PARITY OF THE Θ^+ PENTAQUARK AND SPIN OBSERVABLES FOR THE REACTION $NN \to Y\Theta^+$

PLAN:

- Θ^+ -?
- **Phenomenology $NN \to Y\Theta^+$ at the threshold**
- **Nonstandard method, $C_{y,y}$**
- **General method, arbitrary spins**
- **$\boldsymbol{\sigma}$-representation $\frac{1}{2}+\frac{1}{2} \to \frac{1}{2}+\frac{1}{2}$**
- **Conclusion**

Θ^+ is explicitly exotic

M $q\bar{q}$

B qqq

Θ^+ $B=1$ $S=+1$

$\bar{s}$ + ... (non-strange)

$B_{\bar{s}} = -\frac{1}{3}$

$1 = -\frac{1}{3} + \left(\frac{4}{3}\right)$

$\bar{s}\,uudd$

$\Xi^{--}_{3/2}$ $B=1$ $S=-2$ $Q=-2$

$ss + \cdots$

$B = +\frac{2}{3} + \left(\frac{1}{3}\right) = 1$

$Q = -\frac{2}{3} + \left(-\frac{4}{3}\right) = -2$

$ss\,dd\bar{u}$

New arena for low energy QCD

Does Θ^+ exist?

Positive: $-$**LEPS, DIANA, CLAS, SHAPIR, ZEUS, SVD, COSY-TOF,** $\ldots > 10$

* Θ^+ **S=+1 , B=1** nK^+, pK^0 $M = 1520 - 1540 MeV/c^2$, $\Gamma < 25$, 9, **or** $< 1 MeV$

(Θ^{++} **not observed** $\Longrightarrow I_\Theta = 0$)

* **S=** $-$**2** $\approx 1862 MeV/c^2$, $\Gamma \leq 18 MeV/c^2$

$\Xi^{--}(sdsd\bar{u}), \Xi^0$, $I = \frac{3}{2}$ **NA49** $pp \to \Xi^- \pi^- + \ldots$ $\sqrt{s} = 17.2$ GeV

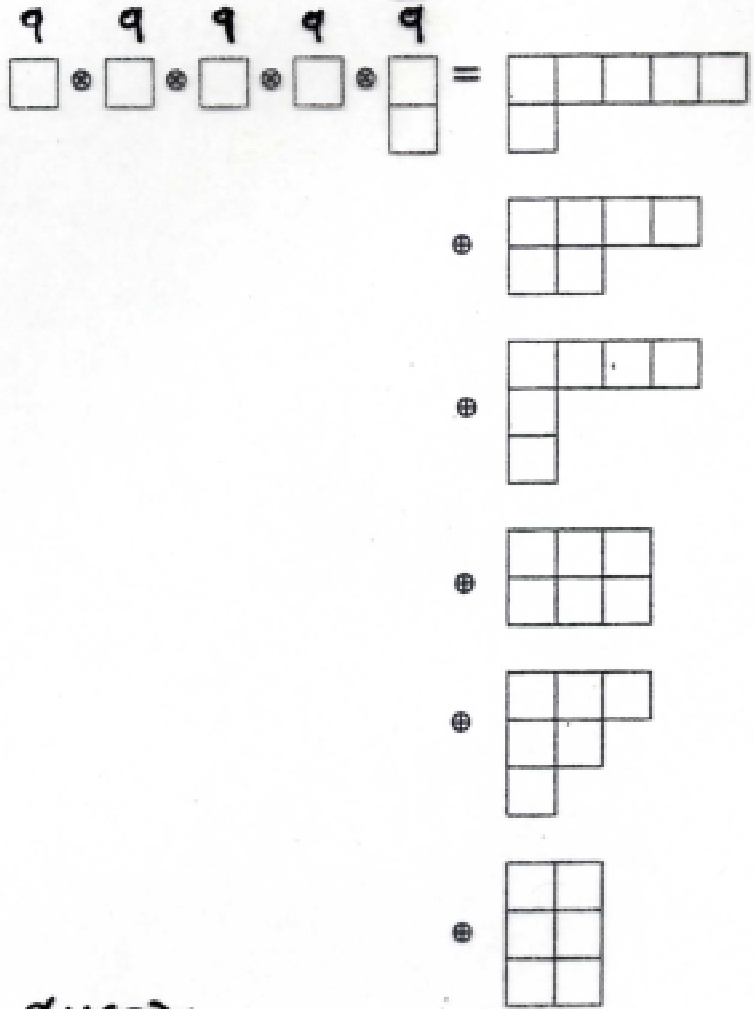
* **uudd**$\bar{c}$ $3099\, MeV/c^2$ **H1 hep-exp/0403017** $ep \to D^{*-}p$, $D^{*+}\bar{p}$

