THE JAFFE-WITTEN MASS GAP AND COLOR CONFINEMENT

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What QCD is about?

$$L_{QCD} = L_{YM} + L_{qg}$$

$$L_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + L_{g.f.} + L_{gh.}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$L_{qg} = i\bar{q}^j_\alpha D_{\alpha\beta} q^j_\beta + \bar{q}^j_\alpha m^j_0 q^j_\beta$$

$$D_{\alpha\beta} q^j_\beta = (\delta_{\alpha\beta}\partial_\mu - ig(1/2)\lambda^a_{\alpha\beta} A^a_\mu)\gamma_\mu q^j_\beta \qquad (covariant\ derivative)$$

$$\alpha, \beta = 1, 2, 3. \qquad j = 1, 2, 3, ...N_f$$

$$a = N_c^2 - 1 = 1, 2, 3, ...8, \quad N_c = 3$$

and λ^a are $SU(N_c)$ matrices.

Repeated indices are always summed over.

QCD Phase Transitions

I The Confinement Phase Transition.

The absence of colored gluons and quarks in asymptotical states (at large distances)

All physical states are color-singlets (colorless)

II The Chiral Phase Transition or usually PCAC

Dynamical (spontaneous) chiral symmetry breaking

$SU_L(N_f) \times SU_R(N_f)$ is spontaneously broken to $SU(N_f)$ in the ground state. The Goldstone theorem then implies $N_f^2 - 1$ plet of massless pseudoscalar particles (bosons)

III No Higgs phase

$SU(N_c)$ is exact and gluons remain massless.

COLOR CONFINEMENT

Gluon Confinement

Quark Confinement

I Necessary condition

$$S(p) \neq \frac{const.}{\hat{p} - m},$$

i.e., quarks are always off mass-shell objects,

(Preparata, Pocsik et al., ...).

II Sufficient condition

Discrete spectrum only (no continuum) in bound states,

('t Hooft, Pagels, ...).

Color Confinement is absolute and permanent

Deconfinement phase transition does not exist

What is known as the Deconfinement phase transition at non zero temperature is in fact the Dehadronization phase transition

A Jaffe and E Witten

http://www.claymath.org/prize-problem/

http://www.arthurjaffe.com/

Theorem Yang Mills Existence And Mass Gap

Prove that for any compact simple gauge group G, quantum Yang Mills theory on R^4 **exists and has a mass gap** $\Delta > 0$

Theorem: Yang Mills Existence, Mass Gap And Gluon Confinement

If quantum Yang Mills theory with compact simple gauge group $G = SU(3)$ exists on R^4, then it exhibits a mass gap and confines gluons

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The main tool of our investigation is the so-called Schwinger-Dyson (SD) equation of motion for the full gluon propagator. Its solution(s) reflect(s) the quantum-dynamical structure of the true QCD ground state. There is a close intersection between the color confinement mechanism and the structure of the true QCD vacuum, and the other way around.

" Progress towards 21 century is impossible without solution of the dynamical equations of motion for particles and fields"

A. Salam

QED Photon propagator

$$D_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d(q^2,\xi) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2},$$

$$T_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} = g_{\mu\nu} - L_{\mu\nu}(q)$$

$$D^0_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}.$$

The gauge invariance condition TI)

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi,$$

II Photon SD equation

$$D(q) = D^0(q) - D^0(q)\Pi(q)D(q),$$

$$\Pi_{\alpha\beta}(q) = -g^2\int\frac{id^4p}{(2\pi)^4}Tr[\gamma_\alpha S(p-q)\Gamma_\beta(p-q,q)S(p)],$$

$$\Pi^s_{\alpha\beta}(q) = \Pi_{\alpha\beta}(q) - \Pi_{\alpha\beta}(0) = \Pi_{\alpha\beta}(q) - \delta_{\alpha\beta}\Delta^2(\lambda),$$

$$\Pi^s_{\alpha\beta}(q) = T_{\alpha\beta}(q)q^2\Pi^s_1(q^2) + q_\alpha q_\beta(q)\Pi^s_2(q^2),$$

where both invariant functions $\Pi^s_n(q^2)$ at $n = 1, 2$ are dimensionless ones and regular at small q^2, since $\Pi^s(0) = 0$, by definition.

