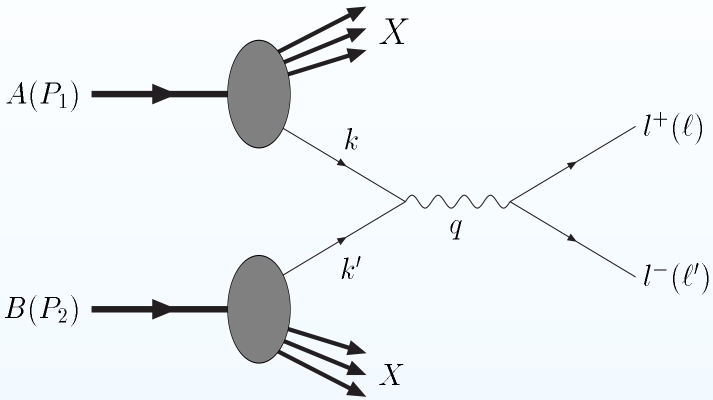
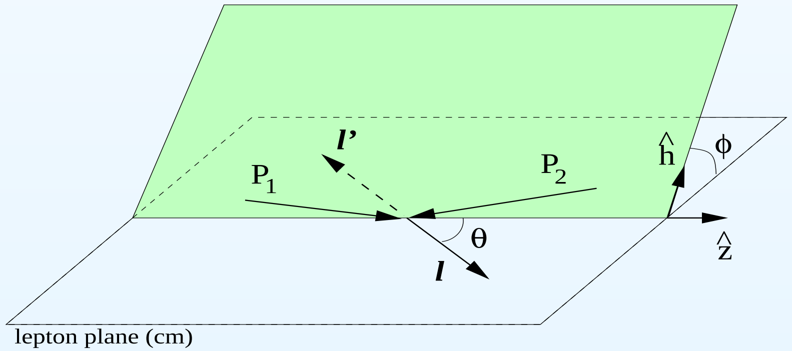
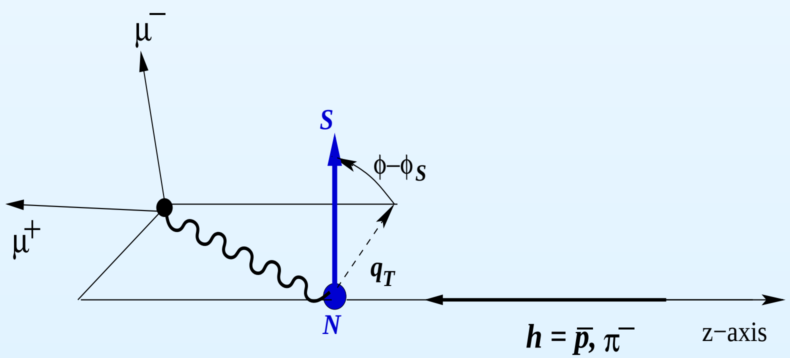
TRANSVERSITY AND T-ODD PDF's at PAX

O. Shevchenko

Kinematics







- $x_1 = \frac{Q^2}{2p_1 q}, \quad x_2 = \frac{Q^2}{2p_2 q}$ **– fractions of the longitudinal momentum of the hadrons A and B carried by the quark and antiquark which annihilate into virtual photon**
- $s = (p_1 + p_2)^2 \simeq 2p_1 p_2$ **– the center of mass energy squared**

  $Q^2 = M^2 \simeq x_1 x_2 s \equiv \tau s$

  $y = \frac{1}{2} \ln \frac{x_1}{x_2}$

  $x_F = x_1 - x_2$

  $x_1 = \frac{\sqrt{x_F^2 + 4\tau} + x_F}{2} = \sqrt{\tau} e^{y}$

  $x_2 = \frac{\sqrt{x_F^2 + 4\tau} - x_F}{2} = \sqrt{\tau} e^{-y}$
- θ **– production angle in the dilepton rest frame – polar angle of the lepton pair in the dilepton rest frame**
- ϕ **– azimuthal angle of lepton pair**
- ϕ_S **– azimuthal angle of the hadron polarization measured with respect to lepton plane**

Cross-sections

QPM: (D. Boer, PRD 60 (1999) 014012)

Unpolarized DY: $H_1 H_2 \longrightarrow l^+ l^- X$

$$\frac{d\sigma^{(0)}(H_1 H_2 \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{12Q^2} \sum_q e_q^2 \Big\{ (1+\cos^2\theta)\mathcal{F}[\bar{f}_{1q} f_{1q}] +$$

$$\sin^2\theta\cos(2\phi)\mathcal{F}\left[(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\,\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T})\frac{\bar{h}_{1q}^{\perp} h_{1q}^{\perp}}{M_1 M_2} \right] \Big\}$$

Single polarized DY: $H_1 H_2^{\uparrow} \longrightarrow l^+ l^- X$

PAX: $\bar{p}p^{\uparrow} \longrightarrow e^+ e^- X$

COMPASS, J-PARC: $\pi^- p^{\uparrow} \longrightarrow \mu^+ \mu^- X$

$$\frac{d\sigma^{(1)}(H_1 H_2^{\uparrow} \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{12Q^2} \sum_q e_q^2 \Big\{ (1+\cos^2\theta)\mathcal{F}[f_1 \bar{f}_1]$$

$$+\sin^2\theta\cos(2\phi)\mathcal{F}\left[(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\,\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T})\frac{h_1^{\perp}\bar{h}_1^{\perp}}{M_1 M_2} \right]$$

$$+(1+\cos^2\theta)\sin(\phi-\phi_{S_1})\mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\frac{f_{1T}^{\perp}\bar{f}_1}{M_1} \right] - \sin^2\theta\sin(\phi+\phi_{S_1})\mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{k}_{2T}\frac{h_1\bar{h}_1^{\perp}}{M_2} \right] \Big\}$$

$$\hat{h} \equiv \mathbf{q}_T/|\mathbf{q}_T|$$

$$\mathcal{F}[f\bar{f}] \equiv \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, \delta^2(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T) f_q(x_1,\mathbf{k}_{1T}^2)\bar{f}_q(x_2,\mathbf{k}_{2T}^2)$$

