A New Application of a Lamb-shift Polarimeter

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Abstract. With the use of a spinfilter as the most important component of a Lamb-shift polarimeter, a beam of metastable atoms in one hyperfine state ($\alpha 1$ or $\alpha 2$) can be produced. By induced transitions it seems possible to observe any transition between the $2S_{1/2}$ metastable hyperfine states or into the short-lived states $2P_{1/2}$ and $2P_{3/2}$ of the hydrogen (deuterium) atom separately as a function of the magnetic field. According to our estimate, the Breit-Rabi diagrams for these states can be measured with a precison of about 1 kHz (4.2×10^{-12} eV) or even better. Furthermore, the hyperfine splittings and the Lamb shift can be observed as well.

Keywords: Lamb shift, Breit-Rabi diagram, Hyperfine splitting.
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INTRODUCTION

In 1952 Rabi [1] suggested that the *Atomic Beam Resonance Method* [2] could be used to study also the exited states of atoms. This was realized in 1955 by Heberle et al. [3] with a modified setup for the metastable hydrogen state $2S_{1/2}$ for a magnetic field strength B = 0 G and in 1956 by Senitzky and Rabi [4] for the alkaline atoms.

When Lamb and Retherford [5] measured the Lamb shift, they were not able to separate the different hyperfine levels of the $2S_{1/2}$, $2P_{1/2}$, and the $2P_{3/2}$ fine structure states completely. When they induced transitions from the metastable $2S_{1/2}$ hyperfine states (HFS) into the short-lived $2P$ HFS, the measured half-width of the resonances exceeded 100 MHz (4.2×10^{-7} eV) due to the short lifetime of $\tau = 1.6 \times 10^{-9}$ s of the $2P$ states and technical limitations.

Since then, different methods have been applied to measure the hyperfine splitting and the Lamb shift of the excited $n = 2$ states of the hydrogen atom for B = 0 G. In 2005 Hall and Hänsch received the Nobel Prize for their work [6] on two-photon laser spectroscopy. The best values today are $\Delta E_{HFS(2S_{1/2})} = 177.556785(29)$ MHz [7] and $\Delta E_{Lamb} = 1057.8446(29)$ [8] or $\Delta E_{Lamb} = 1057.858(2)$ [9].

However, the complete Breit-Rabi diagrams [10], i.e. the energies of the single $n = 2$ HFS as a function of the magnetic field (see Fig. 1), were not measured. The only precisely known details of the Breit-Rabi diagrams are in addition to the B = 0 G measurements the crossing points of the β states of the $2S_{1/2}$ state and the e states of the $2P_{1/2}$ state measured by Lamb and Retherford [11], located at B=535 G and B=605 G.

On the other hand, the binding energies for the different HFS can be calculated as a