$$\Pi_{\alpha\beta}(q) = T_{\alpha\beta}(q)q^2\Pi_1(q^2) + q_\alpha q_\beta(q)\Pi_2(q^2),$$

Let us now impose the transversality condition on the gluon self-energy

$$q_\alpha\Pi_{\alpha\beta}(q) = q_\beta\Pi_{\alpha\beta}(q) = 0,$$

which comes from the current conservation condition, one obtains

$$\Pi_2(q^2) = 0,$$

so it should be purely transversal, indeed

$$\Pi_{\alpha\beta}(q) = T_{\alpha\beta}(q)q^2\Pi_1(q^2).$$

On the other hand, from the transversal condition and the above-shown subtraction, it follows

$$\Pi^s_2(q^2) = -\frac{\Delta^2(\lambda)}{q^2},$$

however, this impossible, since $\Pi^s_2(q^2)$ is regular at small q^2.

$$\Pi_2^s(q^2) = \Delta^2(\lambda) = 0.$$

Thus $\Pi_{\alpha\beta}(q) = \Pi^s_{\alpha\beta}(q)$, and it has no pole at $q^2 = 0$, indeed.

So due to the current conservation in QED,

$$\Pi(q) \to \Pi^s(q) = O(q^2), \quad \Delta^2(\lambda) \to 0.$$

while totally discarding the quadratically divergent constant $\Delta^2(\lambda)$ from all equations. In fact, the current conservation condition lowers the quadratical divergence of the corresponding integral to a logarithmic one, which can be still present. Thus in QED there is no mass gap.

The transversality of the gluon self-energy and the gauge invariance for the full photon propagator

$$q_\alpha \Pi_{\alpha\beta}(q) = 0, \quad q_\mu q_\nu D_{\mu\nu}(q) = i\xi$$

should be maintained at every stage of the calculations, since photon is a physical state. If some QED process includes the full photon propagator, then the corresponding S-matrix element is proportional to the combination $j_1^\mu(q) D_{\mu\nu}(q) j_2^\nu(q)$. The current conservation condition $j_1^\mu(q) q_\mu = j_2^\nu(q) q_\nu = 0$ implies that unphysical (longitudinal) component of the full photon propagator does not change the physics of QED. The transversality condition is important, since $\Pi_{\mu\nu}(q)$ itself is a correction to the amplitude of the physical process, for example such as electron-electron scattering.

QCD Gluon propagator

$$D_{\mu\nu}(q) = i \left\{ T_{\mu\nu}(q) d(q^2, \xi) + \xi L_{\mu\nu}(q) \right\} \frac{1}{q^2},$$

$$T_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} = g_{\mu\nu} - L_{\mu\nu}(q)$$

$$D^0_{\mu\nu}(q) = i \left\{ T_{\mu\nu}(q) + \xi L_{\mu\nu}(q) \right\} \frac{1}{q^2}.$$

The color gauge invariance condition (STI)

$$q_\mu q_\nu D^{ab}_{\mu\nu}(q) = i\xi\delta^{ab}$$

II Gluon SD equation

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i \Pi_{\rho\sigma}(q; D) D_{\sigma\nu}(q).$$

Thus $\Pi_{\rho\sigma}(q; D)$ is the sum of a few terms, namely

$$\Pi_{\rho\sigma}(q; D) = -\Pi^{q}_{\rho\sigma}(q) - \Pi^{gh}_{\rho\sigma}(q)$$

$$+\Pi^{t}_{\rho\sigma}(D) + \Pi_{(1)\rho\sigma}(q; D) + \Pi_{(2)\rho\sigma}(q; D) + \Pi'_{(2)\rho\sigma}(q; D).$$

Let us note that like in QED these skeleton loop integrals are in general quadratically divergent.

