Is there an $\eta\,^3$He quasi–bound state?

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Abstract

The observed variation of the total cross section for the $dp \to {}^3\mathrm{He}\,\eta$ reaction near threshold means that the magnitude of the s–wave amplitude falls very rapidly with the η centre–of–mass momentum. It is shown here that recent measurements of the momentum dependence of the angular distribution imply a strong variation also in the phase of this amplitude. Such a behaviour is that expected from a quasi–bound or virtual η^3He state. The interpretation can be investigated further through measurements of the deuteron or proton analysing powers and/or spin–correlations.

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New and very precise data on the $dp \to {}^3\mathrm{He}\,\eta$ reaction near threshold [1,2], taken at the COSY accelerator of the Forschungszentrum Jülich, confirm the energy dependence of the total cross section found in earlier experiments [3,4], but with much finer steps in energy over an extended range. The measurements at the lowest excess energy Q (the centre–of–mass kinetic energy in the $\eta\,{}^3\mathrm{He}$ system) are of especial interest. The very rapid rise and levelling off of the cross section in this region, shown in Fig. 1 for the COSY–ANKE data [1], suggests that there is a nearby bound or virtual state of the $\eta\,{}^3\mathrm{He}$ nucleus [5,6].

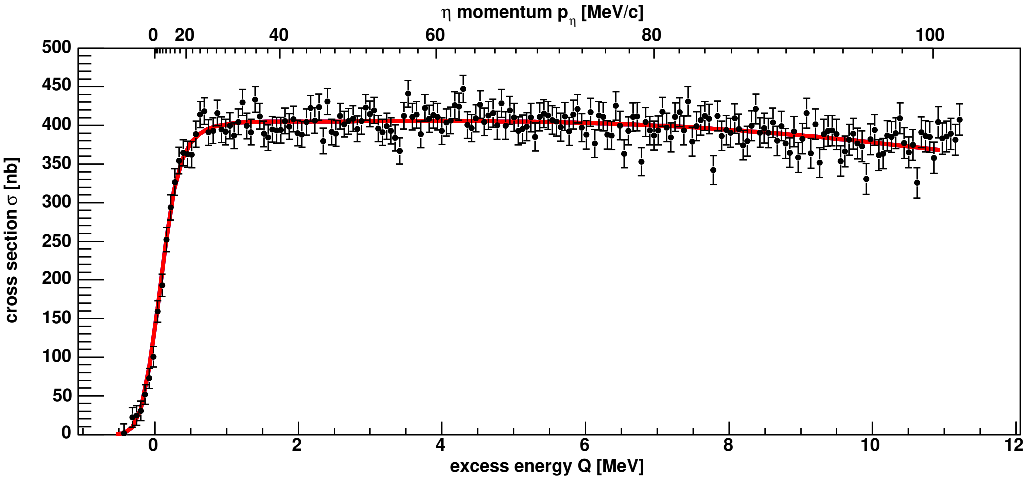

Fig. 1. Total cross section for the $dp \to {}^3\mathrm{He}\,\eta$ total cross section measured at COSY–ANKE [1] in terms of the excess energy Q and η c.m. momentum p_η. The fits with and without the p–waves, as discussed in the text, are indistinguishable and so they are not presented separately.

The concept of η–mesic nuclei was introduced by Liu and Haider [7]. Since the η–meson has isospin–zero, the attraction noted for the η–nucleon system should add coherently when the meson is introduced into a nuclear environment. On the basis of the rather small η–nucleon scattering length $a_{\eta N}$ assumed, they estimated that the lightest nucleus on which the η might bind would be ${}^{12}\mathrm{C}$. Experimental searches for the signals of such effects have generally proved negative, as for example in the ${}^{16}\mathrm{O}(\pi^+,p){}^{15}\mathrm{O}^*$ reaction [8]. The larger $Re(a_{\eta N})$ subsequently advocated [9] means that the η should bind tightly with such heavy nuclei, generating large and overlapping widths, and thus be hard to detect [10]. On the other hand, it also leads to the possibility of binding even in light systems, such as $\eta\,{}^3\mathrm{He}$.

