# $a_0(980)$-$f_0(980)$ mixing and isospin violation in the reactions $pN \to da_0$, $pd \to {}^3\mathrm{He}/{}^3\mathrm{H}\, a_0$ and $dd \to {}^4\mathrm{He}\, a_0$

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**Abstract**

It is demonstrated that $f_0$-$a_0$ mixing can lead to a comparatively large isospin violation in the reactions $pN \to da_0$, $pd \to {}^3\mathrm{He}/{}^3\mathrm{H}\, a_0$ and $dd \to {}^4\mathrm{He}\, a_0$ close to the corresponding production thresholds. The observation of such mixing effects is possible, e.g., by measuring the forward-backward asymmetry in the reaction $pn \to da_0^0 \to d\eta\pi^0$.

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As it was suggested long ago in Ref. [1] the dynamical interaction of the $a_0(980)$- and $f_0(980)$-mesons with states close to the $K\bar{K}$ threshold may give rise to a significant $a_0(980)$-$f_0(980)$ mixing. Different aspects of this mixing and the underlying dymanics as well as the possibilities to measure this effect have been discussed in Refs. [2–6]. Furthermore, it has been suggested recently by Close and Kirk [7] that the new data from the WA102 collaboration at CERN [8] on the central production of $f_0$ and $a_0$ in the reaction $pp \to p_s X p_f$

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provide evidence for a significant $f_0$-$a_0$ mixing intensity as large as $|\xi|^2 = 8 \pm 3\%$.

In this letter we discuss possible experimental tests of this mixing in the reactions

$$pp \to d a_0^+ \quad (a), \quad pn \to d a_0^0 \quad (b),$$
$$pd \to {}^3\mathrm{H}\, a_0^+ \quad (c), \quad pd \to {}^3\mathrm{He}\, a_0^0 \quad (d)$$

and

$$dd \to {}^4\mathrm{He}\, a_0^0 \quad (e)$$

near the corresponding thresholds. We recall that the $a_0$-meson can decay to $\pi\eta$ or $K\bar{K}$. In this paper we consider only the dominant $\pi\eta$ decay mode.

Note that the isospin violating anisotropy in the reaction $pn \to d a_0^0$ due to the $a_0(980)$-$f_0(980)$ mixing is very similar to that what may arise from $\pi^0$-$\eta$ mixing in the reaction $pn \to d\pi^0$ (see Ref. [9]). Recently charge-symmetry breaking was investigated in the reactions $\pi^+ d \to pp\eta$ and $\pi^- d \to nn\eta$ near the $\eta$ production threshold at BNL [9]. A similar experiment, comparing the reactions $pd \to {}^3\mathrm{He}\pi^0$ and $pd \to {}^3\mathrm{H}\pi^+$ near the $\eta$ production threshold, is now in progress at COSY-Jülich (see e.g. Ref. [10]).

## 1 Reactions (a) and (b)

### *1.1 Phenomenology of isospin violation*

In reactions (a) and (b) the final $da_0$ system has isospin $I_f = 1$, for $l_f = 0$ ($S$-wave production close to threshold) it has spin-parity $J_f^P = 1^+$. The initial $NN$ system cannot be in the state $I_i = 1$, $J_i^P = 1^+$ due to the Pauli principle. Therefore, near threshold the $da_0$ system should be dominantly produced in $P$-wave with quantum numbers $J_f^P = 0^-$, $1^-$ or $2^-$. The states with $J_i^P = 0^-$, $1^-$ or $2^-$ can be formed by an $NN$ system with spin $S_i = 1$ and $l_i = 1$ and 3. Neglecting the contribution of the higher partial wave $l_i = 3$ we can write the amplitude of reaction (a) in the following form

$$T(pp \to d\, a_0^+) = \\ = \alpha^+\ \mathbf{p}\cdot\mathbf{S}\ \mathbf{k}\cdot\boldsymbol{\epsilon}^* + \beta^+\ \mathbf{p}\cdot\mathbf{k}\ \mathbf{S}\cdot\boldsymbol{\epsilon}^* + \gamma^+\ \mathbf{S}\cdot\mathbf{k}\ \mathbf{p}\cdot\boldsymbol{\epsilon}^*, \qquad (1)$$

where $\mathbf{S} = \phi_N^T \sigma_2\, \boldsymbol{\sigma} \phi_N$ is the spin operator of the initial $NN$ system; $\mathbf{p}$ and $\mathbf{k}$ are the initial and final c.m. momenta; $\boldsymbol{\epsilon}$ is the deuteron polarization vector; $\alpha^+$, $\beta^+$, $\gamma^+$ are three independent scalar amplitudes which can be considered as constants near threshold (for $k \to 0$).

Due to the mixing the $a_0^0$ may also be produced via the $f_0$. In this case the $da_0^0$ system will be in $S$-wave and the amplitude of reaction (b) can be written as:

$$T(pn \to d\, a_0^0) = \\ = \alpha^0\ \mathbf{p}\cdot\mathbf{S}\ \mathbf{k}\cdot\boldsymbol{\epsilon}^* + \beta^0\ \mathbf{p}\cdot\mathbf{k}\ \mathbf{S}\cdot\boldsymbol{\epsilon}^* + \gamma^0\ \mathbf{S}\cdot\mathbf{k}\ \mathbf{p}\cdot\boldsymbol{\epsilon}^* + \xi F\ \mathbf{S}\cdot\boldsymbol{\epsilon}^*, \tag{2}$$

where $\xi$ is the mixing parameter and $F$ is the $f_0$-production amplitude. In the limit $k \to 0$, $F$ is again a constant. The scalar amplitudes $\alpha$, $\beta$, $\gamma$ for reactions (a) and (b) are related to each other by a factor $\sqrt{2}$, i.e., $\alpha^+ = \sqrt{2}\alpha^0$, $\beta^+ = \sqrt{2}\beta^0$, $\gamma^+ = \sqrt{2}\gamma^0$.

