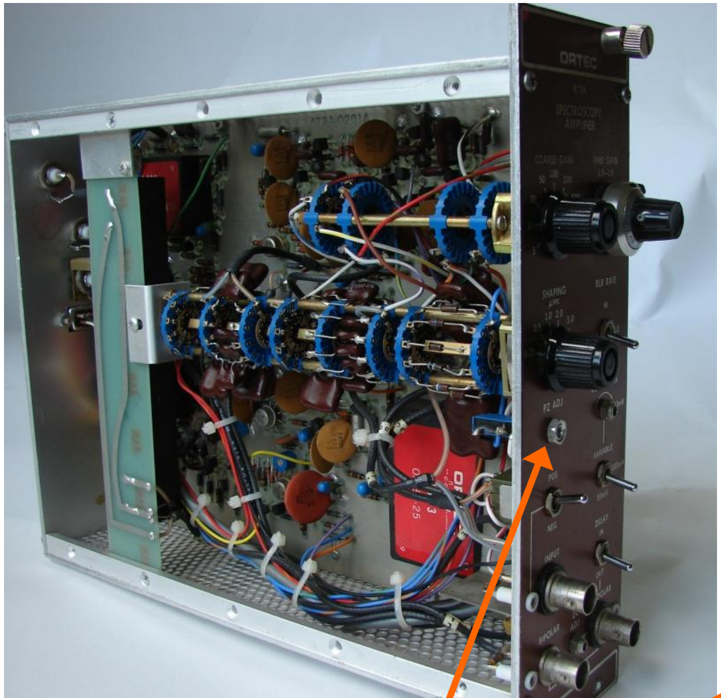
Outline

- **Some Problems from my Diploma Thesis**
- **Understanding Transfer Functions**
- **Why does P/Z-Cancellation Work ?**
- **Why gives a „Bell-Shaped“ Signal the best S/N-Ratio ?**
- **Summary**
- **Questions**
- **Appendices**

Electronics
The Amplifier Chain



Electronics
The Main Amplifier



Potentiometer reads P/Z

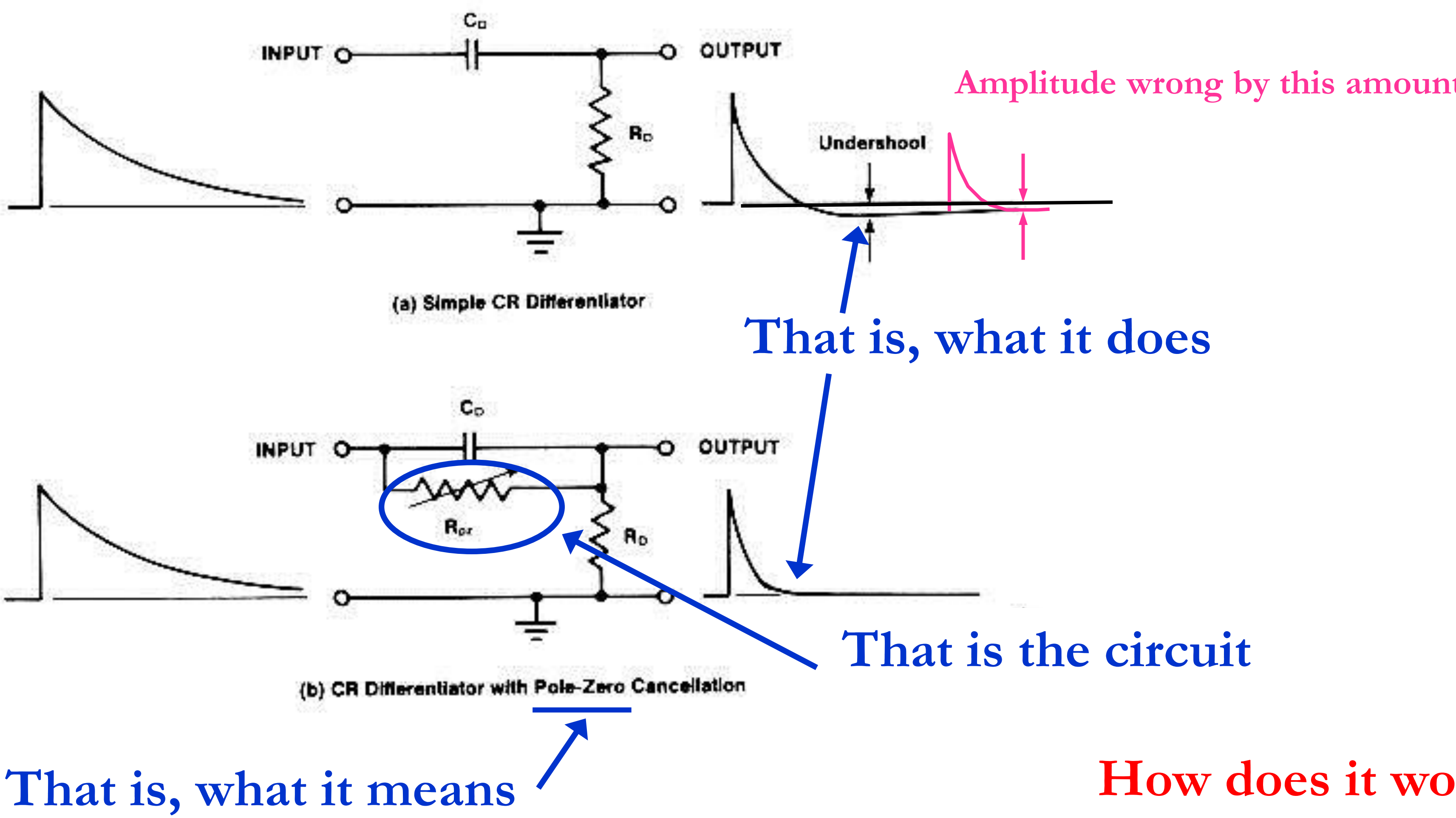
What does it ?

What does it mean ?

How does the circuit look like ?

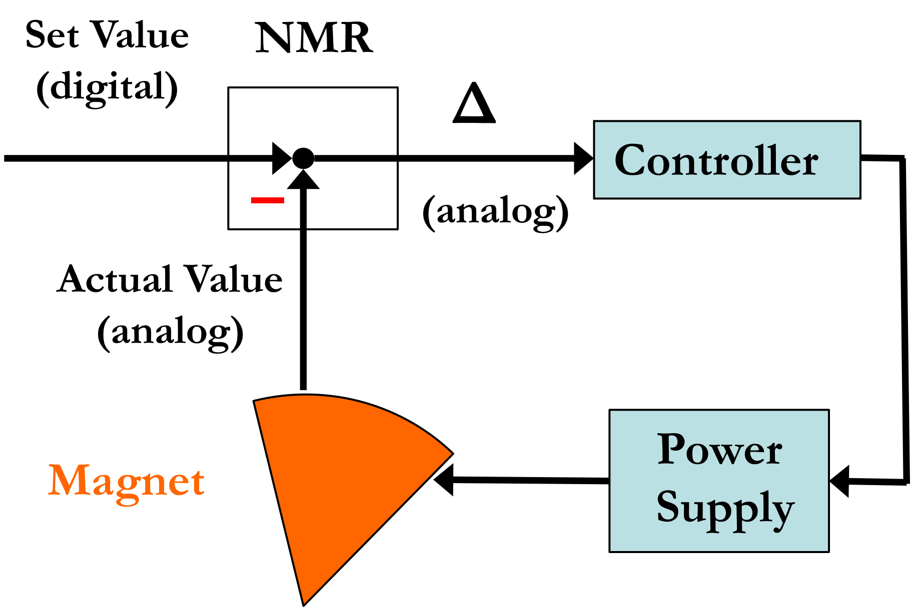
How does it work ?

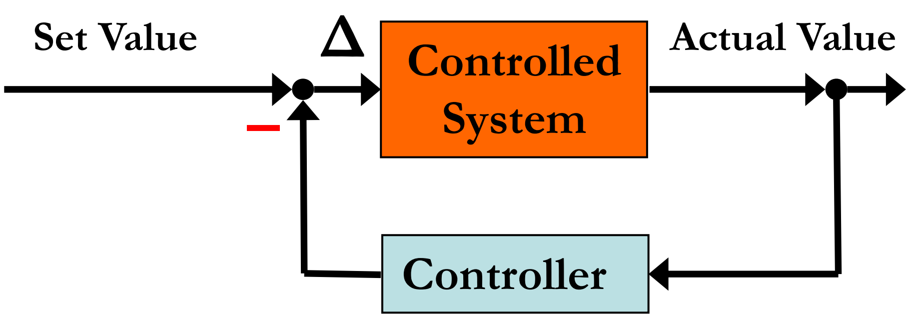
Electronics
The Main Amplifier Input Stage

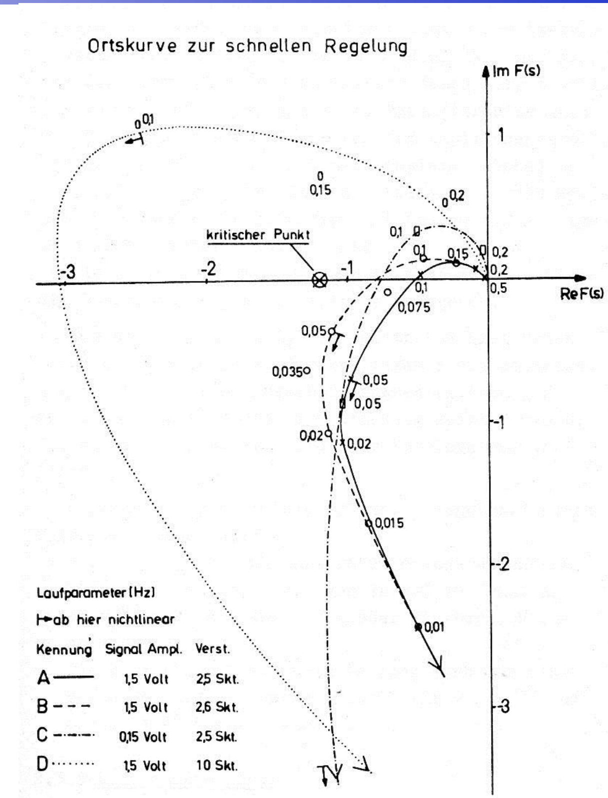


How does it work ?

