

Meson Production in pp , pn , dp , and dd Interactions

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I am supposed in half an hour to give the flavour of this vast topic without creating lots of enemies among the subsequent speakers by stealing all the juicy morsels from their talks. The hors d'oeuvre should not spoil the main course — though my Georgian food book tells me that the hors d'oeuvres can here be eaten throughout the whole meal!

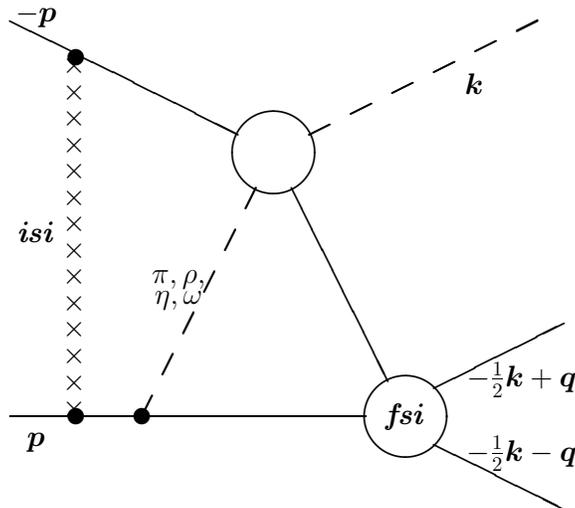


Figure 1: One-boson exchange contribution to meson production illustrating the isi and fsi .

It is important to note that the models that describe meson production in nucleon-nucleon collisions can be very different from those used to understand production in pd or dd interactions. In all cases though, the main degrees of freedom relevant for threshold production seem still to be mesons and nucleons rather than quarks and gluons and most estimates of production in the NN sector in the literature are based upon some version of the one-boson-exchange model of Fig. 1. A

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pion or other meson is emitted by one nucleon and scatters from the second nucleon to give the observed meson.

- The bulk of the cross section for π^+ production is provided by pion exchange. For the production of more massive mesons, where the momentum transfers get much larger and one is sensitive to the NN interaction at much shorter distances, it is likely that the exchange of the ρ or other heavy mesons will grow in importance.
- The final-state interaction (*fsi*) between the nucleons is crucial in any understanding of the phenomena. This distorts the energy dependence in a significant way.
- NN *fsi* are generally much stronger than *fsi* between mesons and nucleons. The simple proof of this is the existence of nuclei. Only for the η is there evidence that the η -nucleus scattering length is of the order of fermi.
- The initial state interaction (*isi*) is expected to produce a relatively slowly varying damping of the initial flux of particles but precise estimates are not easy. At medium energies the very strong production of the Δ isobar through $pp \rightarrow \Delta^{++}n$ removes a large amount of the pp incident wave. If you simply delete this flux through a damping factor then you can dramatically reduce the predictions unless you include also the possibility of a $\Delta^{++}n \rightarrow \eta pp$ transition in your calculation as well.

Since one is using the same basic mechanism to describe the production of different mesons with a mass m_X , it should come as no real surprise that the total cross sections for different mesons look rather similar in terms of the energy with respect to the threshold value. If W is the total c.m. energy in the system, it is convenient to define an excess (or excitation) energy

$$Q = W - 2m_p - m_X \approx \frac{q^2}{m_p} + \frac{k^2}{2m_R},$$

where, non-relativistically, the reduced mass in the final state is

$$m_R = \frac{2m_p m_p}{m_X + 2m_p}.$$

The alternative variable that is much used for pion production is to quote cross sections in terms of the maximum value of the meson momentum k in units of the meson mass. When the excitation in the pp system $Q_{pp} = q^2/m_p = 0$, this gives

$$\eta \approx \sqrt{2m_R Q/m_X}.$$

Note that a three-particle phase space varies like Q^2 or η^4 and so the η variable has the advantage of expanding the near-threshold region.

What really affects the phase-space energy dependence is if there are nearby poles in the NN *fsi* corresponding to a bound state (*e.g.* the deuteron) or virtual state (*e.g.* 1S_0). These can be incorporated into the theory by evaluating the NN final wave function with some potential or by using a Watson-Migdal fudge factor. The third way that Göran Fäldt and I took exploits a theorem in quantum mechanics that links scattering and bound states. If $v(q, r)$ is the S -wave scattering wave function at wave vector q and $u(\alpha, r)$ is a corresponding bound-state wave function with binding energy $\varepsilon = \alpha^2/m$ then, independent of the details of the potential,

$$\lim_{q \rightarrow i\alpha} \left\{ \sqrt{2\alpha(\alpha^2 + q^2)} v(q, r) \right\} = -u(\alpha, r). \quad (1)$$

To use rigorously the theorem in Eq.(1) in the np spin-triplet case would mean that one would have to extrapolate the experimental data as a function of the np excitation energy from positive values down to -2.22 MeV. Fortunately, however, the theorem is very robust and numerical studies with the Paris or other realistic potentials show that, provided that q and r are *not too large*,

$$v(q, r) \approx -\frac{1}{\sqrt{2\alpha(\alpha^2 + q^2)}} u(\alpha, r). \quad (2)$$

Now, because of the large momentum transfers, the meson production operator in Fig. 1 is sensitive to the short-range part of the pn wave function and so the *not-too-large* part of the r -condition is satisfied. Independent of the details of the operators, apart from their short-range nature, the production amplitudes \mathcal{M} are linked for small q by

$$\mathcal{M}(NN \rightarrow \{NN\}_q X) \approx -\mathcal{M}(NN \rightarrow \{NN\}_{bs} X) / \sqrt{2\alpha(q^2 + \alpha^2)}. \quad (3)$$

If, as in Fig. 1, we neglect the rescattering of meson- X from the final nucleus (nucleons), integration of the square of the matrix elements over the appropriate phase spaces allows one to link the production of np spin-triplet final states to that where the deuteron is observed instead:

$$\sigma_{I=0}(pn \rightarrow pnX) \approx \frac{1}{4} \left(\frac{Q}{\varepsilon} \right)^{3/2} \frac{1}{\left(1 + \sqrt{1 + Q/\varepsilon}\right)^2} \sigma(pn \rightarrow dX). \quad (4)$$

There is no bound state in the spin-singlet 1S_0 channel, but the pp system is almost bound with a virtual state at $\varepsilon \approx 0.5$ MeV (but with α negative). The energy dependence for the production of a meson plus a 1S_0 system is similar to that of Eq. (4) but without deuteron data to normalise the cross section. Thus one expects

$$\sigma_{I=1}(NN \rightarrow NNX) \approx C \left(\frac{Q}{\varepsilon} \right)^2 \frac{1}{\left(1 + \sqrt{1 + Q/\varepsilon}\right)^2}, \quad (5)$$

where C reflects the matrix element corresponding to Fig. 1.