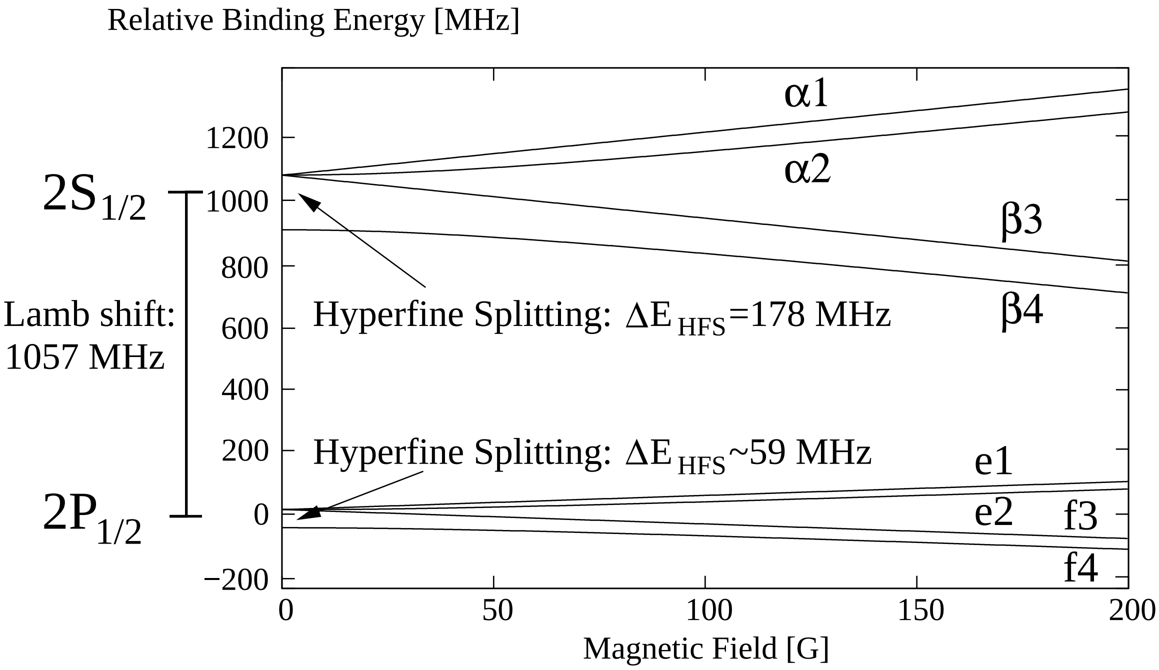


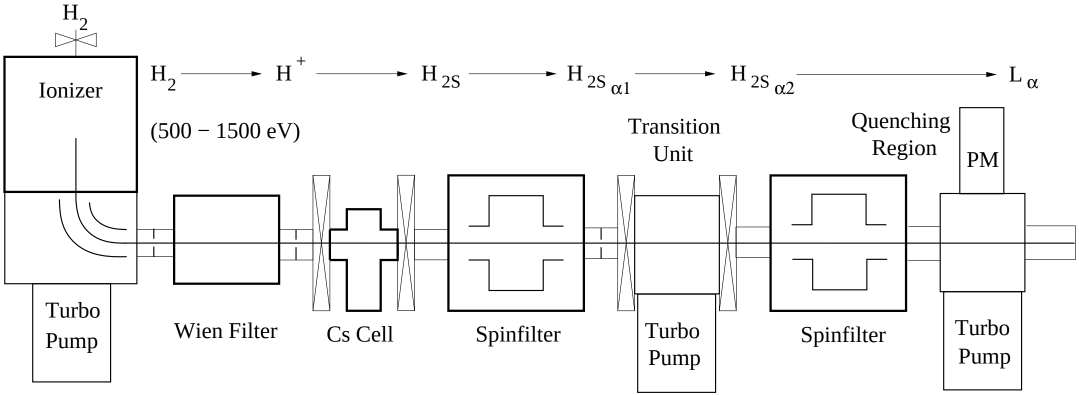
FIGURE 1. The Breit-Rabi diagram for the hydrogen $2S_{1/2}$ and $2P_{1/2}$ hyperfine states up to 200 G.

function of the magnetic field with the method of QED [12]. However, the accuracy of these calculations is limited by the uncertainties of a number of parameters in the expressions. For example, the g factors of the electrons in the $1S_{1/2}$, $2S_{1/2}$, $2P_{1/2}$, and $2P_{3/2}$ states are different and the charge radius of the proton must be taken into account. With a fit to precision data these values could be determined.

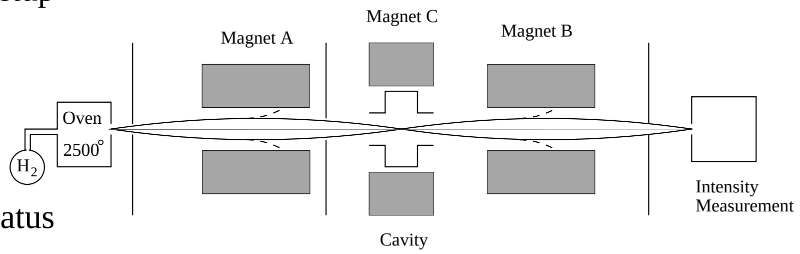
SETUP FOR THE METASTABLE HYPERFINE STATE $2S_{1/2}$

With a spinfilter [13], the most important component of a Lamb-shift polarimeter [14], it is possible to produce an intense beam of hydrogen atoms in the ground state $1S_{1/2}$ and in single metastable HFS $\alpha1$ or $\alpha2$ of the $2S_{1/2}$ state only. This possibility can be used to establish a setup similar to that of the *Atomic Beam Resonance Method* (Fig. 2). In our setup, the *analyzing magnets* of that method are replaced by spinfilters.