Electronics
NMR Control of a Bending Magnet to $3 \cdot 10^{-6}$



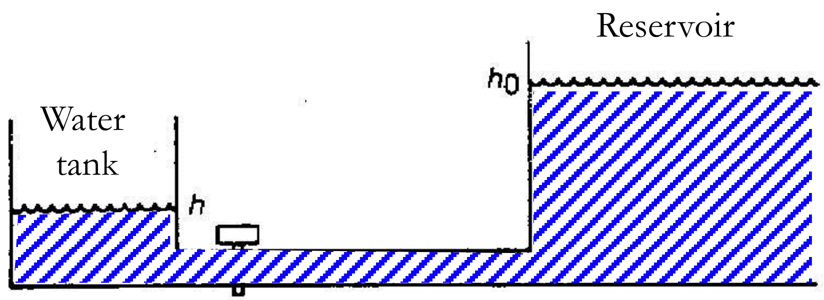


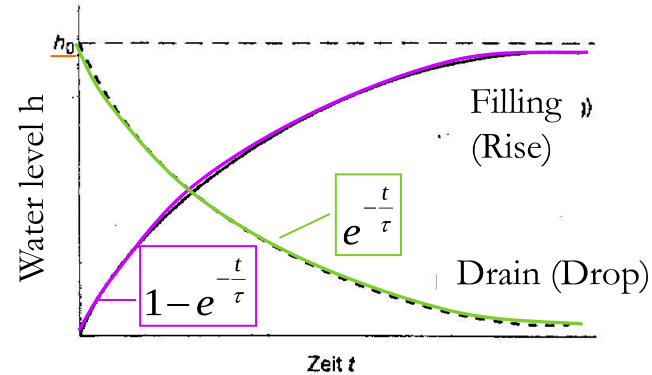


Difficult to interpret

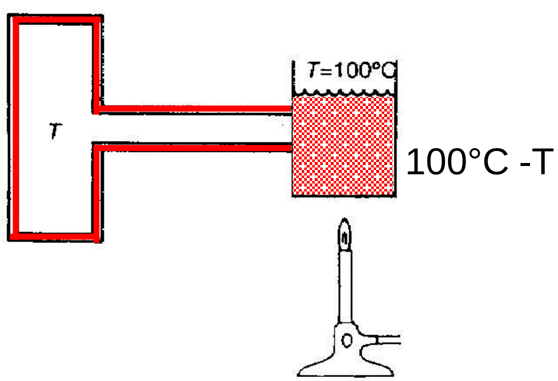
Is there a more compact way to display the dynamics of a system ?

Electronics
Understanding Transfer Functions

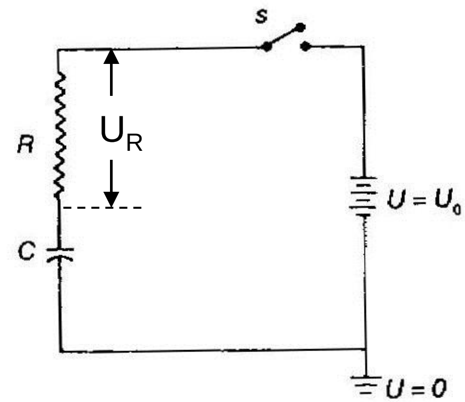




- The water tank is filled from a reservoir of unlimited capacity;
- The water level approaches **exponetially** h_0.



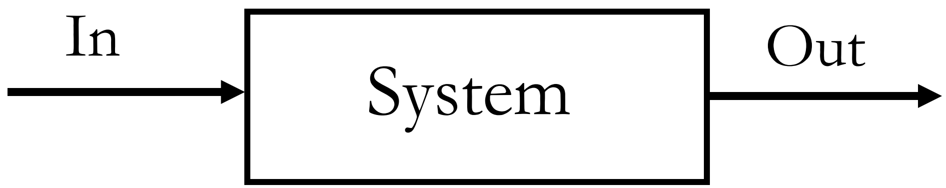
The temperature of a piece of metal approaches exponetially 100°C.



The voltage at the capacitor approaches exponetially U_0

Understanding Transfer Functions

How can the dynamics of a linear system be described most efficiently ?



Refinery
Aeroplane
Musical instrument
Electrical Circuit
Nucleus

Electronics
Understanding Transfer Functions

- Is a „generalized“ Fourier Transform $f(t) \rightarrow F(s)$ with $s=\rho+i\omega$
- Is an integral transform, and therefore linear
- „Algebraizes“ linear differential equations
- A convolution in the time domain corresponds to a multiplication of the corresponding Laplace transforms

Scheme:

$$f(t) \xrightarrow{\mathscr{L}} F(s)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\int_0^t f(u)g(t-u)du \qquad\qquad F(s)\cdot G(s)$$

$$\downarrow \qquad\qquad \downarrow$$

$$h(t) \xleftarrow{\mathscr{L}^{-1}} H(s)$$

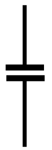
Electronics
Understanding Transfer Functions

F(s)	f(t)	Remark
$a\,F_1(s) + b\,F_2(s)$	$a\,f_1(t) + b\,f_2(t)$	Linearity
$s\,F(s) - f(0)$	$f\,'(t)$	Derivative
$s^n F(s) - s^{(n-1)} f(0) - s^{(n-2)} f'(0) \ldots - f^{(n-1)}(0)$	$f^{(n)}(t)$	n^{th} derivative
$\frac{F(s)}{s}$	$\int_0^t f(u)du$	Integral
$\frac{F(s)}{s^n}$	$\int_0^t \ldots \int_0^t f(u)du^n = \int_0^t \frac{(t-u)^{n-1}}{(n-1)!} f(u)du$	n-fold integral
$F(s) \cdot G(s)$	$\int_0^t f(u)g(t-u)du$	Convolution in the time domain

Understanding Transfer Functions

F(s)	f(t)
$\frac{1}{s}$	1
$\frac{1}{s^2}$	t
$\frac{1}{s^n} \quad n = 1,2,3,\ldots$	$\frac{t^{n-1}}{(n-1)!}, \quad 0! = 1$
$\frac{1}{s-a}$	e^{at}
$\frac{1}{s^2+a^2}$	$\frac{\sin at}{a}$
$\frac{s}{s^2+a^2}$	$\cos(at)$
$\frac{1}{s^2-a^2}$	$\frac{\sinh at}{a}$
$\frac{P(s)}{Q(s)}; \quad Q(s) = (s-\alpha_1)\ldots(s-\alpha_n)$	$\sum_{k=1}^{n} \frac{P(\alpha_k)}{Q'(\alpha_k)} e^{\alpha_k t}; \quad Q'(\alpha_k) = \frac{dQ(\alpha_k)}{d(s-\alpha_k)}$

Electronics
Understanding Transfer Functions



$Q = C\,U;\ \dot{Q} = I = C\dot{U}$

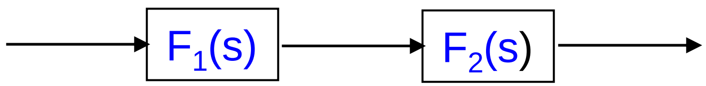
$I(s) = sCU(s)$ $R_C = \frac{U(s)}{I(s)} = \frac{1}{sC}$

$U = L\,\dot{I}$

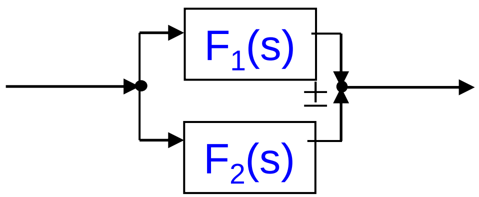
Recall: $s = \varrho + i\omega$

$U(s) = sLI(s)$ $R_L = \frac{U(s)}{I(s)} = sL$

$T(s) \stackrel{\text{def}}{=} \frac{\text{Output}(s)}{\text{Input}(s)}$

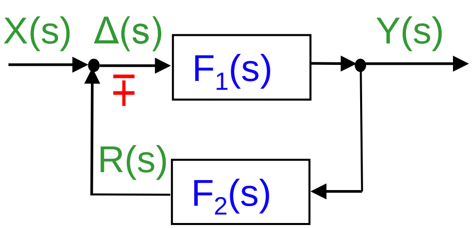


$T(s) = F_1(s) \cdot F_2(s)$ (Convolution)



$T(s) = F_1(s) \pm F_2(s)$ (Linearity)

Feedback (especially: negative feedback)



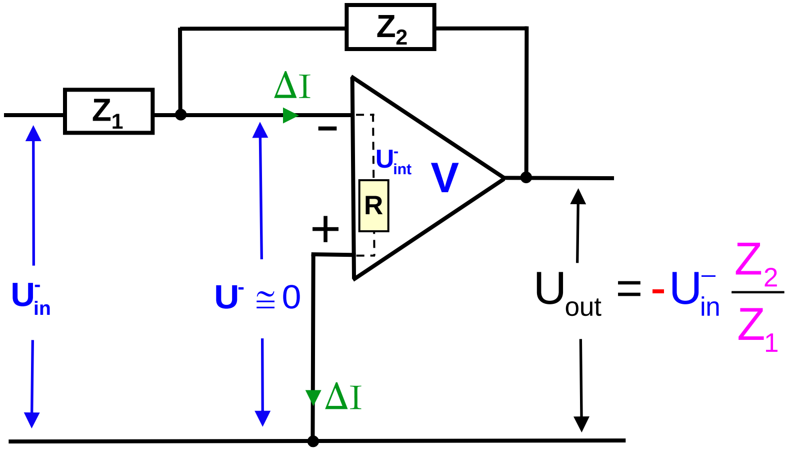
$T(s) = \frac{Y(s)}{X(s)} = \frac{F_1(s)}{1 \pm F_1(s) \cdot F_2(s)}$

