Theory and History of the $(\vec{d}, 2p)$ Reaction

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I first became interested in the $p(d, 2p)$ reaction about 30 years ago while working on the 1 GeV/nucleon data taken with the Dubna hydrogen bubble chamber [1]. I am delighted to say that two of my old collaborators Mikhail Nioradze and Viktor Glagolev are with us today and, in fact, Viktor will describe the much more recent Dubna charge-exchange programme later in this session.

If the proton-proton force had been a bit stronger, then the di-proton would have been bound like the deuteron. As it is, there is a virtual $^1\mathrm{S}_0$ state so close to threshold that one always sees a very sharp enhancement in the pp effective mass spectrum at the lowest excitation (excess) energy $Q_{pp} = k^2/m_p$, where $\boldsymbol{k}$ is the pp relative momentum. In homage to the Dubna experiment, let me start in Fig. 1 by showing the excitation spectrum for the $dp \to (pp)n$ reaction in two regions of momentum transfer $t = -q^2$ between the initial proton and final neutron. It is hard work with a bubble chamber to get sufficient statistics in fine steps of Q_{pp} to show the $^1\mathrm{S}_0$ enhancement clearly but its effects are shown here in the spectrum with *fsi* (solid curve) and without (chain curve). Andro Kacharava in the following talk will discuss data from ANKE analysed in only the first 2-3 MeV of excitation energy.

At these low momentum transfers one can think of the reaction as being a simple charge exchange of the neutron in the deuteron on the proton in the target, $np \to pn$, with the proton in the deuteron acting as a kind of spectator. The trouble is that at small t and low Q_{pp} one often doesn't know which is the spectator and which results from the struck particle.

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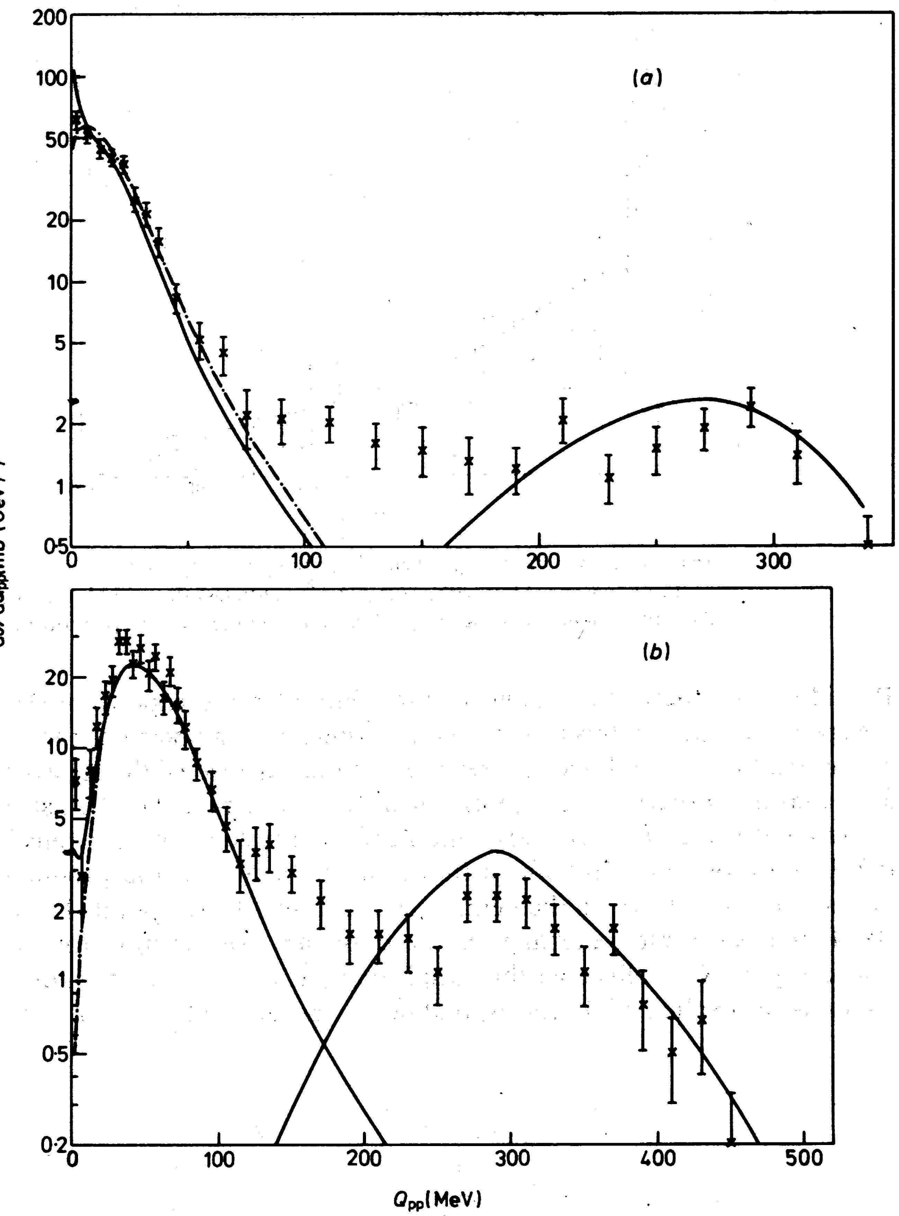