Note that for $Q \ll \varepsilon$ the cross section behaves like the phase-space Q^2 whereas for $Q \gg \varepsilon$ the energy variation is more like Q^1 . The linear region in Q is seen for a wide variety of mesons and the formulae in Eqs. (4), (5) generally work **TOO** well. This could be accidental and it may well be that the NN S -waves fall off quicker with energy than the formulae and that the deficiency is made up by the P -waves that are not included in the model.

To compare with experimental data for pion production, we must take into account that π^+ production is dominated by the excitation of the Δ isobar and that this also gives significant energy dependence in the coefficient C . Including this, and also Coulomb effects, gives the fits shown in Fig. 2.

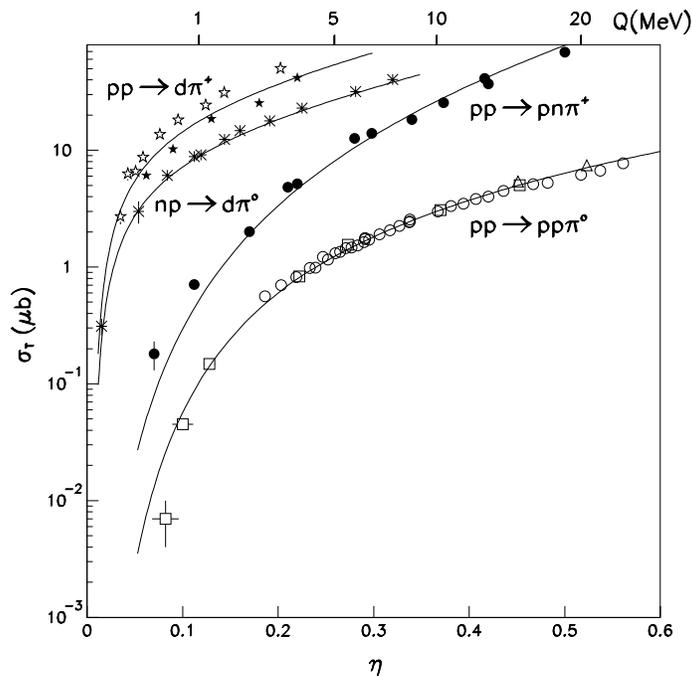


Figure 2: Total cross section for pion production in nucleon-nucleon scattering near threshold as a function of η , the maximum pion c.m. momentum in units of the pion mass (lower scale) and excess energy Q (upper scale).

There are some discrepancies for $Q \leq 1$ MeV, but one then has to be very careful that the beam energy is known rather precisely, and this is one of the many challenges of near-threshold experiments. All of the mass of data shown in Fig. 2 has, to a first approximation, been described by only **TWO** parameters, *i.e.* the C coefficients for $pp \rightarrow pp\pi^0$ and $pp \rightarrow d\pi^+$. Most of the spectacular variation in the picture arises from the *fsi* which should be present in any respectable model. This means that one really has relatively little sensitivity to the differences between various theoretical models provided that they give sufficient π^0 production strength. It is still not completely settled theoretically why C_{π^0} is so strong for π^0 production but the original IUCF data gave a real push for hadronic theory.

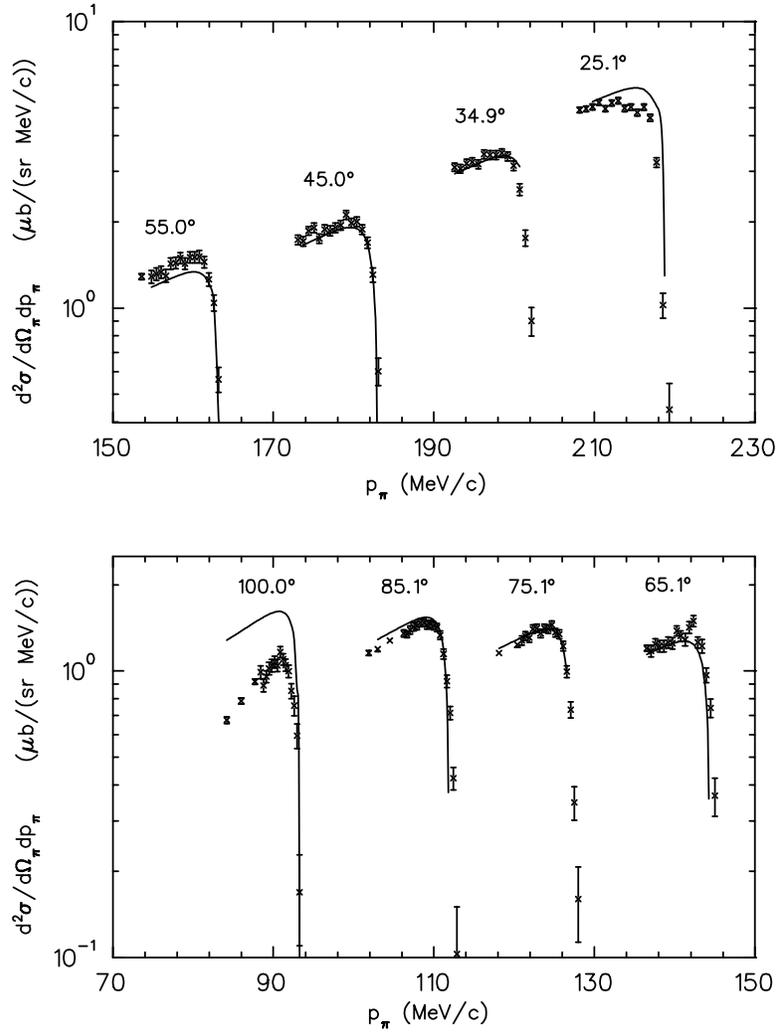


Figure 3: TRIUMF $pp \rightarrow pn\pi^+$ differential cross section at 420 MeV as a function of the pion angle and momentum compared to expectations on the basis of Eq. (6) transformed to the laboratory frame. The upper end points correspond to $Q_{pn} = 0$ and the data are typically shown over a range of ≈ 20 MeV in excitation energy.