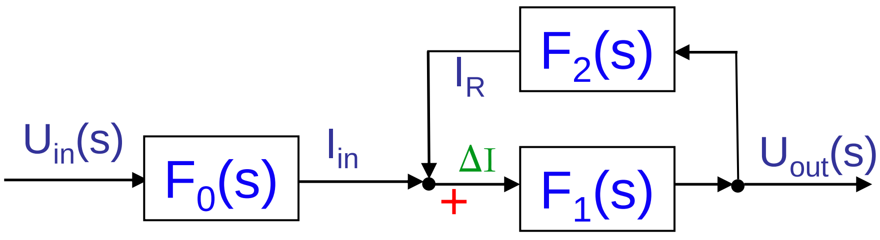
$$\left.\begin{aligned} Y(s) &= F_1(s) \cdot \Delta(s) \\ \Delta(s) &= X(s) \mp R(s) \\ R(s) &= F_2(s) \cdot Y(s) \end{aligned}\right\} \rightarrow Y(s) = F_1(s) \cdot \{X(s) \mp (F_2(s) \cdot Y(s))\}$$

Trick: $F_1(s) \rightarrow \infty \Rightarrow T(s) = \pm \frac{1}{F_2(s)} = \pm F_2^{-1}(s)$

Understanding Transfer Functions

Inverter





$$T(s) = F_0(s)\cdot\frac{F_1(s)}{1 - F_1(s)\cdot F_2(s)} \xrightarrow{F_1(s)\to\infty} \frac{F_0(s)}{-F_2(s)} = -\frac{Z_2}{Z_1}$$

with:

$$F_0(s) = I_{in} / U_{in} = 1/Z_1$$

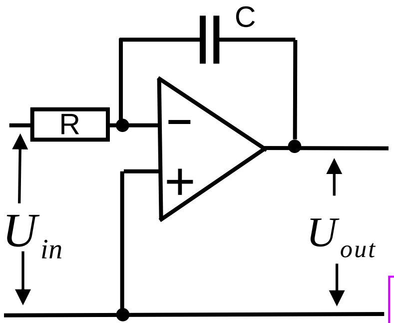
$$F_1(s) = -R\cdot V$$

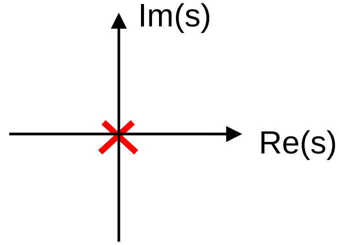
$$F_2(s) = I_R / U_{out} = 1/Z_2$$

; R can be considered to convert the Input Current ΔI to the internal Input Voltage U^{-}_{int}.

Understanding Transfer Functions

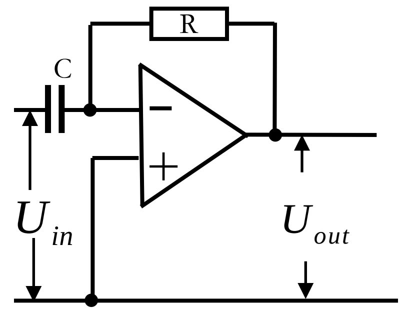
Integrator (U_{off} = 0)

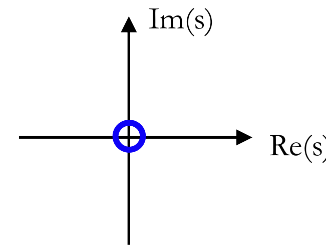




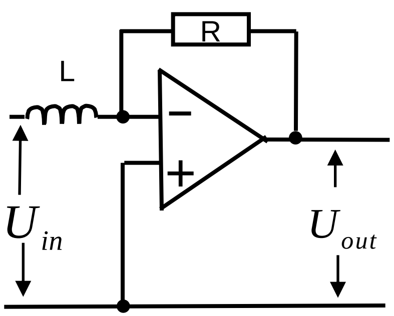
$$U_{out}(s) = -U_{in}(s)\frac{1}{RC}\cdot\frac{1}{s}$$

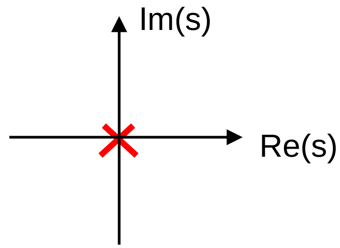
Differentiator





$$U_{out}(s) = -U_{in}(s)\cdot RC\cdot s$$



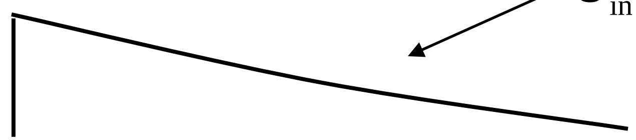


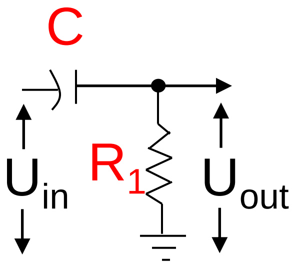
$$U_{out}(s) = -U_{in}(s)\frac{R}{L}\cdot\frac{1}{s}$$

The „s“ in the denominator and nominator determine the dynamics of a system

Understanding Transfer Functions

$$U_{in}(t) = U_0 e^{-\frac{t}{T_0}} \Rightarrow U_{in}(s) = U_0 \frac{1}{s + 1/T_0}$$

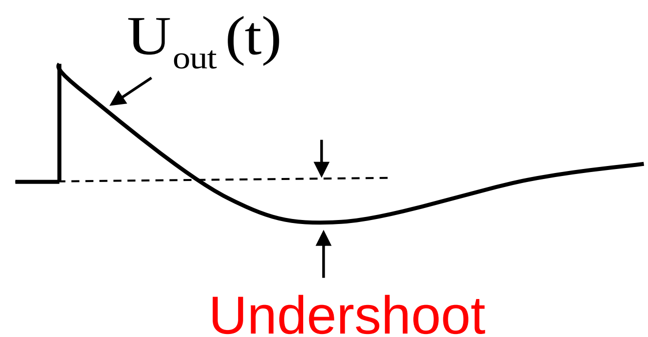




$$T(s) = \frac{U_{out}(s)}{U_{in}(s)} = \frac{R_1}{\frac{1}{sC} + R_1}$$

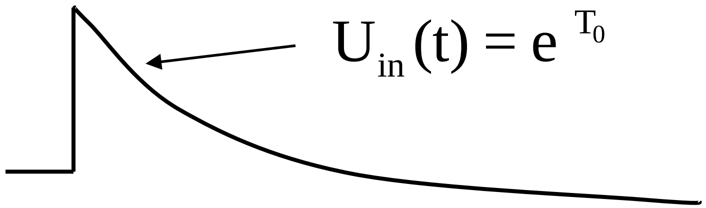
$$U_{out}(s) = T(s) \cdot U_{in}(s) = U_0 \frac{s}{(s + 1/CR_1)(s + 1/T_0)}$$

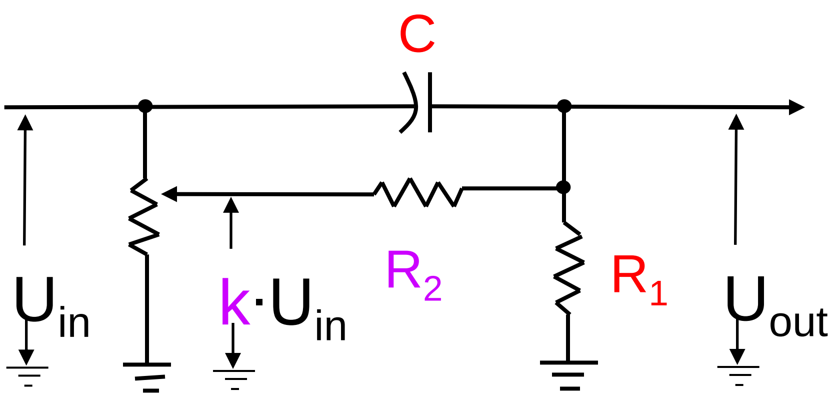
$$U_{out}(t) = U_0 \frac{1}{(CR_1 - T_0)} \left(CR_1 e^{-\frac{t}{T_0}} - T_0 e^{-\frac{t}{CR_1}} \right)$$



Electronics
The Principle of Pole-Zero Cancellation

$$U_{in}(t) = e^{-\frac{t}{T_0}}$$



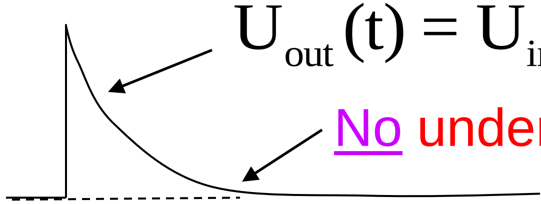


$$R_P = R_1 \parallel R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$T(s) = \frac{s + \frac{k}{R_2 C}}{s + \frac{1}{R_P C}}$$

$$U_{out}(t) = U_{in}(t)$$

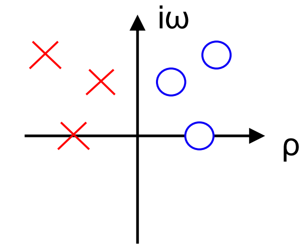
No undershoot



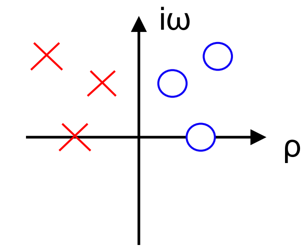
That is , how it works

Understanding Transfer Functions

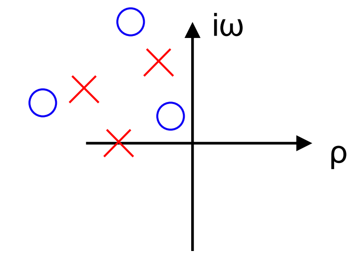
Stable Systems have Poles only in the negative half plane (imaginary axis included), since otherwise ρ>0



Allpasses stand out due to the symmetric position of Poles and Zeros with respect to the imaginary axis. |T(s)| = const.; but the phase changes.

