Subthreshold $K^+$-meson production in proton-nucleus interactions


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A new method of investigating subthreshold $K^+$-meson production, based on detection of the $\mu^+$ mesons from the decay of $K^+$ mesons stopped in a target and on the use of the temporal structure of a proton beam. Results are reported of the measurement of the total cross sections for the production of $K^+$ mesons in proton-nucleus interactions at proton energies 800–1000 MeV for the nuclei Be, C, Al, Ti, Cu, Zr, Sn, Ta, Pb, and U. The data are compared with models based on direct and two-stage mechanisms of $K^+$ production. It is shown that for initial proton energies 900–1000 MeV the mechanism connected with production of a pion in the intermediate state can account for the observed cross-section values and does not require allowance for a high-momentum component in the distribution function of the nuclear nucleons.

1. INTRODUCTION

One of the main recent trends in the investigation of hadron-nucleus interactions is the study of processes that cannot be adequately explained by the traditional nuclear-physics premises, that the nucleon motion in the nucleus has a single-particle character, and involve the mechanism of the nuclear reactions. It is customary to refer to such processes as cumulative, and the degree of cumulation is determined either by the ratio of the inelastic to the elastic interactions necessary for the considered reaction to the Fermi momentum, or by the ratio of the mass of the nuclear nucleon cluster participating in the reaction to the mass of one nucleon. It is assumed that by increasing the degree of cumulation it is possible to obtain new information on the processes occurring at short distances and at short time intervals. This information is qualitatively new and extremely important both for nuclear physics and for elementary-particle physics, in connection with the study of the spatio-temporal picture of a strong interaction.

Cumulative processes are experimentally studied as a rule in reactions with formation of fast particles at large (close to 180°) emission angles with threshold creation of particles (antiparticles) $\pi^0$, $\pi^+$, $\pi^-$, $K^0$, $K^\pm$, $\mu^+$, $\mu^-$, $\nu$, $\bar{\nu}$, reactions in which no particle production could occur for interactions between nucleons, if the interacting nucleons are regarded as quasifree particles moving inside the nucleus and having momenta not larger than Fermi momentum. An advantage of the latter is their simpler physical interpretation, for in contrast to reactions with large-angle-particle emission, a much lesser role should be played here by rescattering processes that lower the energy of the particles and accordingly decrease the probability of subthreshold processes.

The purpose of the present study is to measure the dependences of the cross sections for $K^+$ mesons in proton-nucleus interactions on the primary-proton energy in the range 800–1000 MeV, for a large group of nuclei. Recall that the energy threshold of $K^+$ production in proton-proton interactions is 1582 MeV. We also analyze the data theoretically from the viewpoint of various models. We consider the direct mechanism of $K^+$ production (in the process $NN \rightarrow K^\pi\pi\Lambda$) and the two-step mechanism, wherein a fast pion is produced during the first stage of the interaction of the initial proton with the nucleons, and the $K^+$ mesons are produced during the second stage in the $\pi p \rightarrow K^+\Lambda$ process. Note that a mechanism based on pion production in the intermediate state was successfully used to explain processes with formation of cumulative nucleons in nucleon-nucleus interactions at energies 0.6–10 GeV (Refs. 9–11) and in reactions of subthreshold production of antiprotons and $K^+$ mesons in collisions of heavy ions. In the case of cumulative interactions, we can find a 3–4 GeV energy dependence. It must be emphasized that the highest degree of cumulation (≈ 6–7) attainable in contemporary experiments can be reached in $K^+$ meson production in proton-nucleus interactions at energy ≈ 800 MeV.

2. EXPERIMENT

In investigations of reactions with $K^+$-meson production we recorded in our experiments not the $K^+$ mesons themselves, but the muons from the decay ($K^+ \rightarrow \mu^+\nu$) of the $K^+$ mesons produced and stopped in the very same target. A proton beam from an accelerator was focused on the investigated meson-producing target (Fig. 1) whose size was chosen such as to stop in it the greater part of the produced $K^+$ mesons.