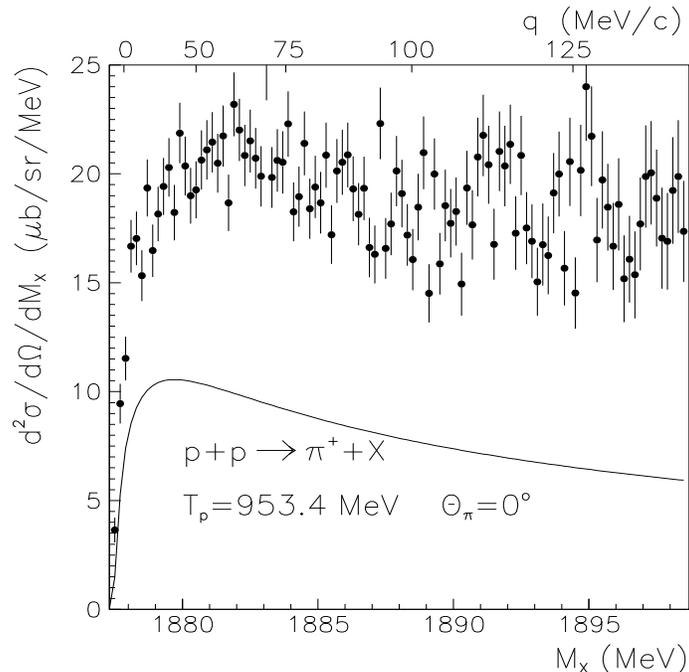


Figure 4: Big Karl data on the $p(p, \pi^+)X$ reaction at 953 MeV and 0° . The proton-neutron continuum shown is predicted from the deuteron counts using Eq. (6) and neglecting the spin-singlet final state.

The formula can be generalised to relate spin-triplet differential cross sections:

$$\frac{d^2\sigma}{d\Omega dx}(pp \rightarrow \{np\}_t\pi^+) \approx \frac{k(x)}{k(-1)} \frac{\sqrt{x}}{2\pi(x+1)} \frac{d\sigma}{d\Omega}(pp \rightarrow d\pi^+), \quad (6)$$

where $x = Q_{pn}/\varepsilon$, and $k(x)$ and $k(-1)$ are the momenta of the pion in the three and two-body reactions respectively. This works well for pion production up to 5–600 MeV, as illustrated by the old TRIUMF data in Fig. 3. There is a problem with the smallest and largest angles but my mother always taught me to be suspicious of experimentalists’ first and last points!

There are, however, some new and unpublished data on $pp \rightarrow \pi^+X$ from Big Karl (shown in Fig. 4) that bring this approach into question. The missing-mass resolution of a few hundred keV allows a clear separation of the deuteron peak from the pn continuum. Though the predicted shape is perfect, the normalisation is too low by a factor of two. This cannot be due to spin-singlet production because that would show a sharp spike in the missing-mass distribution. If this factor of two is genuine, the only way out theoretically would be if the np D -wave had an enormous importance here — the contribution changes sign from the bound-state to the scattering regions.

For η production in $pp \rightarrow pp\eta$, the purely S -wave fsi formula of Eq. (4) describes the bulk of the energy dependence of the total cross section up to $Q \approx 60$ MeV, as shown in Fig. 5a. Higher partial waves are clearly necessary above about 60 MeV. Close to threshold the data seem to be a bit high compared to the fit that has been normalised in the 20 MeV region. This could well be due to a fsi of the η with the two protons being driven by the strong η - p interaction, a point that should be discussed further by Volker or Timo (or both). Such interactions are expected to be far less

for η' production; the measured $pp \rightarrow pp\eta'$ cross section is about three orders of magnitude down on the corresponding η case, though the energy dependence shown in Fig. 5b is still well reproduced by the simple *fsi* formula.

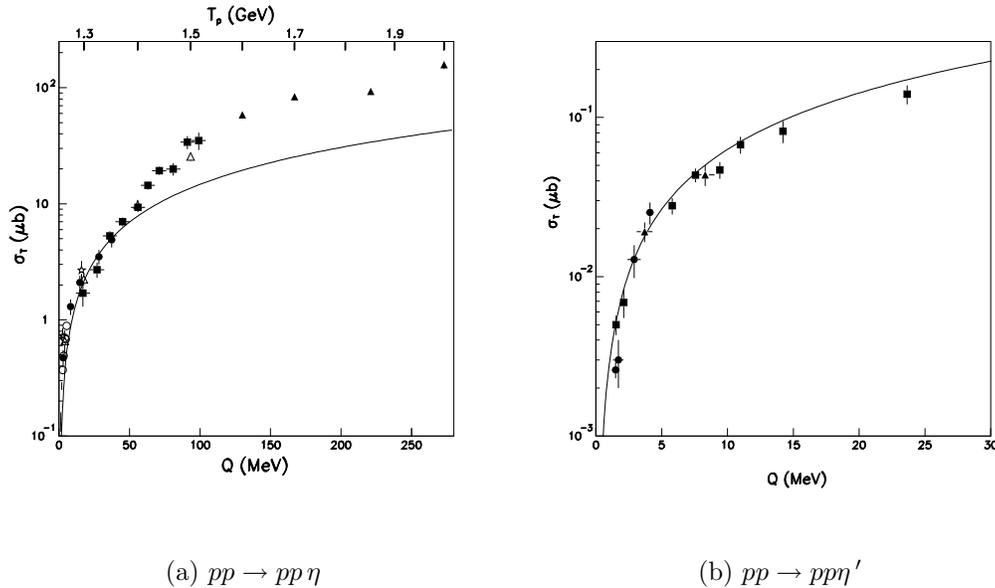
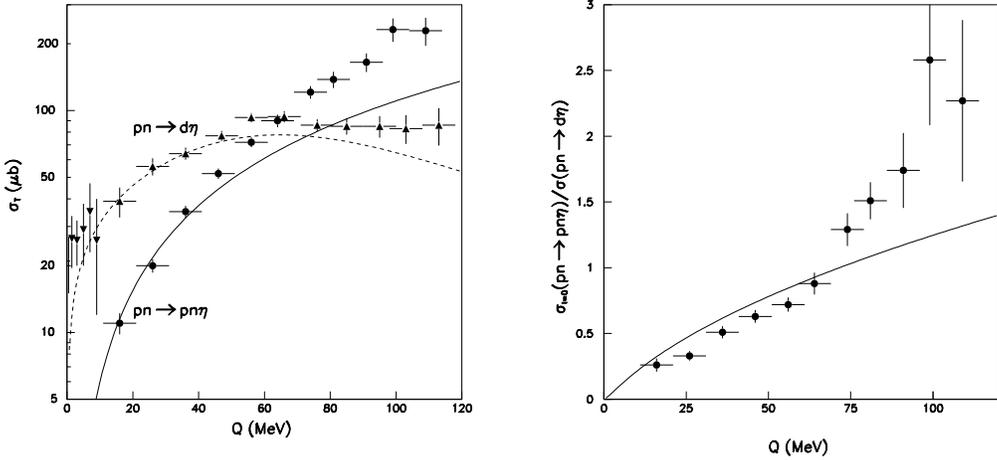


Figure 5: Comparison of the total cross sections for η and η' production in proton-proton collisions with the energy dependence given by the *fsi* formula of Eq. (5).