Unpolarized DY $H_1 H_2 \rightarrow l^+ l^- X$

$$\frac{d\sigma^{(0)}(H_1 H_2 \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{12Q^2}\sum_q e_q^2 \Big\{ (1+\cos^2\theta)\mathcal{F}[\bar{f}_{1q} f_{1q}] +$$

$$\sin^2\theta\cos(2\phi)\mathcal{F}\Big[\big(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\,\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T}\big)\frac{\bar{h}_{1q}^{\perp} h_{1q}^{\perp}}{M_1 M_2}\Big]\Big\}$$

$$\boxed{\begin{aligned} R &= \frac{3}{16\pi}(1+\lambda\cos^2\theta + \mu\sin 2\theta\cos\phi \\ &+ k\sin^2\theta\cos 2\phi), \quad (\lambda\simeq 1, \mu\simeq 0, k\equiv \nu/2) \end{aligned}}$$

$$h_{1q}^{\perp}(x,\mathbf{p}_T^2) = \frac{\alpha_T}{\pi}\frac{M_C M_H}{\mathbf{p}_T^2 + M_C^2}\mathrm{e}^{-\alpha_T \mathbf{p}_T^2} f_{1q}(x) \qquad (\mathrm{M}_c = 2.3\,GeV, \alpha_T = 1 GeV^{-2})$$

$\mathbf{q}_T$ integration approach

- e^+e^- **annihilation:** **D. Boer, R. Jakob, P.J. Mulders, NPB 504, 345 (1997); PLB 424, 143 (1998)**
- **SIDIS:** **A.M. Kotzinian, P.J. Mulders, Phys. Lett. B406 (1997) 373**
- **Single-polarized DY (Sivers function investigation) A. Efremov et al, Phys. Lett. B612 (2005)**

We introduce [Phys. Rev. D 72 (2005) 054027; Eur. Phys. J. C46 (2006) 147]

$$\hat{R} = \frac{\int d^2\mathbf{q}_T [|\mathbf{q}_T|^2/M_1 M_2][d\sigma^{(0)}/d\Omega]}{\int d^2\mathbf{q}_T \sigma^{(0)}},$$

$$\hat{R} = \frac{3}{16\pi}(\gamma(1+\cos^2\theta) + \hat{k}\,\sin^2\theta\cos 2\phi)$$

Factorization

$$\hat{k} = \frac{\int d^2\mathbf{q}_T [\mathbf{q}_T^2/M_1M_2] \sum_q e_q^2 \mathcal{F}[(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T})\frac{\bar{h}_1^\perp h_1^\perp}{M_1M_2}]}{\int d^2\mathbf{q}_T \sum_q e_q^2 \mathcal{F}[\bar{f}_1 f_1]}$$

$$\mathcal{F}[f\bar{f}] \equiv \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, \delta^2(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T) f_q(x_1,\mathbf{k}_{1T}^2)\bar{f}_q(x_2,\mathbf{k}_{2T}^2)$$

$$\delta^2(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T)(2\frac{(\mathbf{q}_T\mathbf{k}_{1T})(\mathbf{q}_T\mathbf{k}_{2T})}{\mathbf{q}_T^2} - \mathbf{k}_{1T}\mathbf{k}_{2T})\mathbf{q}_T^2$$

$$= 2\,\mathbf{k}_{1T}^2\mathbf{k}_{2T}^2 + \underbrace{\mathbf{k}_{1T}^2(\mathbf{k}_{1T}\mathbf{k}_{2T})}_{\downarrow\; 0} + \underbrace{\mathbf{k}_{2T}^2(\mathbf{k}_{1T}\mathbf{k}_{2T})}_{\downarrow\; 0} + 2(\mathbf{k}_{1T}\mathbf{k}_{2T})^2 - 2(\mathbf{k}_{1T}\mathbf{k}_{2T})^2$$

$$\hat{k} = 8\frac{\sum_q e_q^2(\bar{h}_{1q}^{\perp(1)}(x_1)h_{1q}^{\perp(1)}(x_2)+(1\leftrightarrow 2))}{\sum_q e_q^2(\bar{f}_{1q}(x_1)f_{1q}(x_2)+(1\leftrightarrow 2))}$$

$$h_{1q}^{\perp(n)}(x) \equiv \int d^2\mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M^2}\right)^n h_{1q}^\perp(x,\mathbf{k}_T^2)$$

Single polarized DY process $H_1 H_2^{\uparrow} \to l^+ l^- X$

$$\frac{d\sigma^{(1)}(H_1H_2^{\uparrow}\to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{12Q^2}\sum_q e_q^2 \Big\{ (1+\cos^2\theta)\mathcal{F}[f_1\bar{f}_1]$$

$$+\sin^2\theta\cos(2\phi)\mathcal{F}\left[(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\,\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T})\frac{h_1^{\perp}\bar{h}_1^{\perp}}{M_1M_2}\right]$$

$$+(1+\cos^2\theta)\sin(\phi-\phi_{S_1})\mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\frac{f_{1T}^{\perp}\bar{f}_1}{M_1}\right]$$

$$-\sin^2\theta\sin(\phi+\phi_{S_1})\mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{k}_{2T}\frac{h_1\bar{h}_1^{\perp}}{M_2}\right]\Big\}$$