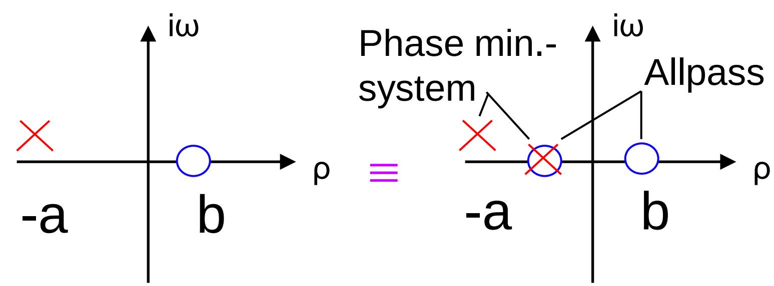


Phase minimum systems do not have any Zeros in the right half-plane.

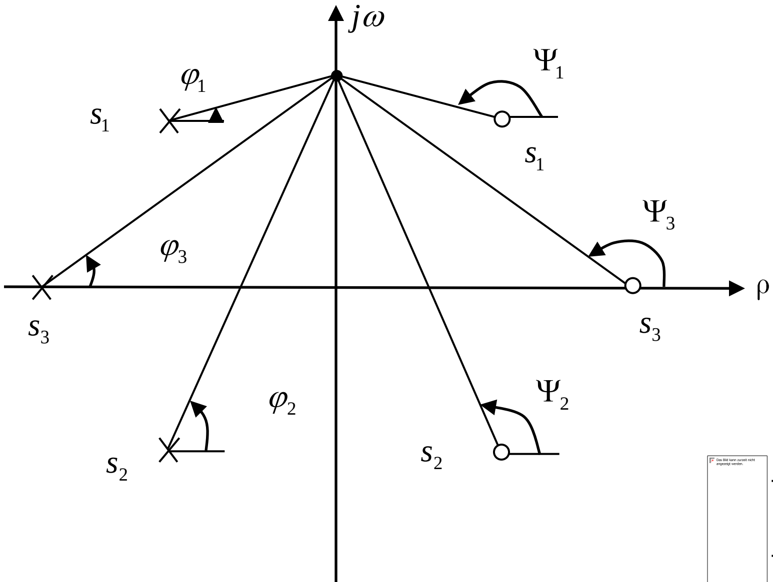


Inferences:

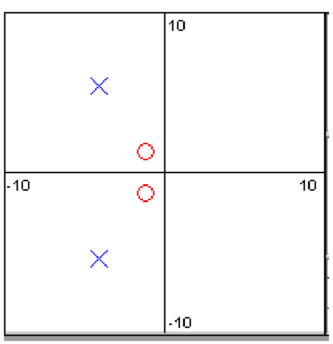
All stable linear systems can be set up by allpasses and phase minimum systems.

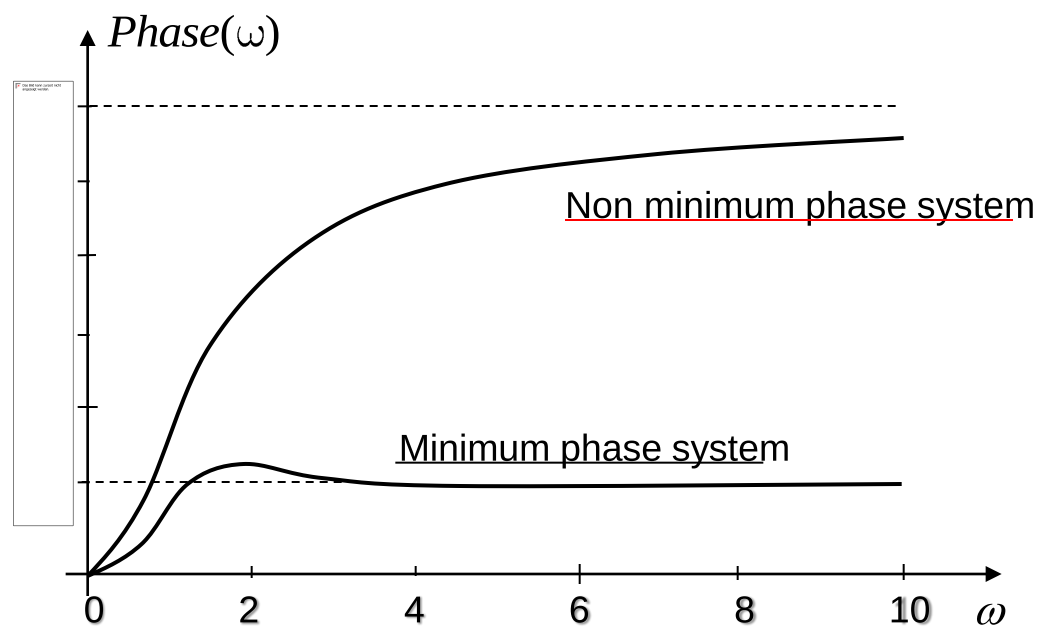


Electronics
PZ-Scheme of an 3rd Order Allpass

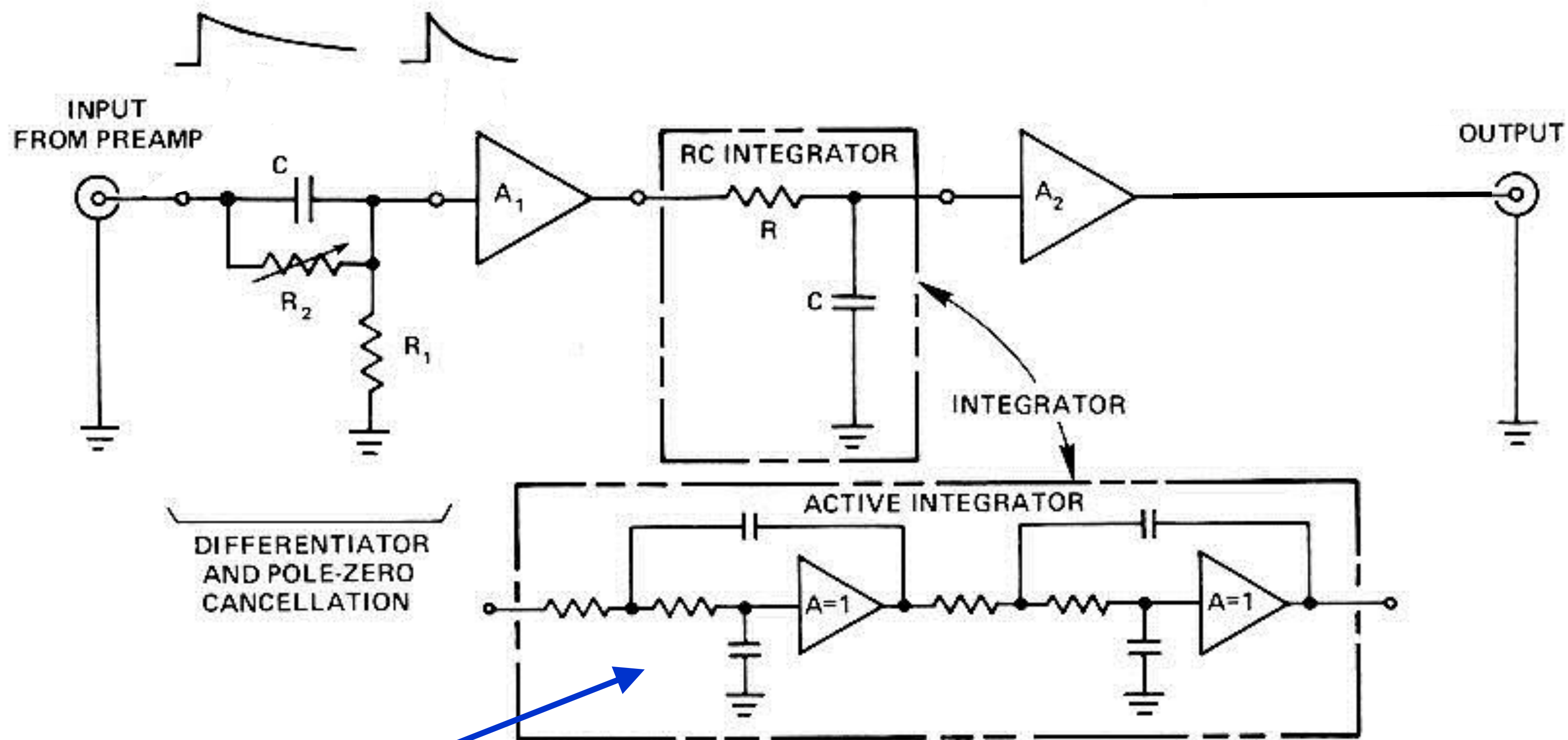


Pole/Zero Applet





Electronics
Pulse Shaping

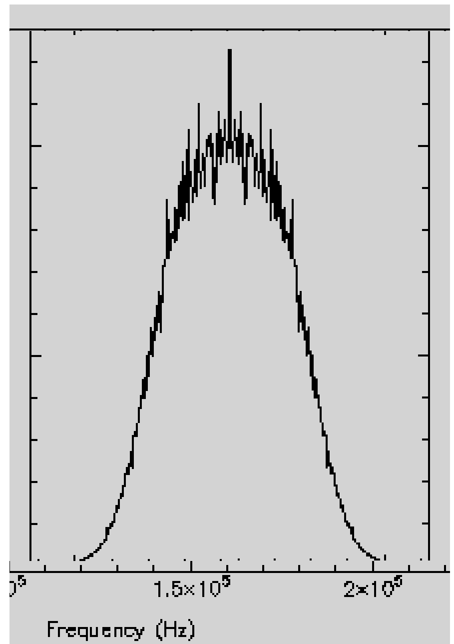


2 Stage Sallen-Key Amplifier

Pulse Shaping in the Semi-Gaussian Shaping Amplifier.

