Spin Constraints in the $pp \to pp\phi$ Reaction close to Threshold

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(Internal report to the COSY ZDF group, July 13th 1995)

(Revised version, eliminating some errors in first draft, August 8th 1995)

The letter of intent No 35 envisages measuring the initial spin correlation in $\vec{p}\vec{p} \to pp\phi$ about 7 MeV above threshold. The aim is to investigate the suggestion that ϕ production would be much more copious from the spin–triplet initial state than the spin–singlet. I do not want here to discuss why this might be interesting but rather to see what the consequences are of carrying out a measurement very close to threshold or, more generally, in the region where one produces a low–mass pp pair in the forward direction. The conclusion will be that if one detects the ϕ through its K^+K^- decay with good efficiency then the C_{NN} spin parameter carries no information extra to that which is obtained from the kaon angular distribution. On the other hand, if the kaon acceptance is poor then this would necessarily distort seriously any C_{NN} information.

Close to threshold the two protons must be in the ${}^1\mathrm{S}_0$ state (d^*) and the ϕ in a relative S–state with respect to this pair. The final state has therefore to be $J^P = 1^-$. To conserve parity the initial state must be in a p–wave and the <u>unique</u> possibility is ${}^3\mathrm{P}_1$. This means that the threshold values of the initial spin correlation parameters are fixed by conservation laws and do not depend upon the detailed dynamics, though the absolute threshold rate does. This threshold result was found independently and almost simultaneously by Rekalo, Arvieux and Tomasi [LNS/Ph/95–10 (July 1995)], though their formalism is rather different to that used here.

Let us look at the slightly more general problem <u>above</u> threshold where the two final protons are in still in the d^* peak and where this is moving in the beam direction. This is of some relevance for the Zero Degree Facility.

To carry out the spin algebra, it is best to write the amplitudes out explicitly in invariant form. Let $\vec{\epsilon}_\phi$ be the polarisation vector of the ϕ, $\vec{\epsilon}_{pp}$ the polarisation vector of the triplet pp state, Φ_{pp} the initial spin–zero singlet pp state function and $\phi^{\dagger}_{pp}$ the produced spin–zero singlet pp (d^*) state function. In the forward direction there are two independent amplitudes which may be written as

$$F = iA\phi^{\dagger}_{pp}\,(\vec{\epsilon}_{pp}\times\vec{\epsilon}^{\,\dagger}_{\phi})\cdot\vec{p} + B\phi^{\dagger}_{pp}\,\Phi_{pp}(\vec{\epsilon}^{\,\dagger}_{\phi}\cdot\vec{k})\,, \tag{1}$$

where $\vec{p}$ is the momentum of the initial proton in the c.m. system and $\vec{k}$ that of the ϕ. This amplitude respects the proton antisymmetry since $\vec{p}$ changes sign under interchange of the two initial protons but the vector $\vec{k}$ does not. It should be noted that the spin/isospin structure of $pp\to d^*\phi$ in the forward direction is identical to that for $pp\to d\pi^+$ [Germond & Wilkin, J.Phys. **G16** (1990) 381, Eqn.(2.1).] In one case the final state is $J^p(T) = 0^+(1)1^-(0)$ whereas in the other it is $1^+(0)0^-(1)$. Of course, when coming to apply the formulae one must remember that in the forward direction $\vec{k}\propto\vec{p}$.

The polarisation vectors of the ϕ for $\lambda=\pm1$ and 0 are

$$\vec{\epsilon}\,(+) = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\ -i\\ 0\end{pmatrix}\quad \vec{\epsilon}\,(0) = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}\quad \vec{\epsilon}\,(-) = \frac{1}{\sqrt{2}}\begin{pmatrix}+1\\ -i\\ 0\end{pmatrix}\,, \tag{2}$$

and similarly for the pp triplet polarisation vector.