The differential cross sections for the reactions (a) and (b) have the form (up to terms linear in $\xi$)

$$\frac{\mathrm{d}\sigma(pp \to d\, a_0^+)}{\mathrm{d}\Omega} = 2\,\frac{k}{p}\left(C_0 + C_2\cos^2\Theta\right) \tag{3}$$

$$\frac{\mathrm{d}\sigma(pn \to d\, a_0^0)}{\mathrm{d}\Omega} = \frac{k}{p}\left(C_0 + C_2\cos^2\Theta + C_1\cos\Theta\right)\ , \tag{4}$$

where

$$C_0 = \frac{1}{2}\,p^2k^2\left[|\alpha^0|^2 + |\gamma^0|^2\right],\ C_1 = p\,k\left[\mathrm{Re}((\xi F)^*(\alpha^0 + 3\,\beta^0 + \gamma^0))\right]\ , \\ C_2 = \frac{1}{2}\,p^2k^2\left[3\,|\beta^0|^2 + 2\,\mathrm{Re}(\alpha^0\beta^{0*} + \alpha^0\gamma^{0*} + \beta^0\gamma^{0*})\right]\ . \tag{5}$$

Similarly, the differential cross section of the reaction $pn \to df_0$ can be written as

$$\frac{\mathrm{d}\sigma(pn \to df_0)}{\mathrm{d}\Omega} = \frac{3\,k}{2\,p}\,|F|^2\ . \tag{6}$$

The mixing effect — described by the term $C_1\cos\Theta$ in Eq.(4) — then leads to an isospin violation in the ratio $R_{ba}$ of the differential cross sections for reactions (b) and (a),

$$R_{ba} = \frac{1}{2} + \frac{C_1\cos\Theta}{C_0 + C_2\cos^2\Theta}, \tag{7}$$

and in the forward-backward asymmetry for reaction (b):

$$A_a(\Theta) = \frac{\sigma_a(\Theta) - \sigma_a(\pi-\Theta)}{\sigma_a(\Theta) + \sigma_a(\pi-\Theta)} = \frac{C_1\cos\Theta}{C_0 + C_2\cos^2\Theta}\ . \tag{8}$$

The latter effect was already discussed in Ref. [11] where it was argued that the asymmetry $A_a(\Theta = 0)$ can reach $5 \div 10\%$ at an energy excess of $Q = (5 \div 10)$ MeV. However, if we adopt a mixing parameter $|\xi|^2 = (8 \pm 3)\%$, as indicated by the WA102 data, we can expect a much larger asymmetry. We note explicitly, that the coefficient $C_1$ in (5) depends not only on the magnitude of the mixing parameter $\xi$, but also on the relative phases with respect to the amplitudes of $f_0$ and $a_0$ production which are unknown so far. This uncertainty has to be kept in mind for the following discussion.

In case of very narrow $a_0$ and $f_0$ states, the differential cross section (3), dominated by $P$-wave near threshold, would be proportional to $k^3$ or $Q^{3/2}$, where $Q$ is the c.m. energy excess. Due to $S$-wave dominance in the reaction $pn \to df_0$ one would expect that the cross section increases as $\sigma \sim k$ or $\sim \sqrt{Q}$. In this limit the $a_0$-$f_0$ mixing leads to an enhancement of the asymmetry $A_a(\Theta)$ as $\sim 1/k$ near threshold. In reality, however, both $a_0$ and $f_0$ have a finite width of about 40–100 MeV. Therefore, at fixed initial momentum their production cross section should be averaged over the corresponding mass distributions, which will significantly change the threshold behavior of the cross sections. Another complication is that broad resonances are usually accompanied by background lying underneath the resonance signals. These problems will be discussed explicitly in Sects. 1.2 and 1.3.

### *1.2 Model calculations*

In order to estimate the isospin-violation effects in the ratio $R_{ba}$ of the differential cross-section and in the forward-backward asymmetry $A_a$ we use the two-step model (TSM), which has successfully been applied to the description of $\eta$-, $\eta'$-, $\omega$- and $\phi$-meson production in the reaction $pN \to dX$ in Refs. [12,13]. Recently, this model has been also used for an analysis of the reaction $pp \to da_0^+$ [14].

The diagrams in Fig. 1 describe the different mechanisms of $a_0$- and $f_0$-meson production in the reaction $NN \to da_0/f_0$ within the TSM. In the case of $a_0$ production the amplitude of the subprocess $\pi N \to a_0 N$ contains three different contributions: i) the $f_1(1285)$-meson exchange (Fig. 1 a); ii) the $\eta$-meson exchange (Fig. 1b); iii) $s$- and $u$-channel nucleon exchanges (Fig. 1c and d). As it was shown in Ref. [14] the main contribution to the cross section for the reaction $pp \to da_0^+$ stems from the $u$-channel nucleon exchange (i.e. from the diagram of Fig. 1d and all other contributions can be neglected in a leading order approximation. In order to preserve the correct structure of the amplitude under permutations of the initial nucleons (which is antisymmetric for the isovector state and symmetric for the isoscalar state) the amplitudes for $a_0$ and $f_0$ production can be written as the following combinations of the

$t$- and $u$-channel contributions

$$T_{pn\to da_0^0}(s,t,u) = A_{pn\to da_0^0}(s,t) - A_{pn\to da_0^0}(s,u)$$
$$T_{pn\to df_0}(s,t,u) = A_{pn\to df_0}(s,t) + A_{pn\to df_0}(s,u), \qquad (9)$$

where $s=(p_1+p_2)^2$, $t=(p_3-p_1)^2$, $u=(p_3-p_2)^2$ and $p_1$, $p_2$, $p_3$, and $p_4$ are the 4-momenta of the initial protons, meson $M$ and the deuteron, respectively. The structure of the amplitudes (9) guarantees that the $S$-wave part vanishes in the case of direct $a_0$ production since it is forbidden by angular momentum conservation and the Pauli principle. Also higher partial waves are included in the model calculations in contrast to the simplified discussion in Sect. 1.1.