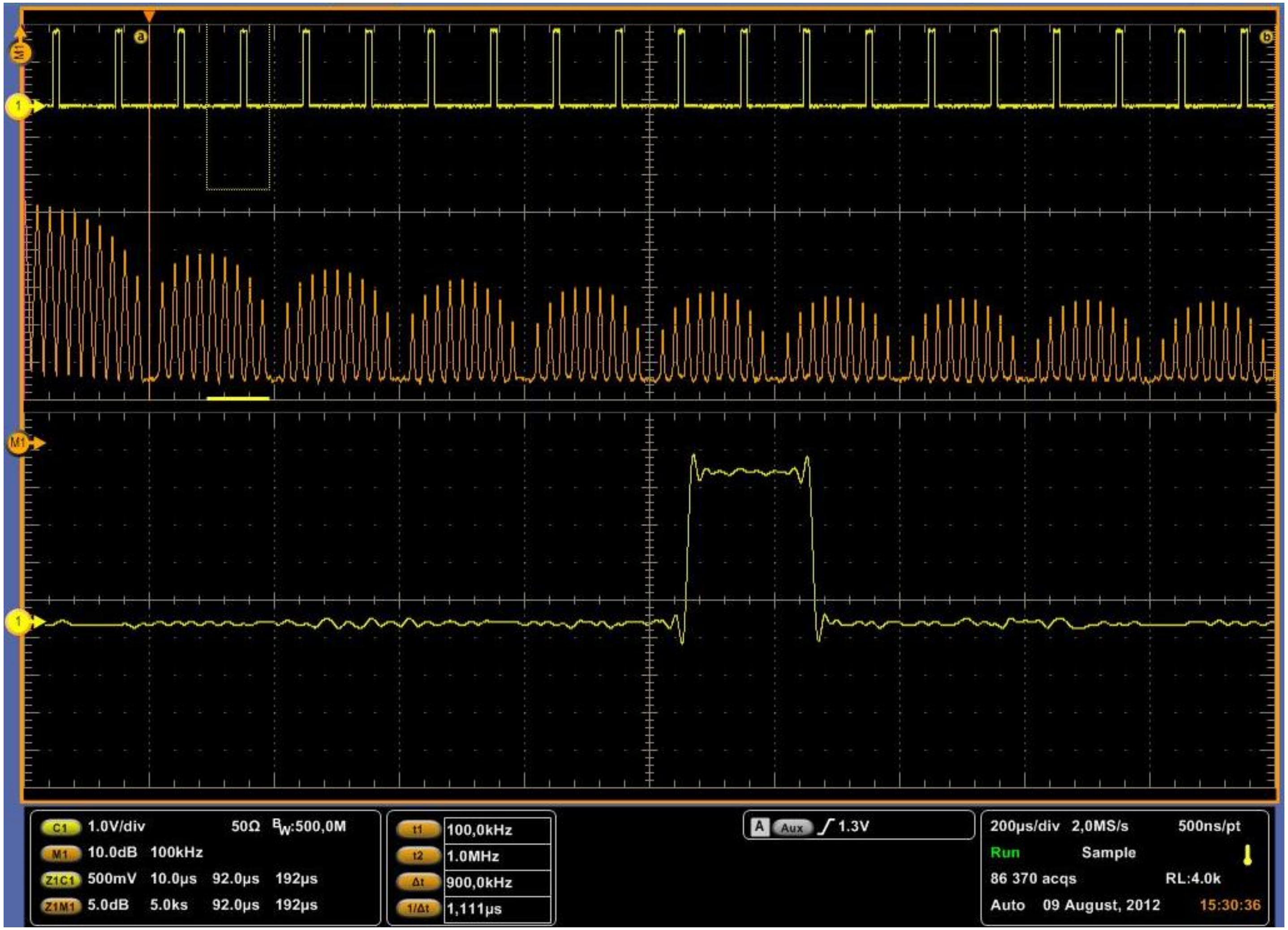
Electronics
The Idea Behind an Optimum S/N-Ratio