Let us consider SSA

$$\hat{A}_{h(f)} = \frac{\int d\Omega d\phi_{S_2}\sin(\phi\pm\phi_{S_2})[d\sigma(\mathbf{S}_{2T})-d\sigma(-\mathbf{S}_{2T})]}{\int d\Omega d\phi_{S_2}[d\sigma(\mathbf{S}_{2T})+d\sigma(-\mathbf{S}_{2T})]}$$

$$A_h = -\frac{1}{4}\frac{\sum_q e_q^2\,\mathcal{F}\left[\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}}{M_1}\bar{h}_{1q}^{\perp}h_{1q}\right]}{\sum_q e_q^2\,\mathcal{F}\left[\bar{f}_{1q}f_{1q}\right]},\quad A_f = \frac{1}{2}\frac{\sum_q e_q^2\,\mathcal{F}\left[\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_{2T}}{M_2}\bar{f}_1^q f_{1T}^{\perp q}\right]}{\sum_q e_q^2\,\mathcal{F}\left[\bar{f}_{1q}f_{1q}\right]}$$

$\downarrow$ A. Sissakian et al, PRD,2005

$\downarrow$ Anselmino et al, PRD, 2003; Efremov et al, PLB, 2005

Factorization

By analogy with $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ considered in ref. A. Efremov et al, PLB **612** (2005) 233, we introduce

$$\hat{A}_h = \frac{\int d\Omega d\phi_{S_2} \int d^2\mathbf{q}_T (|\mathbf{q}_T|/M_1) \sin(\phi+\phi_{S_2}) [d\sigma(\mathbf{S}_{2T}) - d\sigma(-\mathbf{S}_{2T})]}{\int d\Omega d\phi_{S_2} \int d^2\mathbf{q}_T [d\sigma(\mathbf{S}_{2T}) + d\sigma(-\mathbf{S}_{2T})]}$$

so that after integration

$$\hat{A}_h = -\frac{1}{2} \frac{\sum_q e_q^2 [\bar{h}_{1q}^{\perp(1)}(x_1) h_{1q}(x_2) + (1 \leftrightarrow 2)]}{\sum_q e_q^2 [\bar{f}_{1q}(x_1) f_{1q}(x_2) + (1 \leftrightarrow 2)]},$$

$$h_{1q}^{\perp(1)}(x) \equiv \int d^2\mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M^2}\right) h_{1q}^{\perp}(x, \mathbf{k}_T^2)$$

$\bar{p}p \to l^+l^-X$ **and** $\bar{p}p^\uparrow \to l^+l^-X$

By virtue of charge conjugation symmetry:

$$\hat{k}|_{\bar{p}p\to l^+l^-X} = 8\frac{\sum_q e_q^2[h_{1q}^{\perp(1)}(x_1)h_{1q}^{\perp(1)}(x_2)+\bar{h}_{1q}^{\perp(1)}(x_1)\bar{h}_{1q}^{\perp(1)}(x_2)]}{\sum_q e_q^2[f_{1q}(x_1)f_{1q}(x_2)+\bar{f}_{1q}(x_1)\bar{f}_{1q}(x_2)]},$$

$$\hat{A}_h|_{\bar{p}p^\uparrow\to l^+l^-X} = -\frac{1}{2}\frac{\sum_q e_q^2[h_{1q}^{\perp(1)}(x_1)h_{1q}(x_2)+\bar{h}_{1q}(x_1)\bar{h}_{1q}^{\perp(1)}(x_2)]}{\sum_q e_q^2[f_{1q}(x_1)f_{1q}(x_2)+\bar{f}_{1q}(x_1)\bar{f}_{1q}(x_2)]},$$

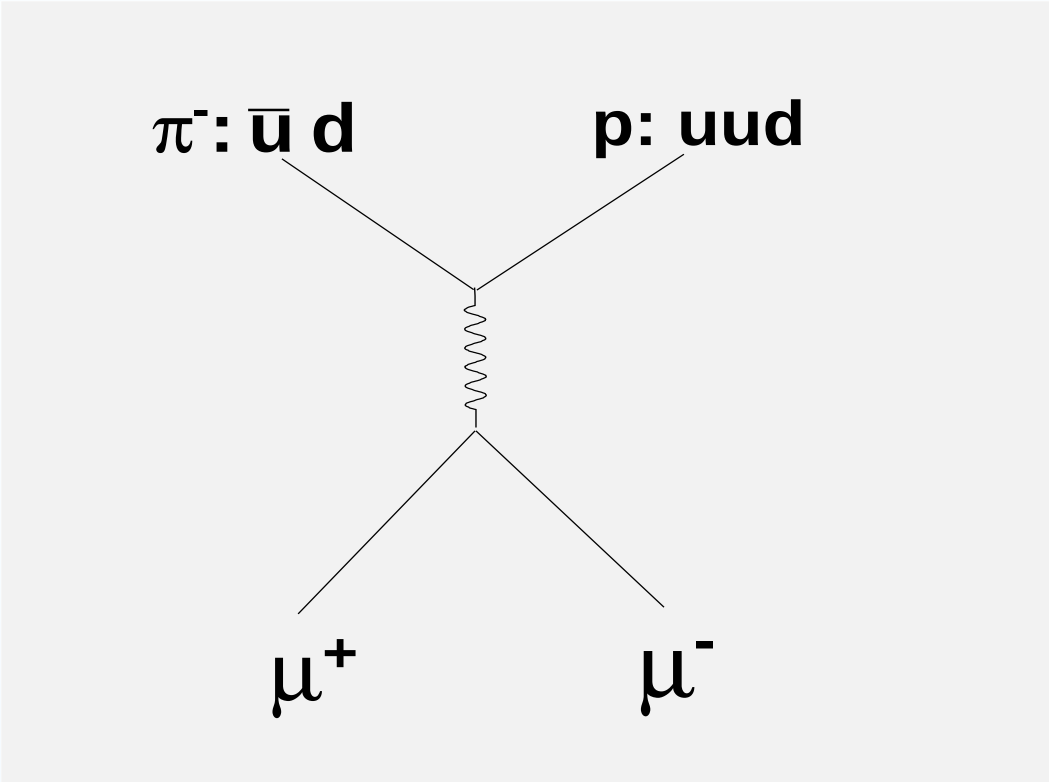
where now all PDF *refer to protons*. Neglecting squared antiquark and strange quark PDF contributions to proton and taking into account the quark charges and u quark dominance at large x, we get