The strength of meson production in proton-neutron collisions can be vastly different from proton-proton and, unless experimentalists provide data on the different isospin channels, theorists have just too much flexibility in interpretation. There are a variety of approaches to get at the pn data, essentially all of them relying on the deuteron as a pseudo-neutron target. The PINOT comparison of inclusive $pd \rightarrow \eta/pp \rightarrow \eta$ already showed that away from threshold the production on the neutron was many times stronger than on the proton. The CELSIUS measurements also relied on detecting the two photons from the η decay but in coincidence with a final deuteron and proton. Exploiting the Fermi motion, this allowed them, through kinematic reconstruction, to determine the total cross sections of both $pn \rightarrow d\eta$ and $pn \rightarrow pn\eta$ as functions of Q while working at one fixed beam energy (corresponding to $Q \approx 40$ MeV). This led to the results shown in Fig. 6a. Due to phase-space considerations, for small Q the two-body reaction is much stronger. Quantitatively the ratio of the three- and two-body data are quite well reproduced up to $Q \approx 60$ MeV by the *fsi* theorem shown in Fig. 6b.



(a) CELSIUS data on the $pn \rightarrow d\eta$ and $pn \rightarrow pn\eta$ total cross sections.

(b) CELSIUS cross section ratio compared to the prediction of the *fsi* theorem.

Figure 6: η production in proton-neutron collisions.

The most striking result is that

$$\frac{\sigma_T(pn \rightarrow pn\eta)}{\sigma_T(pp \rightarrow pp\eta)} \approx 6.5,$$

and this strongly suggests that the production amplitude is dominated by isovector meson exchange, *i.e.* π or ρ . Göran Fäldt and I got our best agreement to data (and prejudice) by taking the ρ to be the dominant term with the π interfering destructively in the $pp\eta$ case, but this interpretation is rather model dependent.

The dashed curve in Fig. 6 shows the shape of $\sigma_T(pn \rightarrow d\eta)$ expected for the production of a Breit-Wigner $N^*(1535)$ resonance at rest in the c.m. There must be deviations from the curve at high Q due to the contributions of higher partial waves. However, just as for $pp \rightarrow pp\eta$, the data are also definitely too high at very small Q and I must stress that it is deviations from the curve that is a signal for interesting Physics. In this case it is a signal for a strong ηd scattering length. The shape of the enhancement was confirmed in a different CELSIUS experiment, where the $pd \rightarrow pd\eta$ reaction was measured in complete kinematics at energies well below the quasi-free threshold. There is no η - d quasi-bound state but a very large scattering length — to be discussed perhaps by Volker or Timo.

The next heaviest meson is the ω and, due to its finite width, we now start getting serious background problems from multipion events when we identify the meson just as a missing-mass peak in the $pp \rightarrow ppX$ reaction. This is illustrated by the SPESIII

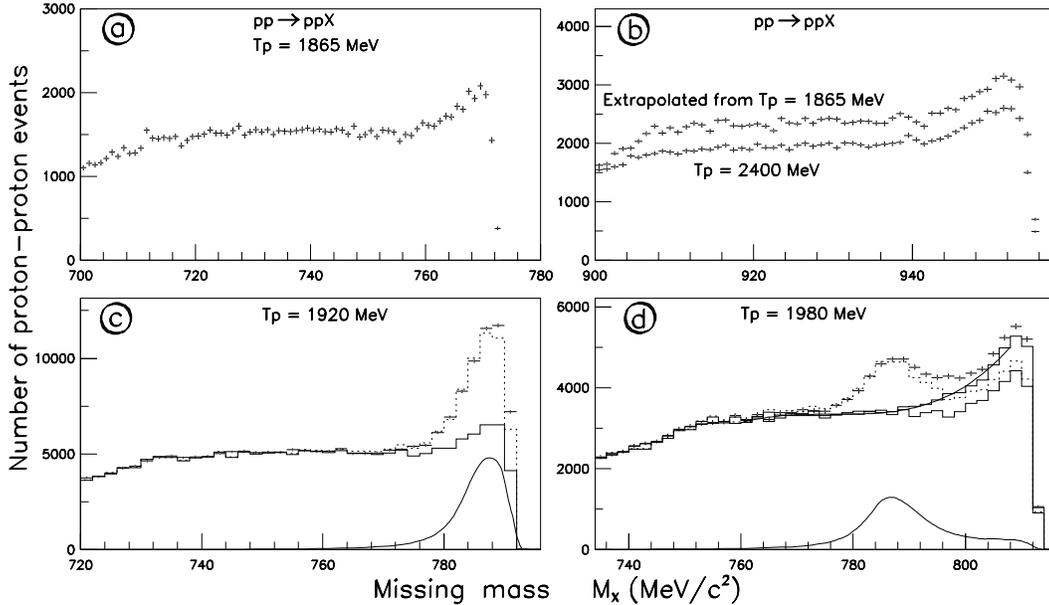


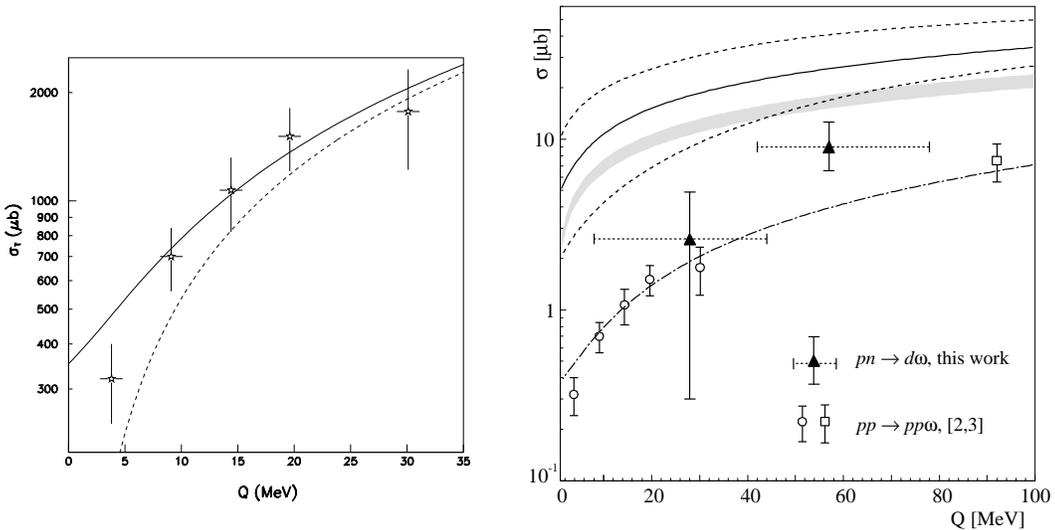
Figure 7: Missing mass spectra of the $pp \rightarrow ppX$ reaction at nominal beam energies of (a) 1865 MeV, (b) 2400 MeV, (c) 1920 MeV, and (d) 1980 MeV.

data in Fig. 7. Fortunately one is saved by the shape of the background varying smoothly with energy, provided that one measures from the maximum missing mass allowed for a particular beam energy. Difficulties of this kind illustrate the point that measuring decay products of the ω may be necessary to beat the background. COSY-TOF looked at the charged pions from the $\omega \rightarrow \pi^+\pi^-\pi^0$ decay but a kinematically more complete channel would have been the $\omega \rightarrow \pi^0\gamma \rightarrow 3\gamma$ branch, which requires electromagnetic calorimetry.