Typical (white) Noise Spectrum

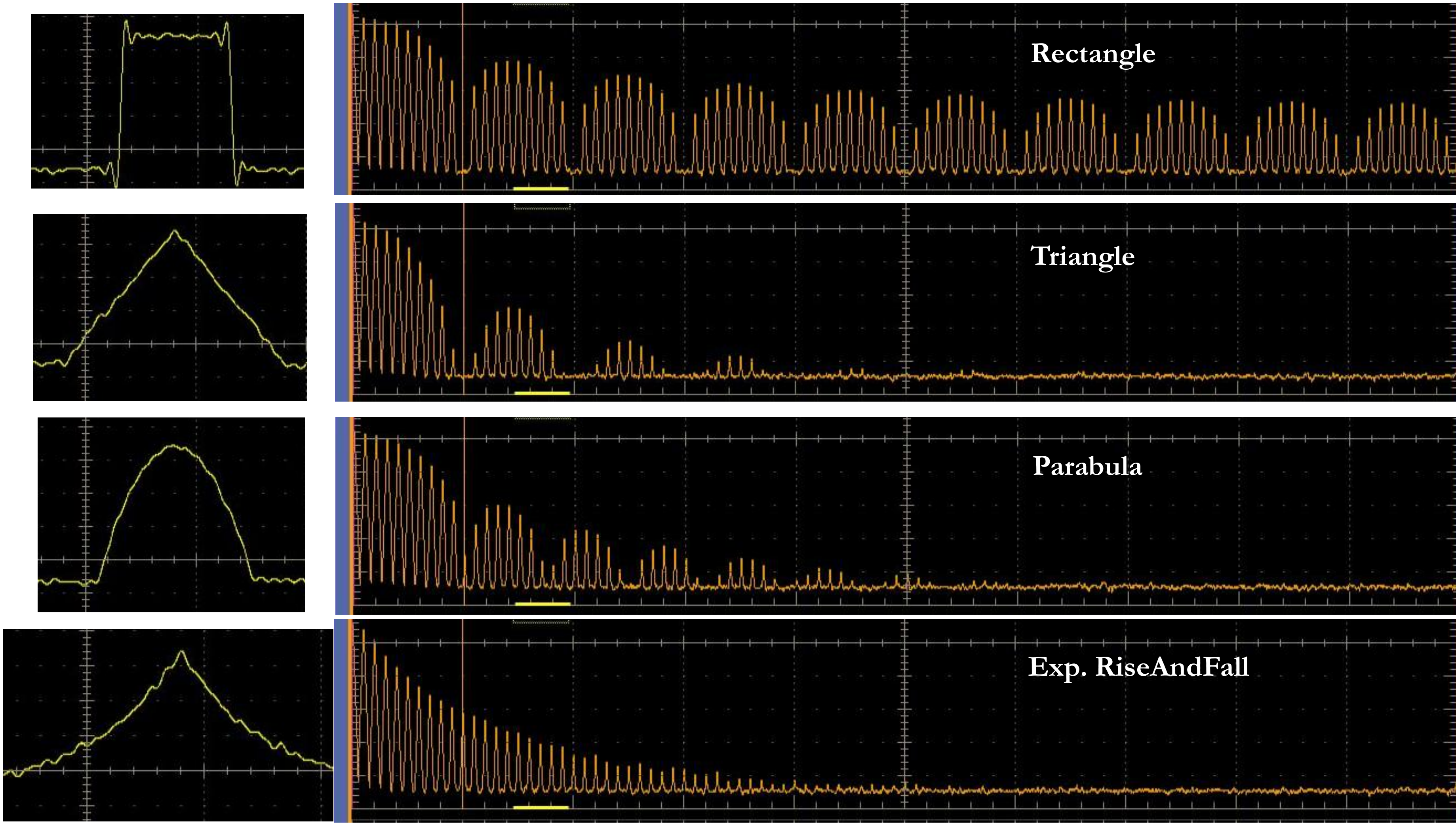


Narrow Bandwidth Noise Spectrum

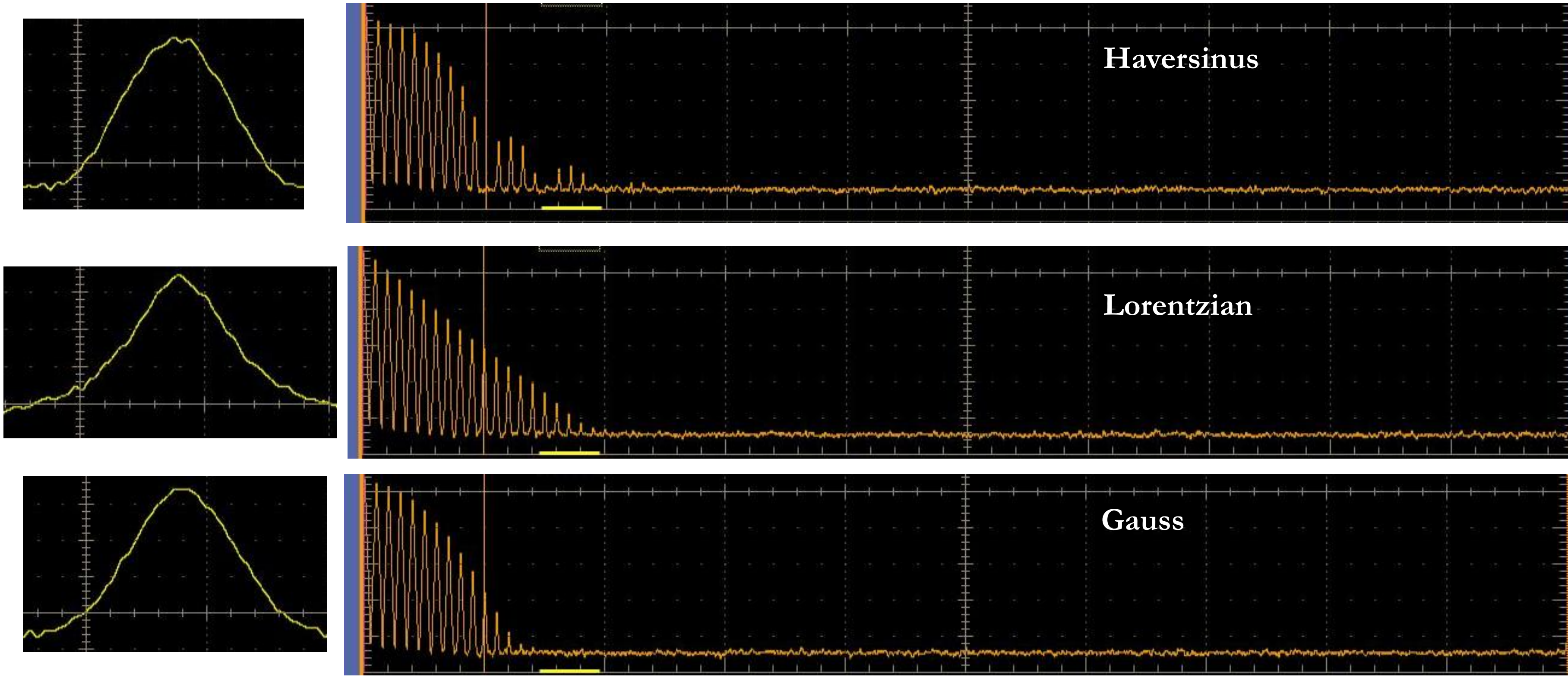
Electronics
Pulse Shaping



Electronics
Pulse Shaping



Electronics
Pulse Shaping



Semi-Gaussian pulse shaping has the fewest harmonics and allows to amplify these pulses with the comparatively smallest bandwidth.

Therefore, all noise outside this bandwidth can be suppressed.

Summary

Transfer Functions:

- Have a wide field of applications
- Are an universal tool to handle even aperiodic signals
- Give a very basic understanding of systems
- Allow to evaluate or predict the dynamical behavior of a system

You can find further informations (and much more):

www.hiskp.uni-bonn.de

↓

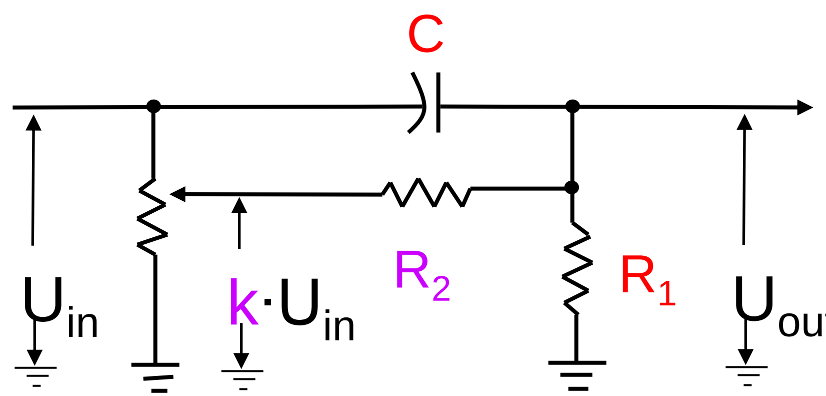
Archive → my lectures

↓

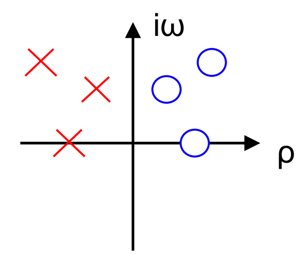
User: student
PW: SSXX
WSXX

Electronics
Questions

— **Give two reasons why the P/Z circuit is termed „principal".**



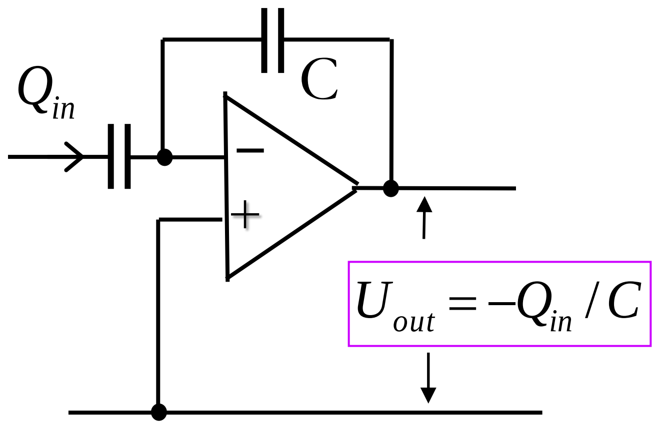
— **Convince yourself „graphically" that the amplitude of an Allpass does not depend on frequency.**



— **Does the „trick" of linearizing (in negative feedback systems) apply only to operational amplifiers ?**

Q=C U

Q/U Converter



The output voltage is changed, until the charge via C compensates Q_{in} from the input

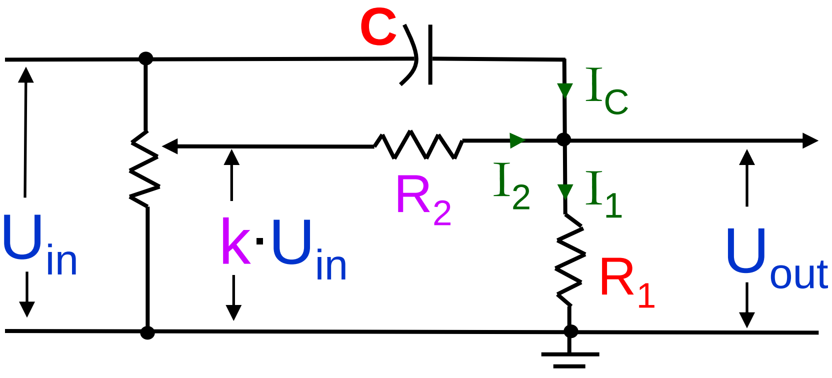
Electronics
Appendix A1 (Backtransform)

$$U_{out}(s) = T(s) \cdot U_{in}(s) = U_0 \frac{s}{(s+1/CR_1)(s+1/T_0)}$$

$$U_{out}(t) = U0 \cdot \left\{ \frac{-1/CR_1}{-1/CR_1 + 1/T_0} e^{-1/CR_1} + \frac{-1/T_0}{-1/T_0 + 1/CR_1} e^{-1/T_0} \right\}$$

$$U_{out}(t) = U0 \cdot \left\{ \frac{-T_0}{-T_0 + CR_1} e^{-1/CR_1} + \frac{-CR_1}{-CR_1 + T_0} e^{-1/T_0} \right\}$$

$$U_{out}(t) = U0 \cdot \frac{1}{CR_1 - T_0} \left\{ CR_1 e^{-1/T0} + T_0 e^{-1/CR1} \right\}$$ q.e.d.



Start with: $I_1 = I_C + I_2$

Recall: $U_{out} = R_1 \cdot I_1 = R_1(I_C + I_2)$

$$U_{out}(s) = R_1 \left\{ \frac{1}{R_2}[k \cdot U_{in}(s) - U_{out}(s)] + C[sU_{in}(s) - sU_{out}(s)] \right\}$$

$$U_{out}(s)\left[1 + \frac{R_1}{R_2} + sCR_1\right] = U_{in}(s)\left[k\frac{R_1}{R_2} + sCR_1\right]$$

$$T(s) = \frac{U_{out}(s)}{U_{in}(s)} = \frac{k\frac{R_1}{R_2} + sCR_1}{1 + \frac{R_1}{R_2} + sCR_1} = \frac{\frac{k}{R_2C} + s}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C} + s} = \frac{s + \frac{k}{R_2C}}{s + \frac{1}{R_pC}} \qquad q.e.d.$$

Appendix A3 (P/Z Circuit Considerations – Reality)

$$T(s) = \frac{s + \frac{k}{R_2 C}}{s + \frac{1}{R_P C}}$$

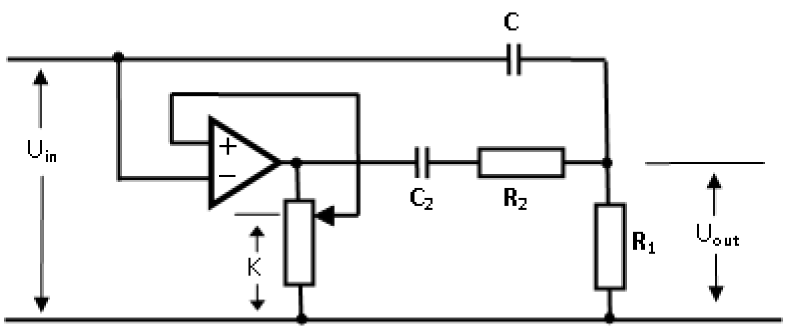
In order to have: $\frac{k}{R_2} = \frac{1}{R_p}$

recall that $R_p \leq R_2$

Therefore: $\frac{k}{R_2} \geq \frac{1}{R_2}$ or: $k \geq 1$

But since k is always a fraction of 1 this constitutes a contradiction.

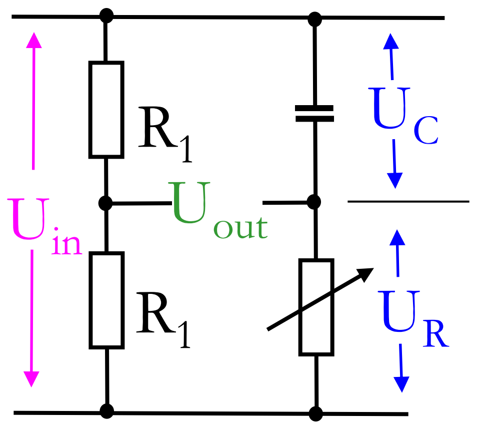
Solution:

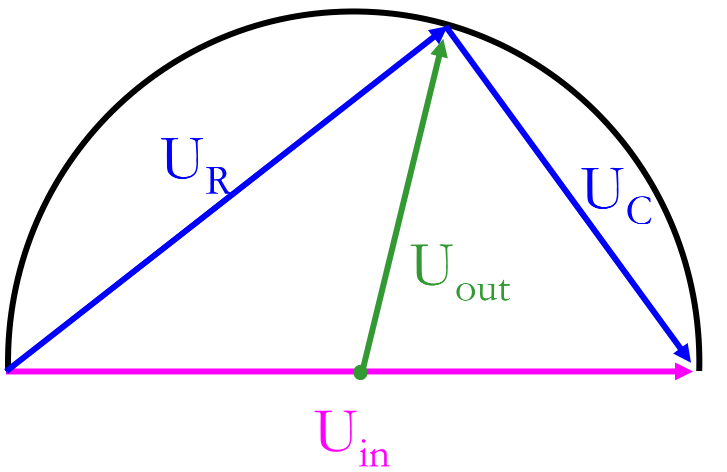


But what is with C_2 ?