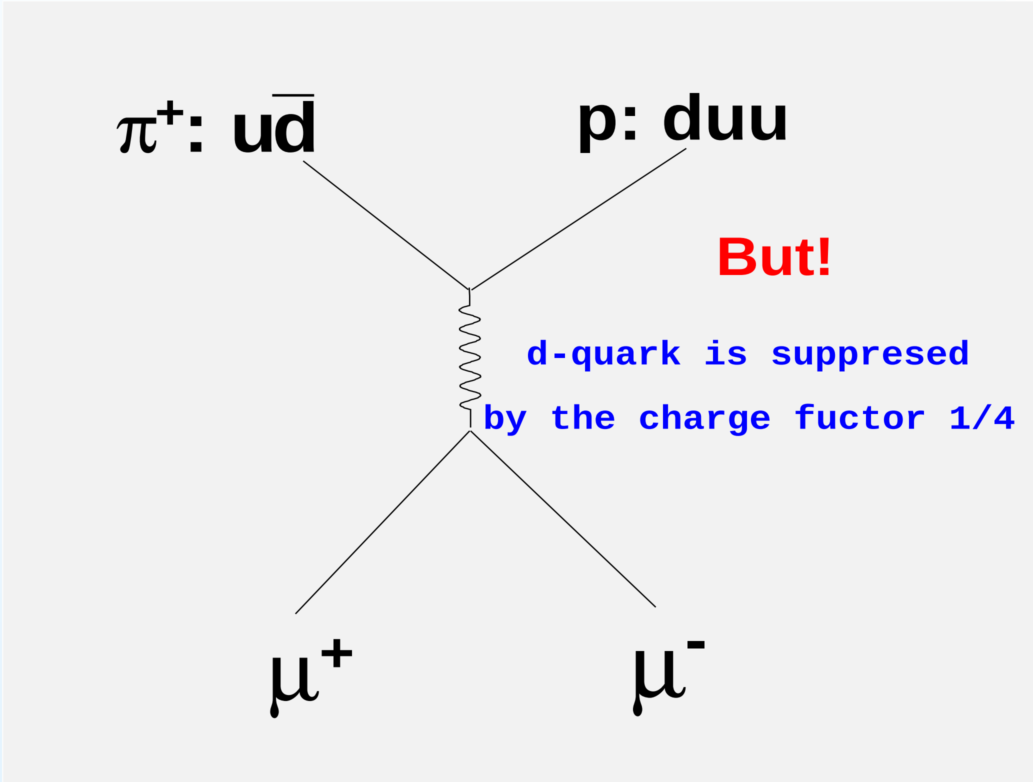
$$\hat{k}(x_1,x_2)|_{\bar{p}p\to l^+l^-X} \simeq 8\frac{h_{1u}^{\perp(1)}(x_1)h_{1u}^{\perp(1)}(x_2)}{f_{1u}(x_1)f_{1u}(x_2)},$$

$$\hat{A}_h(x_1,x_2)|_{\bar{p}p^\uparrow\to l^+l^-X} \simeq -\frac{1}{2}\frac{h_{1u}^{\perp(1)}(x_1)h_{1u}(x_2)}{f_{1u}(x_1)f_{1u}(x_2)}.$$

$\pi^- p \to \mu^+\mu^- X$ and $\pi^- p^\uparrow \to \mu^+\mu^- X$ (COMPASS, J-PARC)

Why $\pi^- p$ but not $\pi^+ p$?





$$\hat{k}(x_\pi, x_p)_{\pi^- p} \simeq 8 \frac{\bar{h}_{1u}^{\perp(1)}(x_\pi)\Big|_{\pi^-} h_{1u}^{\perp(1)}(x_p)\Big|_p}{\bar{f}_{1u}(x_\pi)\Big|_{\pi^-} f_{1u}(x_p)\Big|_p},$$

$$\hat{A}_h(x_\pi, x_p)_{\pi^- p^\uparrow} \simeq -\frac{1}{2} \frac{\bar{h}_{1u}^{\perp(1)}(x_\pi)\Big|_{\pi^-} h_{1u}(x_p)\Big|_p}{\bar{f}_{1u}(x_\pi)_{\pi^-} f_{1u}(x_p)_p}.$$