A quasi–bound state leads to a pole of the $\eta\,{}^3\mathrm{He} \to \eta\,{}^3\mathrm{He}$ scattering amplitude in the complex momentum p plane with $Im(p) > 0$ and in the complex Q plane

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with $Im(Q) < 0$ [1]. Since such a state can decay via the emission of pions or nucleons, it can only be described as being quasi–bound. If the η–nucleus force is not attractive enough, the signs of these imaginary parts are reversed and the state is called virtual. When a pole is close to $Q = 0$ it distorts strongly the energy dependence of the $dp \to {}^3\mathrm{He}\,\eta$ total cross section at low energies. This is precisely what is seen in the experimental data [1–4], with all experiments identifying a pole with $|Q|$ less than a couple of MeV. The ANKE data [1] shown in Fig. 1 include many points in the threshold region and, after taking into account the finite momentum spread of the beam, a pole was identified at $Q_0 = [(-0.30 \pm 0.15_{\mathrm{stat}} \pm 0.04_{\mathrm{syst}}) \pm i(0.21 \pm 0.29_{\mathrm{stat}} \pm 0.06_{\mathrm{syst}})]$ MeV, where the sign of the imaginary part cannot be determined even in principle from such η production data.

The properties of any $\eta\,{}^3\mathrm{He}$ nucleus should be largely independent of the production process but the backgrounds will be reaction–dependent. The only other evidence for the existence of the $\eta\,{}^3\mathrm{He}$ nucleus has come from photoproduction [11]. Though a sharp energy dependence has been seen in the $\gamma\,{}^3\mathrm{He} \to \eta\,{}^3\mathrm{He}$ amplitude, the limited statistics meant that a coarser binning had to be used than for the $dp \to {}^3\mathrm{He}\,\eta$ reaction [1]. A significant improvement in this is to be expected from the new MAMI data, which are currently being analysed [12]. The MAMI–TAPS group also found an anomalous behaviour in the photoproduction of back–to–back (π^-, p) pairs. It was suggested that this is consistent with the existence of a quasi–bound $\eta\,{}^3\mathrm{He}$ state [11], though the interpretation is somewhat controversial [13].

In order to prove that a nearby pole in the complex Q plane is indeed responsible for the unusual energy dependence of the $dp \to {}^3\mathrm{He}\,\eta$ cross section, it is necessary to show that the pole induces a change in the phase as well as in the magnitude of the s–wave amplitude. Since the cross section is proportional to the absolute square of the amplitude, much phase information is thereby lost. However, it is the purpose of the present letter to point out that the interference between the s– and p–waves, as seen in the newly published angular distributions [1,2], leads to the required confirmation.

The $dp \to {}^3\mathrm{He}\,\eta$ differential cross sections were found to be linear in $\cos\theta_\eta$, where θ_η is the c.m. angle between the initial proton and final η. Throughout the range of the new COSY measurements, $Q < 11$ MeV [1,2], there is no sign of the $\cos^2\theta_\eta$ term that is needed for the description of the angular distributions at higher energies [14]. The angular dependence may therefore

[1] The time evolution of the wave function of the state involves a factor $\exp(-iQ_0t) = \exp(-iRe(Q_0)t)\exp(+Im(Q_0)t)$. A quasi–bound state must decay in time, which thus requires that $Im(Q_0) < 0$. In contrast, a virtual state has $Im(Q_0) > 0$ and is also on the second (unphysical) sheet.

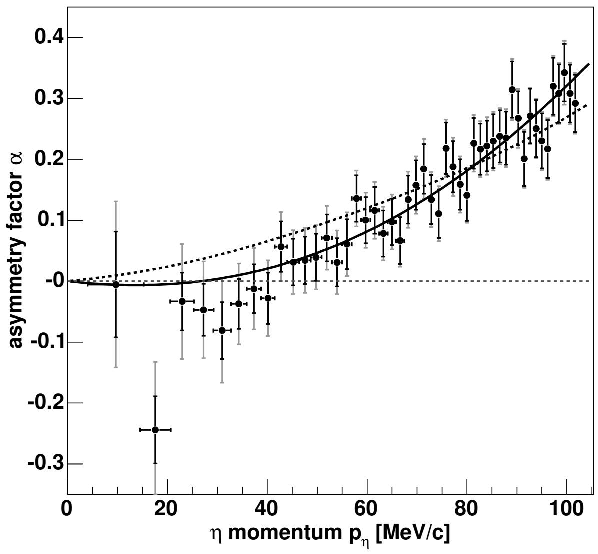

Fig. 2. Variation of the asymmetry parameter α defined by Eq. (1) with the c.m. momentum of the η-meson. The solid line is the result of fitting the ANKE data of Ref. [1] with Eq. (8), where the s–wave amplitude f_s of Eq. (6) was introduced. The dashed line was obtained after neglecting the phase variation of f_s, i.e. letting $f_s \to |f_s|$. In all cases the fits were carried out by considering only the statistical errors shown by dark lines. Combining these quadratically with the systematic uncertainties leads to the total errors, which are represented by the grey lines.

be summarised in terms of an asymmetry parameter α, defined as

$$\alpha = \frac{\mathrm{d}}{\mathrm{d}(\cos\theta_\eta)} \ln\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)\Bigg|_{\cos\theta_\eta=0} . \tag{1}$$

The variation of the ANKE measurements of α with the η momentum p_η is shown in Fig. 2.

