π^+p correlation at 0° and np final state interaction in $pp \to \pi^+pn$ reaction at 495 MeV

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Abstract

In the framework of the one-pion exchange model and Watson approximation of the final state interaction it is shown that for a reliable reconstruction of low-energy np scattering parameters resolutions $\Delta\Theta \leq 0.5^\circ$ and $\Delta p/p \leq 1\%$ are required. 0° facility ANKE is the best choice to investigate the final state interaction.

Contents

1 Introduction

The idea to study low-energy scattering parameters in reactions with three particles in the final state is very old. The most recent interesting results were obtained on the low-energy $p\Lambda$ scattering from the reaction $pp \to K^+ p\Lambda$ at Jülich (COSY-11 facility)[1]. The existing experimental information on $p\Lambda$ scattering is rather poor (fig.1(a)) and the uncertainties for the deduced scattering lengths are very large and model-dependent[2]. It was shown[1], that the data on Λ production are sensitive to the $p\Lambda$ low-energy parameters, but not sensitive to the singlet/triplet ratio (they assumed the singlet and the triplet parameters are the same). The conclusion was that only a polarized beam and target experiment can separate these states. In principal this is a true statement. However, one can show that the correlation experiment is sensitive not only to the low-energy parameters, but also to the singlet/triplet ratio. And the combination of it with a polarized beam and target measurements will be the most efficient way to obtain the low-energy parameters.

The $pp \to \pi^+ pn$ reaction can be a key to this problem. We have a lot of experimental information about this process and there are no serious theoretical problems to describe it. In addition the low-energy np scattering is known rather well (fig.1(b)). So what is necessary is to test all possible observables on its sensitivity to the low-energy np parameters and to confirm it experimentally. By this it may be possible improve our knowledge on the np scattering lengths.

Kinematically all reactions with a three particles in a final state are similar, so there will be no problems to apply the method, describes in the following, to other reactions in order to extract low-energy parameters.

2 Final State Interaction

At low energies np scattering is described by the S-wave parameters $a(^1S_0)$, $r(^1S_0)$ and $a(^3S_1)$, $r(^3S_1)$ (scattering length and effective interaction radius). The same is true for $N\Lambda$ or $N\Sigma$ scattering. These parameters can be extracted from total cross sections (fig.1) close to the threshold $\sigma_{tot}{=}(1/4)\sigma_s+$ $(3/4)\sigma_t$, $\sigma \simeq 4\pi a^2/((qa)^2 + (1 + 0.5arq^2)^2)$. Here the coefficients 1/4 and 3/4 are the statistical spin factors of the singlet and triplet states. For np we see that the situation is very well determined for neutron momentum less than 100 MeV/c. For $p\Lambda$ scattering there is a large uncertainty in the experimental data and an extrapolation to the threshold is ambiguous. As a result there is no unique solution for a_s and a_t.

According to the Watson[3] approximation one can take into account the strong interaction between two nucleons at small momentum in any production reaction ($NN{\rightarrow}\pi NN$), by the final state interaction (FSI) enhancement function, which is expressed in terms of a, r and c.m. momentum of the nucleons in the final state q:

$$\begin{aligned}
FSI(a,r,q) &= X^2/Y \\
X &= 0.5r(q^2+((1+\sqrt{1-2r/a})/r)^2 \\
Y &= q^2+(0.5rq^2-1/a)^2
\end{aligned}$$

The corrected matrix element has to be multiplied by this function. When $q{\rightarrow}\infty$, $FSI(a,r,q){\rightarrow}1$ and this correction vanishes. But a low-energy parametrization is valid only for relative np c.m. energy E_{np} <10 MeV (q <100 MeV/c ($0.5fm^{-1}$), $p_n(lab)$ <200 MeV/c) and one needs to introduce some additional cutting function, which accelerates the decrease above this energy. Some authors just renormalize the FSI to 1 at a certain energy[4]. We prefer the soft cutting from c.m. np momentum $q\sim 50$ MeV/c (0.25 fm^{-1}, E_{np} ~3 MeV, $p_n(lab)$ ~100 MeV/c) with a free parameter, which normalizes the FSI function to 1 around $q\sim 200$ MeV/c (1 fm^{-1}, E_{np} ~40 MeV, $p_n(lab)$ ~400 MeV/c), and is taken from the fit to experimental data.

As there are singlet and triplet states of the nucleons in the final state, the resulting correction looks like:

$$FSI(a,r,q)=W_S\cdot FSI(S,q)+W_T\cdot FSI(T,q)$$

where W_S and W_T are the relative weights (strength) of these states, and $W_S+W_T=1$. They strongly depend on the reaction mechanism. In order to be model-independent and to have the flexibility to check the possible reaction mechanism we decided to introduce an additional degree of freedom by a new free parameter W_S or W_S/W_T. It also can be extracted from the experimental data.

In fig.2 one can see these functions. Above 50-60 MeV/c ($\sim 0.25fm^{-1}$) $FSI(S,q)$ and $FSI(T,q)$ coincide. The most interesting region is below 20 MeV/c, where one can distinguish between them. In the following we will compare two cases, i.e. W_S=0 (W_T=1, it is following from some models, when the reaction is approaching the threshold[5]) and W_S/W_T=1/3 (W_S=0.25, statistical ratio). The ratio of these two functions at q=0 is about 7, so some observables have to clarify this situation. The question is which ones?

3 Pion spectrum: $d^2\sigma/dp_\pi d\Omega_\pi$

Pion cross sections are the best known observables of the $pp \to \pi^+ pn$ reaction. An energy 495 MeV has been chosen for two reasons:

- It is low enough to see strong *FSI*. (And it is regular for COSY facility).
- Here there is the prediction of *Wilkin*[5] for $W_S/W_T \approx 0.1$, about three times less than the statistical ratio.

The double differential cross section at 0° is shown in fig.3. Maximal pion momenta correspond to zero nucleon momenta in the c.m. system of two nucleons in the final state. The momentum axis is proportional to the c.m. energy of the *np* system or the c.m. *np* momenta. The effective range of FSI is in the region from ∼230 up to 311 MeV/c. The contribution of FSI is very important. At a proton energy of 1 GeV it was negligible, on the percent level[6].

Very close to the end of the spectrum one can see the difference for $S = 0$ ($W_S = 0$) and $S/T = 1/3$ ($W_S/W_T = 1/3$, statistical ratio). This is the subject of our investigation. The region is very narrow: from ∼305 up to 311 MeV/c. The 2° acceptance for pions does not change the situation, but a momentum resolution on the level of 1÷1.5% dramatically changes the situation. We are loosing the qualitative difference in the shape of the spectra and can not distinguish these two cases.