The energy dependence of the total cross sections deduced in this experiment and shown in Fig. 8a is again well represented by the *fsi* formula provided that it is smeared over the finite ω width.

Since we have no kinematically complete measurements of the ω from its decay products, we cannot use the CELSIUS η technique to measure $pn \rightarrow d\omega$. Instead we turn to the spectator methodology that was also developed at CELSIUS. Here the low energy proton from the $pd \rightarrow p_s d \omega$ is detected directly in a solid state telescope placed in the vacuum of the ANKE target chamber. The multipion background is as troublesome as in the pp case and things are made worse here by the strong suppression of events away from the maximum missing mass due to the ANKE acceptance.

Nevertheless, by using data away from the ω peak at other energies it is possible to deduce a believable background, and that is what is used in Fig. 8 to isolate the ω signals. This experiment was carried out with a preliminary version of the spectator



(a) Energy dependence of $\sigma_T(pp \rightarrow pp\omega)$ measured at SPESIII. The dashed curve represents the *fsi* formula of Eq. (5) whereas the solid curve is smeared over the finite width of the ω meson.

(b) The $pp \rightarrow pp\omega$ data are those from SATURNE (open circles) and COSY-TOF (open square), whereas the two ANKE $pn \rightarrow d\omega$ points are given by the closed triangles. If the ratio for $d\omega$ to $pp\omega$ were similar to that for η production, one would then obtain the solid curve.

Figure 8: Total cross sections for ω production in proton-nucleon collisions.

telescope and an order of magnitude improvement could be envisaged with the more refined set-up that is now operational. Though this will reduce the error bars and allow finer bins in Q , the background is always going to be a problem in such a missing-mass experiment.

The total cross sections are shown in Fig. 9 together with those for $pp \rightarrow pp\omega$. It is important to note that the range of theoretical predictions is typically a factor of 2–3 higher than the ANKE measurements and this information is crucial for any theoretical model builder.

Since Irakli and Yoshi are going to talk about ϕ production and Yury K^+ , let me say a few words about the ρ . Though this has a mass similar to that of the ω , its width is enormous in comparison and this makes it even difficult to define what one means by near-threshold production. More troublesome, even in an exclusive measurement, is the fact that there is an enormous background arising from $pp \rightarrow \Delta\Delta \rightarrow pp\pi^+\pi^-$. Thus in the $\pi^+\pi^-$ invariant mass distribution obtained by the DISTO collaboration and presented in Fig. 10, it is a little hard to quantify the ρ signal even though all four final particles were identified.

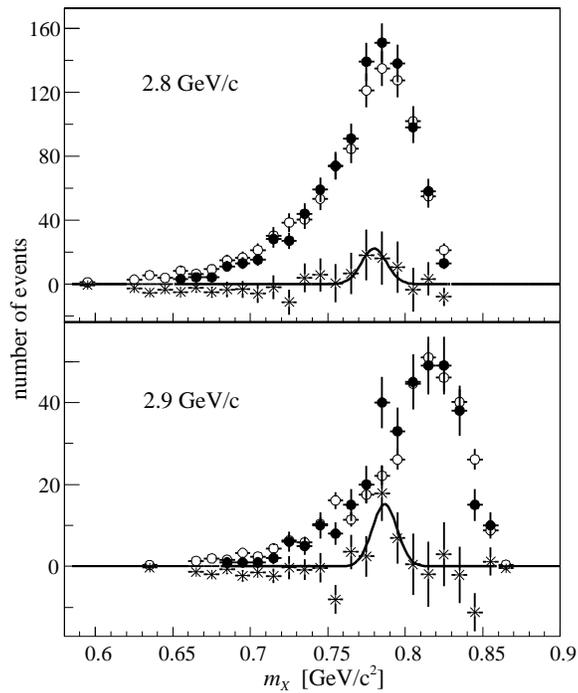


Figure 9: Missing-mass spectra of the $pd \rightarrow p_s d X$ reaction. The closed circles are the experimental data at 2.8 and 2.9 GeV/c whereas the open circles represent the data at the other momenta kinematically shifted as in the SPESIII pp experiment. The differences between the two sets (stars) are fitted to the expected ω -peak shape to yield the measured production cross section.