Figure 1: Dubna data [1] on the $dp \to (pp)n$ reaction at 1 GeV/nucleon as a function of the excitation energy in the pp system for (a) $|t| < 0.1$ (GeV/c)^2, and (b) $0.1 < |t| < 0.4$ (GeV/c)^2. The impulse approximation predictions are shown with (solid curve) and without (chain curve) the $^1S_0$ *fsi*.

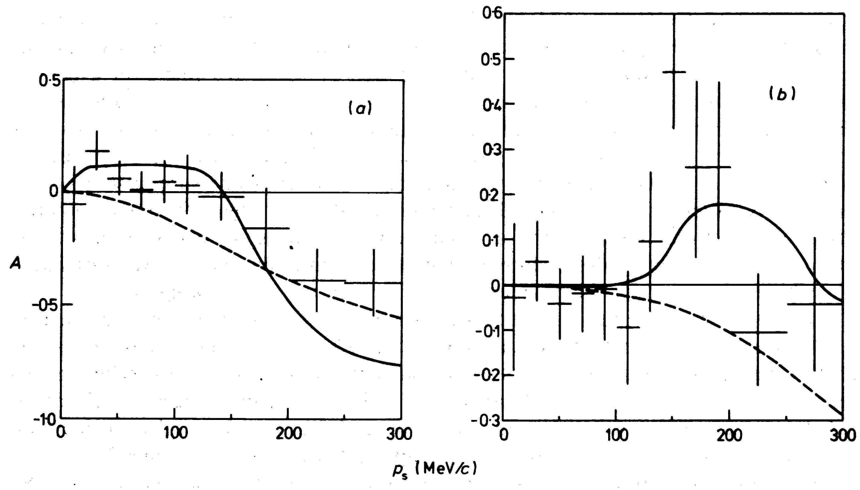

Figure 2: Angular asymmetry parameter for (a) $|t| < 0.1$ (GeV/c)^2, and (b) $0.1 < |t| < 0.4$ (GeV/c)^2 [1]. The impulse approximation predictions are shown with (solid curve) and without (chain curve) the ${}^1\mathrm{S}_0$ *fsi*.