If one observes nothing of the decay of the ϕ, then summing over its spin projection yields

$$Z = \sum_{\lambda_\phi}|F|^2 = |A|^2(\vec{\epsilon}_{pp}\times\vec{p})\cdot(\vec{\epsilon}^{\,\dagger}_{pp}\times\vec{p}) + |B|^2k^2\Phi^{\dagger}_{pp}\Phi_{pp}\,. \tag{3}$$

There is no interference term between A and B — this would only come about in spin–transfer measurements.

Simplifying the vector product,

$$Z = |A|^2[(\vec{\epsilon}_{pp}\cdot\vec{\epsilon}^{\,\dagger}_{pp})p^2 - (\vec{\epsilon}_{pp}\cdot\vec{p})(\vec{\epsilon}^{\,\dagger}_{pp}\cdot\vec{p})] + |B|^2k^2\Phi^{\dagger}_{pp}\Phi_{pp}\,. \tag{4}$$

If we first quantise along the beam direction, it means that we consider states with $\vec{p}$ lying along the z–direction. It is then straightforward to evaluate $Z(m_1,m_2)$, where $m_{1(2)}$ are the spin projections of the two incident protons. Thus

$$\begin{aligned} Z(+\tfrac{1}{2},+\tfrac{1}{2}) &= |A|^2p^2\,,\\ Z(+\tfrac{1}{2},-\tfrac{1}{2}) &= \tfrac{1}{2}|B|^2k^2\,. \end{aligned} \tag{5}$$

The asymmetry coefficient is

$$C_{LL}=\frac{Z(+\frac{1}{2},+\frac{1}{2})-Z(+\frac{1}{2},-\frac{1}{2})}{Z(+\frac{1}{2},+\frac{1}{2})+Z(+\frac{1}{2},-\frac{1}{2})}=\frac{2|A|^2p^2-|B|^2k^2}{2|A|^2p^2+|B|^2k^2}\,. \tag{6}$$

The experimental conditions at COSY dictate that one rather measures the transverse spin correlation parameter C_{NN}. To work this out, one keeps the same form for the polarisation vectors but lets the proton momentum lie along the y–direction. In this case

$$\begin{aligned} Z(+\tfrac{1}{2},+\tfrac{1}{2}) &= \tfrac{1}{2}|A|^2p^2\,, \\ Z(+\tfrac{1}{2},-\tfrac{1}{2}) &= \tfrac{1}{2}|A|^2p^2+\tfrac{1}{2}|B|^2k^2\,. \end{aligned} \tag{7}$$

The asymmetry coefficient is

$$C_{NN}=\frac{Z(+\frac{1}{2},+\frac{1}{2})-Z(+\frac{1}{2},-\frac{1}{2})}{Z(+\frac{1}{2},+\frac{1}{2})+Z(+\frac{1}{2},-\frac{1}{2})}=\frac{-|B|^2k^2}{2|A|^2p^2+|B|^2k^2}\,. \tag{8}$$

Note that the denominator is the same in C_{NN} and C_{LL} since the unpolarised cross section does not care about the quantisation axis. Note further that C_{NN} is always negative, even if the triplet state is completely dominant. This is completely at variance with the hand–waving arguments in the proposal. In fact, from eqns.(6) and (7), one derives the relation

$$C_{LL}-2C_{NN}=1\,. \tag{9}$$

This clearly means that, independent of any dynamics, one could <u>never</u> have both C_{NN} and C_{LL} vanishing simultaneously. Quite naturally it is the same relation which holds for the forward $pp\to d\pi^+$ reaction [Wilkin, J.Phys.**G6** (1980) L5], which was checked experimentally by the Geneva group at PSI.

At threshold the ϕ momentum $\vec{k}$ is zero in the c.m. system so we obtain the absolute predictions

$$C_{LL}=1 \;\text{ and }\; C_{NN}=0 \quad \text{at threshold.} \tag{10}$$

This corresponds to having the unique ${}^3\mathrm{P}_1$ initial state.