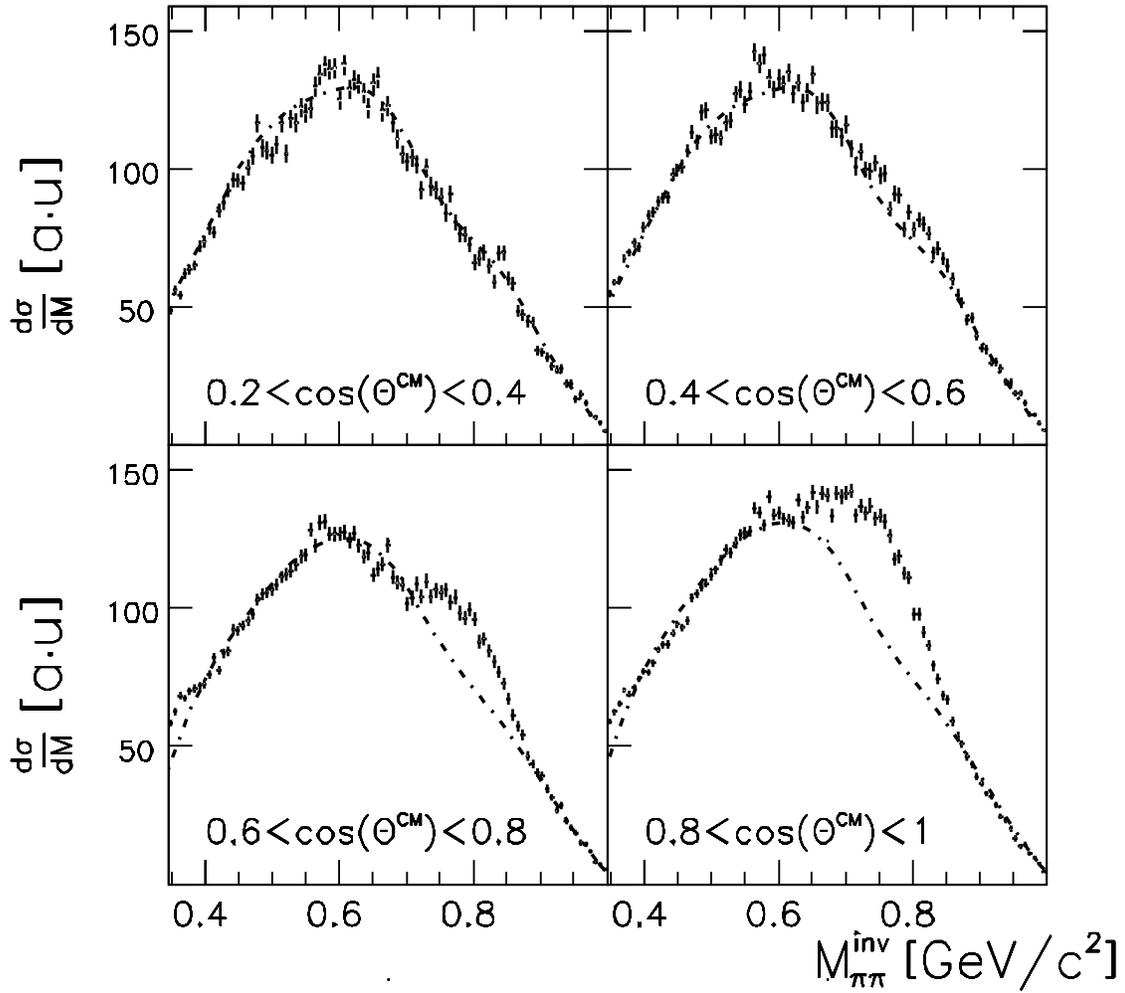


Figure 10: DISTO measurement of $pp \rightarrow pp\pi^+\pi^-$ at 3.67 GeV/c.

Let me turn now to meson production in proton-deuteron collisions. Timo will discuss in detail the ${}^3\text{He}\eta$ system but I must talk a little about $pd \rightarrow {}^3\text{He}\eta$ because it has been measured several times and the problems here are typical of many reactions. The amplitude for the reaction is as strong as that for pion production at threshold despite the much larger momentum transfer. This very large momentum transfer makes models where there is a spectator nucleon, as illustrated in Fig. 11a, underpredict the $pd \rightarrow {}^3\text{He}\eta$ cross section. A contribution is required where all three final nucleons are involved in the reaction mechanism. Kilian and Nann suggested that the diagram of Fig. 11b, with a pion produced on one nucleon *via* $pp \rightarrow d\pi^+$ followed by $\pi^+n \rightarrow p\eta$, might fill the gap. The near-threshold kinematics are very favourable for the final proton and deuteron sticking to form a ${}^3\text{He}$ nucleus.

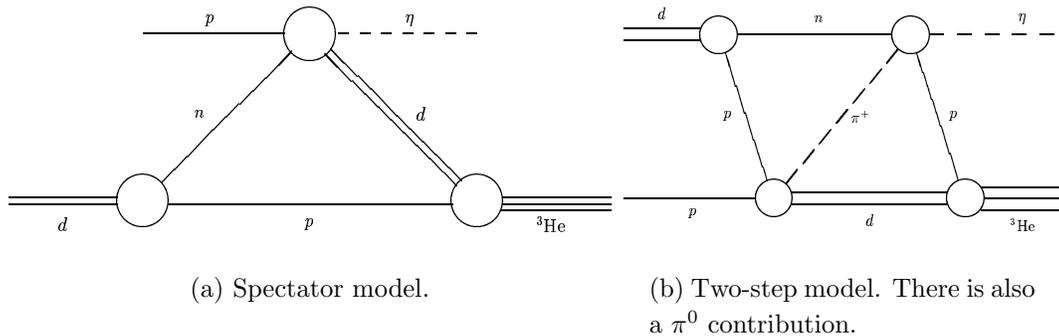


Figure 11: Models for the $pd \rightarrow {}^3\text{He}\eta$ reaction.

Timo will doubtless present the $pd \rightarrow {}^3\text{He}\eta$ data but, having taken the *fsi* into account, the two-step model is too low by a factor of 2.4. There is a similar underestimate for η production in $dd \rightarrow \eta{}^4\text{He}$, that is one of the subjects of Volker's talk. Instead of stepping further on these peoples' toes, let me show you in Fig. 12 the values of the threshold amplitudes for heavier meson production as well as for the η .

It should be noted that, whereas η production in $pd \rightarrow {}^3\text{He}\eta$ is about 2×10^{-3} of that with pion beams, this figure drops by almost two orders of magnitude for the η' . In the two-step model, this is mainly due to the steep fall of the $pp \rightarrow d\pi^+$ amplitude on the high side of the Δ resonance.

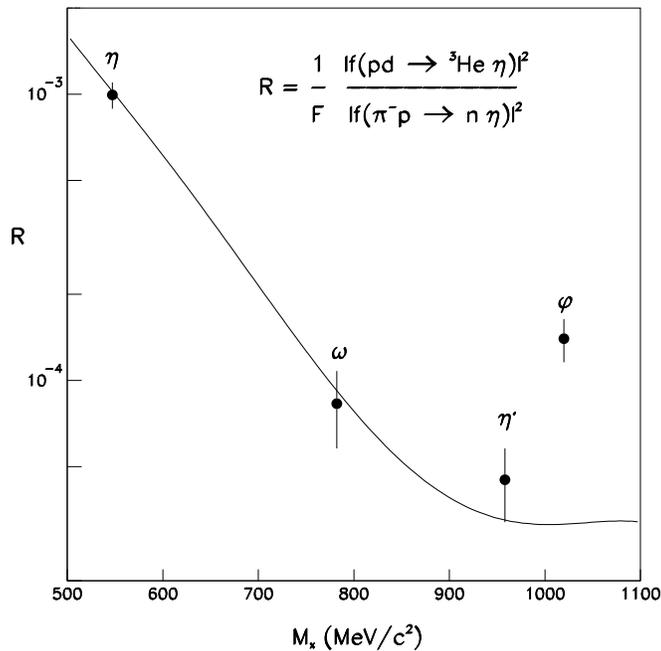


Figure 12: Ratio of $pd \rightarrow {}^3\text{He} X^0$ to $\pi^- p \rightarrow n X^0$ threshold amplitudes squared, assuming a constant normalisation factor $N = 2.4$ and that the *fsi* enhancement factor F is only important in the η case.