If one takes the plausible approach of calling the slower proton in deuteron rest frame the *spectator*, one often gets it wrong and this mistake is made worse by the *pp fsi*. If the deuteron is unpolarised then the spectator distribution should be essentially isotropic in the deuteron rest frame but, with the practical definition of *spectator*, it need not be. The distribution in the angular asymmetry parameter

$$A \equiv \frac{N(\cos\alpha > 0) - N(\cos\alpha < 0)}{N(\cos\alpha > 0) + N(\cos\alpha < 0)}\,, \tag{1}$$

where α is the angle between the spectator momentum and the momentum transfer vector, is shown in Fig. 2. Such effects can easily be treated theoretically since they depend primarily upon the low energy nucleon-nucleon interaction. This is important because, as Ralf Schleichert will discuss later in this session, such studies might be possible at COSY using purely the spectator telescopes.

The transition from a deuteron to the ${}^1\mathrm{S}_0$ state of two protons flips both the spin and isospin of the NN system and it is a perfect example of a Gamow-Teller transition. Such transitions can have strong polarisation dependence and this led David Bugg and myself in the mid 1980s to wonder whether the $\vec{d}p \to (pp)n$ reaction could have strong enough analysing powers to justify its use as the basis of a deuteron polarimeter [2]. The diagram that we were considering was that of the impulse approximation of Fig. 3 where, at small momentum transfers between the initial proton and final neutron, one has a simple charge exchange on the neutron, with the final proton possibly being affected by the *pp fsi*. The overall amplitude must therefore be proportional to $f(np \to pn)$ times a form factor reflecting the overlap of the spatial parts of the initial deuteron and final pp wave functions.

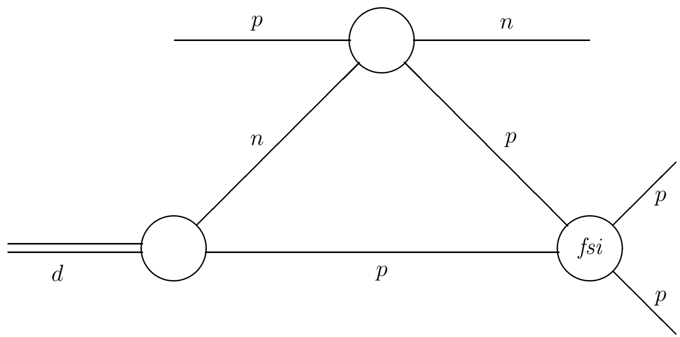

Figure 3: Impulse approximation diagram for proton charge-exchange on the deuteron at small momentum transfers.

To understand the spin dependence of the deuteron charge exchange, one must first describe the situation in the NN system. The amplitude of the elementary $np \to pn$ reaction in the cm system can be written in terms of five scalar amplitudes

$$f_{np} = \alpha + i\gamma(\boldsymbol{\sigma}_n + \boldsymbol{\sigma}_p)\mathbf{n} + \beta(\boldsymbol{\sigma}_n \cdot \mathbf{n})(\boldsymbol{\sigma}_p \cdot \mathbf{n}) + \delta(\boldsymbol{\sigma}_n \cdot \mathbf{m})(\boldsymbol{\sigma}_p \cdot \mathbf{m}) + \varepsilon(\boldsymbol{\sigma}_n \cdot \mathbf{l})(\boldsymbol{\sigma}_p \cdot \mathbf{l}), \quad (2)$$

where $\boldsymbol{\sigma}_n$ and $\boldsymbol{\sigma}_p$ are the Pauli matrices for neutron and proton, respectively. The orthogonal unit vectors are defined in terms of the initial ($\mathbf{k}$) and final ($\mathbf{k}'$) momenta as

$$\begin{aligned} \boldsymbol{n} = \hat{\boldsymbol{y}} &= \frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}\times\mathbf{k}'|}\,, \\ \boldsymbol{m} = \hat{\boldsymbol{x}}' &= \frac{\mathbf{k}'-\mathbf{k}}{|\mathbf{k}'-\mathbf{k}|}\,, \\ \boldsymbol{l} = \hat{\boldsymbol{z}}' &= \frac{\mathbf{k}'+\mathbf{k}}{|\mathbf{k}'+\mathbf{k}|}\,. \end{aligned} \quad (3)$$