Assumption:

$$\frac{\bar{h}_{1u}^{\perp(1)}(x)_{\pi^-}}{h_{1u}^{\perp(1)}(x)_p} = C_u \frac{\bar{f}_{1u}(x)_{\pi^-}}{f_{1u}(x)_p}.$$

is consistent with the Boer's model, where

$$C_u = M_p c_\pi^u / M_\pi c_p^u$$

The simulations results show that $C_u \simeq 1$

Upper bounds on $\hat{k}$, $h_{1u}^{\perp(1)}$ and $\hat{A}_h$

$x_1 \simeq x_2 \simeq \sqrt{Q^2/s}$ **(**$x_F \simeq 0$**)**

- $(|\mathbf{k}_T|/M) h_1^{\perp}(x, \mathbf{k}_T^2) \leq f_1(x, \mathbf{k}_T^2)$ (A. Bacchetta et al. PRL. **85**, 712 (2000))

  $\langle k_T \rangle \simeq 0.8\, GeV$ (A.V. Efremov et al, PLB **612**, 233 (2005))

$$\boxed{h_{1u}^{\perp(1)} \lesssim 0.4 f_{1u}(x).}$$

- Soffer inequality:

$$\boxed{|h_{1u}| \leq (f_{1u} + g_{1u})/2}$$

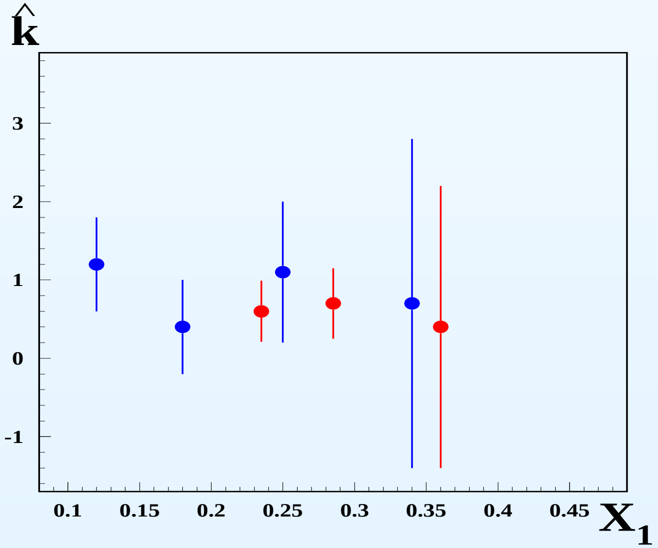
Collider mode		Fixed target mode	
$h_{1u\,(max)} \simeq$ 2.3	$h_{1u\,(max)}^{\perp(1)} \simeq$ 1.2	$h_{1u\,(max)} \simeq 1.5$	$h_{1u\,(max)}^{\perp(1)} \simeq 0.8$
$\hat{k}_{(max)} \simeq 1.2$	$\lvert\hat{A}_{h\,(max)}\rvert \simeq 0.14$	$\hat{k}_{(max)} \simeq 1.4$	$\lvert\hat{A}_{h\,(max)}\rvert \simeq 0.17$

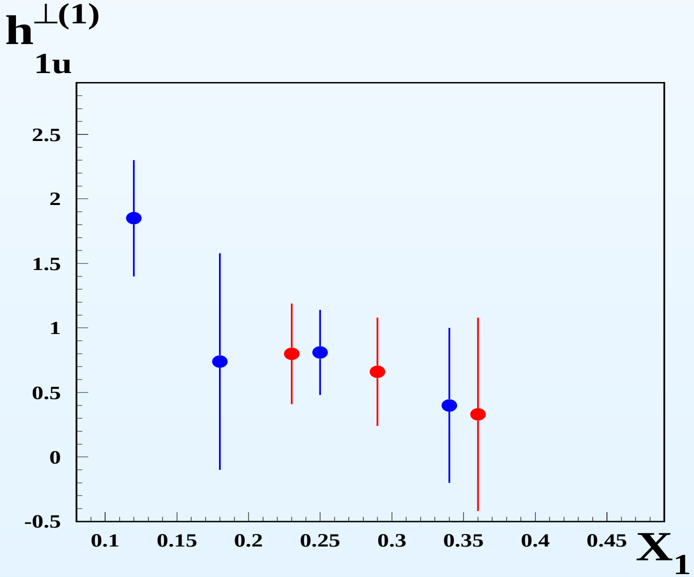
Simulations results (unpolarized DY, $\bar{p}p \to l^+l^-X$)