Thus the following negative conclusion can be drawn :

- From the pion spectrum one obtains information only on the long-range part of FSI and one can not distinguish singlet and triplet components.

The same conclusion was made in [7], after the measurements of π^+ cross sections at 400 and 450 MeV.

4 Proton spectrum: $d^2\sigma/dp_p d\Omega_p$

The proton spectrum for 0° is shown in fig.4. The shape of this spectrum is completely determined by the FSI. Again the effect of FSI is very large. Two strong peaks at ∼400 and ∼600 MeV/c are responsible for the kinematical situations when the two nucleons are going together approximately in the same direction and with the same momentum. At 400 MeV/c the pions are also going forward. At 600 MeV/c pions are going in backward direction. In principle one can distinguish the cases of $S = 0$ and $S/T = 1/3$.

The angular acceptance $\Delta\Theta_p = 2^\circ$ and momentum resolution up to 3% does not kill the effect, but a direct extraction of low-energy parameters is not possible. Here at each proton momenta we have an integral over different np c.m. momentum. And only a small part (close to zero) is responsible for these peaks. This spectrum could be a very good test for different models.

Unfortunately in an experiment we have contribution to the inclusive proton spectrum from two reactions: $pp{\to}\pi^+pn$ and $pp{\to}\pi^\circ pp$. And the protons from the second one will change this spectrum. Only the correlation with a π^+ can help us on this way.

5 π^+p correlation: $d^3\sigma/dp_pd\Omega_pd\Omega_\pi$

If we measure the coincidence of the forward protons and pions, the proton spectra will look like in fig.5. There is only one strong peak at ~400 MeV/c. The ~600 MeV/c peak has disappeared.

This is a pure matrix element, there is no integration. And each proton momenta is corresponding to a certain np c.m. momenta (right axis of fig.5). We can transform the proton momentum laboratory axis in the np c.m. momentum axis. The zero $q_{CM}(np)$ is exactly at 385.9 MeV/c protons in the laboratory system. And we have the experimental distribution of the FSI function, multiplied by some factor.

If there would be no FSI, we would have only a smooth and flat "platform" (shape). The model dependence has "vanished". The absolute value depends, of course, on this "platform", but we are interested only in the relative numbers, or in the shape of this spectrum. In this narrow momentum interval the shape of the spectrum is totally determined by FSI.

If $S = 0$ $(T = 1)$ we have the same number, around the 7, for the ratio of the statistical and the pure triplet cases in the maximum position. For $S/T = 1/10$ it is about 3. The ratio of the maximum to the values at, for example, 340 or 440 MeV/c (where $FSI(S,q){\approx}FSI(T,q)$) will give us the the real singlet component. This is a direct way to determine it. And we should not worry about systematical errors. What is necessary above all is very good statistics.

In fig.6(a) one can see this cross section as a function of np c.m. momenta. The squared invariant energy of np system is $s_{np} = m_p^2 + m_n^2 + 2(\sqrt{q^2+m_p^2}\sqrt{q^2+m_n^2} + q^2)$, so we have the missing mass spectra of np system as in the case of pion double differential cross sections (fig.3). But in this case $q = 0$ $(\sqrt{s_{np}} = m_p + m_n)$ is not at the limit of the laboratory phase space that is in the center of the proton momentum scale. The sign minus has only symbolic sense that the data are taken below the maximum. It looks like d^* at threshold. By inverting the left part of the curves around $q = 0$, we can estimate the influence of the "model-dependence" on the shape of the spectra (fig.6(b)). If original "platform" is a constant, the curves have to coincide. In our case this difference is negligible. It confirms our conclusion on the "model-independence" of these measurements. But if there will be a small asymmetry in the left and the right wings, it is not dangerous. It means the model-depended "platform" has the linear dependence in the narrow momentum interval and one can correct it by taken the mean value of the left and the right wings. In this sense the model-dependence is the asymmetry in the **shape** of this spectra.

Let's look more carefully to fig.7, what will happen, if we have some real angular and momentum resolutions for protons and pions. The pion angular acceptance $\Delta\Theta_\pi = 1^\circ$ practically does not change the spectra. Additional proton angular acceptance $\Delta\Theta_p = 2^\circ$ will reduce the effect by a factor of 2. And the additional proton momentum resolution $\Delta p_p/p = 3\%$ decreases the maximum up to about 1/3 of the initial value.

In order to save the situation, we have to set the following requirements:

- the angular acceptances must be better then 0.5°,
- the proton momentum resolution must be not worse then $\Delta p_p/p = 1\%$.

These numbers are reasonable and can be obtained experimentally. When taking them into account, one can extract the low-energy pn parameters from these spectra. The shape of this distribution is sensitive to a_s, a_t, r_s and r_t. If for example the singlet component would be equal to zero, it would be very easy to extract the triplet parameters from the data.

6 Angular $\pi^+ p$ correlation: $d^2\sigma/d\Omega_p d\Omega_\pi$

If the previous cross sections are integrated over the proton momentum, the angular correlation cross section is obtained. For forward pions it is shown in fig.8(a). Again one can see a big difference for the $S/T = 1/3$ and the $S = 0$ cases at 0°. The ratio of these functions at 0° is about 2. We lost the a factor of 3.5, because of the integration. The maximal angle for protons in this reaction at 495 MeV is 39.7°. The difference of our functions is vanishing at angles of the protons of about $5^\circ \div 6^\circ$. This spectrum can give us the information on the singlet component, if we compare the cross sections at 0° and 10°, where there is no difference in our functions. Again, we are interested only in the relative numbers, or in the shape of the spectra.

One can improve the situation, if taking into account the information from the previous section that the biggest difference appears at the peak at

386 MeV/c. So, if we apply the constraint for proton momentum to be only between 340÷420 MeV/c, we would obtain fig.8(b). Here we lost in the value of cross section, but got the advantage in the ratio at 0°, from 2 to 3. At the same time we have much lower cross section(∼4 times), if $FSI = 0$. This is good, because it has a model dependence. Now it is negligible.

Finally, one needs to take into account experimental acceptances. The influence of the the pion acceptance ($\Delta\Theta_\pi$=1°, 2°, 3°) is shown in fig.9(a). It would be enough to have $\Delta\Theta_\pi$=0.5° for reliable separation of $S = 0$ and $S/T = 1/3$ cases. The requirement for proton acceptance is more strictly (fig.9(b)). One needs at least $\Delta\Theta_p$=0.5° acceptance for reliable reconstruction of the singlet component. The data are two times more sensitive to the proton acceptance at 0°.