First, H_2 molecules are dissociated and ionized in a Glavish-type ionizer. The energy of the produced protons lies between 500 and 1500 eV, and a beam intensity up to 10 μA can be achieved. After deflection to a horizontal beam direction, a Wien filter is used to separate the protons from the ions, which stem from the residual gas. The width of the proton-velocity distribution is minimized by two diaphragms at both ends of the Wien filter. By charge exchange with cesium vapour [15], metastable hydrogen atoms in the state $2S_{1/2}$ are produced from more than 15% of the protons. In the first spinfilter all metastable atoms in three out of the four HFS are quenched into the ground state and only metastable atoms in the HFS $\alpha1$ or $\alpha2$ remain in the beam. In a homogeneous magnetic field, radio frequency (rf) transitions are induced with the electric field vector



a.) Spinfilter Setup



b.) Rabi Apparatus

FIGURE 2. **a.)** The setup with two spinfilters for the measurement of the Breit-Rabi diagram for the metastable $2S_{1/2}$ HFS. **b.)** The analogous setup of the Rabi apparatus for the $1S_{1/2}$ HFS.

always parallel to the velocity of the hydrogen atoms to avoid Doppler shifting and broadening. The direction of the magnetic field, which is produced by two Helmholtz-like coils for $B \leq 150$ G, can be parallel or perpendicular to the velocity direction of the atoms. Thus, the same transitions as in a standard polarized atomic beam source (ABS) can be induced. The necessary magnetic dipole transition rules for transitions within the $2S_{1/2}$ states ($B_{crit.}$ = 63.4 G) are:

Magnetic field	Rules	mag. field direction	Transition
weak field ($B < B_{crit.}$)	$\Delta m_F = 0$ / $\Delta F = \pm 1$	parallel	$\alpha 2 \leftrightarrow \beta 4$
weak field ($B < B_{crit.}$)	$\Delta m_F = 1$ / $\Delta F = 0, \pm 1$	vertical	$\alpha 1 \leftrightarrow \alpha 2$
weak field ($B < B_{crit.}$)	$\Delta m_F = 1$ / $\Delta F = 0, \pm 1$	vertical	$\alpha 2 \leftrightarrow \beta 3$
weak field ($B < B_{crit.}$)	$\Delta m_F = 1$ / $\Delta F = 0, \pm 1$	vertical	$\alpha 1 \leftrightarrow \beta 4$
strong field ($B > B_{crit.}$)	$\Delta m_J = 0$ / $\Delta m_I = \pm 1$	vertical	$\alpha 1 \leftrightarrow \alpha 2$
strong field ($B > B_{crit.}$)	$\Delta m_I = 0$ / $\Delta m_J = \pm 1$	vertical	$\alpha 1 \leftrightarrow \beta 4$
strong field ($B > B_{crit.}$)	$\Delta m_I = 0$ / $\Delta m_J = \pm 1$	vertical	$\alpha 2 \leftrightarrow \beta 3$

For example, the first spinfilter is used to depopulate the HFS $\alpha 2$, $\beta 3$ and $\beta 4$. So, only metastable atoms in the HFS $\alpha 1$ are found in the beam. With the magnetic field in the transition unit perpendicular to the beam direction the rf frequency is ramped across the resonance inducing transitions from the HFS $\alpha 1$ to the HFS $\alpha 2$. A subsequent spinfilter is tuned to transmit only $\alpha 2$. After quenching the metastables into the ground state by the Stark effect the change is registered by counting the Lyman α photons detected by a photomultiplier in the quenching region as a function of the rf to find the resonance frequency.