Saturne and later CELSIUS produced data on the $pd \rightarrow pd\eta$ reaction near threshold. Although there is no evidence in the data for any *fsi* between the proton and deuteron, one does see the reflection of the large ηd scattering length. To get a model where one neglects all *fsi*, one just chops off the last blobs in Fig. 11 to get the spectator and two-step terms of Fig. 13.

However, there is a third possibility that is shown in Fig. 14, where just one of the nucleons from a quasi-free $pn \rightarrow pn\eta$ reaction is captured on the spectator proton to produce the observed deuteron. This diagram could in fact be responsible for the couple of events of $pd \rightarrow K^+\Lambda d$ below the NN threshold that Yury Valdau might talk about before the *coffee* break.

As one approaches the free NN threshold ($Q \approx 200$ MeV) it is clear that the spectator model will be utterly dominant because it is not then kinematically suppressed and it only involves a single interaction. This is seen in Fig. 15 as the dashed curve rising fast with increasing Q . To get agreement with the data, the intermediate pion contribution has to be multiplied by a factor of 2.5. Though such a factor is not terribly different from that required for the $pd \rightarrow {}^3\text{He}\eta$ case, unfortunately the deuteron angular distribution is completely wrong, with deuterons being produced

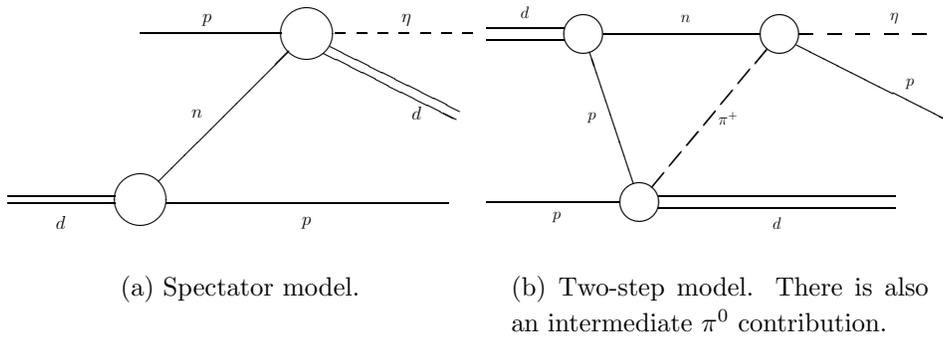


Figure 13: Models for the $pd \rightarrow pd\eta$ reaction.

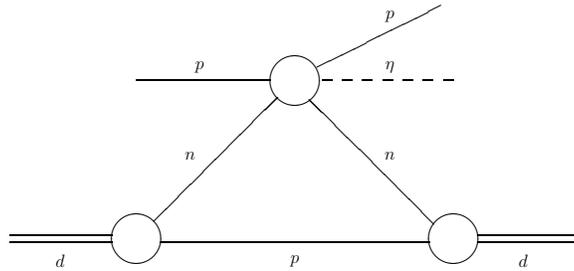


Figure 14: Impulse approximation for the $pd \rightarrow pd\eta$ reaction. There is also a contribution where the reaction takes place on the proton in the deuteron.

preferentially in the forward direction. This is possibly due to the neglect of the impulse approximation diagram of Fig. 14, which mainly produces slow deuterons in the laboratory frame because this minimises the momentum transfer between the initial and final deuteron. The evaluation of this will have to await until the return from Tbilisi.

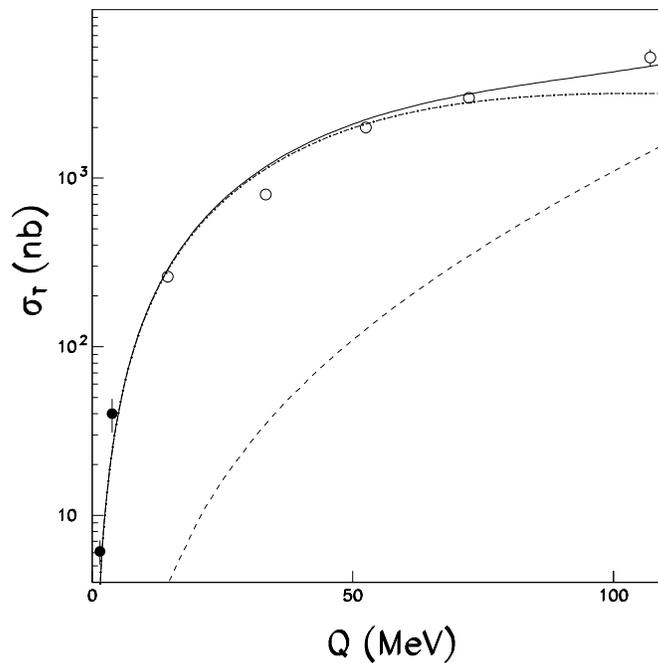
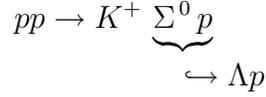


Figure 15: Spectator model (Fig. 13a) for the $pd \rightarrow pd\eta$ reaction (dashed curve) compared to the contribution from the two-step model of Fig. 13b [Tengblad *et al.*], multiplied by a factor of 2.5. The solid curve represents the incoherent sum of these two terms. Note that the impulse approximation contribution of Fig. 14 has not yet been included.

I have not had time to describe the limited information on the angular distributions on η production. Furthermore, rather than spending the last few minutes talking about two-pion production (which I could discuss for hours), I want to address the general question of why it might be useful to study meson production with proton and deuteron probes rather than photon or pion probes. Of course one has first to build up a database of information for a particular meson, as will be discussed by the later speakers this morning. However, here are a few suggestions to which I hope the workshop participants could add a few ideas of their own.