In the case of $f_0$ production the amplitude of the subprocess $\pi N \to f_0 N$ contains two different contributions: i) the $\pi$- meson exchange (Fig. 1 b); ii) $s$- and $u$-channel nucleon exchanges (Fig. 1 c and d). Our analysis has shown that similarly to the case of $a_0$ production the main contribution to the cross section of the reaction $pn \to df_0$ is due to the $u$-channel nucleon exchange (Fig. 1 d); the contribution of the combined $\pi\pi$ exchange (Fig. 1 b) as well as the $s$-channel nucleon exchange can be neglected. In this case we obtain for the ratio of the squared amplitudes

$$\frac{|A_{pn\to df_0}(s,t)|^2}{|A_{pn\to da_0}(s,t)|^2} = \frac{|A_{pn\to df_0}(s,u)|^2}{|A_{pn\to da_0}(s,u)|^2} = \frac{|g_{f_0NN}|^2}{|g_{a_0NN}|^2}. \qquad (10)$$

If we take $g_{a_0NN} = 3.7$ (see e.g. Ref. [15]) and $g_{f_0NN}$ =8.5 [16] then we find for the ratio of the amplitudes $R(f_0/a_0) = g_{f_0NN}/g_{a_0NN} = 2.3$. Note, however, that Mull and Holinde give a different value for the ratio of the coupling constants $R(f_0/a_0) = 1.46$, which is about 37% lower. In the following we thus use $R(f_0/a_0) = 1.46 \div 2.3$.

The forward differential cross section for reaction (a) as a function of the proton beam momentum is presented in Fig. 2. The bold dash-dotted and solid lines (taken from Ref. [14] and calculated for the zero width limit $\Gamma_{a_0} = 0$) describe the results of the TSM for different values of the nucleon cut-off parameter, $\Lambda_N = 1.2$ and 1.3 GeV/c, respectively.

In order to take into account the finite $a_0$ width we use a Flatté mass distribution with the same parameters as in Ref. [18]: K-matrix pole at 999 MeV, $\Gamma_{a_0\to\pi\eta} = 70$ MeV, $\Gamma(K\bar{K})/\Gamma(\pi\eta) = 0.23$ (see also [19] and references therein). The thin dash-dotted and solid lines in Fig. 2 are calculated within the TSM using this mass distribution with a cut $M(\pi^+\eta) \geq 0.85$ GeV and for $\Lambda_N = 1.2$ and 1.3 GeV, respectively. The corresponding $\pi^0\eta$ invariant mass distribution for the reaction $pn \to da_0^0 \to d\pi^0\eta$ at 3.4 GeV/c is shown in Fig. 3 by the dashed line.

In case of the $f_0$, where the branching ratio $BR(K\bar{K})$ is not yet known [19], we use a Breit-Wigner mass distribution with $m_R = 980$ MeV and $\Gamma_R \simeq \Gamma_{f_0\to\pi\pi} = 70$ MeV.

The calculated total cross sections for the reactions $pn \to da_0$ and $pn \to df_0$ (as a function of the beam energy $T_{\rm lab}$ for $\Lambda_N = 1.2$ GeV ) are shown in Fig. 4. The solid and dashed lines describe the calculations with zero and finite widths, respectively. In case of $f_0$ production in the $\pi\pi$ decay mode we choose the same cut in the invariant mass of the $\pi\pi$ system, i.e. $M_{\pi\pi} \geq 0.85$ GeV. The lines denoted by 1 and 2 are obtained for $R(f_0/a_0) = 1.46$ and 2.3, respectively. Comparing the solid and dashed lines it is obvious that near threshold the finite width corrections to the cross sections are quite important in particular for the energy behavior of the $a_0$-production cross section (see also bold and thin curves in Fig. 2).

In principle, $a_0$-$f_0$ mixing can modify the mass spectrum of the $a_0$ and $f_0$. However, in the $a_0$-$f_0$ case the effect is expected to be less pronounced as for the $\rho$-$\omega$ case, where the widths of $\rho$ and $\omega$ are very different (see e.g. the discussion in Ref. [9] and references therein). Nevertheless, the modification of the $a_0^0$ spectral function due to $a_0$-$f_0$ mixing can be measured by comparing the invariant mass distributions of $a_0^0$ with that of $a_0^+$. According to our analysis, however, a much cleaner signal for isospin violation can be obtained from the measurement of the forward-backward asymmetry in the reaction $pn \to da_0^0 \to d\pi^0\eta$ integrating over the full $a_0$ mass distribution. For the following calculations, the strengths of the $a_0$ and $f_0$ thus will be integrated over the mass interval 0.85–1.02 GeV.

The magnitude of the isospin violation effects is shown in Fig. 5, where we present the differential cross section of the reaction $pn \to da_0^0$ at $T_p = 2.6$ GeV as a function of $\Theta_{\rm c.m.}$ for different values of the mixing intensity $|\xi|^2$ from 0.05 to 0.11. For reference, the solid line shows the case of isospin conservation, i.e. $|\xi|^2 = 0$. The dashed-dotted curves include the mixing effect. Note that all curves in Fig. 5 were calculated assuming maximal interference of the amplitudes describing the direct $a_0$ production and its production through the $f_0$. The maximal values of the differential cross section may also occur at $\Theta_{\rm c.m.} = 0^\circ$ depending on the sign of the coefficient $C_1$ in Eq.(4).