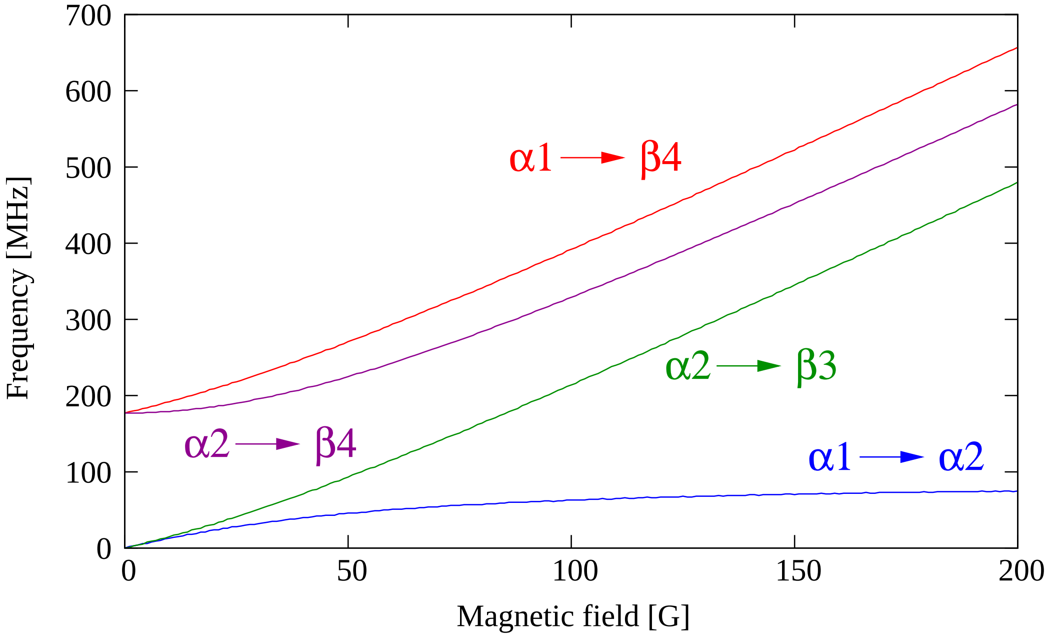


FIGURE 3. Calculated resonance frequencies[17] for the different HFS transitions in the metastable 2S state as a function of the magnetic field. Typical errors of the calculated frequencies are 3 kHz.

With some minor modifications of the electric field inside the cavity of the spinfilters one can produce combinations of the HFS $\alpha1$ and $\beta4$ or $\alpha2$ and $\beta3$ or determine the occupation numbers of the HFS $\beta3$ and $\beta4$. Thus, all possible transition frequencies can be determined.

POSSIBLE RESULTS AND ERRORS

Due to the long lifetime (0.14 s) of the metastable 2S state [16], the natural half-width of the resonance is about 1.1 Hz (4.6×10^{-15} eV).

The longitudinal Doppler effect is suppressed in first order, because the beam direction of the metastable atoms is perpendicular to the rf-wave vector. Second order effects (caused by misalignment of the beam axis) of the Doppler shift will be around 250 Hz for 1 keV protons from the ionizer. By varying the proton energy, the Doppler shift is changed, too, and can be corrected. Another method would be to exchange the direction of the rf wave vector, because in this case the Doppler shift will be in opposite direction. The Doppler broadening of the resonance peak will be on the order of 10 Hz due to the velocity distribution of the incoming metastable atoms of about 10 eV. The transverse Doppler shift will be around 200 Hz for a 1 keV and around 50 Hz for a 500 eV beam energy. The broadening in this case is less than 10 Hz.

The dominating source of uncertainties is the homogeneity and the absolute value of the magnetic field strength in the transition region. An absolute accuracy of 0.01 G for the magnetic field in the transition region leads to an error in the resonance frequency of up to 20 kHz. An inhomogeneity of 0.01 G produces a broadening of the resonance