It is important for this procedure that the $K^+$ mesons have a two-particle decay channel $K^+ \rightarrow \mu^+\nu$ with 0.63 relative probability. The muons have then an isotropic angular distribution and a fixed momentum $p_\nu = 256$ MeV/c. These muons are selected by magnetic spectrometers, namely the pion channels of the synchrocyclotron of our Institute (LINP) at an angle 60° with 5% momentum resolution and at 0° with 2% resolution. The particles leaving the spectrometer were analyzed with a scintillation-counter telescope. Counters $S_1$, $S_2$, and $S_3$ with plastic scintillators 5 mm thick provided the coincidence multiplicity needed to decrease the random-coincidence background. Counter $S_2$ (scintillator thickness 3 cm) and $S_3$ (15 cm) analyzed the selected particles by the ($\Delta E, E$) method. A thin (2.5 g/cm²) copper filter $t_1$ stopped protons and heavier particles from entering the detector. Copper filter $t_2$ (of variable thickness) was chosen for each of the spectrometer momenta so as to maximize the
ΔE losses for the pions in the $S_x$ counter and maximize the energy release $E$ for the muons in the $S_y$ counter. The use of a magnetic spectrometer and of the (ΔE, E) method turned out to be sufficient to separate the needed channel for the separation of the muons from the background due to positrons, $\pi^+$ mesons, and other particles having the same momentum.

To separate the muons produced in the decays of the $K^+$ mesons stopped in the target from the muon background due to the $\pi^+\rightarrow\mu^+$ meson decay in flight, which exceeds by $10^4\sim10^5$ times the useful effect, we used a time criterion. This criterion can be used with accelerators having a discrete temporal structure of the proton beam. In the LINP proton synchrotron the proton beam is grouped in time, in synchronism with the high-frequency (hf) phase of the accelerator, into microclusters having a period 75 ns. If the muon detector records only muons due to $\pi^+\rightarrow\mu^+$ meson decay in flight, the spectrum of the time intervals between the instant of muon registration by the detector and certain phases of the accelerator hf signal constitutes a peak whose width and shape are determined by the time distribution of the beam intensity in the microcluster and by the finite momentum resolution of the spectrometer (Fig. 2).

For the LINP synchrocyclotron, the average duration of the microclusters is 5 nsec, and the background due to proton extraction in a time interval $\sim$ 40 nsec in the interval between the microclusters does not exceed $10^{-2}$ of the intensity of the main beam. The time spectrum of the muons due to the decay of the stopped $K^+$ mesons has an exponentially decreasing distribution determined by the lifetime of the $K^+$ meson at rest ($\tau = 12.4 \pm 0.3$ ns). If the muons are recorded only in time windows located between microcluster (Fig. 2), it is possible to suppress substantially the muon background from the $(\pi^+\rightarrow\mu^+\nu)$ decays in flight. The use of all the selection criteria has made it possible to get rid almost completely of the background-particle contribution and to separate muons due to $(K^+\rightarrow\mu^+\nu)$ decays with an error not worse than 3%.

The positions and widths of the peaks of the muon momentum distributions obtained using all the selection criteria are determined, as seen from Fig. 3, by the kinematics of the $(K^+\rightarrow\mu^+\nu)$ decay, by the momentum resolution of the spectrometer, and by the energy loss in the target; they do not depend on the initial-proton energy. A confirmation of the fact that the observed momentum distribution is indeed formed in the $K^+\rightarrow\mu^+\nu$ decay is the absence of an analogous peak in the $\mu^-\nu_e$-meson spectra (Fig. 3). This is due, first, to the peculiarities of the recording procedure, since the $K^+$ mesons stopped in the target should be captured by the nuclei prior to the instant of decay. Second, observation of $K^+$ meson production of proton energies $T_p \leq 1$ GeV is impossible in view of the substantially higher threshold of this reaction.