* N^0 **or** Ξ^0 $I = \frac{1}{2}$ $1734 MeV/c^2$ ΛK^0_s

STAR RHIC, hep-ex/0406032 Au+Au $\sqrt{s_{NN}} = 200$ GeV

Null results for Θ^+, $\Xi^{--}_{3/2}$, Θ_c: $-$

BES, HERA B, PHENIX, SPHENIX, WA89 hep-ex/0405042 hep-ex/0401014



Dim.	multi.	(λ,μ)
35	(1)	(4,1)
27	(3)	(2,2)
10	(4)	(3,0)
$\overline{10}$	(2)	(0,3)
8	(8)	(1,1)
1	(3)	(0,0)

SU(3):

$$N(\lambda,\mu) = \frac{1}{2}(\lambda+1)(\mu+1)(\lambda+\mu+2)$$

$$3 \otimes 3 \otimes 3 \otimes 3 \otimes \bar{3} = 3^5 =$$

$$= 35 \oplus (3)\,27 \oplus (2)\,\overline{10} \oplus (4)\,10 \oplus (8)\,8 \oplus (3)\,1$$

$$243 = 243$$

$$N_f = 3, 4, 5, 6$$

MODELS

Soliton model

$(ud)-(ud\bar{s})$ Karliner-Lipkin

$(ud)-(ud)-\bar{s}$ Jaffe-Wilczek

QCD sum rules

Large N_c QCD

Lattice QCD

KN or $K\pi N$ - Bound states

Soliton model

"rotational band":

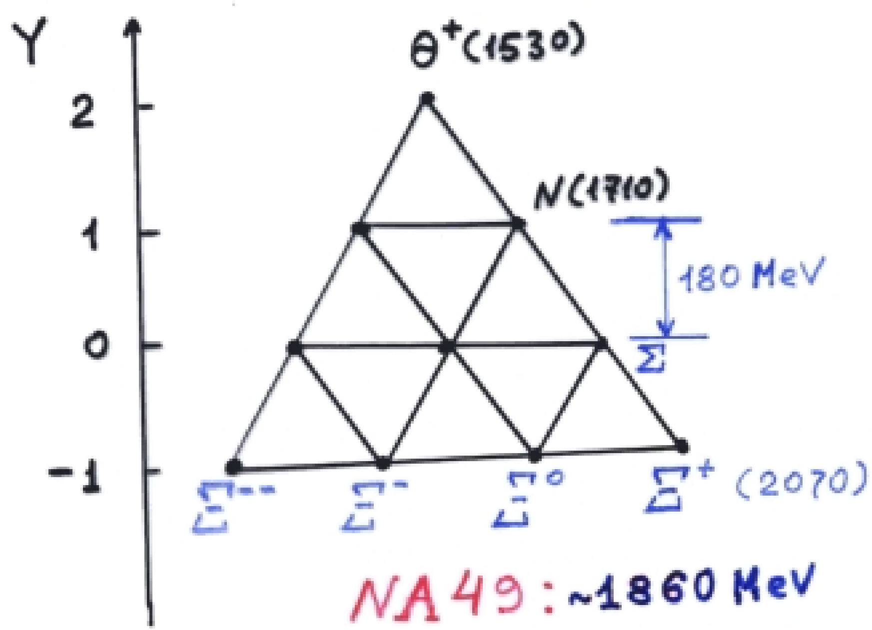
$(8_f, \frac{1}{2}^+)$ $(10_f, \frac{3}{2}^+)$ $(\overline{10}_f, \frac{1}{2}^+)$ ...