$$x \equiv x_1 \simeq x_2 \;(x_F \simeq 0 \pm 0.4)$$

$$h_{1u}^{\perp(1)}(x) = f_{1u}(x)\sqrt{\frac{\hat{k}(x,x)}{8}}$$

$$h_{1u(x)} = -4\sqrt{2}\frac{\hat{A}_h(x,x)}{\sqrt{\hat{k}(x,x)}}f_{1u}(x)$$

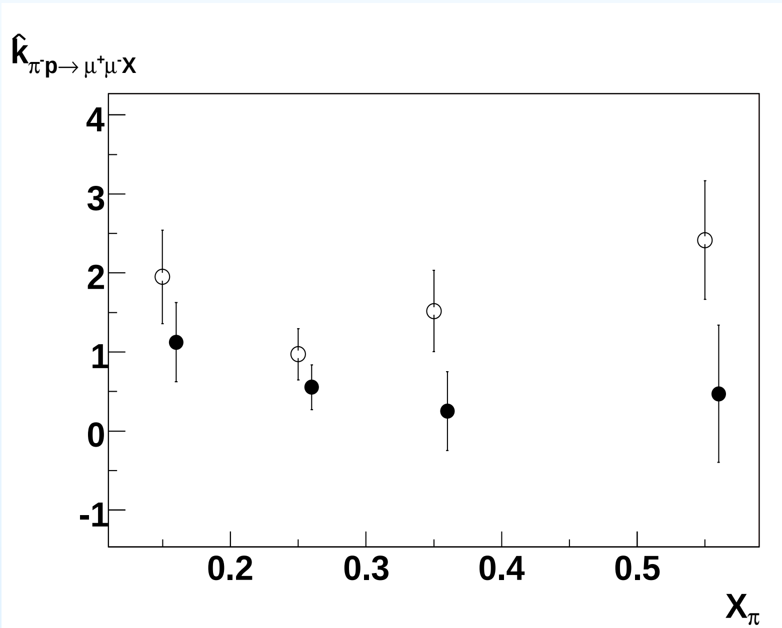


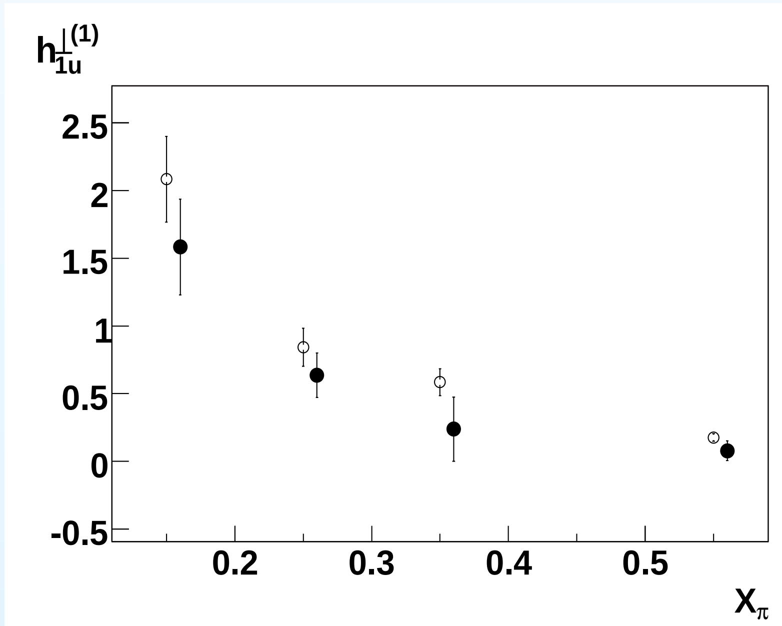


blue points: collider mode, red points: fixed target mode

Simulation results (unpolarized DY, $\pi^- p \to l^+ l^- X$)

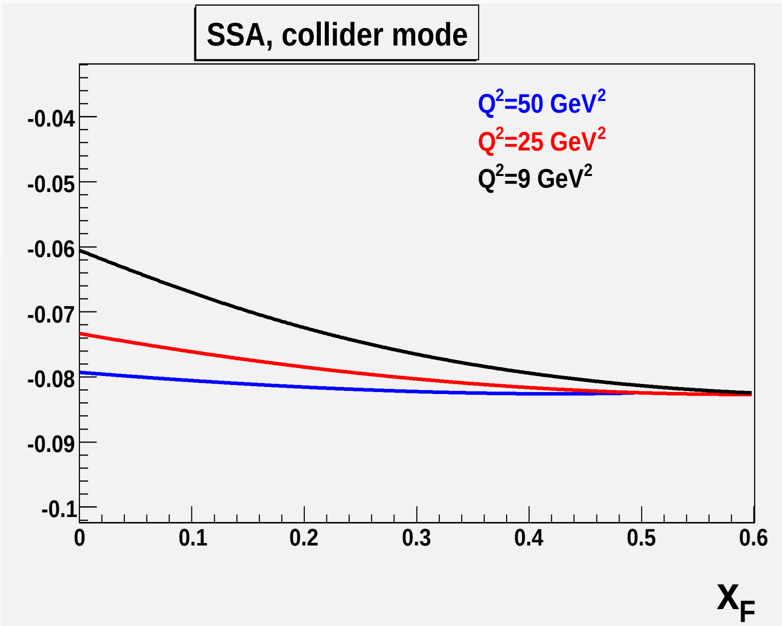
$$x \equiv x_1 \simeq x_2 \ (x_F \simeq 0 \pm 0.4)$$
$$h_{1u}^{\perp(1)}(x) = f_{1u}(x)\sqrt{\frac{\hat{k}(x,x)}{8}}$$
$$h_{1u(x)} = -4\sqrt{2}\frac{\hat{A}_h(x,x)}{\sqrt{\hat{k}(x,x)}} f_{1u}(x)$$