The amplitudes are normalised such that the elementary $np \to pn$ differential cross section has the form

$$\left(\frac{d\sigma}{dt}\right)_{np\to pn} = I_{np} = |\alpha|^2 + |\beta|^2 + 2|\gamma|^2 + |\delta|^2 + |\varepsilon|^2. \quad (4)$$

As shown in Fig. 4, the spin-flip amplitudes charge-exchange amplitudes are dominantly real, with β and ϵ varying quite smoothly with momentum transfer q $(=\sqrt{-t})$. However the δ amplitude falls off rapidly and even changes sign at about $q = m_\pi$. This is because the very long range one-pion-exchange amplitude contributes here and this strong variation will give rise to similar behaviour in some of the polarisation observables. Note that in the forward direction, kinematics forces us to have $\beta(0) = \delta(0)$ because we cannot tell there the x from the y direction.

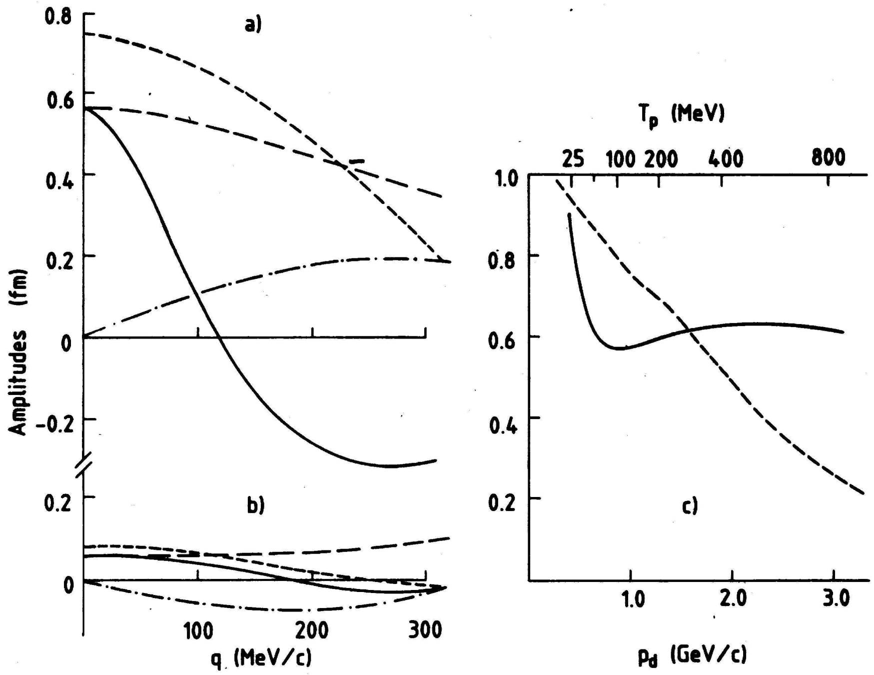

Figure 4: Real (a) and imaginary (b) parts of the np charge exchange amplitudes at 142 MeV: β (long dashes), $-i\gamma$ (chain), δ (solid curve), and ε (short dashes). (c): Interpolations of the values of $|\beta(0)| = |\delta(0)|$ (solid curve) and $|\varepsilon(0)|$ (dashed curve) as a function of deuteron momentum or proton kinetic energy. Both figures are taken from Ref. [2].

The general structure of these amplitudes seems to be remarkably stable as one varies the beam energy but what does change is the relative size of the spin-longitudinal (ϵ) and the spin-transverse (β, δ) terms, as illustrated also in Fig. 4.

The impulse approximation predictions are easy to write down if we confine ourselves to just the ${}^1\mathrm{S}_0$ component of the final pp state.