$SU_f(3)$ chiral soliton: 1530 MeV, Γ=15 MeV (!)
(30 MeV)

1997 Diakonov, Petrov, Polaykov

1987 Praszalowicz, Skyrme model

$I=0$ $Y=2$ K^+n



N(1646) ?

$\Sigma(1754)$?

$\Xi(1862)$

NA49: ~1860 MeV

V. Kopeliovich

J.R. Ellis, M. Karliner, M. Praszalowicz, hep-ph/0401127

D. Diakonov, hep-ph/0406043

But! I. Klebanov, P. Ouyang hep-ph/0408251 rigid rotator appr.

P-parity of the Θ^+ (models)

Chiral soliton $\frac{1}{2}^+$

Non-correlated quarks
$(1s)^5$ $\pi = -1$

Chiral qq-interaction
$\pi = +1$ $\frac{1}{2}, \frac{3}{2}$

Color-magnetic qq-forces
$(ud)^2 \bar{s}$ $\frac{1}{2}^+$

Color-electric forces
$(ud)(ud\bar{s})$ $\frac{1}{2}^+$

Lattice QCD $\pi = -1$

– " – " – $(ud)^2 \bar{s}$ $\pi = +1$

T. W. Chiu, T. H. Hsieh
hep-ph/0403020

$J^\pi = \frac{1}{2}^+$: $m_N [(ud)^2 \bar{d}] = 1460\ (51)$

$m_\Theta [(ud)^2 \bar{s}] = 1539\ (95)$

$m_\Xi [(ds)^2 \bar{u}] = 1826\ (87)$

P_Θ -Parity determination in $NN \to Y\Theta^+$
(model-independent)

A.W. Thomas, K.Hicks, A.Hosaka,

Prog.Theor.Phys. 111 (2004) 291

$\vec{p}\,\vec{p} \to \Sigma^+ \Theta^+$

C.Hanhart et al. Phys.Lett. 590 (2004) 39;

$C_{x,x}$, $j_\Theta = \frac{1}{2}$

Yu.N.U. , Phys.Lett. B595(2004) 277; hep-ph/0402216.

$\vec{p}n \to \vec{\Lambda}^0\Theta^+$ **arbitrary spin** j_Θ $\qquad \vec{\Lambda}^0 \to p + \pi^-$

$\vec{p}p \to \vec{\Sigma}^+\Theta^+$, $\quad K_y^y$

M.Rekalo, E.Tomasi-Gustaffson, Phys.Lett. B591 (2004) 225; hep-ph/0402227.

C.Hanhart, J.Haidenbauer, K.Nakayama, U.-G.Meissner, hep-ph/0407107 $\quad$ $K + K^*$ - exch.

The reaction $1 + 2 \to 3 + 4$ at the threshold

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \sum_{\substack{J\,M \\ S\,M_S\,L\,m}} (j_1\mu_1\, j_2\mu_2|S M_S) \times$$

$$(j_3\mu_3\, j_4\,\mu_4|J\,M)\,(S\,M_S\,L\,m|J\,M)\,Y_{Lm}(\hat{\mathbf{k}})a^{LS}_J$$