One can always find for any given ω a C, so that $R_C = \frac{1}{sC} \xrightarrow{\rho=0} \frac{1}{i\omega C} \ll R_2$

Electronics
Appendix B (Scheme of an 1st Order Allpass)





$$U_{out} = U_{R_1} - U_C = \frac{1}{2} U_{in} - \frac{R_C}{R + R_C} U_{in}$$

$$T_C(s) = \frac{U_{out}}{U_{in}} = \frac{1}{2} \frac{R + R_C - 2R_C}{R + R_C} = \frac{1}{2} \cdot \frac{R - 1/sC}{R + 1/sC} = \frac{1}{2} \cdot \frac{sRC - 1}{sRC + 1}$$

Appendix C (Sallen-Key Amplifier)

$$U_\Sigma \cdot \frac{sC}{sRC+1} = I_\Sigma = \frac{U_{in}-U_\Sigma}{R} + (U_{out}-U_\Sigma)\cdot sC$$

$$= \frac{U_{in} - U^+(1+sRC)}{R} + (U_{out} - U^+(1+sRC))\cdot sC$$

$$U^+\left[(1+sRC)\cdot\frac{sC}{sRC+1} + \frac{1+sRC}{R} + (1+sRC)\cdot sC\right] = \frac{U_{in}}{R} + U_{out}\cdot sC$$

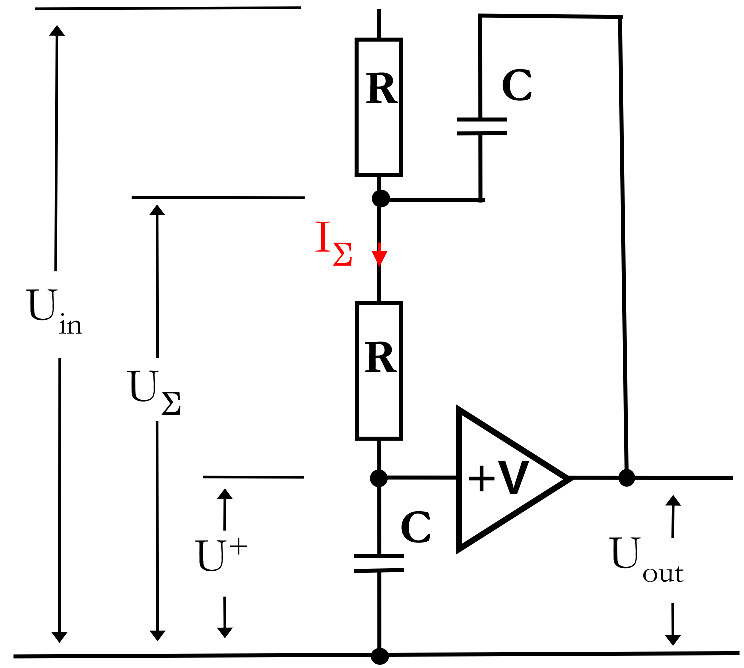
$$U_{out} = V\cdot \frac{U_{in}/R + U_{out}\cdot sC}{sC + \frac{1+sRC}{R} + (1+sRC)\cdot sC} \stackrel{def}{=} \frac{U_{in}/R + U_{out}\cdot sC}{Nenner}$$

$$U_{out}\left(1-\frac{VsC}{Nenner}\right) = \frac{V}{R}\cdot\frac{U_{in}}{Nenner}$$

$$T(s) = \frac{V}{R}\cdot\frac{1}{Nenner}\cdot\left(\frac{Nenner-VsC}{Nenner}\right)$$

$$T(s) = \frac{V}{R}\cdot\frac{1}{Nenner-VsC} = \frac{V}{sRC+1+sRC+(1+sRC)\cdot sRC-VsRC}$$

$$= \frac{V}{1+3sRC-VsRC+(sRC)^2} = \frac{V}{1+sRC\cdot(3-V)+(sRC)^2}$$



Given:

Set of m non-linear differential equations:

$$\vec{y} = f(\vec{x}, \vec{u}) \quad \text{with:} \quad \begin{array}{l} \vec{x} - n \text{ Variables} \\ \vec{u} - \ell \text{ Parameters} \end{array}$$

Expand f for the (operating) point A:

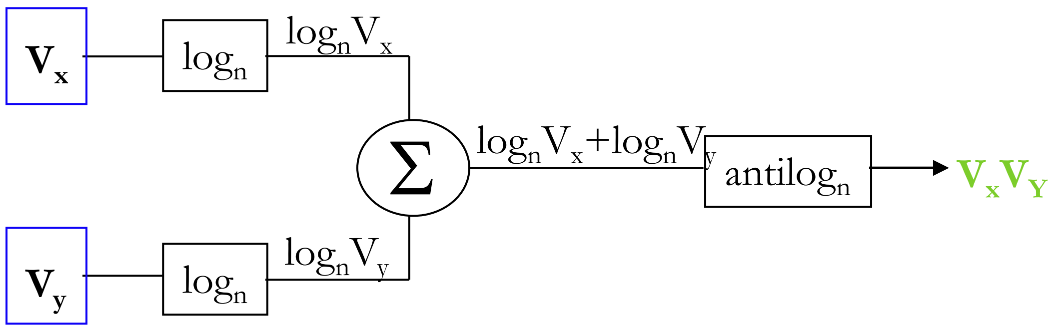
$$\vec{y} \approx f(\vec{x}_A, \vec{u}_A) + \underbrace{\frac{\partial f}{\partial \vec{x}_{|A}}}_{m \times n \text{ matrix } \mathbf{F}} \cdot \delta\vec{x} + \underbrace{\frac{\partial f}{\partial \vec{u}_{|A}}}_{m \times \ell \text{ matrix } \mathbf{G}} \cdot \delta\vec{u} + \text{Higher order terms;} \quad m \le n$$

$$= \mathbf{f}(\vec{\mathbf{x}}_A, \vec{\mathbf{u}}_A) + \mathbf{F} \cdot \delta\vec{\mathbf{x}} + \mathbf{G} \cdot \delta\vec{\mathbf{u}}$$

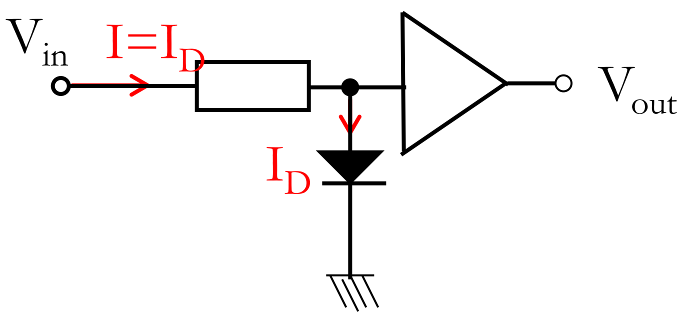
This is a system of coupled linear differential equations, which can be algebraized by means of the Laplace transform.

Electronics

Example Ia (Principle of a Log-Antilog Multiplier)



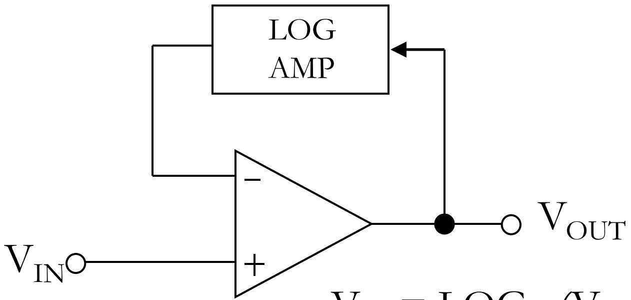
Log - Amp



$$I_D \propto e^{-\frac{U}{U_T}} \Rightarrow U \propto U_T \ln I_D$$

Example Ib (Principle of a Log-Antilog Multiplier)

Log Amp as a negative feedback element
results in a **Anti-Log-Amplifier**:



$V_{IN} = LOG_A\,(V_{OUT})$
$A^{V(in)} = V_{OUT}$

$$T(s) = \frac{F(s)}{1+F(s)G(s)}$$

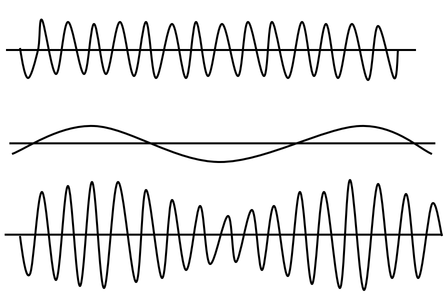
$$= \frac{1}{1/F(s)+G(s)} \xrightarrow{F(s)\to\infty} \mathbf{1/G(s) = G^{-1}(s)}$$