To measure the energy dependence of the $K^+$ meson production cross section, the energy of the protons in the beam was lowered by a beryllium absorber of appropriate thickness. The additional energy spread in the proton beam incident on the meson-producing target did not exceed in this case 8 MeV at 200 meV energy applied to the target, and the proton-beam intensity decreased by not more than a factor of 10. Proton passage through the target produces in the latter pions and other particles. If the energy of the pions

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**FIG. 1.** Experimental setup: $M_1$-$M_2$ turning magnets; L 11-L 17) quadrupole lenses; T) meson-producing target; W) protecting wall; D1 and $\Delta p$, $\pi$3) muon and pion detectors; (t1-t5) copper filters; $S_1$-$S_5$ scintillation counters.

**FIG. 2.** Time distribution of muons. The dashed line shows the redistribution due to pion decay in flight.
reaching the meson-producing target exceeds 550 MeV, $K^+$ mesons can be produced by the background reaction $\pi d \rightarrow K^+ \ldots$. The most dangerous in the baryon component of the secondary particles are the high-energy neutrons produced in a charge-exchange reaction. Estimates show that even on application of 200 MeV of proton energy the contribution of these processes does not exceed 3–5%. Nonetheless, additional measurements were made at 900 MeV proton energy, using absorbers of two different metals (Be and C) located at different distances from the meson-producing target (25 and 300 cm). In all cases the results agreed within 5%.

We investigated experimentally the probability of $K^+$ meson stopping in the meson-producing target by measuring the dependence of the yields of pions and decay muons on the target dimensions. The number $N_\pi$ of pions should increase linearly with increase of the target length, while the ratio $N_\mu / N_\pi$ should remain constant. The target dimensions (see Table 1) were optimized so as to ensure that at least 80% of the produced $K^+$ are stopped.

We measured simultaneously the $\pi^+$ and $\mu^+$-meson yields $N_\pi$ and $N_\mu(P)$. The total cross sections for $K^+$-meson production were then calculated from

$$\sigma_{tot} = \frac{d^2\sigma}{d\Omega} N_\pi \frac{4\pi}{dE/dP} N_\mu 8\pi \varepsilon_{\pi} \varepsilon_{\mu} 0.63\alpha,$$

(1)

where $N_\pi$ is the number of events (area) under the peak of the muons from the decay of the $K^+$ mesons stopped in the target,

$$N_\pi = \int N_\pi(P) dP;$$

(2)

$\alpha$ is the fraction of muons landing in a time interval 40 ns (points on Fig. 2) relative to all the muons from the $K^+$-meson decay. The coefficient $\alpha = 0.25$ was calculated with allowance for the time distribution of the protons in the microbunch (it coincides with the dashed curve of Fig. 2); $\varepsilon_{\pi}$ and $\varepsilon_{\mu}$ are coefficients that take into account the decay of muons and $\pi^+$ mesons on the path from the target to the detector; $d^2\sigma_{\pi}/dEdP$ stands for the cross sections, measured by us earlier, for $\pi^+$-meson production.

Thus, by simultaneously recording the $\pi^+$ mesons and $\mu^+$ mesons at a momentum $P = 230$ MeV/c it is possible to solve the problem of the absolute normalization of the cross sections, while the relative monitoring in the measurement of $N_\pi(P)$ was carried out by recording the $\pi^+$ mesons at an angle $0^\circ$.

To determine the energy dependence of the cross sections for $K^+$-meson production it was found convenient to use the relation

$$a_{K^+}(T) - a_{\pi^+}(T) = \frac{N_\pi(T)}{N_\pi(T_0)} \frac{d^2\sigma_{\pi^+}(T)}{d^2\sigma_{\pi^+}(T_0)},$$

(3)

where $T$ is the proton energy, if the cross section at $T = T_0$ is measured directly, using relation (1). Such measurements were made at $T_0 = 1$ GeV. The factor $d^2\sigma_{\pi^+}(T)/d^2\sigma_{\pi^+}(T_0)$ takes into account the dependence of the $\pi^+$-meson production cross section on the proton energy, which was specially measured, and it was shown (see Ref. 15) that the cross sections decrease smoothly with decrease of energy, at practically the same rate for light and heavy nuclei; the difference between the cross sections for 1 and 0.85 GeV does not exceed 25%.