This cross section is very useful to understand the influence of the angular resolution on the final results, but triple differential cross section (see previous section) is more informative.

7 Conclusions

The 0° facility **ANKE** is the best choice to investigate the FSI. Here we have maximal cross sections for any observables. The π^+p correlation cross sections enable us to get information on low energy parameters of np scattering and S/T ratio. The systematical errors are not very important.

These experiments are very challenging and the main requirements to the **FSI spectrometer** are following:

- angular resolution $\Delta\Theta \leq 0.5°$,
- momentum resolution $\Delta p/p \leq 1\%$.

In fig.10 one can see the predictions for two correlation cross sections at 495 MeV for three different values of $S/T = 0, 0.1(Wilkin), 1/3(statistical)$ with these resolutions. The broken line represents the case when all resolutions are equal to zero. The main difference appears because of the $\Delta\Theta_p$ and $\Delta p_p/p$.

It would be interesting to perform these measurements in a wide energy range, from the threshold region (∼300 MeV) up to 1 GeV. S/T at 0° as a function of proton energy. It is sensitive to the π^+ production models. The kinematical conditions are very different in this region. At threshold all particles are monoenergetic and going only in forward direction. We have no spectra. But in our method one needs to have $q_{cm}(np)$ at least up to ∼50 MeV/c to examine the **shape** of the spectra. It means we have to be a "little" above the threshold. May be that 495 MeV is not the best choice. It is necessary to check carefully the requirements to the FSI spectrometer at different energies. Again from the common consideration, the lower the energy, the greater the FSI effect. But we have some real experimental restrictions and can not measure anywhere. Very close to the threshold the cross sections are getting very small, and it takes a lot of time to get the reliable data.

If this program will be successful and we get self-consistent results with existing low energy np parameters, the essential extension of it will be the investigation of $p\Lambda$, $p\Sigma$, ... low energy interactions in reactions $pp \rightarrow K^+ p\Lambda$, $pp \rightarrow K^+ p\Sigma^\circ$, ...

The most difficult correlation experiments with a polarized beam and target could be the final stage of this program.

Our conclusions are based on the Watson FSI approximation. As any approximation it has some limits and problems. One of the main is the off-shell effect in FSI[8]. The $\pi^+ p$ correlations give us opportunity to check it. This is unique chance to verify some of the most advanced models of the meson production[9].

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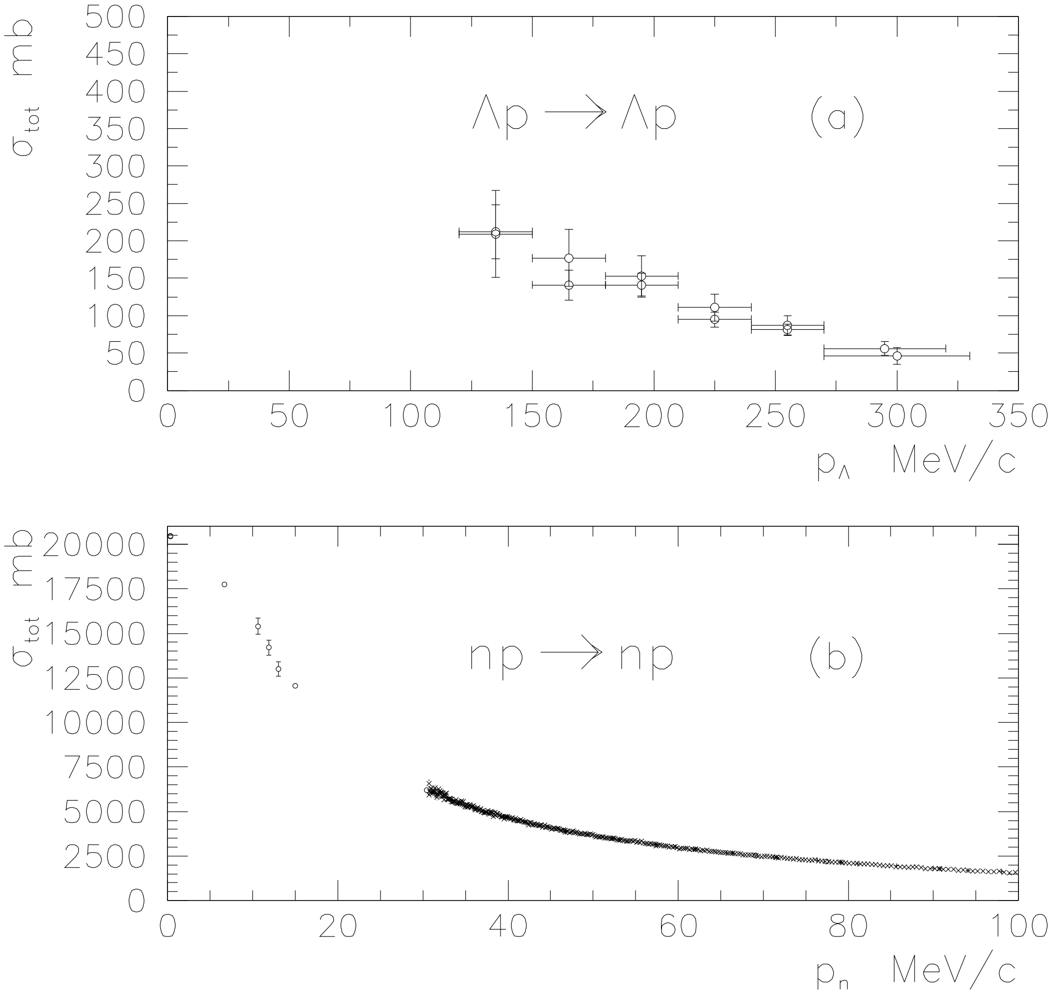