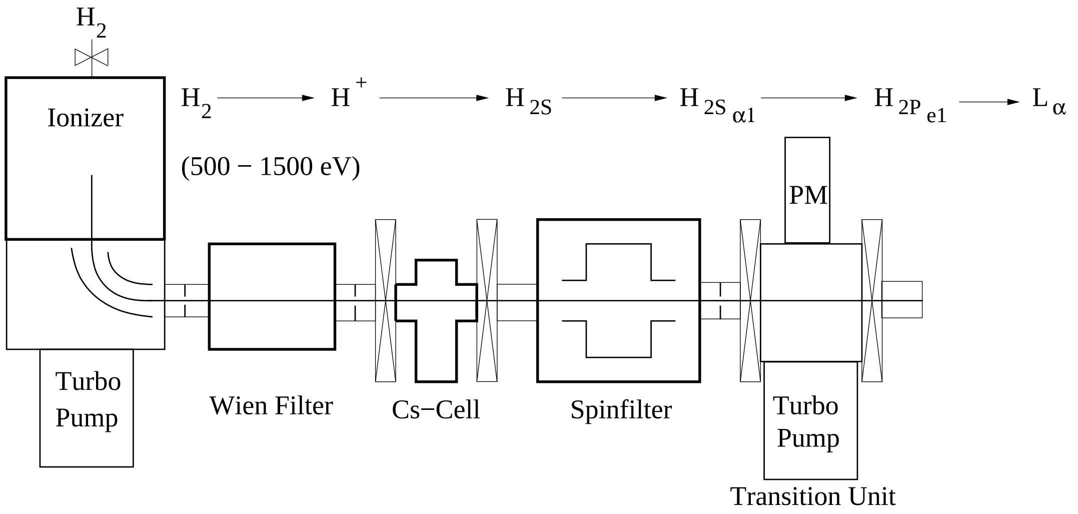


FIGURE 4. The setup for the measurement of the Breit-Rabi diagram for the $2P_{1/2}$ hyperfine states.

peak by a similar value, and the shape of the peak can be substantially distorted. The center of the peak, however, should be stable as long as the average magnetic field is not changed. The large count rate (10^7 photons/s are collected in the photomultiplier) provides good statistics and the center of the peak can be determined with an error of about 20 Hz. In view of the expected absolute inaccuracy of 0.01 G in measurements with different magnetic field settings, it seems impossible to achieve the accuracy of the transition frequencies aimed at.

A higher experimental accuracy can be achieved for the level splitting of the Breit-Rabi diagram of the $2S_{1/2}$ HFS, if the calculated QED values [17] (see Fig. 3) of the transitions in one branch (e.g. those of the $\alpha 1 \rightarrow \alpha 2$ transitions) are used as reference data. The $\alpha 1 \rightarrow \alpha 2$ transition frequency at a certain magnetic field strength can be measured with an error of about 20 Hz. If the magnetic field is kept constant and stable, then the frequencies of the other transitions between the $2S_{1/2}$ HFS can be measured with the same accuracy by a change of the rf frequency in the transition unit only. By varying the magnetic field strength the complete Breit-Rabi diagram for the $2S_{1/2}$ HFS can be measured. The errors in the ratios of the transition frequencies are expected to be at least in the order of those in the results of the QED calculations (Fig. 3).

SETUP FOR THE HYPERFINE STATE $2P_{1/2}$

With a similar setup the classical Lamb shift, i.e. the energy splitting between the $2S_{1/2}$ and the $2P_{1/2}$ states, the hyperfine splitting ΔE_{HFS} and the Breit-Rabi diagram for the HFS $2P_{1/2}$ can be measured (Fig. 4).

The transition unit can be used to induce transitions from the metastable HFS $\alpha 1$ or $\alpha 2$ into the HFS of the short-living $2P_{1/2}$ state ($\tau = 10^{-9}$ s), with transition frequencies around 1 GHz. In this case these atoms will directly decay to the ground state $1S_{1/2}$ and the produced Lyman α photons can be used to tune the resonance frequency.