On kinematic grounds, the angular dependence near threshold might be expected to develop like $\vec{p}_p \cdot \vec{p}_\eta = p_p p_\eta \cos\theta_\eta$, where $\vec{p}_p$ and $\vec{p}_\eta$ are the c.m. momenta of the incident proton and final η–meson, respectively. However, one striking feature of Fig. 2 is that, although α rises sharply with p_η, it only does so from about 40 MeV/c instead of from the origin, as one might expect on the basis of the above kinematic argument. At low values of p_η the error bars are necessarily large and α might even to go negative in this region. This feature is not inconsistent with the results of other measurements [2,4] that have different systematic uncertainties and so it is possibly a genuine effect. Part of this non–linear behaviour arises from the steep decrease in the magnitude of the s–wave amplitude with momentum. However, the size of the effect observed can only arise through the rapid variation of the phase of this amplitude, of the type generated by a nearby pole in the complex Q plane.

There are two independent $dp \to {}^3\mathrm{He}\,\eta$ s–wave amplitudes (A and B) [15] and five p–wave though, to discuss the data phenomenologically, we retain only the two (C and D) that give a pure $\cos\theta_\eta$ dependence in the cross section. The production operator

$$\hat{f} = A\,\vec{\varepsilon}\cdot\hat{p}_p + iB\,(\vec{\varepsilon}\times\vec{\sigma})\cdot\hat{p}_p + C\,\vec{\varepsilon}\cdot\vec{p}_\eta + iD\,(\vec{\varepsilon}\times\vec{\sigma})\cdot\vec{p}_\eta \tag{2}$$

has to be sandwiched between ^{3}He and proton spinors. Here $\vec{\varepsilon}$ is the polarisation vector of the deuteron. The corresponding unpolarised differential cross section depends upon the spin-averaged value of $|f|^2$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{p_\eta}{p_p}\,\overline{|f|^2} = \frac{p_\eta}{3p_p}\,I\,. \tag{3}$$

Using the amplitudes of Eq. (2) this yields

$$I = |A|^2 + 2|B|^2 + p_\eta^2|C|^2 + 2p_\eta^2|D|^2 + 2p_\eta Re(A^*C + 2B^*D)\cos\theta_\eta, \tag{4}$$

which has the desired linear dependence on $\cos\theta_\eta$, with an asymmetry parameter

$$\alpha = 2p_\eta\,\frac{Re(A^*C + 2B^*D)}{|A|^2 + 2|B|^2 + p_\eta^2|C|^2 + 2p_\eta^2|D|^2}\,. \tag{5}$$

The strong η^3He final–state interaction that gives rise to the quasi–bound pole should affect the two s–wave amplitudes A and B in a similar way and some evidence for this is to be found from the deuteron tensor analysing power t_{20}, which is small and seems to change little, if at all, from near threshold to their measurement limit 16.6 MeV above threshold [3]. As a consequence, $|A| \propto |B|$ throughout our range of interest and it is plausible to represent the data using a spin–average amplitude.

In the original fit to the whole of the ANKE $dp \to {}^3\mathrm{He}\,\eta$ total cross section data [1] shown in Fig. 1, any influence of p–waves was neglected and the data represented by

$$f_s = \frac{f_B}{(1 - p_\eta/p_1)(1 - p_\eta/p_2)}\,, \tag{6}$$

with

$$f_B = (50 \pm 8)\,(\mathrm{nb/sr})^{1/2}\,,$$

$$
\begin{aligned}
p_1 &= [(-5 \pm 7^{+2}_{-1}) \pm i(19 \pm 2 \pm 1)]\,\mathrm{MeV/c}\,, \\
p_2 &= [(106 \pm 5) \pm i(76 \pm 13^{+1}_{-2})]\,\mathrm{MeV/c}\,.
\end{aligned} \tag{7}
$$

The first error bar is statistical and the second, where given, systematic. The error on f_B is dominated by the 15% luminosity uncertainty [1]. Note that only the first pole (at $p_\eta = p_1$) is of physical significance and for this unitarity requires that $Re(p_1) < 0$. The signs of the imaginary parts of the pole positions are not defined by the data. As will be seen later, the position of the first pole remains stable when fitting simultaneously the angular dependence and the total cross section. In contrast, the second pole is introduced to parametrise the residual energy dependence, which can arise from the reaction mechanism as well as from a final–state interaction.