Figure 1: Total cross sections

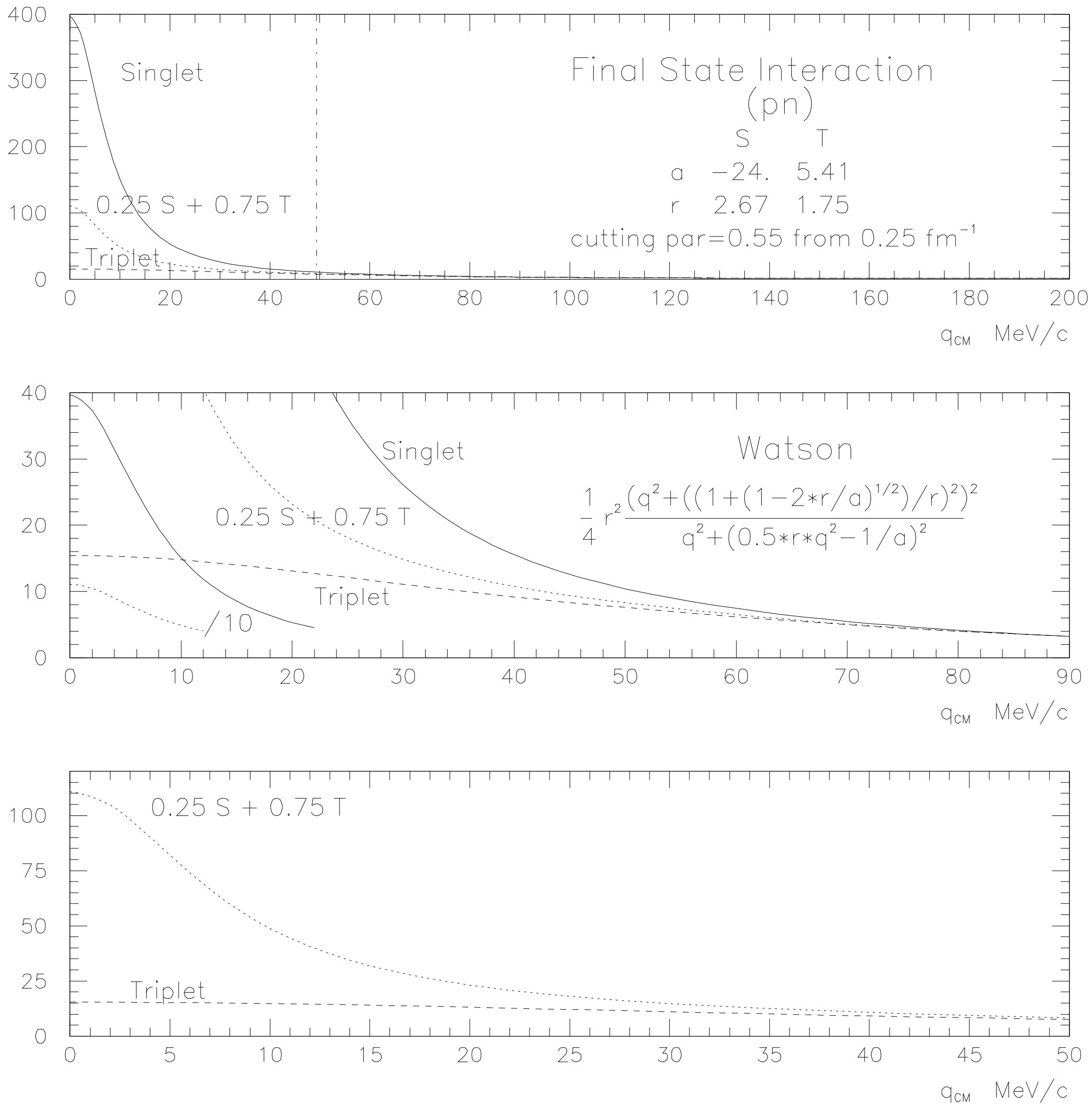

Figure 2: Final State Interaction

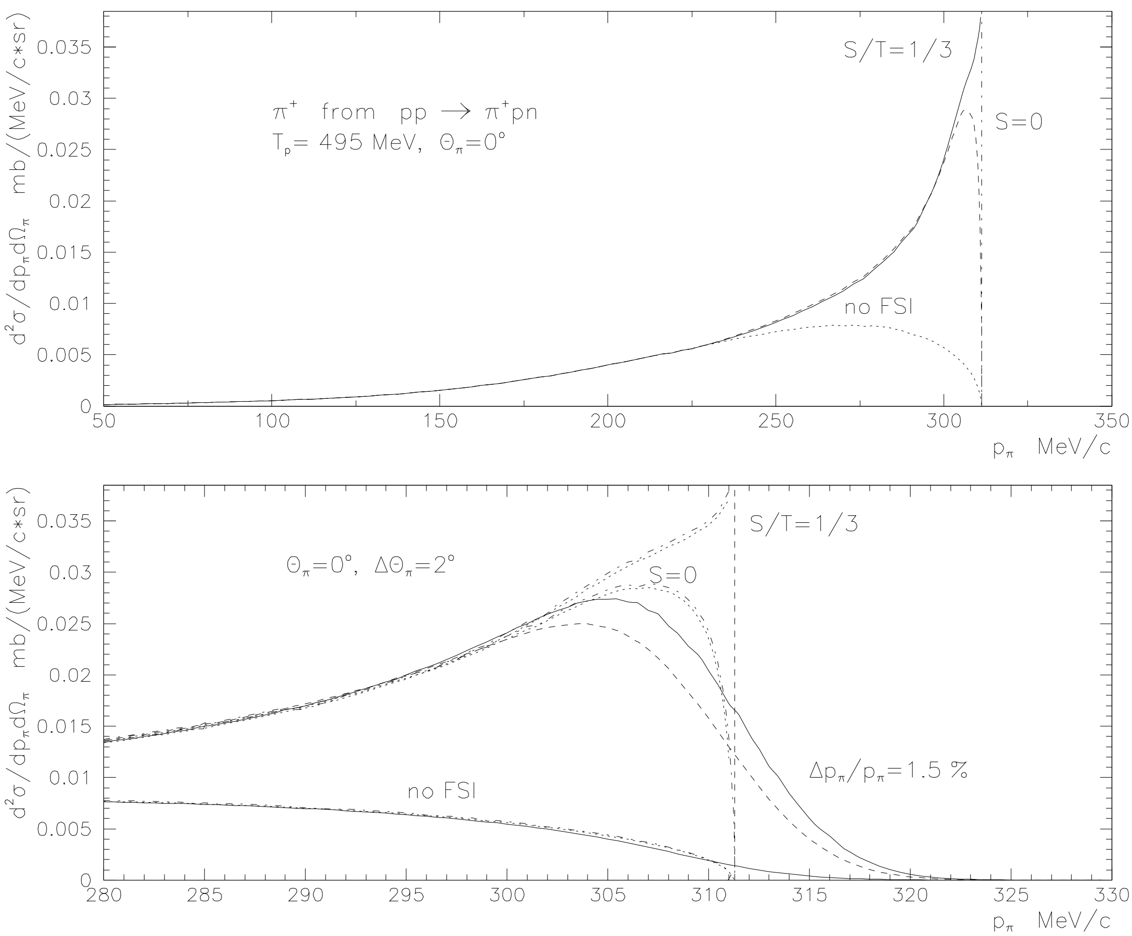