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}t\,\mathrm{d}^3k} = \tfrac{1}{3}\left\{\left(|\beta|^2+|\gamma|^2+|\varepsilon|^2\right)\left|S^-(k,\tfrac{1}{2}q)\right|^2 + |\delta|^2\left|S^+(k,\tfrac{1}{2}q)\right|^2\right\}\,, \tag{5}$$

$$A_{x'x'}\,\frac{\mathrm{d}^4\sigma}{\mathrm{d}t\,\mathrm{d}^3k} = \tfrac{1}{3}\left\{\left(|\beta|^2+|\gamma|^2+|\varepsilon|^2\right)\left|S^-(k,\tfrac{1}{2}q)\right|^2 - 2|\delta|^2\left|S^+(k,\tfrac{1}{2}q)\right|^2\right\}\,, \tag{6}$$

$$A_{yy}\,\frac{\mathrm{d}^4\sigma}{\mathrm{d}t\,\mathrm{d}^3k} = \tfrac{1}{3}\left\{\left(-2|\beta|^2-2|\gamma|^2+|\varepsilon|^2\right)\left|S^-(k,\tfrac{1}{2}q)\right|^2 + |\delta|^2\left|S^+(k,\tfrac{1}{2}q)\right|^2\right\}\,, \tag{7}$$

$$A_{x'z'} = A_y = 0\,. \tag{8}$$

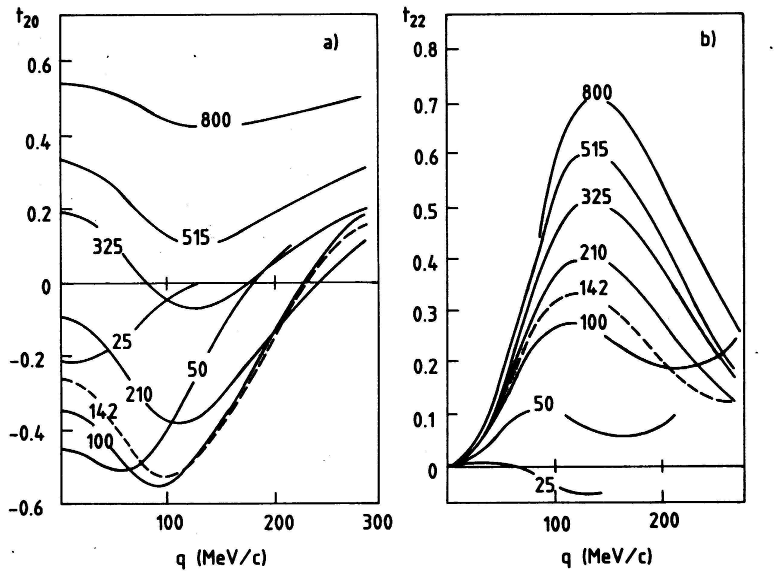

Figure 5: Predictions for the spherical tensor analysing powers for the $\vec{d}p \to (pp)_{^1\mathrm{S}_0}n$ reaction in impulse approximation [2].

Here the transition form factors involve the S and D deuteron radial functions:

$$\begin{aligned} S^{+}(k,q) &= \langle\, ^1\mathrm{S}_0\, | j_0(qr) |\, S\,\rangle + \sqrt{2}\,\langle\, ^1\mathrm{S}_0\, | j_2(qr) |\, D\,\rangle\;, \\ S^{-}(k,q) &= \langle\, ^1\mathrm{S}_0\, | j_0(qr) |\, S\,\rangle - \langle\, ^1\mathrm{S}_0\, | j_2(qr) |\, D\,\rangle\,/\sqrt{2}\,. \end{aligned} \tag{9}$$