Example of an analog multiplier: Amplitude modulation

Carrier frequency: V_x

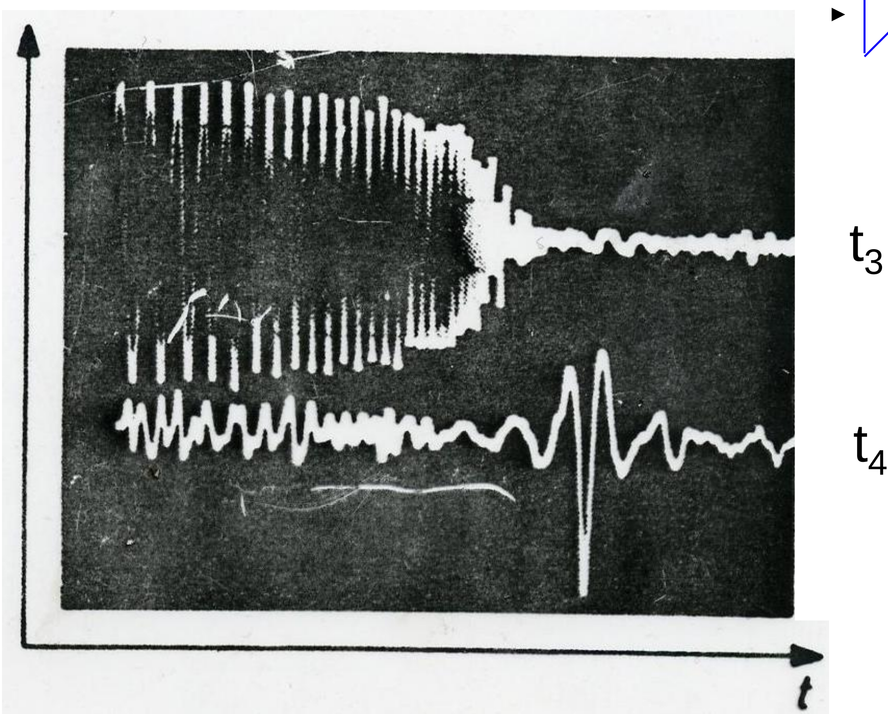
Modulated signal: V_y

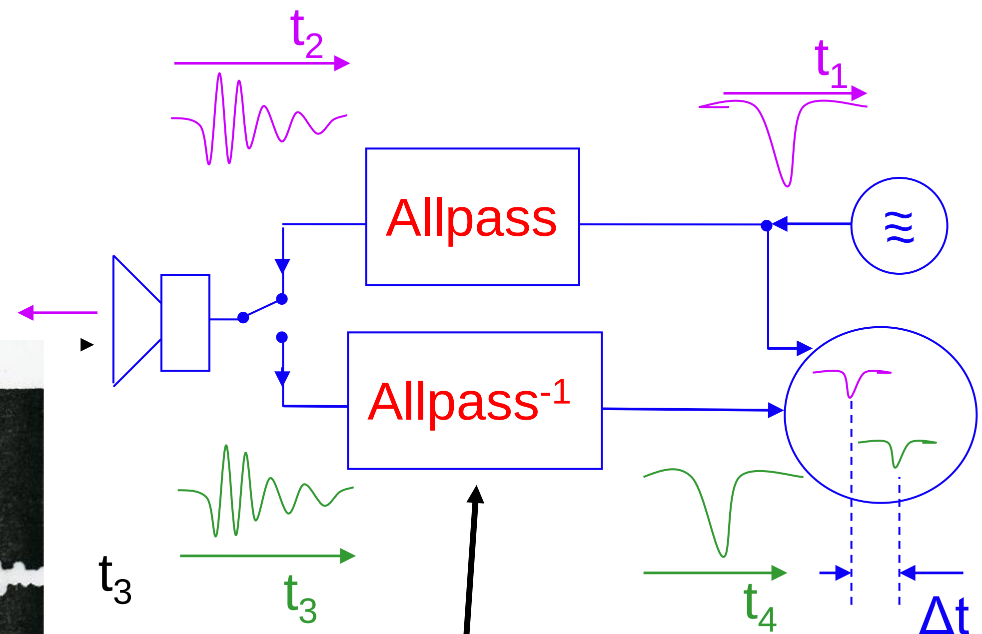
Output (Modulated carrier): V_xV_y



Electronics

Example IIa (Principle of a Sonar)



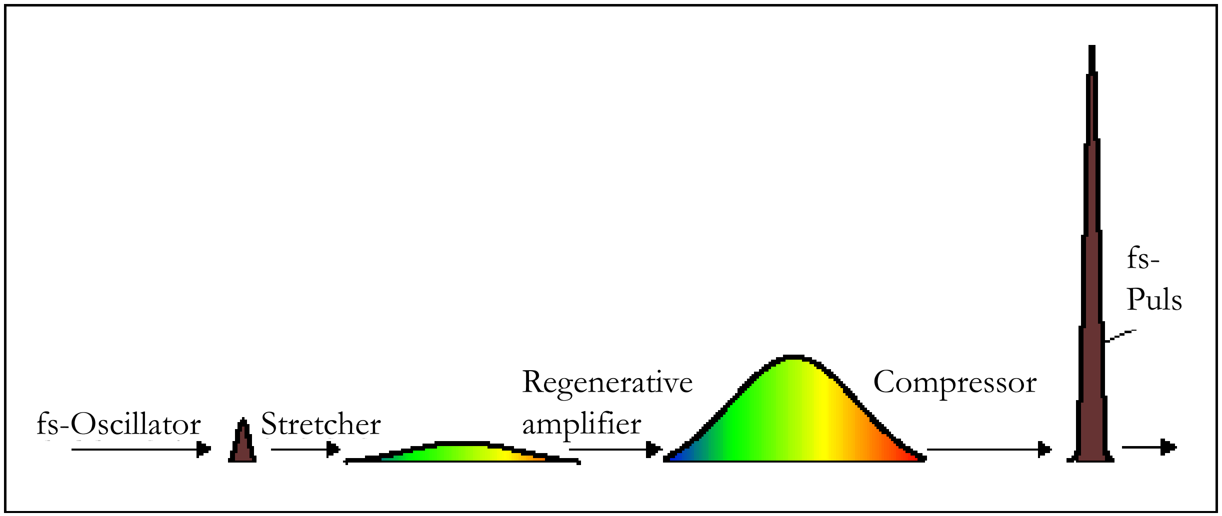


$$\Delta t = t_4 - t_1$$

Unfortunately this does not work!

Electronics

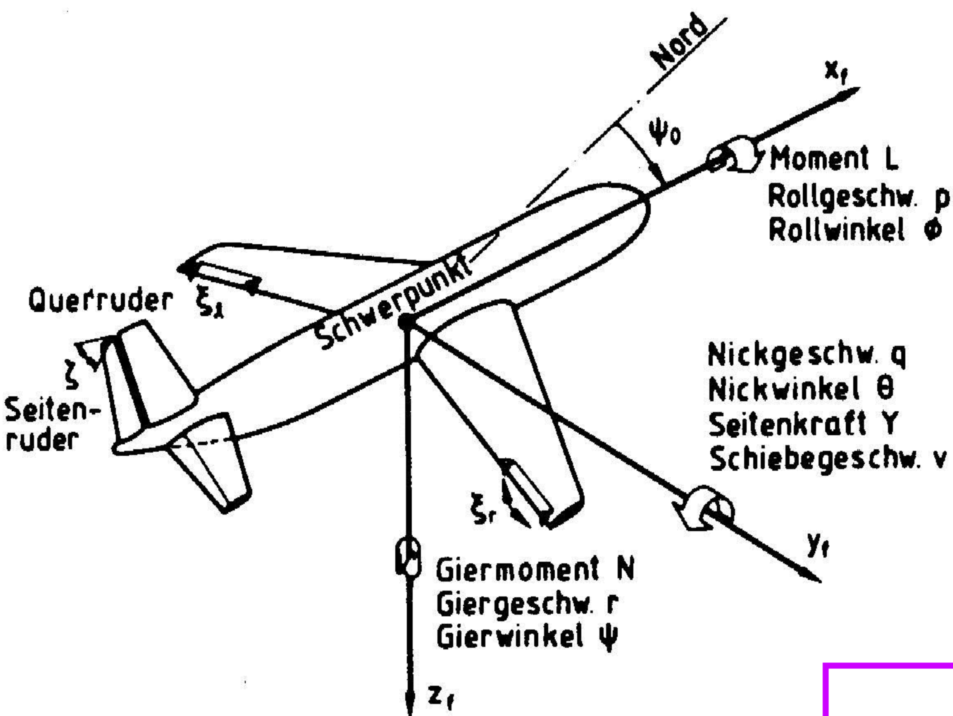
Example IIb (Chirped Pulse Amplification (CPA))

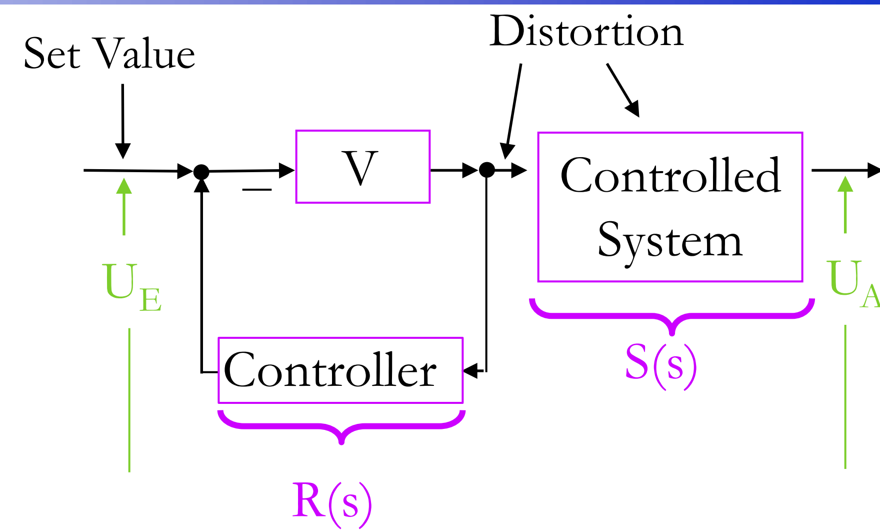


Principle of the CPA : High peak intensities within the amplifier optics are avoided by timewise and spatial dilatation of the pulses.

Electronics

Example III (Linearizing Systems)





$$T(s) = \frac{V}{1+V\cdot R} S = \frac{V\cdot S}{1+V\cdot R} \xrightarrow{V\to\infty} \frac{S}{R}$$

→ the dynamics of T(s) can be determined by the position of the Pole/Zero-distribution of R(s).