Figure 3: π^+ cross section at 0°

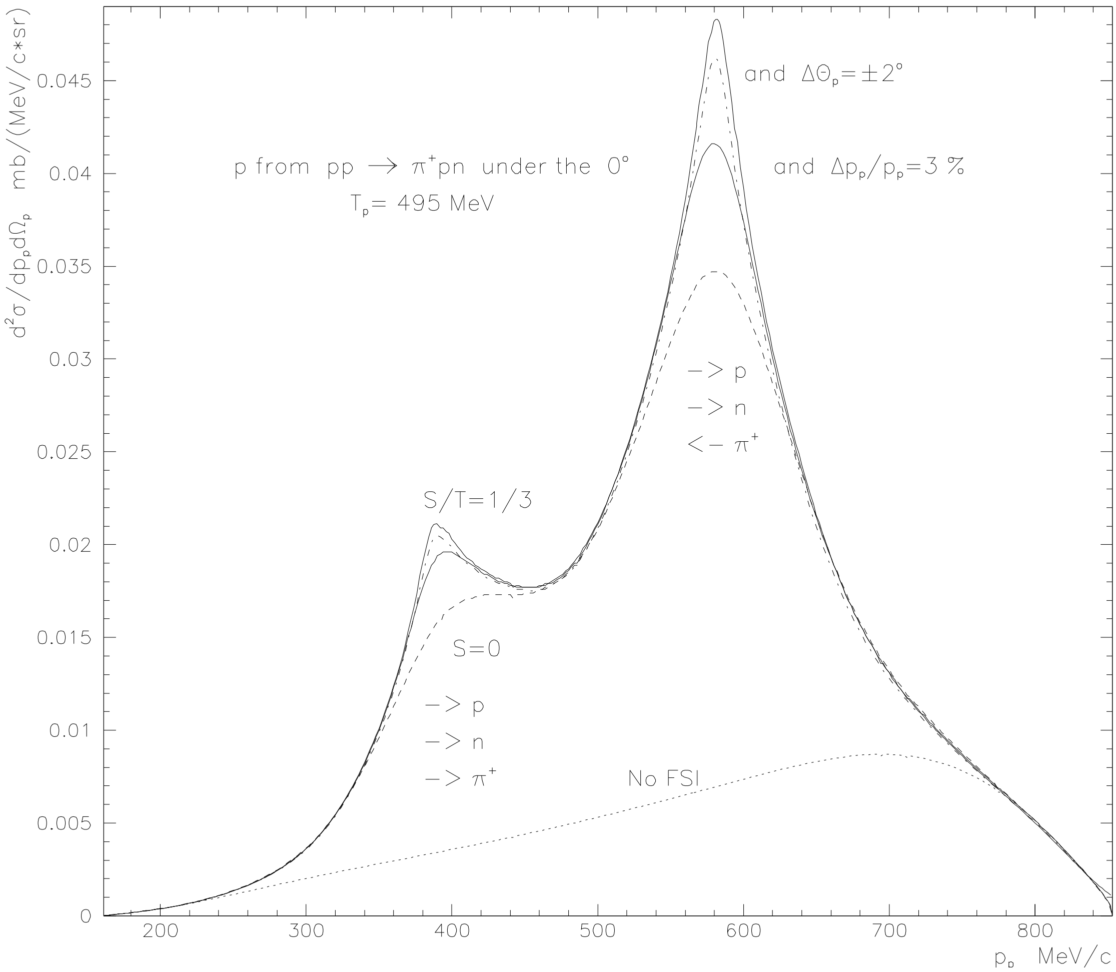

Figure 4: Proton cross section at 0°

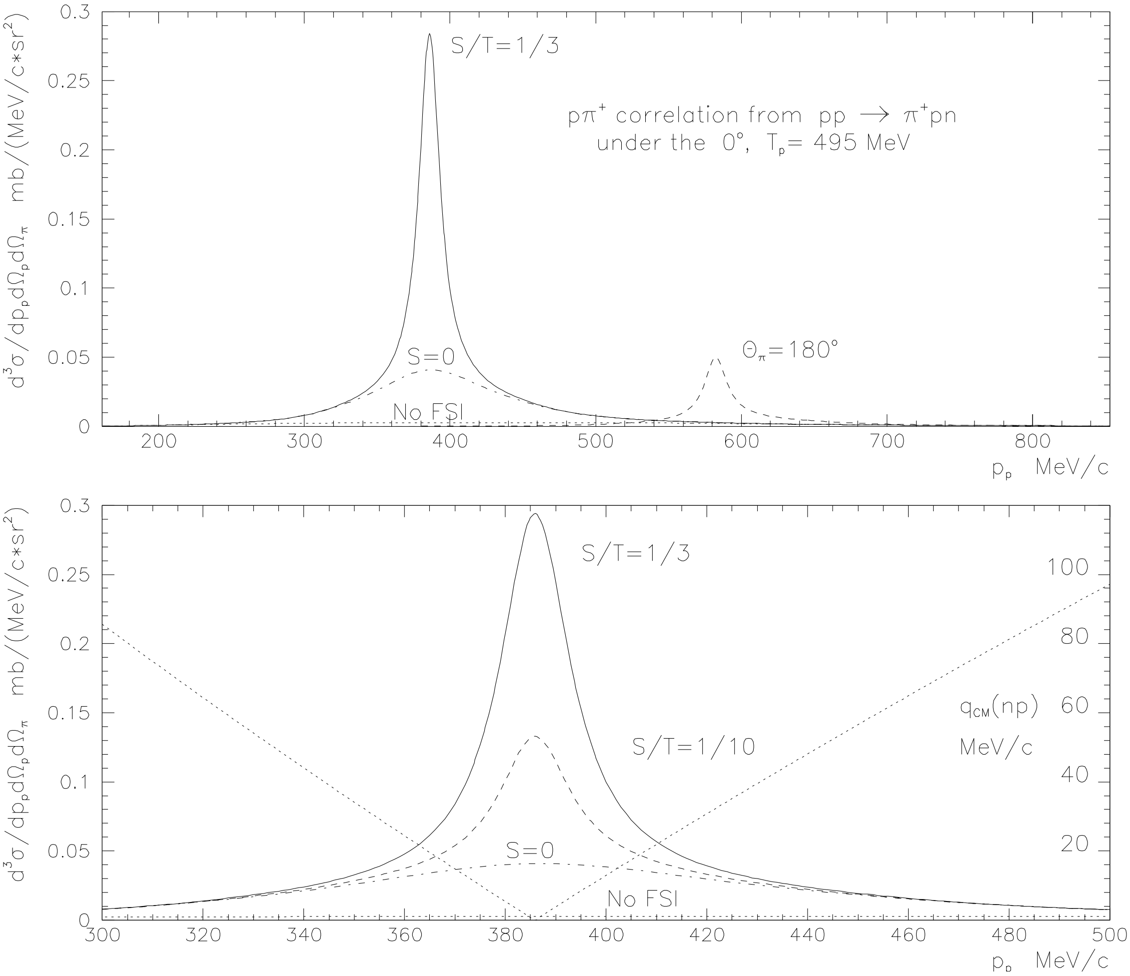