The description of spin-1 analysing powers is much more complicated than that for spin-$\frac{1}{2}$ and the above formulae are written down in a system where one quantises along the mean of the initial and final c.m. proton and neutron momenta. To compare Andro's experimental data with theory away from the forward direction one has to do a kinematic Wigner rotation which mixes some of the components [3]. The second complication is that there are two alternative descriptions of analysing powers. That in Eqs. (6-8) is in the Cartesian basis but one also uses the spherical tensors $t_{\ell m}$ for which the Wigner rotation is slightly more transparent. The relation between the two is

$$\begin{aligned} A_{xx} &= -\sqrt{\tfrac{1}{2}}\,t_{20} + \sqrt{3}\,\mathrm{Re}\,(t_{22})\;, \\ A_{yy} &= -\sqrt{\tfrac{1}{2}}\,t_{20} - \sqrt{3}\,\mathrm{Re}\,(t_{22})\;, \\ A_{y} &= 2\sqrt{\tfrac{1}{3}}\,\mathrm{Re}\,(it_{11})\;. \end{aligned} \tag{10}$$

The predictions for the tensor analysing powers from the original paper with David Bugg [2] are shown in Fig. 5. The strong energy variation comes principally from the variation of $|\beta(0)|/|\varepsilon(0)|$ but the dramatic angular dependence arises mainly from the one-pion-exchange pole in the δ-amplitude. Note for example that t_{20} peaks for $q \approx m_\pi$.

I will not discuss here corrections arising from multiple scattering or from the variation of effective beam energy with momentum transfer, both of which tend to affect the cross section more than the analysing powers. It should be noted that to extract the cross section experimentally one has to have a very good determination of the excitation energy Q_{pp}, and this is one of the major challenges. I also have no time to discuss sum rules, where one integrates over all Q_{pp}.

The formulae were tested in an extensive series of experiments up to $T_d = 2$ GeV at the SPESIV spectrometer at Saclay [4], where they also investigated the $\vec{d}\,^{12}\mathrm{C} \to (pp)_{^1\mathrm{S}_0}\,^{12}\mathrm{B}^*$ reaction. However, the study of $\vec{d}p \to (pp)_{^1\mathrm{S}_0}\Delta^0$ did not live up to its promise because the form factor $S(k,q)$ drops off too fast with momentum transfer to give sufficiently high counting rates in the small aperture SPESIV spectrometer.

Let me show instead the comparison of theory with experiment at the lower energy of 350 MeV by the Grenoble group [5]. The differential cross section in Fig. 6 is well reproduced, though it should be noted that both experiment [5] and theory [6] should be multiplied by a factor of two!

The analysing powers of Fig. 7 are equally successful, though some of the small deviations could be due to effects of multiple scatterings that have not been included. However, I want to insist that one gets extra information from regions away from the $^1\mathrm{S}_0$ peak through looking at the angular distribution of the *pp* relative momentum. This is particularly important in the higher Q_{pp} regions, where P-waves are significant. Although we have really only examined the ANKE data with a 2 MeV cut in Q_{pp}, there are also a lot of events at higher Q_{pp}.

The results of the Grenoble group were used as the basis for the construction of a deuteron polarimeter (POLDER) which was employed at Saclay to measure the polarisation of the scattered deuteron in the $d\,^{12}\mathrm{C} \to \vec{d}\,^{12}\mathrm{C}^*(12.7\,\mathrm{MeV})$ [7]. More interestingly, it was used at J-Lab for the measurement of the polarisation of the recoil deuteron from elastic electron-deuteron scattering [8]. This allowed the separation of the spherical and quadrupole form factors out to the highest momentum transfers yet achieved.