It follows from Fig. 5 in either case that the isospin-violation parameter $A_a(\Theta)$ for $\Theta_{\rm c.m.} = 180^\circ$ may be quite large, i.e.

$$A_a(180^\circ) = 0.86 \div 0.96 \quad \text{or} \quad 0.9 \div 0.98 \tag{11}$$

for $R(f_0/a_0) = 1.46$ or 2.3, respectively. Note that the asymmetry depends rather weakly on $R(f_0/a_0)$. It might be more sensitive to the relative phase of $a_0$ and $f_0$ contributions, which has to be settled experimentally.

### *1.3 Background*

The dash-dotted line in Fig. 3 shows our estimate of the possible background from nonresonant $\pi^0\eta$ production in the reaction $pn \to d\pi^0\eta$ at $T_{\rm lab} = 2.6$ GeV (see also Ref. [20]). The background amplitude is described by the diagram shown in Fig. 1 e, where the $\eta$ and $\pi$ mesons are created through the intermediate production of a $\Delta(1232)$ (in the amplitude $\pi N \to \pi N$) and a $N(1535)$ (in the amplitude $\pi N \to \eta N$). The total cross section for the nonresonant $\pi\eta$ production due to this mechanism was found to be $\sigma_{\rm BG} \simeq 0.8$ $\mu$b for a cut-off in the one-pion exchange of $\Lambda_\pi = 1$ GeV.

We point out that the background is charge-symmetric and cancels in the difference of the cross sections $\sigma(\Theta) - \sigma(\pi - \Theta)$. Therefore, a complete separation of the background is not crucial for a test of isospin violation due to the $a_0$-$f_0$ mixing. There will also be some contribution from $\pi$-$\eta$ mixing as discussed in Refs. [9,10]. According to the results of Ref. [9] this mechanism yields a charge-symmetry breaking in the $\eta NN$ system of about 6%:

$$R = d\sigma(\pi^+ d \to pp\eta)/\sigma(\pi^- d \to nn\eta) = \ 0.938 \pm 0.009.$$

A similar isospin violation due to $\pi$-$\eta$ mixing can also be expected in our case.

The best strategy to search for isospin violation due to $a_0$-$f_0$ mixing is a measurement of the forward-backward asymmetry for different intervals of $M_{\eta\pi^0}$. It follows from Fig. 3 that $\sigma_{a_0}(\sigma_{\rm BG}) = 0.3(0.4),\ 0.27(0.29)$ and $0.19(0.15)$ $\mu$b for $M_{\eta\pi^0} \geq 0.85,\ 0.9$ and $0.95$ GeV, respectively. For $M_{\eta\pi^0} \leq 0.7$ GeV the resonant contribution is rather small and the charge-symmetry breaking will dominantly be related to $\pi$-$\eta$ mixing and, therefore, be small. On the other hand, for $M \geq 0.95$ GeV the background does not exceed the resonance contribution and we expect a comparatively large isospin-breaking signal due to $a_0$-$f_0$ mixing.

### *1.4 The reaction $pn \to df_0 \to d\pi\pi$*

The isospin-violation effects can also be measured in the reaction

$$pn \to df_0 \to d\pi^+\pi^-, \tag{12}$$

where, due to mixing, the $f_0$ may also be produced via the $a_0$. The corresponding differential cross section is shown in Fig. 6. The differential cross section for $f_0$ production is expected to be substantiatially larger than for $a_0$ production, but the isospin violation effect turns out to be smaller than in

the $\pi\eta$-production channel. Nevertheless, the isospin violation parameter $A$ is expected to be about $10 \div 30\%$ and can be detected experimentally.

## 2 Reactions (c) and (d)

We continue with $pd$ reactions and compare the final states ${}^3\mathrm{H}\, a_0^+$ (c) and ${}^3\mathrm{He}\, a_0^0$ (d). Near threshold the amplitudes of these reactions can be written as

$$T(pd \to {}^3\mathrm{H}\ a_0^+) = \sqrt{2} D_a\, \mathbf{S_A} \cdot \boldsymbol{\epsilon} \tag{13}$$

$$T(pd \to {}^3\mathrm{He}\ a_0^0) = (D_a + \xi D_f)\, \mathbf{S_A} \cdot \boldsymbol{\epsilon}\ , \tag{14}$$

with $\mathbf{S_A} = \phi_A^T \sigma_2\ \boldsymbol{\sigma} \phi_N$. Here $D_a$ and $D_f$ are the scalar $S$-wave amplitudes describing the $a_0$ and $f_0$ production in case of $\xi$=0. The ratio of the differential cross sections for reactions (d) and (c) is then given by

$$R_{dc} = \frac{|D_a + \xi D_f|^2}{2|D_a|^2} = \frac{1}{2} + \frac{2\mathrm{Re}(D_a^* \xi D_f) + |\xi D_f|^2}{|D_a|^2}\ . \tag{15}$$

The magnitude of the ratio $R_{dc}$ now depends on the relative value of the amplitudes $D_a$ and $D_f$. If they are comparable $|D_a| \sim |D_f|$ or $|D_f|^2 \gg |D_a|^2$ the deviation of $R_{dc}$ from 0.5 (which corresponds to isospin conservation) might be 100% or more. Only in the case $|D_f|^2 \ll |D_a|^2$ the difference of $R_{dc}$ from 0.5 will be small. However, this seems to be very unlikely.