Equation (6) shows an s–wave amplitude whose phase and magnitude vary quickly with p_η, but we expect that, apart from the momentum factor, the p–wave amplitudes should be fairly constant. In the absence of detailed analysing power information, we take $A = B = f_s$ and $C = D$ to be a complex constant. With these assumptions the total cross section and asymmetry parameter become:

$$
\begin{aligned}
\sigma &= \frac{4\pi p_\eta}{p_p}\left[|f_s|^2 + p_\eta^2|C|^2\right], \\
\alpha &= 2p_\eta\,\frac{Re(f_s^* C)}{|f_s|^2 + p_\eta^2|C|^2}\,.
\end{aligned} \tag{8}
$$

If the phase variation of the s–wave amplitude is neglected, by replacing f_s by $|f_s|$, the best fit to the asymmetry parameter does display a little curvature due to the falling of $|f_s|^2$ with p_η. Nevertheless, as shown by the dashed line in Fig. 2, it fails badly to reproduce shape of the low–momentum data.

On the other hand, when the phase variation of f_s given by Eq. (6) is retained, the much better description of the data given by the solid line in Fig. 2 is achieved, with no degradation in the description of the total cross section presented in Fig. 1. Furthermore, the difference in the behaviour of α in the low and not–so–low momentum regions can now be easily understood. The parameters of the fit are

$$
\begin{aligned}
f_B &= (50 \pm 8)\,(\mathrm{nb/sr})^{1/2}\,, \\
C/f_B &= [(-0.47 \pm 0.08 \pm 0.20) + i(0.33 \pm 0.02 \pm 0.12)]\,(\mathrm{GeV/c})^{-1}\,, \\
p_1 &= [(-4 \pm 7^{+2}_{-1}) - i(19 \pm 2 \pm 1)]\,\mathrm{MeV/c}\,, \\
p_2 &= [(103 \pm 4) - i(74 \pm 12^{+1}_{-2})]\,\mathrm{MeV/c}\,.
\end{aligned} \tag{9}
$$

The systematic error in the value of C was estimated by moving all the points in Fig. 2 collectively up and down by one standard deviation in the systematic uncertainty. Since the overall phase is unmeasurable, it is permissable to take the f_B of Eq. (6) to be real. Furthermore, because of the interference between the s– and p–waves, the relative phases of C, p_1, and p_2 do now influence the observables, though the differential cross section remains unchanged if the signs of all the imaginary parts are reversed. Compared to the original solution, where the effects of the p–waves were neglected [1], the position of the nearby pole p_1 is little changed. This is hardly surprising because this parameter is mainly fixed by the data from a region which is dominated by the s–waves. Less expected is the very modest change in the position of p_2, which could have been affected more by the introduction of C. As a consequence, the $\eta\,^3$He scattering length is also changed only marginally to $a = (\pm 10.9 + 1.0\,i)\,$fm, where the two signs of $Re(a)$ again reflect the possibility of either a quasi–bound or a virtual state.

The ANKE data indicate that the s–wave amplitude for $dp \to\ ^3\text{He}\,\eta$ undergoes a very rapid change of phase in the near–threshold region of the type expected from the presence of a quasi–bound or virtual $\eta\,^3$He state. When the fits to the COSY-11 results of Ref. [2] are generalised to include the angular dependence, it is also found that a reasonable description of data requires that one takes the fast phase variation due to the nearby pole into account [16].

It is clearly important to try to justify our interpretation further through the study of other observables, such as the deuteron and proton analysing powers, which can also be expressed in terms of the four chosen amplitudes:

$$\begin{aligned}
\sqrt{2}\,I\,t_{20} &= 2\left(|B|^2-|A|^2\right)+\left(|D|^2-|C|^2\right)p_\eta^2(3\cos^2\theta_\eta-1)\\
&\quad + 4p_\eta\cos\theta_\eta\,Re\,(B^*D-A^*C)\ ,\\
I\,t_{21} &= \sqrt{3}\,\left(Re\,(A^*C-BD^*)\,p_\eta\sin\theta_\eta+\ (|C|^2-|D|^2)p_\eta^2\sin\theta_\eta\cos\theta_\eta\right)\ ,\\
2I\,t_{22} &= \sqrt{3}\,\left(|D|^2-|C|^2\right)p_\eta^2\sin^2\theta_\eta\ ,\\
I\,it_{11} &= \sqrt{3}\,Im\,(A^*C-BD^*)\,p_\eta\sin\theta_\eta\ ,\\
I\,t_{10} &= 0\ ,\\
IA_y^p &= 2\,Im\,(A^*D-B^*D+CB^*)\ p_\eta\sin\theta_\eta\ ,
\end{aligned}\tag{10}$$

where the y direction is taken along $\vec{p}_\eta\times\vec{p}_p$.