$\mathbf{J = j_3 + j_4,\ j_3 + j_4 - 1,\ \ldots,\ |j_3 - j_4|}.$

$\vec{S} = \vec{j}_1 + \vec{j}_2$

$(-\mathbf{1})^{\mathbf{L}} = \boldsymbol{\pi}$, where $\boldsymbol{\pi} = \boldsymbol{\pi}_1\,\boldsymbol{\pi}_2\,\boldsymbol{\pi}_3\,\boldsymbol{\pi}_4$

Pauli: $(-\mathbf{1})^{\mathbf{L+S+T}} = -\mathbf{1}$

$(-\mathbf{1})^{\mathbf{S}} = \pi(-\mathbf{1})^{\mathbf{T+1}}$

T=1 $\quad (-1)^S = \pi \quad$ **S=0 if** $\pi = +1$

* $\quad$ **S = 1 if** $\pi = -1$

T=0 $\quad (-1)^S = -\pi \quad$ **S=1 if** $\pi = +1$

* $\quad$ **S = 0 if** $\pi = -1$

At given T, S $\Rightarrow$ π independently of j_3, j_4

NONSTANDARD METHOD

The polarized cross section

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = \Phi \sum_{\mu_3\,\mu_4} |T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 =$$

$$= \frac{1}{4\pi} \sum_M (\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | S M)^2 \sum_{J\,M\,L\,L'} \sqrt{(2L+1)(2L'+1)} \times$$
$$\times (S\,M\,L\,0|J\,M)(S\,M\,L'\,0|J\,M) a^{LS}_J \,(a^{L'S}_J)^*,$$

Using the relations

$$(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | 00) = \chi^+_{\mu_1} \frac{i\sigma_y}{\sqrt{2}} \chi^{(T)+}_{\mu_2}$$

$$(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | 1\lambda) = \chi^+_{\mu_1} \sigma_\lambda \frac{i\sigma_y}{\sqrt{2}} \chi^{(T)+}_{\mu_2}$$

$\sigma_i \,(i = y, \lambda)$ is the Pauli matrix

$$(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | 00)^2 = \frac{1}{4}(1 - \mathbf{p}_1 \cdot \mathbf{p}_2), \qquad (3)$$

$$(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | 1M)^2 = \begin{cases} \frac{1}{4}(1 + \mathbf{p}_1 \cdot \mathbf{p}_2 - 2p_{1z}p_{2z}), & M = 0, \\ \frac{1}{4}[1 \pm (p_{1z} + p_{2z}) + p_{1z}p_{2z}], & M = \pm 1, \end{cases} \qquad (4)$$

The spin singlet initial state

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = d\sigma_0(1 - \mathbf{p}_1 \cdot \mathbf{p}_2), \qquad (5)$$

$$C_{x,x} = C_{y,y} = C_{z,z} = -1$$

.

The unpolarized cross section is given as

$$d\sigma_0 = \Phi \frac{1}{4} \sum_{\mu_1 \mu_2 \mu_3 \mu_4} |T^{\mu_3 \mu_4}_{\mu_1 \mu_2}|^2 = \frac{1}{16\pi} \Phi \sum_{J,L} (2J+1) |a^{L\,S}_J|^2. \qquad (6)$$

Spin-spin correlation for $\vec{\frac{1}{2}} + \vec{\frac{1}{2}} \to j_3 + j_4$

$S = 1$

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = d\sigma_0\,(1 + C_{x,x}\,p_{1x}\,p_{2x} + C_{y,y}\,p_{1y}p_{2y} + C_{z,z}\,p_{1z}\,p_{2z})$$

for arbitrary j_3 and j_4:

$$C_{x,x} = C_{y,y} = \frac{\sum_J |\sqrt{J}\,a_J^{J-1} - \sqrt{J+1}\,a_J^{J+1}|^2}{\sum_{J\,L}(2J+1)\,|a_J^L|^2}, \geqslant 0$$

$$C_{z,z} = 1 - 2\,C_{y,y}$$

General method

S.M. Bilenky, L.L. Lapidus, L.D. Puzikov and R.M. Ryndin, Nucl. Phys. **7** (1958) 646.