To decrease the statistics-acquisition time in measurements of the energy dependences of the $K^+$-meson production cross sections, a simplified measurement variant was used. The value of $N_\pi$ can be determined from the relation

$$N_\pi = N_{\pi}^{max}(P) \Delta P_p,$$

(4)

where $\Delta P_p$ is the width at half-maximum of the muon momentum spectrum, and

$$N_{\pi}^{max} = \frac{1}{2} \left[ N_\pi(200 \text{ MeV/s}) + N_\pi(260 \text{ MeV/s}) \right].$$

(5)

where the term in the square brackets takes background effects into account. As seen from Fig. 3, a change of the proton-beam energy is not accompanied by deformation of the muon momentum peak, and $\Delta P_p$ is independent of the proton energy. It is possible in this case to replace $N_\pi$ in (3) by $N_{\pi}^{max}$, and the measurement of the energy dependence of the cross section is substantially simplified.

Let us list the principal errors that determine the accuracy with which the cross sections are determined. Note that the relative error of the energy dependence of the cross sections is statistical. The absolute value of the cross section is determined accurate to ±20%, due mainly to the error in the calculation of $\alpha$ (15%) and to the error in the absolute values of the cross section for $\pi^+$-meson production at an angle $60^\circ$ (Ref. 16). No account is taken here of the corre-

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**Table 1.** Parameters of targets used to measure the total cross sections for the process $pd \rightarrow K^+$.

<table>
<thead>
<tr>
<th>Material</th>
<th>Dimension, mm</th>
<th>$\rho$, $g/cm^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>$\phi$ 70 x 20</td>
<td>1.85</td>
</tr>
<tr>
<td>C</td>
<td>$\phi$ 60 x 20</td>
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</tr>
<tr>
<td>Ca</td>
<td>$10 x 30 x 50$</td>
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<td>Sg</td>
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</tr>
<tr>
<td>Pb</td>
<td>$10 x 30 x 50$</td>
<td>11.35</td>
</tr>
</tbody>
</table>

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tion necessitated by the probability that the produced \( K^+ \) mesons will not remain in the target. This can systematically decrease the experimental cross sections by \( \sim 20\% \).

### 3. MEASUREMENT RESULTS

We have thus presented here the measured total cross section for \( K^+ \)-meson production in photon-nucleus interactions, at a primary proton energy \( T_p = (997 \pm 5) \text{ MeV} \), for \( ^7\text{Be}, ^{12}\text{C}, ^{27}\text{Al}, ^{40}\text{Ti}, \text{Cu, Zr, Sn, Ta, Pb, and U} \), as well as the measured dependences of the total cross sections of the process \( pA \rightarrow K^+ \ldots \) on the primary proton energy for the nuclei \( ^7\text{Be}, ^{12}\text{C}, \text{Cu, Sn, and Pb} \). The measured total cross sections for the nuclei from beryllium to uranium are listed in Table II, while the data on the energy dependences of the cross sections for the proton energy range \( T_p = 800-997 \text{ MeV} \) are given in Table III. The cross sections in Table II were determined from Eq. (3). Tables II and III list only the statistical errors connected with the measurements of \( N_p \) and \( N^\text{meas} \) and with subtraction of the background.

Figure 4 shows the behavior of the total cross sections for \( K^+ \)-meson production in proton-nucleus interactions at proton energies \( T_p = 850-1000 \text{ MeV} \). Note some general features of the behavior of the total cross section. First, the \( K^+ \)-meson production cross sections decrease exponentially for all nuclei when the proton energy is decreased, by approximately one order of magnitude when the proton energy is decreased by \( \Delta T_p = 100 \text{ MeV} \). Second, while the rate of decrease of the cross section for the reaction \( pC \rightarrow K^+ \ldots \) remains unchanged in the energy interval 800-850 MeV, the cross section for \( K^+ \)-meson production on carbon at \( T_p = 800 \text{ MeV} \) is three or four times larger than the upper-bound estimate given in Ref. 17. We note also that the dependence of the total cross sections on the atomic number of the nucleus (the \( A \)-dependence) remains unchanged, at least in the proton energy range \( T_p = 900-1000 \text{ MeV} \).