Using the two-step model for the reactions $pd \to {}^3\mathrm{He}\ a_0^0$ and $pd \to {}^3\mathrm{He}\ f_0$, involving the subprocesses $pp \to d\pi^+$ and $\pi^+ n \to p\ a_0/f_0$ (cf. Refs. [21,22]), we find

$$\frac{\sigma(pd \to {}^3\mathrm{He}\ a_0^0)}{\sigma(pd \to {}^3\mathrm{He}\ f_0)} \simeq \frac{\sigma(\pi^+ n \to p\ a_0^0)}{\sigma(\pi^+ n \to p\ f_0)}\ . \tag{16}$$

According to the calculations in Ref. [14] we expect $\sigma(\pi^+ n \to p a_0) = \sigma(\pi^- p \to n a_0) \simeq 0.5 \div 1$ mb at 1.75–2 GeV/c. A similar value for $\sigma(\pi^- p \to n f_0)$ can be found using the results from Ref. [23]. According to the latter study $\sigma(\pi^- p \to n f_0 \to n K^+ K^-) \simeq 6 - 8\ \mu$b at 1.75–2 GeV/c and $Br(f_0 \to K^+ K^-) \simeq 1\%$, which implies that $\sigma(\pi^- p \to n f_0) \simeq 0.6 - 0.8$ mb. Thus we expect that near threshold $|D_a| \sim |D_f|$ . This would imply that the effect of isospin violation in the ratio $R_{dc}$ can be rather large.

Recently the cross section of the reaction $pd \to {}^3\mathrm{He}\, K^+K^-$ has been measured by the MOMO collaboration at COSY-Jülich. It was found that $\sigma = 9.6 \pm 1.0$ and $17.5 \pm 1.8$ nb for $Q = 40$ and 56 MeV, respectively [24]. The authors note that the invariant $K^+K^-$ mass distributions in those data show broad peaks which follow phase space. However, as it was shown in Ref. [18], the shape of an invariant mass spectrum following phase space cannot be distinguished from an $a_0$-resonance contribution at small values of $Q$. Therefore, the events from Ref. [24] might also be attributed to $a_0$ and/or $f_0$ production. Moreover, due to the phase space behavior near threshold one expects a dominance of two-body reactions. Thus the cross section of the reaction $pd \to {}^3\mathrm{He}\, a_0^0 \to {}^3\mathrm{He}\, \pi^0\eta$ is expected to be not significantly smaller than the upper limit of about $80 \div 150$ nb at $Q = 40 - 60$ MeV which follows from the MOMO data (using $\Gamma(K\bar{K})/\Gamma(\pi\eta) = 0.23$ from [19]).

## 3 Reaction (e)

Any direct production of the $a_0$ in the reaction $dd \to {}^4\mathrm{He}\, a_0^0$ is forbidden. It thus can only be observed due to $f_0$-$a_0$ mixing:

$$\frac{\sigma(dd \to {}^4\mathrm{He}\, a_0^0)}{\sigma(dd \to {}^4\mathrm{He}\, f_0)} = |\xi|^2. \tag{17}$$

Therefore, it will be very interesting to study the reaction

$$dd \to {}^4\mathrm{He}\, (\pi^0\ \eta) \tag{18}$$

near the $f_0$-production threshold. Any signal of the reaction (18) then will be related to isospin breaking. It is expected to be much more pronounced near the $f_0$ threshold as compared to the region below this threshold.

## 4 Summary

In summary, we have discussed the effects of isospin violation in the reactions $pN \to da_0$, $pd \to {}^3\mathrm{He}/{}^3\mathrm{H}\, a_0$ and $dd \to {}^4\mathrm{He}\, a_0^0$ which can be generated by $f_0$-$a_0$ mixing. It has been demonstrated that for a mixing intensity of about $(8\pm3)\%$, the isospin violation in the ratio of the differential cross sections of the reactions $pp \to da_0^+ \to d\pi^+\eta$ and $pn \to da_0^0 \to d\pi^0\eta$ as well as in the forward-backward asymmetry in the reaction $pn \to da_0^0 \to d\pi^0\eta$ not far from threshold may be about 50–100%. Such large effects originate from the interference of direct $a_0$ production and its production via the $f_0$. The former amplitude is

suppressed close to threshold due to the $P$-wave amplitude whereas the latter is large due to $S$-wave production. A similar isospin violation is expected in the ratio of the differential cross sections of the reactions $pd \to {}^3\mathrm{H}\, a_0^+(\pi^+\eta)$ and $pd \to {}^3\mathrm{He}\, a_0^0(\pi^0\eta)$.

Finally, we have also discussed the isospin-violation effects in the reactions $pn \to df_0(\pi^+\pi^-)$ and $dd \to {}^4\mathrm{He}\, a_0$. All reactions together — once studied experimentally — are expected to provide detailed information on the strength of the $f_0$-$a_0$ mixing.

Corresponding measurements are now in preparation for the ANKE spectrometer at COSY-Jülich [25].