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \chi^+_{J3\,\mu_3}\chi^+_{J4\,\mu_4}\hat{F}\chi_{J1\,\mu_1}\,\chi_{J2\,\mu_2}, \tag{14}$$

$$\hat{F} = \sum_{\substack{m_1\,m_2\\ m_3\,m_4}} T^{m_3\,m_4}_{m_1\,m_2}\,\chi^+_{j_1\,m_1}(1)\chi^+_{j_2\,m_2}(2)\chi_{j_3\,m_3}(3)\,\chi_{j_4\,m_4}(4), \tag{15}$$

$$d\sigma_0 = \frac{\Phi}{(2j_1+1)\,(2j_2+1)}SpFF^+. \tag{16}$$

The spin-transfer coefficient

$$K_\lambda^\kappa = \frac{SpF\sigma_\lambda(1)\,F^+\,\sigma_\kappa(3)}{SpF\,F^+}, \qquad (17)$$

where $\lambda, \kappa = 0,\ \pm 1$. For $j_1 = j_3 = \frac{1}{2}$

$$\begin{aligned} 4\,d\sigma_0 K_\lambda^\kappa \equiv \delta_{\lambda,-\kappa}\frac{3}{2\pi} \sum_{\substack{S\,S'\,J\,J'\\ L\,L'\,J_0}} &\sqrt{(2L+1)\,(2L'+1)}\times \\ &\sqrt{(2S+1)\,(2S'+1)}\,(2J+1)\,(2J'+1) \\ &(-1)^{j_2+j_4+S'+J'+L}(1\ -\lambda\,1\,\lambda|J_0\,0)(L'\,0\,L\,0\,|J_0 0) \\ &\left\{\begin{matrix} \frac{1}{2} & j_2 & S \\ S' & 1 & \frac{1}{2}\end{matrix}\right\} \left\{\begin{matrix} \frac{1}{2} & j_4 & J' \\ J & 1 & \frac{1}{2}\end{matrix}\right\} \left\{\begin{matrix} J & S & L \\ J' & S' & L' \\ 1 & 1 & J_0\end{matrix}\right\} a_J^{L\,S}(a_{J'}^{L'\,S'})^* \end{aligned}$$

$$K_{+1}^{-1} = K_{-1}^{+1} = -K_x^x = -K_y^y$$

$$K_i^j = 0\,, \quad i \neq j$$

$S = S' = 0$

$$\begin{Bmatrix} \frac{1}{2} & j_2 & 0 \\ 0 & 1 & \frac{1}{2} \end{Bmatrix} = 0 \Rightarrow \text{all } K_i^j = 0$$

$S = S' = 1$

$$K_x^x = K_y^y \neq 0$$

$$K_z^z \neq 0$$

Spin-spin correlation coefficients

$$C_{\lambda\kappa} = \frac{SpF\sigma_\lambda(1)\,\sigma_\kappa(2)\,F^+}{SpF\,F^+}, \qquad (23)$$

we found for the case of $j_1 = j_2 = \frac{1}{2}$

$$4d\sigma_0\,C_{\lambda,\kappa} = \delta_{\lambda,-\kappa}\frac{3}{2\pi}\sum_{S\,S'\,J}(-1)^{S+J}(2J+1)$$

$$\times\sqrt{(2S+1)(2S'+1)}\sum_{L\,L\,J_0}(-1)^{L'}(2J_0+1)\sqrt{2L'+1}$$

$$(1\lambda\,1-\lambda\,|\,J_0\,0)(J_0\,0\,L'\,0\,|\,L\,0)\begin{Bmatrix} S' & S & J_0 \\ L & L' & J \end{Bmatrix}\begin{Bmatrix} S' & \frac{1}{2} & \frac{1}{2} \\ S & \frac{1}{2} & \frac{1}{2} \\ J_0 & 1 & 1 \end{Bmatrix}\times$$

$$a_J^{L\,S}(a_{J'}^{L'\,S'})^*$$

$C_{+1,-1} = C_{+1,-1} = -C_{x,x} = -C_{y,y} \neq 0$, $C_0^0 = C_z^z \neq 0$ whereas $C_{i,j} = 0$, where $i \neq j$ $(i,j = x,y,z)$.