To interpret the data it may be useful to find for them a

### TABLE III. Total cross sections for \( K^+ \)-meson production in \( pA \rightarrow K^+ \ldots \) reactions.

<table>
<thead>
<tr>
<th>( A )</th>
<th>( T_p ) ( \text{MeV} )</th>
<th>( \sigma \times 10^6 ) ( \text{cm}^2 )</th>
<th>( \sigma \times 10^6 ) ( \text{cm}^2 )</th>
</tr>
</thead>
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<td>Be</td>
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<tr>
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<tr>
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<tr>
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</tr>
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<tr>
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<td>863</td>
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</tr>
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<td>855</td>
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<td>2.4 ( \pm 2.4 )</td>
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<td>2.4 ( \pm 2.4 )</td>
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</table>

*At these points, the proton-beam energy is dumped with the aid of a copper absorbing filter, whereas at all other points a beryllium absorbing filter was used.

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universal formulation. The natural variables for cumulative-type processes are the minimum mass $M_{\min}$ of the nucleon cluster at rest, or the minimum momentum $P_{\min}$ of one proton, both values determined from the condition that the initial proton energy $T_p$ be the threshold for $K^+$ production. Analysis of the data shows that the cross sections decrease exponentially as functions of $M_{\min}$ and $P_{\min}$:

$$
\sigma_{\min} \sim e^{-M_{\min}/M_o}, \quad \sigma_{\min} \sim e^{-P_{\min}/P_o}
$$

(see Fig. 5).

The parameter $P_o$ decreases with increase of the mass number $A$ of the target nucleus, whereas the dependence on $M_{\min}$ has a universal character for $M_{\min} = 3-5$. A deviation from an exponential law is observed at $M_{\min} > 5$, apparently due to the increase of the parameter $M_o$. Note that from the data on the production of antiprotons at values $M_{\min} < 2$ (Ref. 2) one can also obtain a value of $M_o$ which turns out, as seen from Fig. 5, to be smaller than ours. We can thus note the following tendency of the variation of the subthreshold-production cross sections as functions of $M_{\min}$: an exponential behavior is observed on individual sections, and the parameter $M_o$, which is indicative of the slope, increases with increase of $M_{\min}$.

4. COMPARISON WITH THEORETICAL CALCULATIONS

In principle, two approaches are possible to a theoretical description of $K^+$-meson production in photon-nucleus interactions at energies substantially below the threshold:

a) $K^+$-meson production in a single interaction between the primary proton and one ($n = 1$) nuclear nucleon or a group of them ($n > 1$) in the reactions $p(\gamma N) \rightarrow K^+ \Lambda(nN)$ (here $N$ is the a nucleon inside the nucleus). b) $K^+$-meson production with participation of two nuclear nucleons far enough from each other, in cascade processes, with production of a pion in an intermediate state, i.e., in the first stage a pion is produced in the reaction $NN \rightarrow \pi NN$, and in the second a $K^+$-meson is produced in the reaction $NN \rightarrow K^+ \Lambda$.

Simple estimates show that if the nucleon moves with near-Fermi momentum in the nucleus, the threshold of the direct mechanism (a) for $n = 1$ is 1.02 GeV, whereas for the two-stage mechanism (b) the threshold drops to 700 MeV, i.e., the latter mechanism can contribute to the cross section in the entire energy range of the proton for which experimental data have been obtained; in contrast to the one-stage process, there is no need to assume beforehand a high-momentum component in the distribution function of the nuclear nucleons.

We shall disregard hereafter reactions with production of $K^+$ mesons (the process $pA \rightarrow K^+ K^+ \ldots$) or of $\Sigma$ hyperons, for in the former case the reaction threshold exceeds 1 GeV even for a nucleus with infinite mass, and $\Sigma$-hyeron production is energywise less favored, since the differences between the $\Lambda$ and $\Sigma$ hyperon masses is $\approx 80$ MeV, while the thresholds of these reactions differ correspondingly by $\sim 200$ MeV.