As can be seen from Eq. (10), the deuteron spherical analysing powers t_{21} and t_{11} are sensitive, respectively, to the real and imaginary parts of an s–p interference and another combination is to be found in the forward/backward asymmetry of t_{20}. These will be investigated in forthcoming experiments at ANKE [17]. However, if indeed $A\approx B$ and $C\approx D$, which is consistent with the results of the SPESIV experiment [3], then the magnitudes of the signals in the polarised deuteron experiments might be small. Even if this proves to be the case, the proton analysing power A_y^p will not suffer from the same cancellation. This possibility could be eliminated entirely by measuring the proton analysing power with an $m = 0$ deuteron, because such a tensor spin–correlation observable is proportional to $Im(A^*D)$.

Although we have worked with a restricted number of p–wave amplitudes, the basic elements remain in place when all five are considered because the analysis depends upon the rapid variation of the s–wave phase compared to the p–waves, which are assumed to have constant phase. Furthermore, it should be noted that at $\cos\theta_\eta = \pm 1$ there are indeed only two p–wave amplitudes and so our formulae for the cross section and t_{20} are exact at these points. The combinations

$$\begin{aligned}
\tfrac{1}{3}(1 - t_{20}\sqrt{2}) &= \left(|A|^2 + p_\eta^2|C|^2 \pm 2p_\eta\, Re(A^*C)\right) \Big/ \left(|A|^2 + p_\eta^2|C|^2\right) , \\
\tfrac{1}{3}(1 + t_{20}/\sqrt{2}) &= \left(|B|^2 + p_\eta^2|D|^2 \pm 2p_\eta\, Re(B^*D)\right) \Big/ \left(|B|^2 + p_\eta^2|D|^2\right) , \qquad (11)
\end{aligned}$$

where the $\pm$ sign refers to forward and backward production, would then allow one to test the phase variation of A and B separately. Interferences of different nature are to be found in the spin correlation of transversally polarised protons and vector polarised deuterons, for which

$$I\, C_{y,y} = -2Re\left[A^*B + C^*Dp_\eta^2 \pm (A^*D + BC^*)p_\eta\right] . \qquad (12)$$

Experiments to measure both the deuteron tensor analysing powers and spin–correlations will be undertaken at COSY–ANKE [17,18].

If the momentum variation of the forward/backward asymmetry in $\gamma\,{}^3\mathrm{He}\to \eta\,{}^3\mathrm{He}$ could be measured near threshold [11,12], then this observable should be influenced by the same s–wave phase variation as noted here. The size of any effect will, of course, depend upon the strengths of the higher partial waves, and it is possible that these will enter even faster than for the $dp \to {}^3\mathrm{He}\,\eta$ reaction due to quasi–free η production.

In summary, the angular distribution for the $dp \to {}^3\mathrm{He}\,\eta$ reaction near threshold is sensitive to an s-p interference. The variation of both the ANKE and COSY-11 experimental data with η momentum requires an extremely strong dependence of both the phase and magnitude of the s–wave production amplitude on p_η. Such a behaviour is that to be expected from a pole which is very close to the $\eta\,{}^3\mathrm{He}$ threshold, though it is important to stress that no $dp \to {}^3\mathrm{He}\,\eta$ observable can show whether this pole lies on the bound or virtual state plane. Because of the numerous possible decay channels for ${}^3_\eta\mathrm{He}$, this distinction has somewhat less significance than that between the deuteron and the ${}^1S_0$ state of the proton–proton system.

It is reassuring to note that considering the angular distribution in addition to the total cross section in the fitting procedure leads to negligible changes in the position of the nearby pole. This is because it is largely fixed by the very rapid rise in the cross section close to threshold where the p–waves are small. However, it is the behaviour of the angular distribution which shows that the interpretation in terms of a pole to be correct. This belief should be reinforced through the measurement of $dp \to {}^3\mathrm{He}\,\eta$ analysing powers and spin correlations, which will allow us to pursue the investigation without some of the simplifying assumptions which have been made in the current analysis.

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