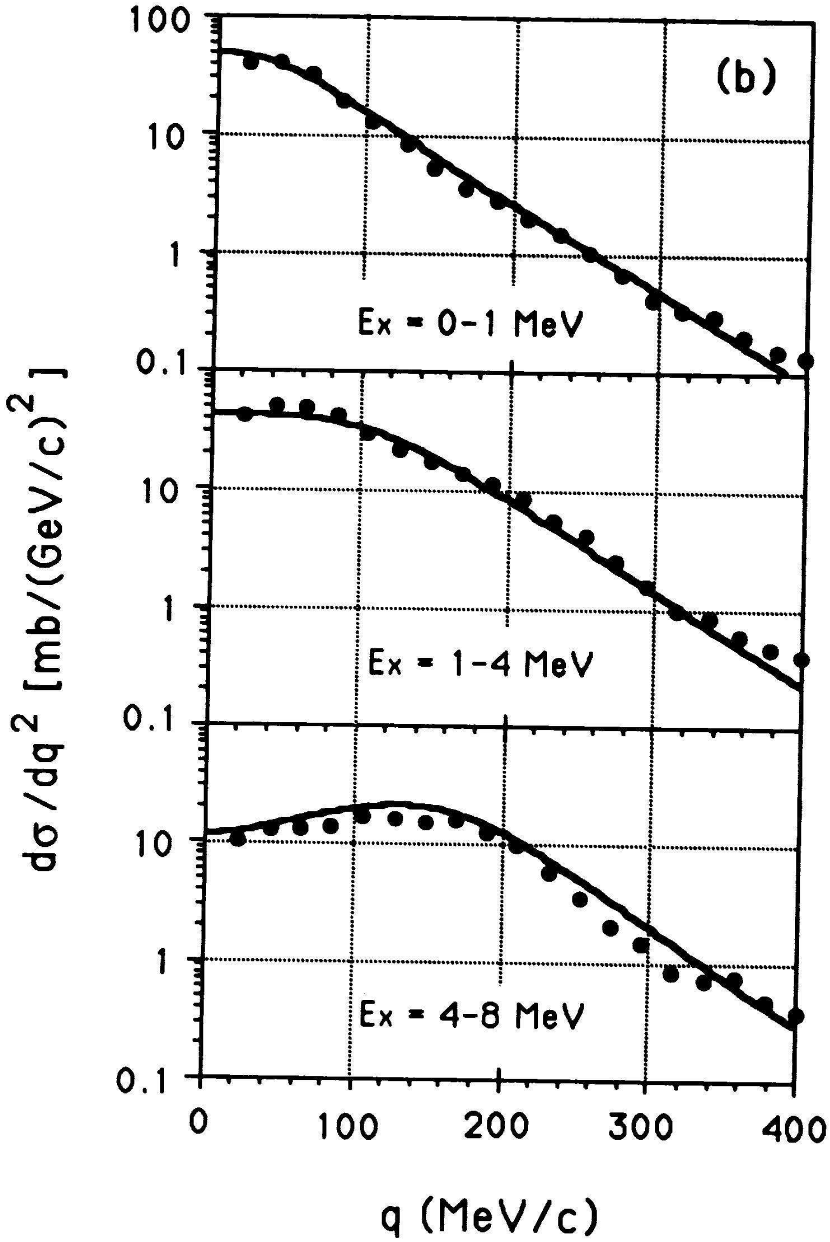

Figure 6: Differential cross section for the $dp \rightarrow (pp)n$ reaction at 350 MeV for different cuts in the pp excitation energy. The experimental data of the Grenoble group [5] is compared with impulse approximation predictions [6]. Note that the cross section scale is too low by a factor of two!