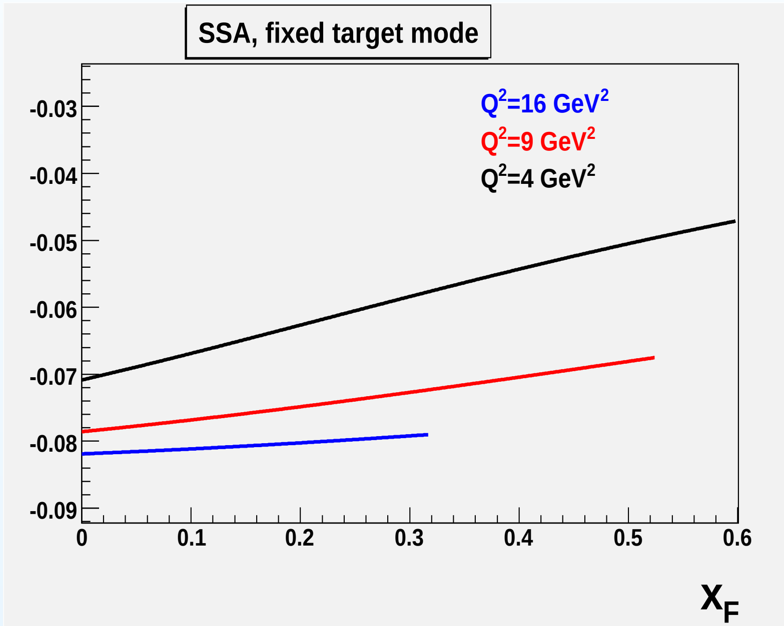




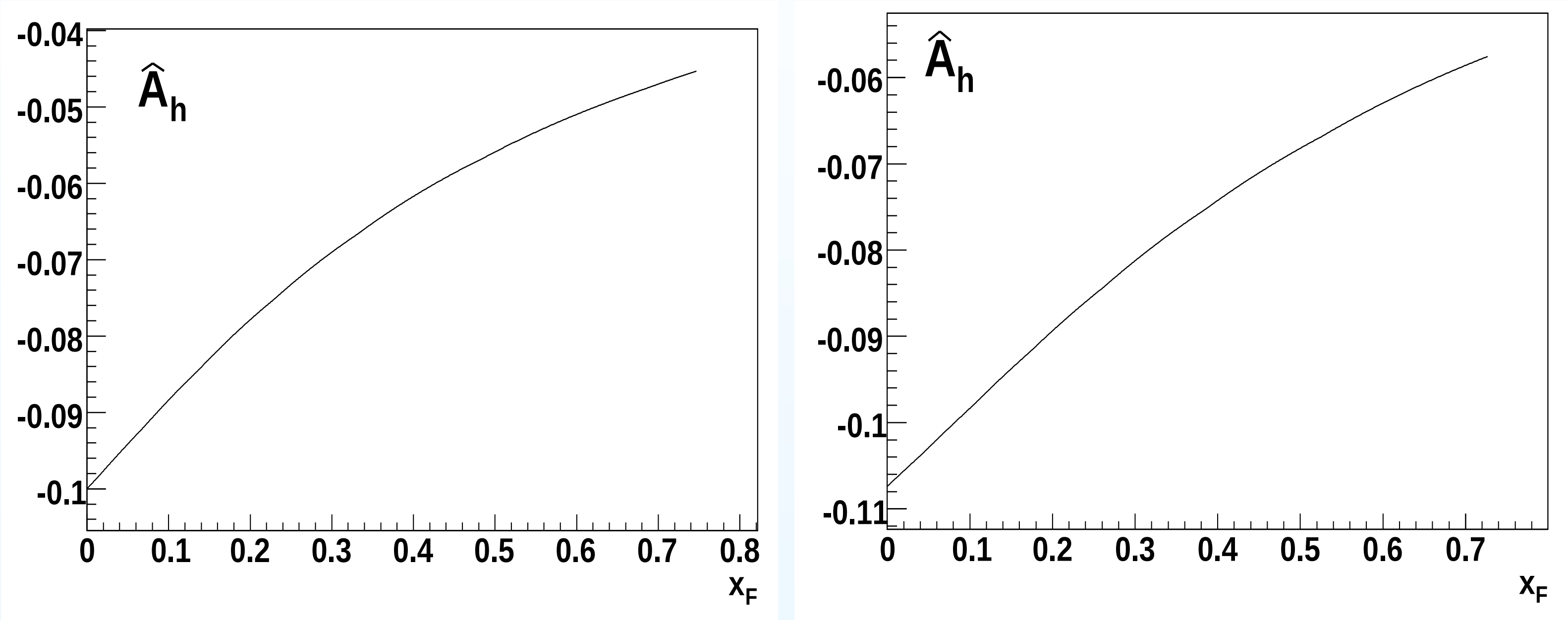
closed circles: 60 GeV, open cirles: 100 GeV

Estimation of SSA $\hat{A}_h$ (single-polarized DY, $\bar{p}p^{\uparrow} \to l^+l^-X$)





Estimation of SSA $\hat{A}_h$ (single-polarized DY, $\pi^- p^\uparrow \to l^+ l^- X$)



Left: 100 GeV ($Q^2_{average} = 6.2\,GeV^2$), Right: 60 GeV ($Q^2_{average} = 5.5\,GeV^2$)

Sivers function from the single-polarized Drell-Yan

A. Efremov et al (Phys. Lett. B612 (2005))

q_T-integrated asymmetry

$$A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}} = \frac{\int d\Omega d\phi_{S_2} \int d^2\mathbf{q}_T (|\mathbf{q}_T|/M_1) \sin(\phi-\phi_{S_2})[d\sigma(\mathbf{S}_{2T}) - d\sigma(-\mathbf{S}_{2T})]}{\int d\Omega d\phi_{S_2} \int d^2\mathbf{q}_T [d\sigma(\mathbf{S}_{2T}) + d\sigma(-\mathbf{S}_{2T})]}.$$

As a result:

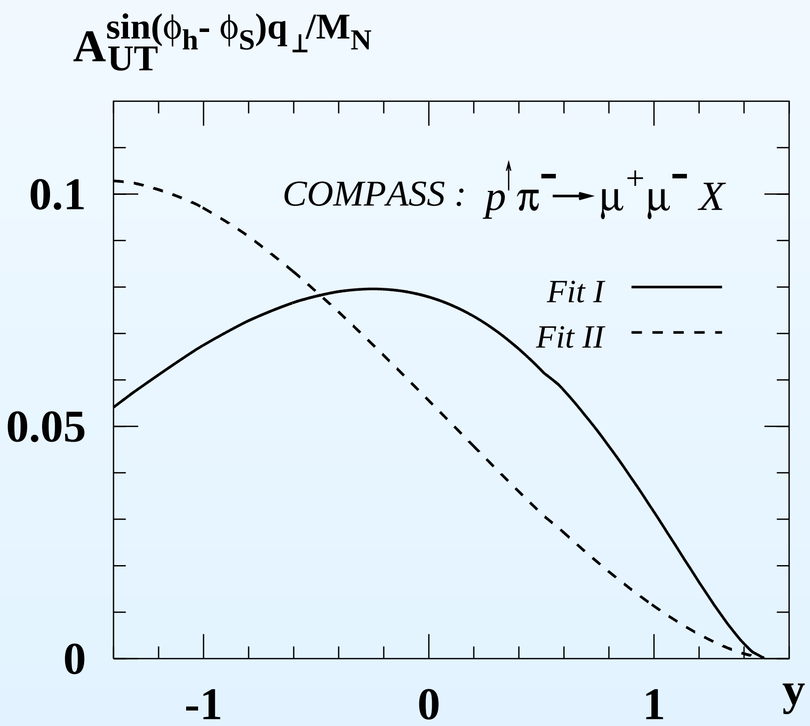
$$A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}} = 2\,\frac{\sum_a e_q^2\, f_{1T}^{\perp(1)q/p}(x_1)\, f_1^{\bar{q}/\bar{p}(\pi^-)}(x_2)}{\sum_a e_q^2\, f_1^{q/p}(x_1)\, f_1^{\bar{q}/\bar{p}(\pi^-)}(x_2)},$$

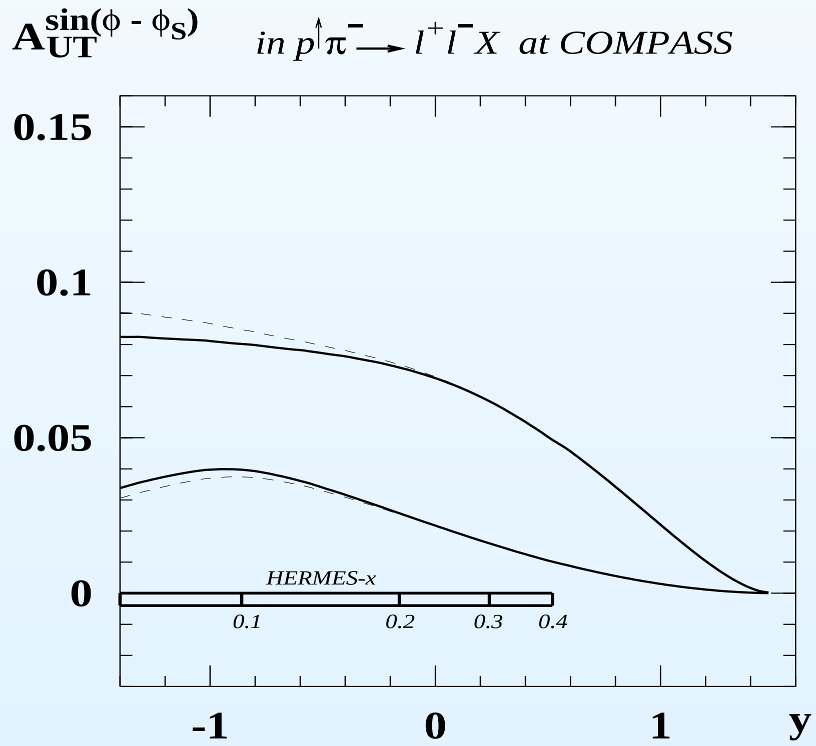
u-quark dominance:

$$A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}} \simeq 2\,\frac{f_{1T}^{\perp(1)u/p}(x_1)\, f_1^{\bar{u}/\bar{p}(\pi^-)}(x_2)}{f_1^{u/p}(x_1)\, f_1^{\bar{u}/\bar{p}(\pi^-)}(x_2)},$$

Estimation of SSA $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$

Two fits for Sivers function extracted from the HERMES data are used:
A. Efremov et al, Phys. Lett. B612 (2005)
J. Collins et al, hep-ph/0511272





Summary

- The procedure of direct (without any model assumptions) extraction of transversity and its accompanying T-odd PDF is proposed
- Both unpolarized and single-polarized DY processes necessary to extract the quantities h_1 and $h_1^{\perp(1)}$
- The preliminary estimations performed for PAX kinematics demonstrate that it is quite real to extract both h_1 and $h_1^{\perp(1)}$ in the processes $\bar{p}p \to l^+l^-X$ and $\bar{p}p^\uparrow \to l^+l^-X$.
- The preliminary estimations performed for PAX kinematics demonstrate that the single-polarized Drell-Yan processes $\bar{p}p^\uparrow \to l^+l^-X$ can provide us the access to the Sivers function.

DY $\pi^-p \to \mu^+\mu^-X$ at COMPASS and J-PARC can provide us by:

- Purely unpolarized DY $\Rightarrow$ first moment of the Boer-Mulders function
- Single-polarized DY $\Rightarrow$ first moment of the Sivers function
- Both unpolarized and single-polarized DY $\Rightarrow$ transversity