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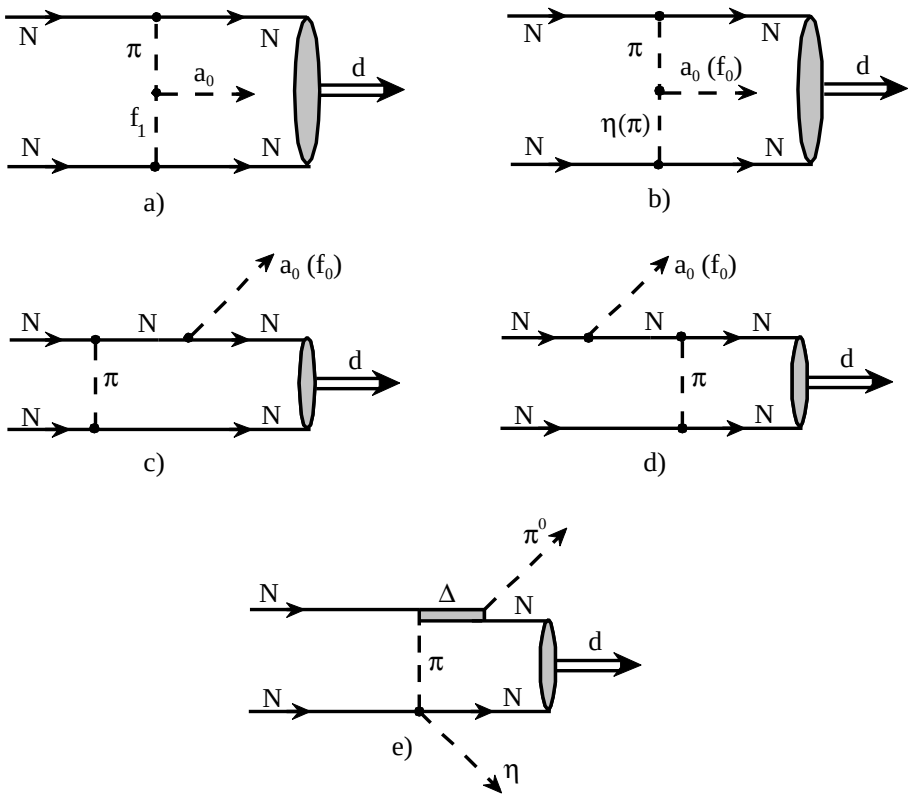


Fig. 1. a)–d) Different mechanisms of $a_0$ and $f_0$-meson production in the reaction $NN \to da_0(f_0)$ within the framework of the two-step model (TSM). The nonresonant $\pi\eta$ production is described by diagram e).

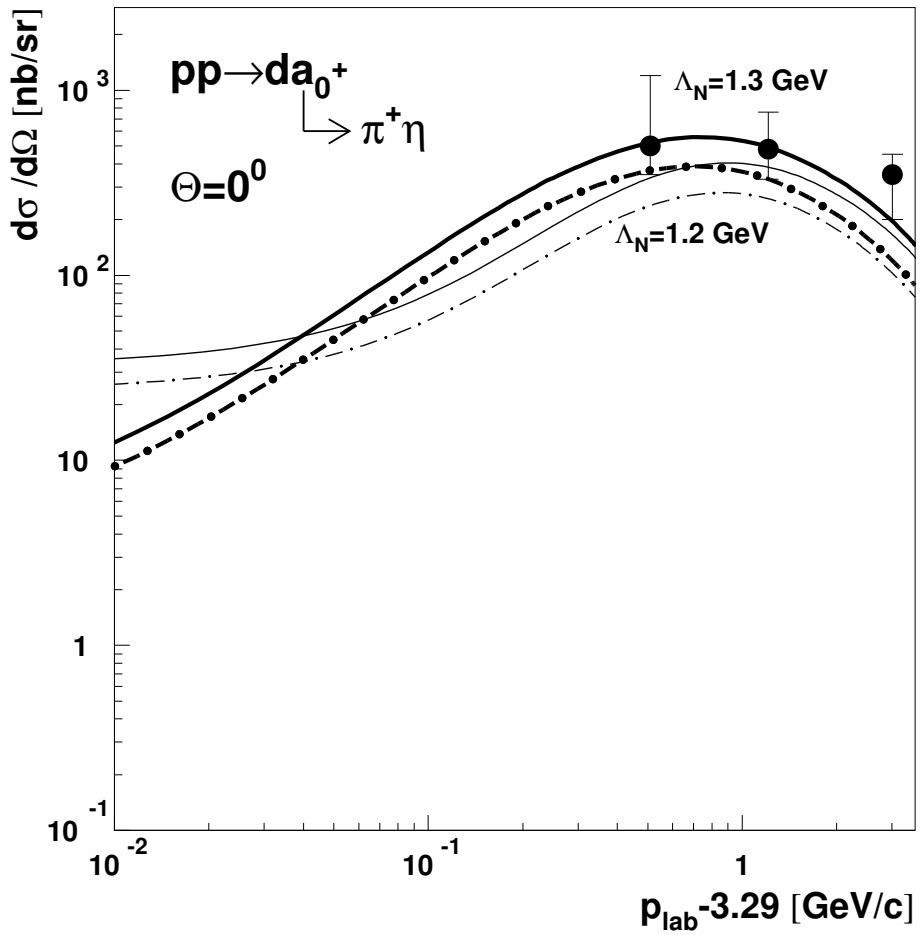


Fig. 2. Forward differential cross section of the reaction $pp \to da_0^+$ as a function of $(p_{\rm lab} - 3.29)$ GeV/c. The full dots are the experimental data from Ref. [17] while the bold dash-dotted and solid lines describe the results of the TSM for $\Lambda_N = 1.2$ and 1.3 GeV, respectively, and $\Gamma_{a_0} = 0$. The thin dash-dotted and solid lines are calculated using the Flatté mass distribution for the $a_0$ spectral function with a cut $M \geq 0.85$ GeV (see text).