We consider first the direct mechanism (a). To calculate the cross section for subthreshold particle production within the framework of the model of single interaction of the incident nucleons with the nuclear nucleons (or with a group of the latter), we must know the behavior of the cross sections for the production of the corresponding particles in nucleon-nucleon interactions near the threshold, specify the form of the function $\rho_p(P, E)$ that describes the momentum and energy distributions of the nucleons in the nucleus, take into account the absorption of the incident nucleon by the nucleus, and determine the probability $W_n$ of formation of a

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**Fig. 4.** Energy dependences of the for $K^+$ production cross sections for various nuclei. Curves—calculation of direct mechanism using Eq. (8). The cross section region obtained by the two-step mechanism is hatched.

**Fig. 5.** Subthreshold-production cross sections vs $M_{\min}$ (a) for $K^+$ mesons (solid line) and antiprotons (dashed line) and vs $P_{\min}$ (b) for $K^+$ mesons. $\bullet$—C, $P_o = 62 \pm 4$ MeV/c, $\Delta$—Cu, $P_o = 56 \pm 3$ MeV/c O—Pb, $P_o = 49 \pm 3$ MeV/c.

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group of nucleons in a superdense state in the nucleus if the production takes place on the corresponding groups. Assuming $W_x$ to be independent of the point of formation of a group of $n$ nucleons, the total cross section can be written in the form

$$
\sigma_{xx} = N_{er} \int \sigma_{p+n,p+n}(P,r) dP dr.
$$

(6)

It is assumed here that the cross sections for production of $K^+$ mesons on a group of nucleon is the same, accurate to the kinematic relations, as for production by nucleons, the latter having the form

$$
\sigma_{K^+} = a(s-t)^{\frac{1}{2}} 26 \times 10^{-36} (s-t)^3 \text{ cm}^2.
$$

(7)

It is assumed that the cross sections for $K^+$ mesons by protons and neutrons to be equal: $\sigma_{p-n,K^+} = \sigma_{n-p,K^+}$. The factor $N_{er}$ takes into account the absorption of protons in the nucleus and is calculated in the Glauber absorption of protons in the nucleus and is calculated in the Glauber model ($N_{er} = 66$, $44$, $28$, $7.3$, and $6.3$ respectively for the nuclei $^{208}\text{Pb}$, $^{110}\text{Sn}$, $^{64}\text{Cu}$, $^{12}$C, and $^{9}$Be for $T_p = 1 \text{ GeV}$). The models differ in the choice of the form of the function $p_x(p,r)$, in the allowance for the kinematics of the process, and in the definition of $W_x$.

Calculations of $p_x(p,r)$ in the one-particle Hartree-Fock model have yielded cross sections $10^3$ times smaller than in experiment. A good description of the experimental data can be obtained phenomenologically by choosing the spectral function in the form

$$
p_x(P) = p_x(p=1 \exp \left( (P(P-P_x)) / (2\sigma_a) \right) )^{-1},
$$

(8)

where $a$ is the diffuseness of the nucleon distribution in the nucleus, $P_x$ is the Fermi momentum, and $\rho_{p,n}$ is determined by the normalization condition $\int p_x(P)dP = 1$. It is seen from Fig. 4 that in this approximation the calculations agree with experiment if $a$ and $P_x$ are chosen to be the experimental values from Refs. 19-21. A difference is observed only for light nuclei at low energies. We note that the phenomenological analysis is ambiguous. Thus, if we use $p_x$, the approximation proposed in Ref. 22 and, when choosing the parameters in $p_x$, carry out the normalization to the cross section at $T_p = 1 \text{ GeV}$, improvement of the agreement between the calculations and the experimental data for the $^{12}\text{C}$ nucleus leads to a discrepancy in the description of the energy dependence for heavy nuclei (see Fig. 6).