$C_{x,x} = C_{y,y} = C_{z,z} = -1$ for $S = S' = 0$.

TENSOR POLARIZED Θ^+ $(j_\Theta \geq \frac{3}{2})$

Spin-tensors for any spin j:

$T_{JM}(j)$, **J=2j, 2j-1, $\cdots$,0**

M=J, J-1, $\cdots$, -J

$$\vec{N} + N \to \vec{Y} + \vec{\Theta}^+$$

$$K^{J_Y\, M_Y, J_\Theta\, M_\Theta}_{J_N\, M_N} = \frac{Sp\left\{T_{J_Y M_Y} T_{J_\Theta M_\Theta}\, F\, T_{J_N M_N}\, \boldsymbol{F}^+\right\}}{SpFF^+},$$

$K^{1\, M_Y, 0\, 0}_{1\, M_N} \to K^{M_Y}_{M_N}$ **spin-transfer**

$K^{1\, M_Y, 1\, M_\Theta}_{00} \to C^f_{M_Y\, M_\Theta}$ **spin-spin correlation**

$K^{00, J_4\Theta\, M_\Theta}_{00} \to t_{J_\Theta\, M_\Theta}$ **tensor polarisation**

For $J_\Theta - even$, $M_\Theta = 0$: $t_{J_\Theta 0}$ **is measurable**

by asymmetries in $\Theta^+ \to M + N$,

/S.M.Berman, M.Jacob, Phys.Rev. 139 (1965) B1023)/

Isospin of the Θ^+

$$\vec{p}\,p \to \vec{\Sigma}^+\,\Theta^+ \qquad (1)$$

$$\Sigma^+ \Rightarrow n + \pi^+,\ p + \pi^0$$

$$p\,n \to \Lambda^0\,\Theta^+ \qquad (2)$$

$$\Lambda^0 \to \pi^- + p$$

* Θ^+ isosinglet ($I_\Theta = 0$) $T = 0$

  pn: $T = 0$ $(-1)^S = -\pi$

  pp: $T = 1$ $(-1)^S = \pi$

* Θ^+ isotriplet ($I_\Theta = 1$) $T = 1$
  (27-plet, ...)

  pn: $T = 1$ identical
  pp: $T = 1$

* Θ^+ isoquintet ($I_\Theta = 2$) $T = 2$

  pn: forbidden

  pp: $T = 1$

Conclusion

Model independent analysis of the $NN \to Y\Theta^+$ at the threshold for arbitrary spin j_Θ $(\frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm)$:

$*$ **S=0**

$C_{x,x} = C_{y,y} = C_{z,z} = -1$

$K_y^y = 0,\ K_{J_N M_N}^{J_Y M_Y, J_\Theta M_\Theta} = 0$

$pp \to \Sigma^+ \Theta^+ \quad \pi = +1$

$pn \to \Lambda^0 \Theta^+ \quad \pi = -1$

(if $I_\Theta = 0$)

$*$ **S=1**

$C_{x,x} = C_{y,y} \geq 0$

$K_y^y \neq 0,\ K_{J_N M_N}^{J_Y M_Y, J_\Theta M_\Theta} \neq 0$

$pp \to \Sigma^+ \Theta^+ \quad \pi = -1$

$pn \to \Lambda^0 \Theta^+ \quad \pi = +1$

$*$ **if $j_\Theta \geq \frac{3}{2}$, then $t_{J0} \Longrightarrow \pi_\Theta$**

$*$ **The isospin T=0,1 of the NN-system acts as the P-parity π_Θ=+1, -1**

The method can be applied for others baryons