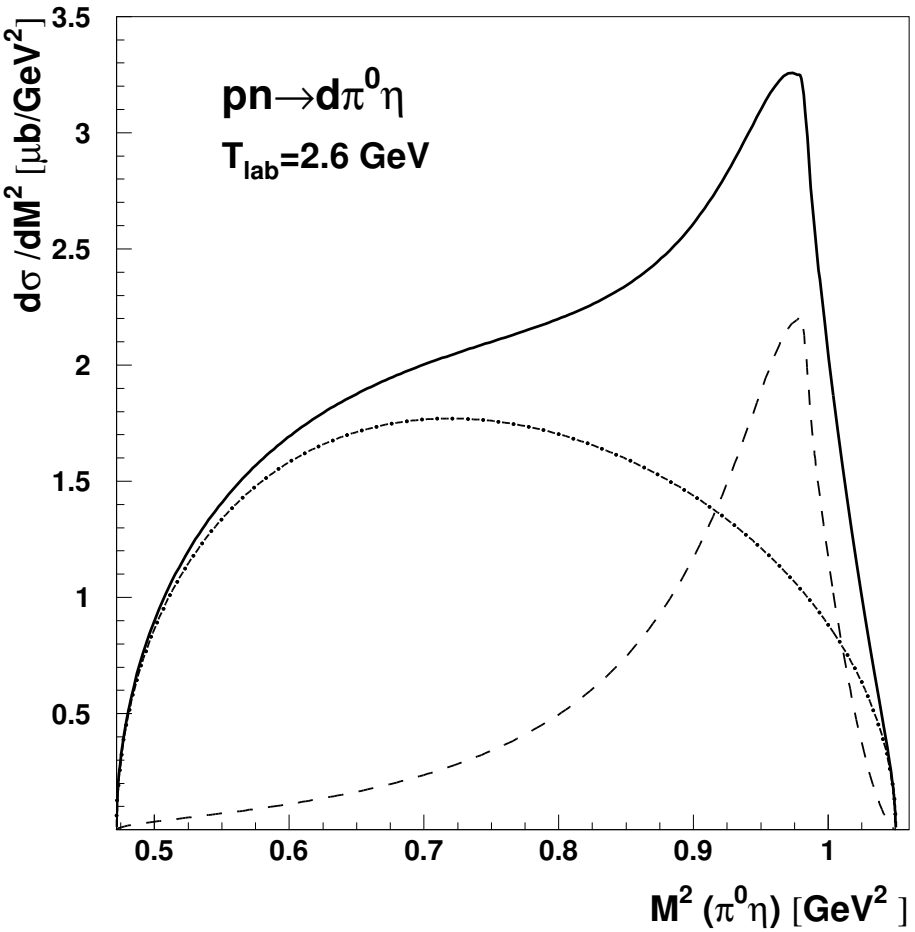


Fig. 3. $\pi^0\eta$ invariant mass distribution for the reaction $pn \to d\pi^0\eta$ at 3.4 GeV/c. The dashed and dash-dotted lines describe the $a_0$-resonance contribution and non-resonant background, respectively. The solid line is the sum of both contributions.

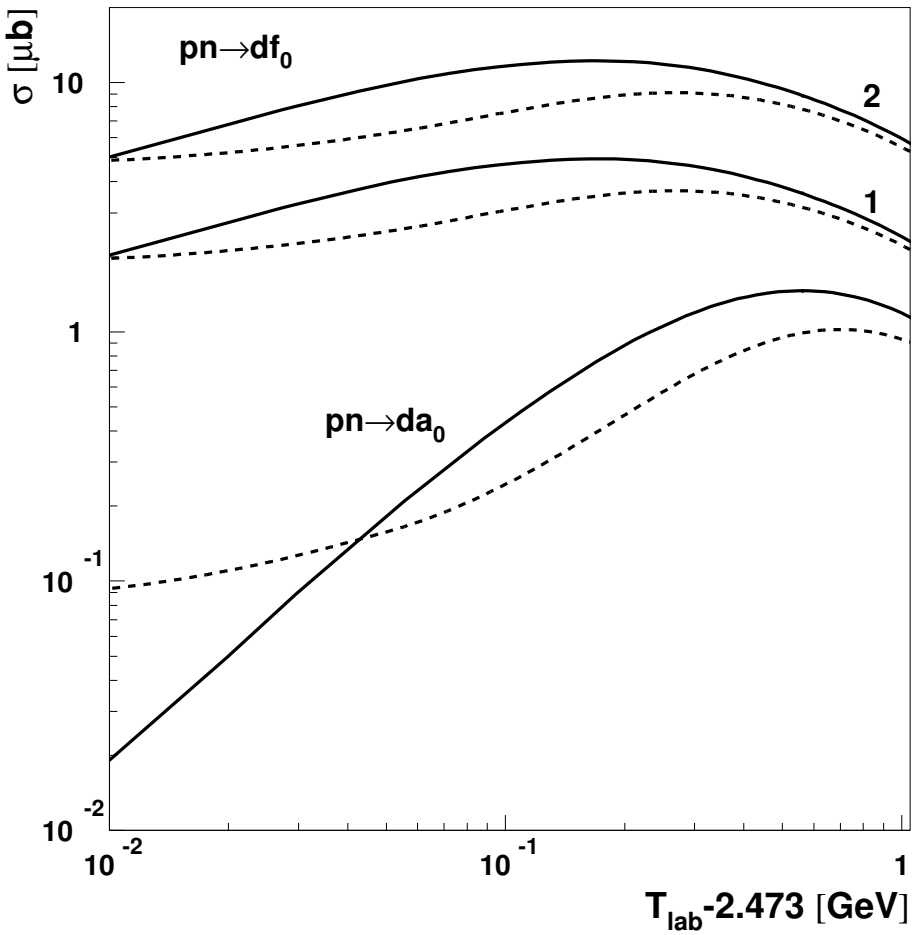


Fig. 4. Total cross sections for the reactions $pn \to da_0$ (lower lines) and $pn \to df_0$ (upper lines) as a function of $(T_{\rm lab} - 2.473)$ GeV. The solid and dashed curves are calculated using narrow and finite resonance widths, respectively. The curves denoted by 1 and 2 correspond to the choices $R(f_0/a_0) = 1.46$ and 2.3.