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The transversality condition for the gluon self-energy can be reduced to the three independent transversality conditions.

$$q_\rho \Pi^q_{\rho\sigma}(q) = q_\sigma \Pi^q_{\rho\sigma}(q) = 0.$$

In the same way the sum of the gluon contributions can be done transversal by taking into account the ghost contribution,

$$q_\rho[\Pi_{(1)\rho\sigma}(q; D) + \Pi_{(2)\rho\sigma}(q; D) + \Pi'_{(2)\rho\sigma}(q; D) - \Pi^{gh}_{\rho\sigma}(q)] = 0.$$

However, there is no such regularization scheme (preserving or not gauge invariance) in which the transversality condition for the constant skeleton tadpole term could be satisfied, i.e.,

$$q_\rho \Pi^t_{\rho\sigma}(D) = q_\rho \delta_{\rho\sigma} \Delta^2_t(D) = q_\sigma \Delta^2_t(D) \neq 0.$$

In any NP approach the transversality condition imposed on the gluon self-energy is not valid, i.e., in general

$$q_\rho \Pi_{\rho\sigma}(q; D) = q_\sigma \Pi_{\rho\sigma}(q; D) \neq 0.$$

Analytically the skeleton tadpole contribution is

$$\Pi_t(D) = g^2 \int \frac{id^4q_1}{(2\pi)^4} T_4^0 D(q_1) \neq 0.$$

In the PT, when the full gluon propagator is always approximated by the free one, i.e., when $D = D^0$,

$$\Pi_t(D^0) = g^2 \int \frac{id^4q_1}{(2\pi)^4} T_4^0 D^0(q_1) = 0$$

within the DRM. So in the PT the transversality condition for the gluon self-energy is always satisfied.

So our proposal is not to impose the transversality condition on the gluon self-energy. The special role of the constant skeleton tadpole term in NP QCD dynamics should be emphasized, since it explicitly violates the transversality condition for the gluon self-energy. The second important observation is that ghosts themselves cannot automatically provide now the transversality of the gluon propagator in NP QCD. However, this does not mean that we need no ghosts at all. Of course, we need them in other sectors of QCD, for example in the quark-gluon Ward-Takahashi identity, which contains the so-called ghost-quark scattering kernel explicitly.

Subtractions

$$\Pi^s_{\rho\sigma}(q;D) = \Pi_{\rho\sigma}(q;D) - \Pi_{\rho\sigma}(0;D) = \Pi_{\rho\sigma}(q;D) - \delta_{\rho\sigma}\Delta^2(\lambda;D).$$

It maintains the gauge structure of the gluon self-energy. The mass gap $\Delta^2(\lambda;D)$ itself is mainly generated by the non-linear interaction of massless gluon modes

$$\Delta^2(\lambda;D) = \Pi^t(D) + \sum_a \Pi^a(0;D) = \Delta^2_t(D) + \sum_a \Delta^2_a(0;D),$$

The transversality condition for the gluon self-energy can be satisfied partially, i.e., to impose it on the quark and gluon (along with ghosts) skeleton loop contributuions, then the mass gap is to be reduced to $\Pi^t(D)$, since all other constants $\Pi^a(0;D)$ can be discarded in this case.