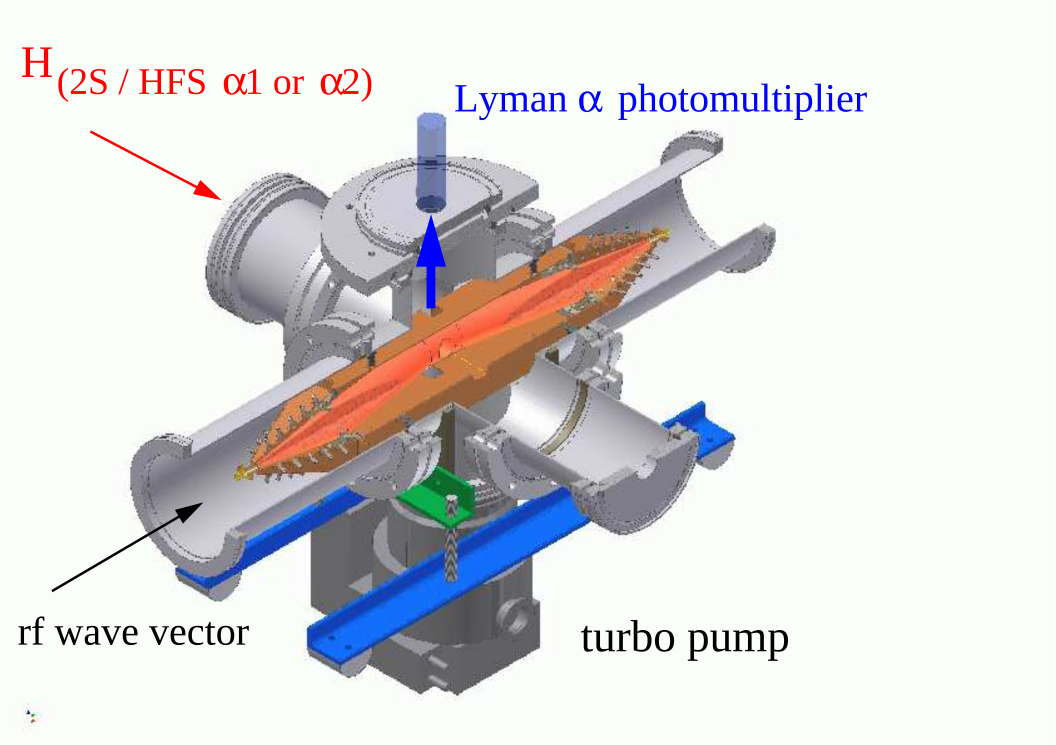


FIGURE 5. The design of the interaction region with the TEM waveguide.

Already Lamb and Retherford [5] supposed, that this should be the best method to measure the Lamb shift. But at that time they saw two technical problems:

- There were no rf generators available to produce all frequencies in the range around 1 GHz with good precision.
- The only way to get the rf power into the vacuum, containing the metastable atoms, would be the use of an antenna. But in this case the rf amplitude would be unstable, when the frequency is changed to find the resonance. The half-width of the resonance peak is about 100 MHz due to the short lifetime of the $2P_{1/2}$ states. When the frequency is varied by several 100 MHz, reflections at the antenna occur and the peak will be distorted.

Both problems can be solved nowadays. Rf generators for the GHz range with a precision better than 1 Hz and a stable amplitude are available. Inducing the transition at a stable amplitude can be achieved by the use of a Lecher (TEM) transmission line (see Fig. 5). The TEM waveguide of a constant characteristic impedance transports the rf through the vacuum and at the other end the amplitude can be controlled. By holes in the conductors around the beam line the metastable atoms can reach the volume between the inner and the outer conductor of the TEM waveguide and there the transitions are induced. With a proper design one can guarantee that the electric field vector of the rf is nearly homogeneous and parallel to the velocity vector of the hydrogen beam. In this case the rf wave vector must be perpendicular to the atomic beam and the

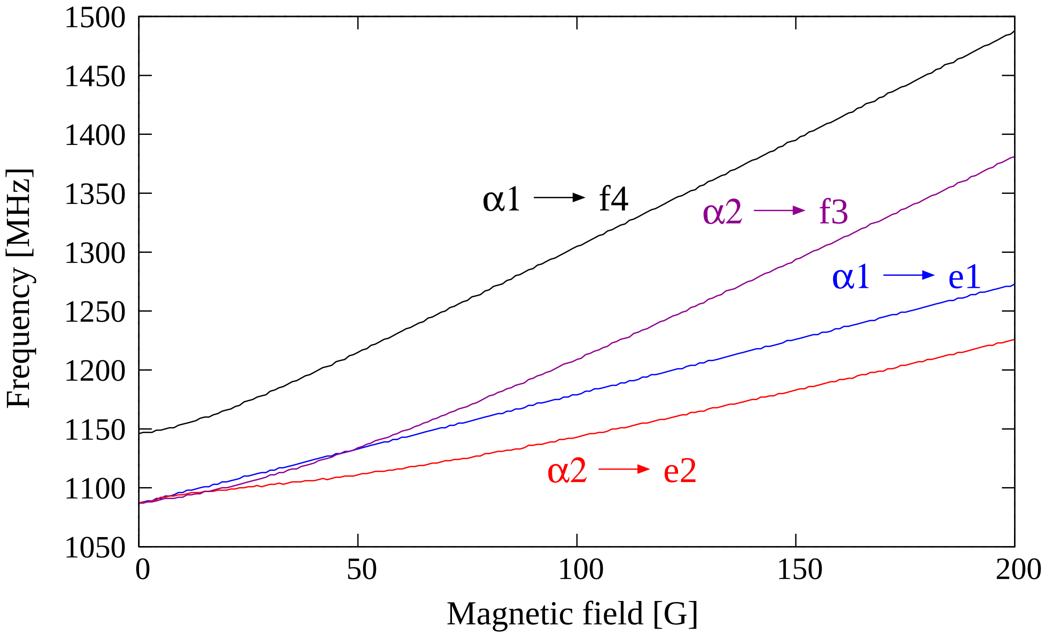


FIGURE 6. Calculated resonance frequencies[17] for the different HFS transitions from the metastable $2S_{1/2}$ HFS into the $2P_{1/2}$ HFS as a function of the magnetic field.