Figure 5: $\pi^+ p$ correlation at 0°

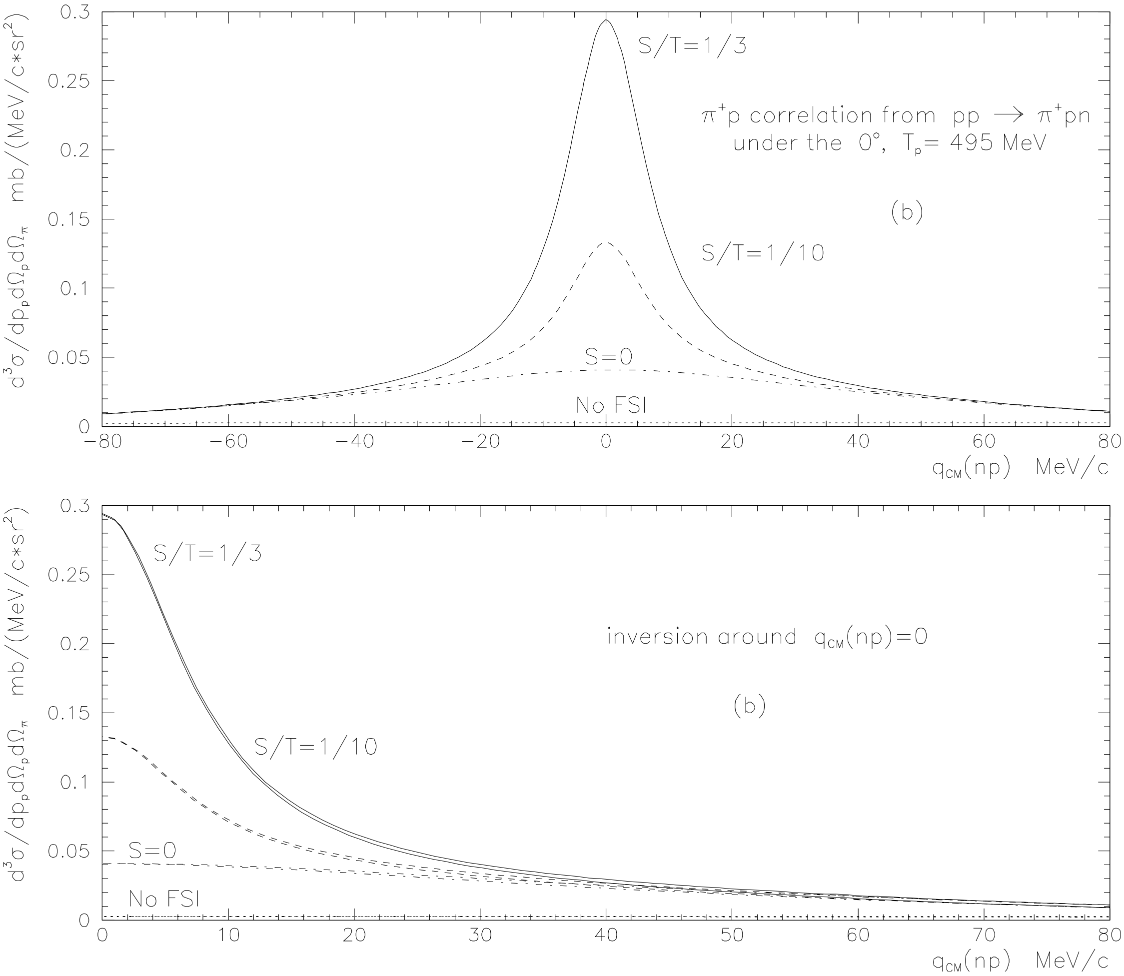

Figure 6: π^+p correlation at 0° as a function of $q_{CM}(np)$.