$$\Pi^s_{\rho\sigma}(q;D) \equiv \Pi^s(q;D) = \sum_a \Pi^s_a(q;D)$$

since

$$\Pi^s_t(D) = \Pi_t(D) - \Pi_t(D) = 0.$$

$$\Pi^{s}_{\rho\sigma}(q;D) = T_{\rho\sigma}(q)q^2\Pi(q^2;D) + q_\rho q_\sigma \tilde{\Pi}(q^2;D)$$

$$\Pi^{s}_{\rho\sigma}(0;D) = 0$$

$$q_\rho \Pi^{s}_{\rho\sigma}(q;D) = q_\sigma \Pi^{s}_{\rho\sigma}(q;D) = 0,$$

which implies

$$\tilde{\Pi}(q^2;D) = 0$$

$$\Pi^{s}_{\rho\sigma}(q;D) = T_{\rho\sigma}(q)q^2\Pi(q^2;D),$$

and it is always of the order q^2 at any D, since the invariant function $\Pi(q^2;D)$ is regular at small q^2 at any D. Thus the subtracted quantities are free from the quadratic divergences, but logarithmic ones can be still present in $\Pi(q^2;D)$ like in QED.

General structure of the full gluon propagator

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) q^2 \Pi(q^2; D) D_{\sigma\nu}(q)$$

$$+ D^0_{\mu\sigma}(q) i \Delta^2(\lambda; D) D_{\sigma\nu}(q),$$

$$D^0_{\mu\nu}(q) = i[T_{\mu\nu}(q) + \xi L_{\mu\nu}(q)] \frac{1}{q^2}.$$

The color gauge invariance condition imposed on the full gluon propagator

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi,$$

implies

$$D_{\mu\nu}(q) = i \left\{ T_{\mu\nu}(q) d(q^2) + \xi L_{\mu\nu}(q) \right\} \frac{1}{q^2}.$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) - T_{\mu\sigma}(q)\Pi(q^2; D)D_{\sigma\nu}(q) - T_{\mu\sigma}(q)\frac{\Delta^2(\lambda; D)}{q^2}D_{\sigma\nu}(q),$$

and its "solution" is

$$d(q^2) = \frac{1}{1 + \Pi(q^2; D) + (\Delta^2(\lambda; D)/q^2)}.$$

The only price we have payed by violating color gauge invariance is the gluon self-energy, while the full and free gluon propagators and the subtracted gluon self-energy always satisfy it. Let us emphasize that the expression for the full gluon form factor cannot be considered as the formal solution for the full gluon propagator, since both the mass gap $\Delta^2(\lambda; D)$ and the invariant function $\Pi(q^2; D)$ depend on D themselves.

Thus, we have established the general structure of the full gluon propagator and the corresponding gluon SD equation in the presence of the mass gap.

Cond usions

In order to realize the mass gap, we propose not to impose the transversality condition on the gluon self-energy, while preserving the color gauge invariance condition for the full gluon propagator. Such a temporary violation of color gauge invariance/symmetry (TVCGI/S) is completely NP effect, since in the formal PT limit $\Delta^2 = 0$ this effect vanishes.

Since a gluon is not a physical state because of color confinement, the TVCGI/S in QCD has no direct physical consequences, i.e., none of physical quantites/processes in low-energy QCD will be directly affected by this proposal.

For the calculations of physical observables from first principles in low-energy QCD we need the full gluon propagator, which transversality has been sacrificed in order to realize a mass gap (despite their general role the ghosts cannot guarantee its transversality in this case). However, we have already pointed out how the transversality of the gluon propagator relevant for NP QCD is to be restored at the final stage (hep-ph/0606010 v3 by V. Gogokhia)

In QED a mass gap is always in the "gauge prison". It cannot be realized even temporarily, since a photon is a physical state. However, a door of the "color gauge prison" can be opened for a moment in order to realize a mass gap, and a key to this "door" is the constant skeleton tadpole term. On the other hand, this "door" can be opened without key (as any door) by not imposing the transversality condition on the gluon self-energy.

In QED a mass gap cannot be "liberated" from the vacuum, while photons and electrons can be liberated from the vacuum in order to be physical states

In QCD a mass gap can be "liberated" from the vacuum, while gluons and quarks cannot be liberated from the vacuum in order to be physical states