It is of greater interest to compare the experimental data with models in which the high-momentum component in $p(p)$ can be directly calculated. Thus, in a mechanism proposed in Ref. 23, the high-momentum component accumulates at the expense of the short-range few-nucleon correlations, and the parameters of the model are chosen to meet the condition of normalization to the experimental spectra of the cumulative nuclei. As seen from Fig. 6, the calculated $K^+$ production cross sections obtained in this approach do not describe the energy behavior and are considerably lower than the experimental values.

The manifestation of superdense states of nuclei in processes with large momenta transferred to the nucleus was considered in the flcuton model, in which the high-momentum component can be related to collective forms of the motion of nuclear matter. It is possible to calculate $W_x$ in this model. The values of $W_x$, however, are quite low, and the cross sections turn out to be considerably lower than the experimental ones (Fig. 6; $n = 2,3$).

It was proposed in Ref. 25 to describe the experimental data on subthreshold $K^+$ meson production by using the color string model based on a microscopic approach. $K^+$ meson production on multiquark configurations was considered. From a comparison with the experimental energy dependences of the cross sections, estimates were presented in Ref. 25 for the probability of the presence of such configurations in the nucleus. This approach, however, requires an arbitrary normalization of the theoretical cross sections, making this analysis not constructive enough, since a substantial contribution to the cross section for $K^+$ production can be made by other mechanisms.

We see thus that the direct mechanism does not explain satisfactorily the experimental data, although it succeeds in a phenomenological description of an appreciable part of the data.

Let us dwell on the possibility of describing the experimental data in the framework of the cascade mechanism (b). Calculation of the cross sections for $K^+$ production is carried out under the following assumptions. The pion production process is described by using the cross sections for free-nucleon collisions. In contrast to Ref. 8, account is taken here, besides the reaction $NN \rightarrow p\pi\pi$, of pion production with a deuterorn in the final state: $pp \rightarrow \pi^0 d$, $pn \rightarrow \pi^0 d$, and $nn \rightarrow \pi^0 d$. The data of Refs. 26 and 27 were used for the cross section of the $pp \rightarrow \pi^0 d$ reaction, while the remaining cross sections were determined from isotropic-invariance considerations. The momentum distribution of the nucleons in the nucleus is specified using the Fermi-gas model. The propagation of the nucleons and protons inside the nucleus is tak-
en into account by a semiclassical approach. These approximations have made it possible to describe well the spectra of the pions produced in proton-nucleus scattering at proton energies 500–1000 MeV.\(^{10,28}\) The cross sections for \(K^+\) production are approximated by an expression that accounts well for the experimental data\(^{19}\):

\[
\sigma(\pi N - K\Lambda) = \begin{cases} 
0, & E < 767\text{ MeV} \\
6.6 \times 10^{-14} (E - 767)^2 \text{ cm}^2, & 767 < E < 877 \text{ MeV} \\
7.5 \times 10^{-14} \text{ cm}^2, & E > 877 \text{ MeV}.
\end{cases}
\]

Here \(E\) is the kinetic energy of the pion in a system in which the nucleon is at rest.

Since we are interested in the total cross sections for the production of \(K^+\) mesons, the \(K^+\)-meson re-scattering in the nucleus can be neglected. In fact, allowance for the \(K^+ = K^0\) charge exchange can change the result by not more than 10%. In the analysis of the energy distributions, however, this approximation can shift the calculated \(K^+\)-meson spectrum towards the higher energies. It is seen from Fig. 7 that the two-stage mechanism accounts for the experimental \(A\)-dependence of the cross sections at \(T_p = 990\) MeV, but when account is taken of pion production in the process \(NN \rightarrow rd\) the calculation results are systematically higher than the experimental data. It is quite difficult to determine unambiguously the cause of this discrepancy. An important role can be played here both by the normalization of the experimental data and by the accuracy of the semiclassical calculation scheme\(^{10}\).