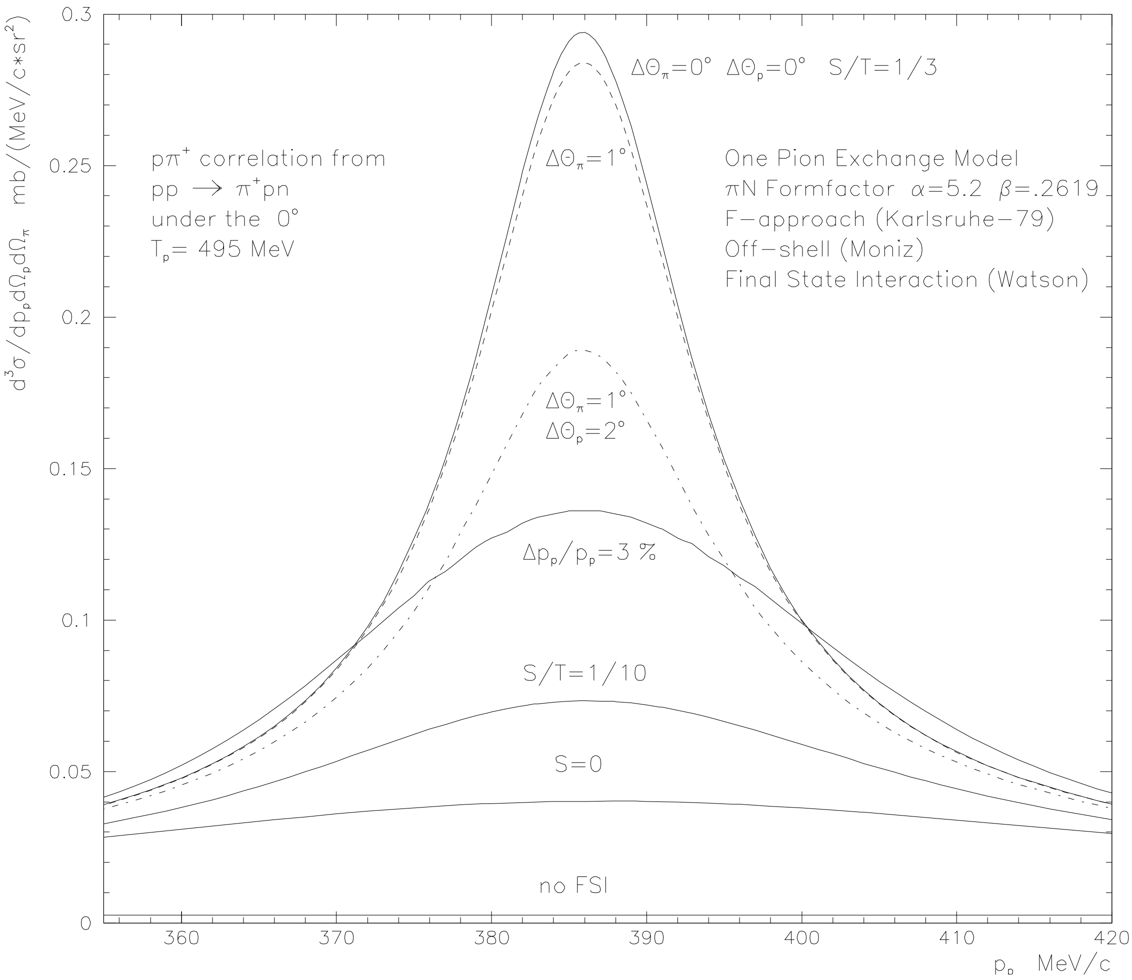

Figure 7: Resolutions in π^+p correlation at 0°

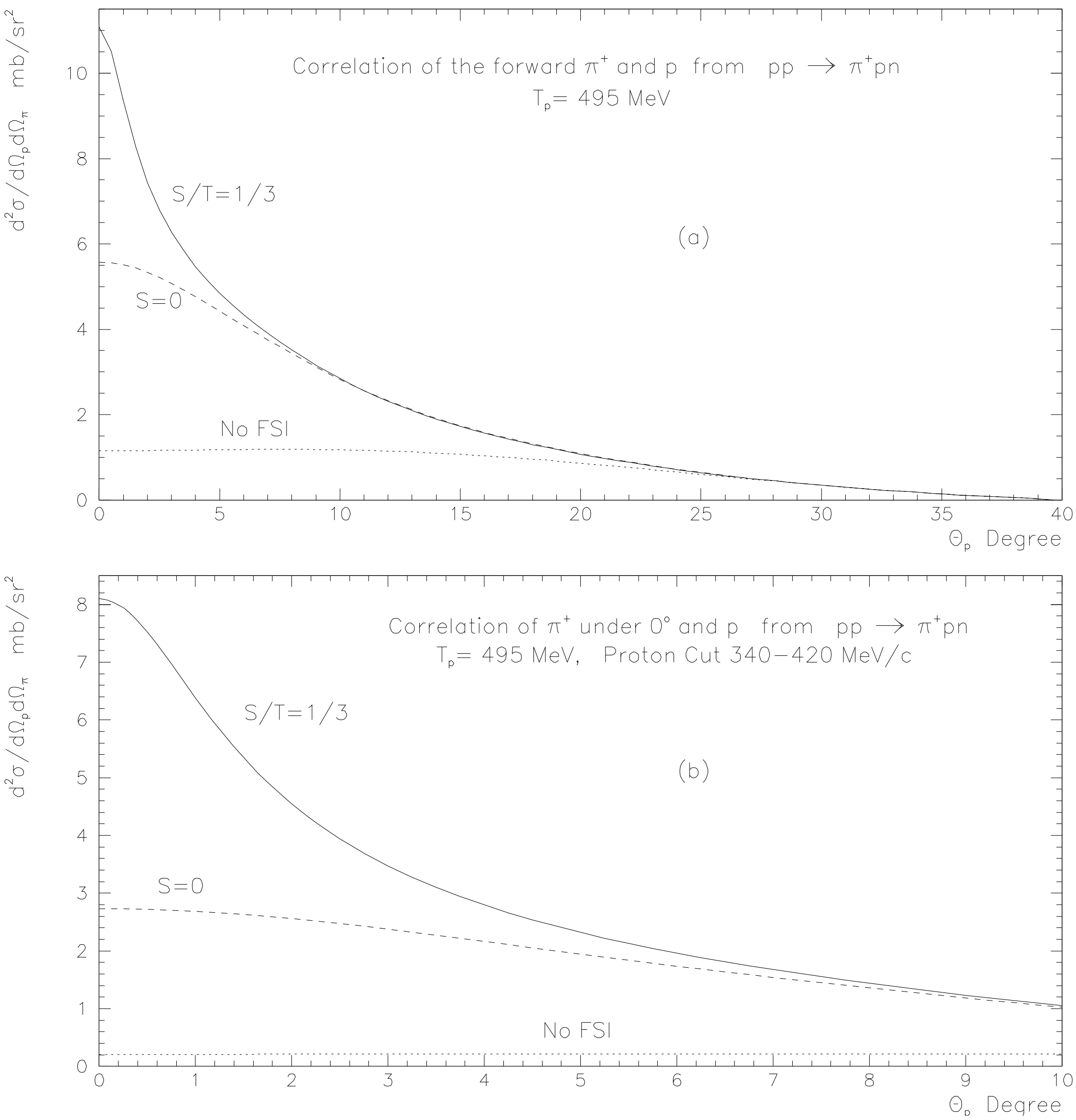

Figure 8: $\pi^+ p$ angular correlation. Pions at 0°

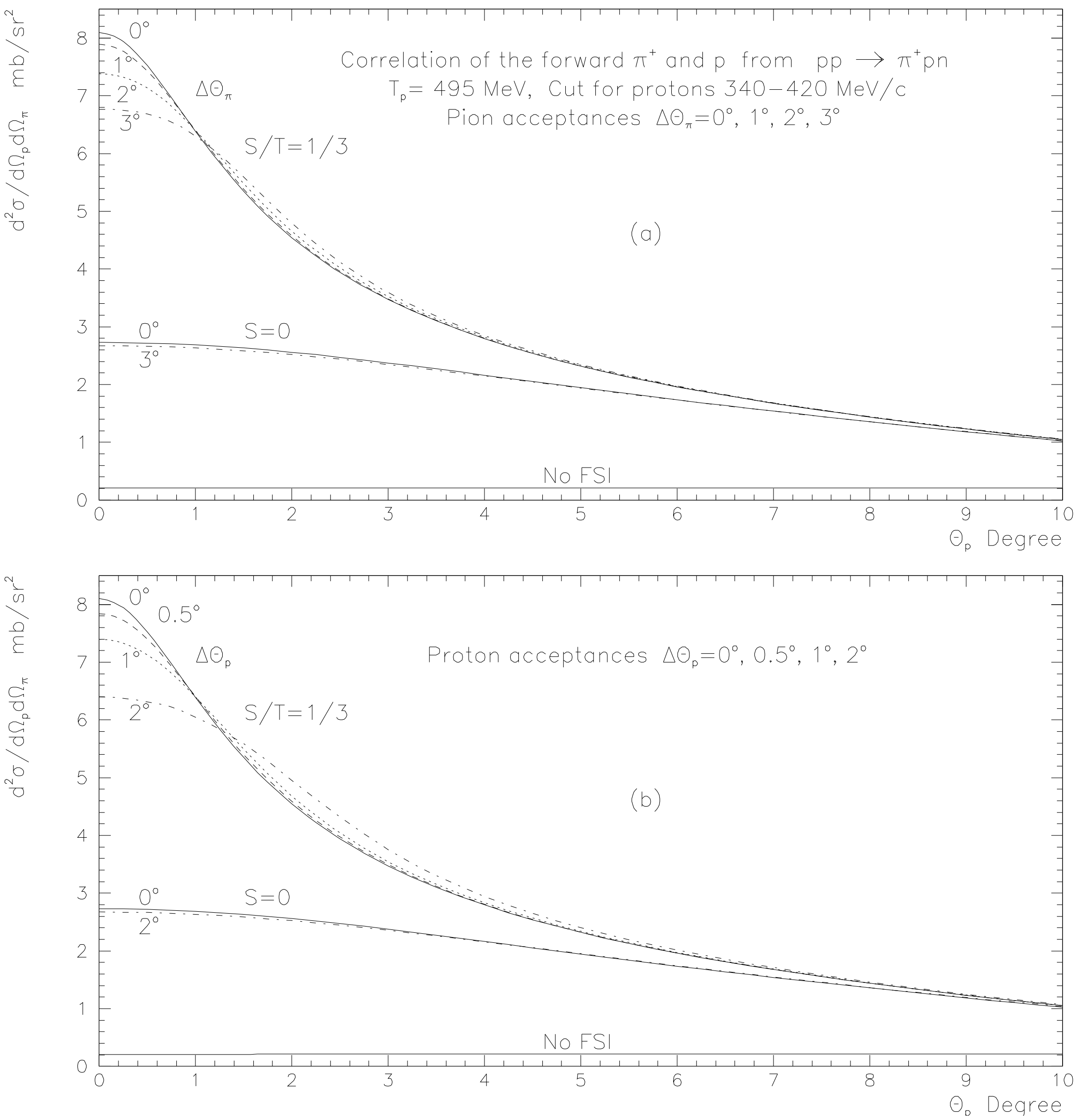