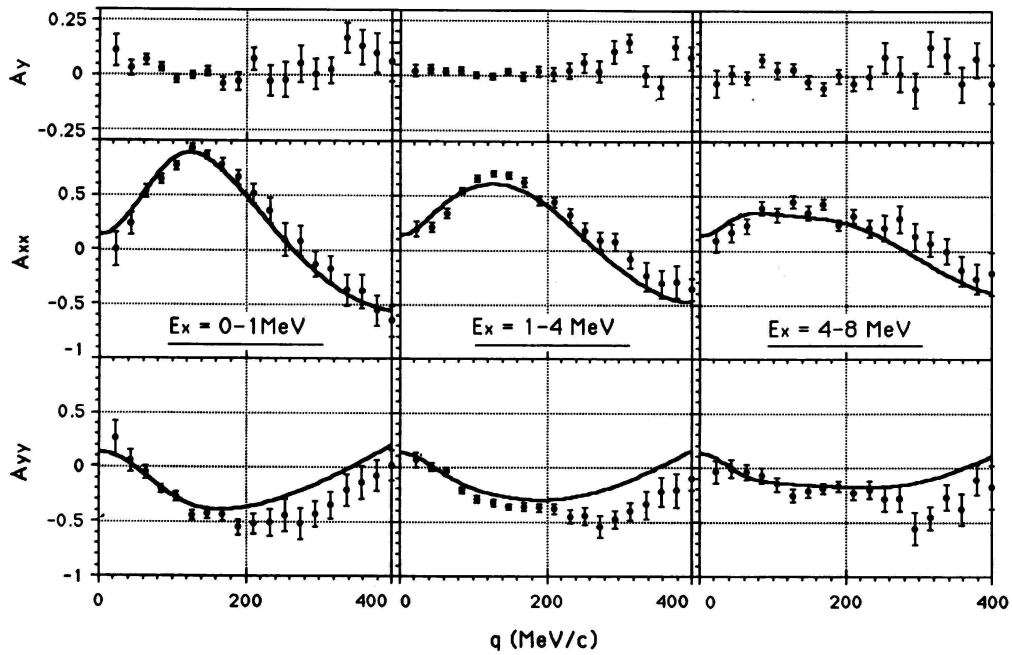

Figure 7: Cartesian analysing powers for the $dp \to (pp)n$ reaction at 350 MeV for different cuts in the pp excitation energy. The experimental data of the Grenoble group [5] is compared with impulse approximation predictions [6].

The spin-selectivity of the reaction is also being used to study nuclear structure. Thus at 260 MeV [9] and 171 MeV [10] there have been $\vec{d}\,^{12}\mathrm{C} \to (pp)_{^1\mathrm{S}_0}\,^{12}\mathrm{B}^*$ experiments to search for Gamow-Teller strengths (particularly 0^- levels) in ^{12}B [11].

In summary, although there are correction terms, impulse approximation gives a very reasonable first description of the $\vec{d}p \to (pp)n$ reaction at low momentum transfers at low pp excitation energies. With known $np \to pn$ amplitudes, it is possible to estimate quite accurately the polarisation observables. Alternatively, if one measures the cross section and analysing powers, it is possible to extract information on the amplitudes which, at the NN level, would require the measurement of a spin-transfer in the $\vec{n}p \to \vec{p}n$ reaction.

The future Physics programme at ANKE will also involve the measurement of spin-correlation coefficients involving the initial deuteron and proton. These will give access to the relative phases of some of the amplitudes. For example, for the production of the $^1\mathrm{S}_0$ state [12],

$$C_{y;y}\,\frac{\mathrm{d}^4\sigma}{\mathrm{d}t\,\mathrm{d}^3k} = -\tfrac{2}{3}\,\mathrm{Re}\,(\varepsilon^*\delta)\;S^-(k,\tfrac{1}{2}q)\,S^+(k,\tfrac{1}{2}q)\,, \tag{11}$$

and this information is very hard to achieve directly in np charge exchange.

Andro Kacharava will now present (in English) the results that we already have on the $(\vec{d}, 2p)$ reaction from a very preliminary but very successful run at ANKE. However, let me draw your attention also to the talk of Ralf Schleichert where he will (I hope) present the possibility of measuring the $\vec{d}(\vec{p}, 2p)n$ reaction by using a polarised gas cell and detecting both low energy protons in separate silicon telescopes. The resolution in both momentum transfer and excitation energy should be excellent and such a configuration would allow us to study deuteron charge exchange up to the maximum COSY proton energy, but I will let Ralf describe this.

This brings me back to the beginning of my talk and the Dubna measurement of charge exchange. One has to be careful about not to be careless in defining which is the spectator and which proton comes from the struck neutron. Otherwise one will generate false angular asymmetries. Leave that problem to the theorists — we can handle it!

References

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