Doppler effect can be suppressed in first order. Through a hole in the outer conductor the Lyman α photons are detected by the photomultiplier. The possible transitions (see Fig. 6) are electric dipole transitions and the selection rules into the $2P_{1/2}$ HFS are now ($B_{crit.}$ = 21 G):

Magnetic field	Rules	magnetic field direction	Transition
weak field ($B<B_{crit.}$)	$\Delta m_F = 0$ / $\Delta F = 0$	parallel	$\alpha 1 \leftrightarrow e1$
weak field ($B<B_{crit.}$)	$\Delta m_F = 0$ / $\Delta F = 0$	parallel	$\alpha 2 \leftrightarrow e2$
weak field ($B<B_{crit.}$)	$\Delta m_F = \pm 1$ / $\Delta F = 0, 1$	vertical	$\alpha 1 \leftrightarrow f4$
weak field ($B<B_{crit.}$)	$\Delta m_F = \pm 1$ / $\Delta F = 0, 1$	vertical	$\alpha 2 \leftrightarrow f3$
strong field ($B>B_{crit.}$)	$\Delta m_J = 0$ / $\Delta m_I = 0$	parallel	$\alpha 1 \leftrightarrow e1$
strong field ($B>B_{crit.}$)	$\Delta m_J = 0$ / $\Delta m_I = 0$	parallel	$\alpha 2 \leftrightarrow e2$
strong field ($B>B_{crit.}$)	$\Delta m_J = \pm 1$ / $\Delta m_I = 0$	vertical	$\alpha 1 \leftrightarrow f4$
strong field ($B>B_{crit.}$)	$\Delta m_J = \pm 1$ / $\Delta m_I = 0$	vertical	$\alpha 2 \leftrightarrow f3$

POSSIBLE RESULTS AND ERRORS

Due to the short lifetime of $\tau = 1.6 \times 10^{-9}$ s, the half-width of the resonance peak is about 100 MHz. Therefore, the error of the center of the peak can only be minimized with a large count rate. After 1000 s about 10^{10} photons are collected by the photomultiplier, which will yield an uncertainty of 1 kHz. The influence of the Doppler effect and the inhomogeneity have been discussed before. In addition the stability of the rf amplitude must be known to a good precision to avoid resonance shifts.

DISCUSSION

With the help of a spinfilter it seems possible to measure the complete Breit-Rabi diagrams of the first excited state of the hydrogen atom with $n = 2$ with good accuracy. The hyperfine splittings ΔE_{HFS} and the Lamb shift can be determined as well.

Up to now, the hyperfine splittings ΔE_{HFS} for the $2P_{1/2}$ and the $2P_{3/2}$ states have not been measured. Only calculations by Bethe and Salpeter [18] exist.

Especially the famous measurements of Lamb and Retherford can be repeated with a better quality and, therefore, a test of modern QED calculations can be performed. The theoretical precision for modern QED predictions are on the level of 3 kHz, dominated by the uncertainty of the Lamb shift. We expect, that the precision for this experiment will be around 1 kHz for one single measurement. With a fit of 100 measurements at different magnetic fields, the achieved error of the Lamb shift should become smaller.

The same method can be used also for deuterium and tritium. Because of the nuclear spin I=1 of the deuteron, three HFS are separated by the spinfilter. With modified spinfilters the metastable 3S and 4S states can be analyzed in the same way. Even spinfilters for other atoms like ^{3}He can be built [19].

This method constitutes an interesting alternative to study antihydrogen. During recombination of antiprotons with positrons the metastable state $2S_{1/2}$ is populated with a probability of 30%. Therefore, in principle one can induce transitions into the $2P_{1/2}$ or $2P_{3/2}$ state with rf in a magnetic trap for antihydrogen.

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