Figure 9: π^+p angular correlation for different pion and proton acceptances.

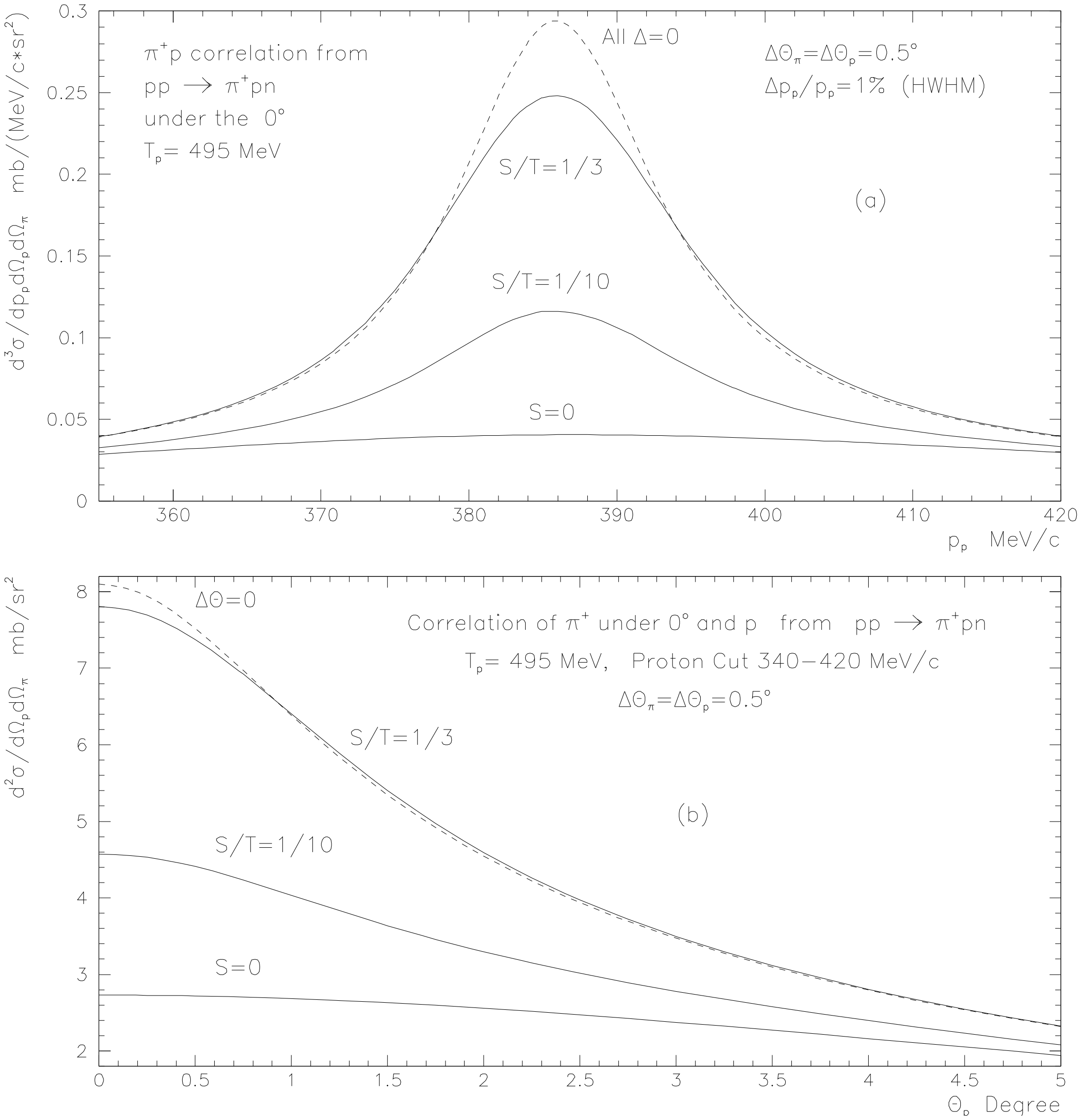

Figure 10: Predictions for π^+p correlation for different S/T and $\Delta\Theta_\pi = \Delta\Theta_p = 0.5^\circ$, $\Delta p/p = 1\%(HWHM)$.