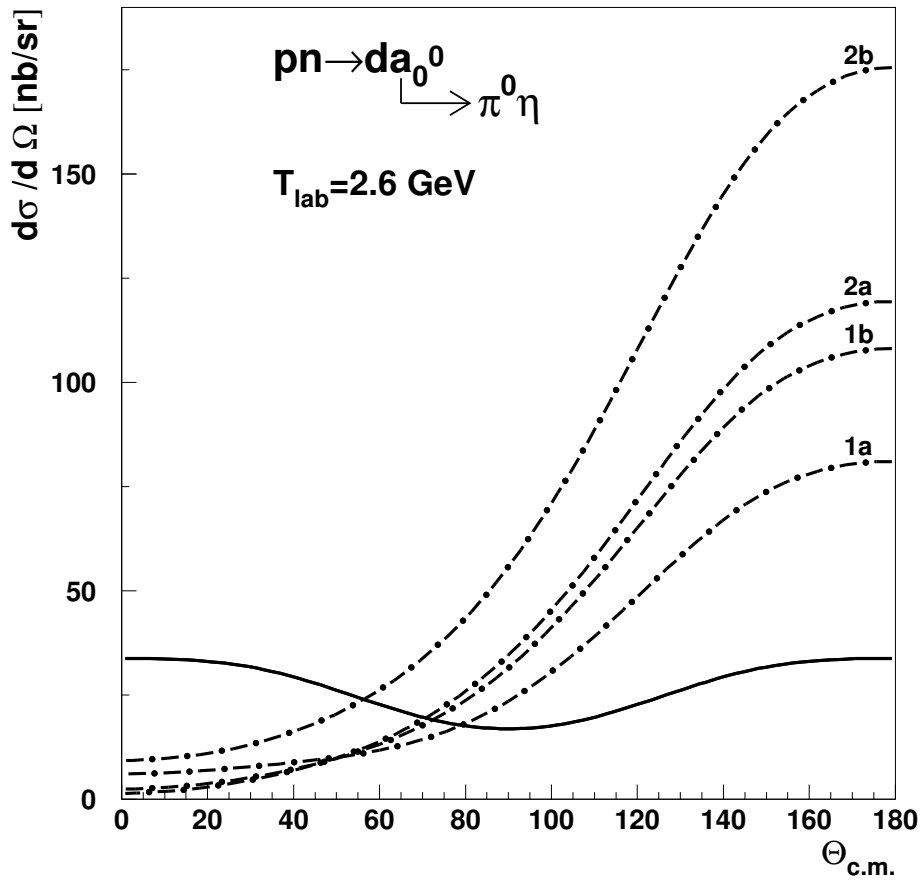


Fig. 5. Differential cross section of the reaction $pn \to da_0^0$ at $T_p = 2.6$ GeV as a function of $\Theta_{\rm c.m.}$. The solid curve corresponds to the case of isospin conservation, i.e. $|\xi|^2 = 0$. The dashed-dotted lines include the mixing effect with $|\xi|^2 = 0.05$ for the lower curves (1a and 2a) and $|\xi|^2 = 0.11$ for the upper curves (1b and 2b). The lines 1a, 1b (2a, 2b) have been calculated for $R(f_0/a_0) = 1.46$ (2.3), respectively.

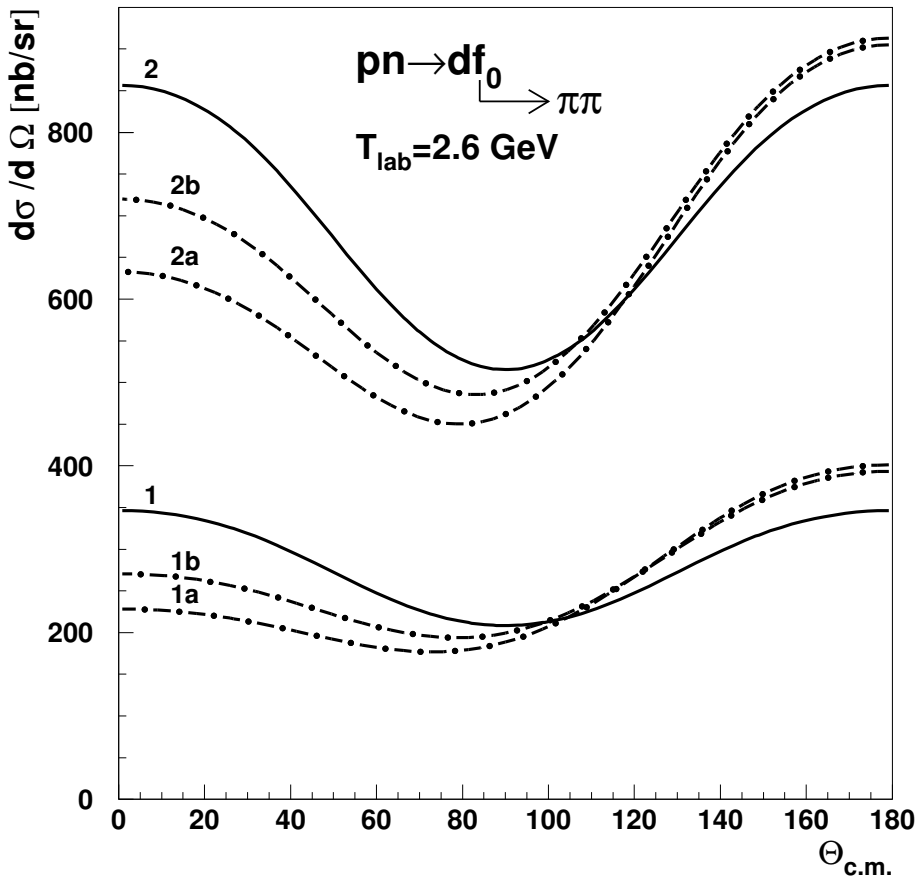


Fig. 6. Differential cross section of the reaction $pn \to df_0$ at $T_p = 2.6$ GeV as a function of $\Theta_{\rm c.m.}$. The notation of the curves is the same as in Fig. 5.