1. Proton beams are far more intense and monochromatic than pion. However, to exploit them fully, we have to have very good triggers because production processes (other than that for the pion) represent a smaller proportion of the total hadronic cross section. This is illustrated by Fig. 12 where it is seen that η' production through $pd \rightarrow \eta' {}^3\text{He}$ is less than 10^{-4} that in $\pi^- p \rightarrow \eta' n$. This is very relevant if we are looking for these reactions as sources for rare decay investigations.
2. Data on meson production in both pp and pn collisions are needed in order that one might interpret similar production in nucleus-nucleus interactions. The relative phase between the two amplitudes may require one to attempt measurements of coherent production on the deuteron, *e.g.* $pd \rightarrow p \eta d$, above threshold in regions with the smallest momentum transfers between the initial and final deuterons.
3. Chiral perturbation theory is making great advances in interpreting systems of pions and nucleons; the $pp \rightarrow pp \pi^0$ is the simplest reaction which tests our understanding of systems of pions and nucleons in nuclear physics.
4. Final-state interactions of mesons with nuclei. The prime example is, of course, the case of η -mesic nuclei and the best evidence here has come from data taken with proton and deuteron beams. These should also be visible in final-state enhancements of say $p {}^6\text{Li} \rightarrow \eta {}^3\text{He} {}^4\text{He}$.
5. Final-state interactions involving hyperons have been investigated in $pp \rightarrow K^+(\Lambda p)$ and there is a COSY proposal to look at the p_K dependence of the proton analysing power at $\theta_K^{\text{cm}} = 90^\circ$ in order to extract the Λp spin-triplet scattering length. I personally think that it might be more promising to look at spin-correlation measurements in $\vec{p}\vec{p} \rightarrow K^+(\Lambda p)$ in the forward direction.

6. TOF has detected cusp effects due to intermediate Σ production



We can then deduce something about the low energy transition $\Sigma N \rightarrow \Lambda p$. If this could be done with polarised beam and target, we could say something about the spin dependence of the reaction. This dispersive force is relevant in hypernuclear studies.

7. Intermediate virtual pion beams, which have been shown to have a crucial importance for reactions such as $pd \rightarrow {}^3\text{He} \eta$ near threshold, are intrinsically interesting because in some sense they represent a three-body force.
8. The transformation of two protons into two Δ , in reactions such as $pp \rightarrow pp\pi^+\pi^-$ or, even more clearly, in $dd \rightarrow {}^4\text{He} \pi\pi$ requires the presence of more than one nucleon!
9. Isospin selection rules can be helpful. For example, the ω is seen very clearly in $dd \rightarrow {}^4\text{He} \omega$ because it has isospin zero. Are there quasi-bound states of the ω with nuclei to parallel those of the η ? Alternatively, is ω production suppressed near threshold, as seems to be indicated by $pd \rightarrow {}^3\text{He} \omega$ data? These reactions could be measured much better at COSY if the ω were identified through its $\pi^0\gamma$ decay mode.
10. The violation of isospin conservation has been identified in the $dd \rightarrow {}^4\text{He} \pi^0$ reaction at IUCF. Though this is one of the best existing tests, the challenge is to take results here, or from the related $dd \rightarrow dd \pi^0$ reaction, and extract what they mean at a more microscopic level.

Let me finish by thanking the organisers of the Workshop and hoping that these few hors d'oeuvres will not give you indigestion. Remember what Alexander Pushkin said:

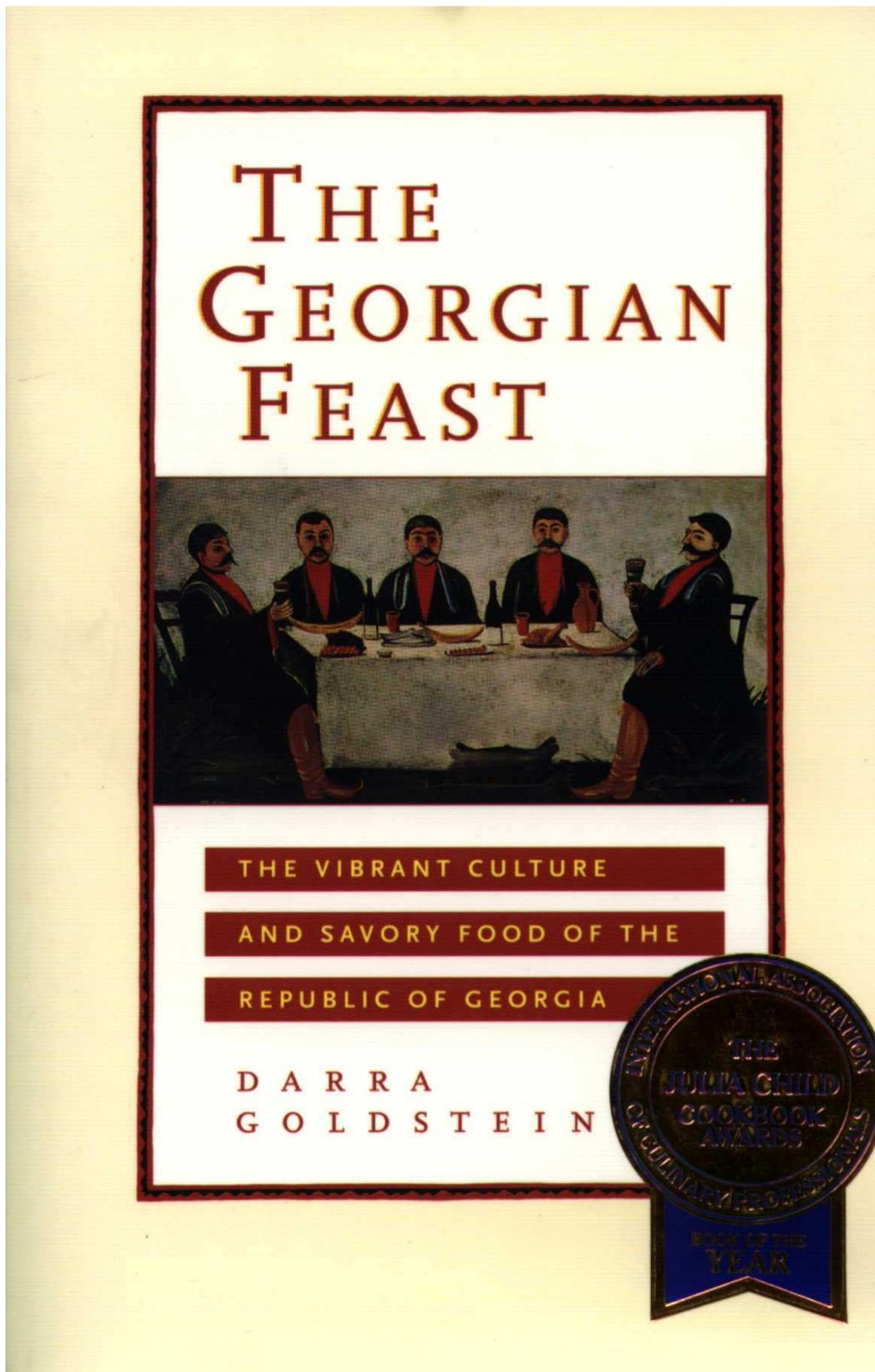


Figure 16: “Every Georgian dish is a poem.”