Nor is the possibility excluded that the \(NN \rightarrow rd\) process is suppressed (for example, owing to the strong distortion of the deuteron wave function) in the central regions of the nucleus, where calculations point to a predominance of the cascade mechanism that leads to \(K^+\)-meson production. A comparison of the dependences of the cross sections for \(K^+\) production on the initial-proton energy, however, shows that at a proton energy lower than 900 MeV the contributions of the processes \(NN \rightarrow \pi N\) and \(NN \rightarrow rd\) become equal, i.e., allowance for the second process is important for a better description of the energy dependence of the cross sections.

Let us analyze the spatial picture of \(K^+\)-meson production, as given by the two-stage mechanism. It is most important here to consider the distribution in the mean free paths of the pions that take part in the \(K^+\)-meson production, i.e., in the distances from the point of pion creation to the point of its annihilation, i.e., of the \(K^+\)-meson production (Fig. 8). It has been found that the fraction of short distances (\(\ell < 0.5\) Fm) is \(\beta = 15\%\) for the lead nucleus and \(\beta = 25\%\) for carbon. It is clear that when such distances are considered the quasiclassical scheme, which is based on treatment of the pion as a quasi-free particle, is incorrect. At the same time, the values obtained for \(\beta\) can be regarded as an upper-bound estimate of the contribution of exotic mechanisms to the \(K^+\)-meson production cross section.

The conclusion that can be drawn from an analysis of the two-stage mechanism is that this mechanism is apparently decisive for the \(K^+\)-meson yields in proton-nucleus interactions for Pb nuclei at \(T_p > 850\) MeV and for C nuclei at \(T_p > 900\) MeV.

To conclude this section, we note that most information in the investigation of the roles of various mechanisms of sub-threshold \(K^+\)-meson production is apparently obtained by directly measuring the \(K^+\)-meson angle and energy distributions and comparing them with the theory. Since, however, this information is still impossible to obtain, the problem might be solvable by correlation measurements of the yield of \(K^+\) mesons in coincidence with protons, neutrons, and deuterons, whose spectra have been shown by calculation to be vital for the verification of various theoretical hypotheses.

5. CONCLUSION

We have developed in this paper a new procedure of measuring \(K^+\)-meson production cross sections, and obtain experimental data on the total cross sections for sub-threshold production of \(K^+\) mesons in proton-nucleus interactions in a wide range of primary proton energies, \(T_p = 800–1000\) MeV, and for a large group of nuclei. A high degree of cumulation (\(\sim 6–7\)) was attained.

The data were theoretically analyzed by two approaches—a direct and a two-stage mechanism of \(K^+\)-meson production. Comparison with calculations has shown that the model with a direct mechanism of sub-threshold production in \(K^+\) mesons in interactions of incident protons with nuclear nucleons or with groups of nucleons does not
lead to satisfactory agreement with the experimental data. A phenomenological account of the high-momentum component as a function of the nuclear distribution shows that for a reasonable explanation of the experimental data it must be assumed that the contribution of the high-momentum component should exceed by three orders of magnitude the level obtained in the single-particle model.

Calculations using the cascade mechanism that connects the subthreshold $K^+\pi^-$ meson production with pion production in an intermediate state have shown that this mechanism is apparently decisive at proton energies 900-1000 MeV and does not require the use of the high-momentum component of the nuclear distribution function.

Thus, processes with pion production in the intermediate state play an important role in investigations of the cumulative problem in inclusive particle production in greatly differing nuclear reactions. The most promising reaction for avoidance of the influence of multistage mechanism is apparently subthreshold production of $K^+\pi^-$ mesons in pion-nucleus interactions. Calculations show that the cross sections for the $\pi^+ \rightarrow K^+$ reaction in the Pb nucleus at a pion energy 600 MeV, without allowance for the high-momentum component of the distribution function, is $5 \times 10^{-29}$ cm$^2$, and if this component is taken into account in the approximation (8) the cross section turns out to be larger by an order of magnitude ($3.6 \times 10^{-28}$ cm$^2$). Cross sections of this size are measurable, and their results would yield quite definite information on the high-momentum component of the distribution function of nucleons and on the mechanism that governs this function.

17C. L. Morris, LA-6994-Pt, Progress at LAMPF, 1981.

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