In the COSY experimental conditions, one is fairly insensitive to the exact parameter values providing $|A|^2p^2$ is much bigger than $|B|^2k^2$. Thus if $|A|^2p^2=2|B|^2k^2$, then $C_{NN}=-0.2$ rather than the 0.0 which one gets when

$|B|^2$ vanishes. The longitudinal variation is exactly twice as much, due to the factor of two in eqn.(9).

A further important complication is that unless one detects all the ϕ's from their K^+K^- decay, then this might generate a false asymmetry. This is because in the beam quantisation direction the ϕ spin projection is equal to the spin projection of the initial pp pair. Note that the ϕ spin projection also determines the angular distribution of the final K^+K^- pair. This then raises the question of whether one could do the experiment by just measuring the kaons with an initial unpolarised system.

Carrying out the same algebra as before,

$$X = \sum_{m_1 m_2} |F|^2 = |A|^2[(\vec{\epsilon}_\phi \cdot \vec{\epsilon}_\phi^\dagger)p^2 - (\vec{\epsilon}_\phi \cdot \vec{p})(\vec{\epsilon}_\phi^\dagger \cdot \vec{p})] + |B|^2(\vec{\epsilon}_\phi \cdot \vec{k})(\vec{\epsilon}_\phi^\dagger \cdot \vec{k})] \; . \quad (11)$$

In this case

$$\begin{aligned} X(\lambda_\phi = \pm 1) &= |A|^2 p^2 \; , \\ X(\lambda_\phi = 0) &= |B|^2 k^2 \; . \end{aligned} \quad (12)$$

The kaon angular distribution is proportional to

$$\left| Y_1^{\lambda_\phi} \right|^2 \; . \quad (13)$$

Hence the total angular distribution must be proportional to

$$\frac{d\sigma}{d\Omega} \propto |A|^2 p^2 \sin^2\theta_K + |B|^2 k^2 \cos^2\theta_K \; . \quad (14)$$

Clearly, if one had a good kaon acceptance then one would not need any initial spin information to separate singlet and triplet production.

On the other hand, if one had a very bad acceptance then this could generate a false asymmetry. As an extreme case, suppose one were only capable of measuring at $\theta_K = 0^o$. That case corresponds to putting $A = 0$, *i.e.* triplet initial states are not capable of giving kaons in the forward direction and one would automatically measure $C_{LL} = -1$.

The original motivation of the experiment was to cast light on the OZI rule, which is normally associated with a comparison of ϕ and ω production. Now one has no idea whether vector mesons should be produced polarised, independent of any $\bar{s}s$ argument. As a test case therefore one should also look at the ω polarisation in pp production. Can one deduce something about this polarisation from looking at the pion distribution from the ω decay into $\pi^+\pi^-\pi^o$? The answer is basically yes! Essentially all one has to do is interchange $\cos\theta_K$ with $\sin\theta_\pi$. However this is further than we need to go at present.

One crucial question, which I have not tried to answer, is whether there are general spin constraints away from the d^* region in a case where one detects both final protons from the $\vec{p}\vec{p} \to pp\phi$ reaction in the forward direction. I suspect that there will be some but rather weaker.

In summary, if one does the experiment by detecting protons at low relative momentum, the final pp spin correlations are as important as the initial ones and completely drown any possible signal. If kaon angular distributions can be measured well then one does not need the initial spin information. If the kaon angular distributions are biased then this would reflect in the initial pp spin correlation, which would therefore be false. This means that it would be dangerous, for example, to detect the K^- merely in the forward direction.

If one worked at say 50 MeV above threshold then one would get mixtures of S and P–waves in the final pp system and the situation would be even more complicated to analyse. The quark annihilation picture is only really clean if there is a tremendous amount of energy available in the final state such that the particular configuration of the particles there would not affect the result. One might also have go away from the collinear dynamics discussed here. These are far from the COSY ZDF conditions.

I am grateful to M. Sapozhnikov, F. Lehar, J. Arvieux and M.Rekalo for pointing out that the ${}^1\mathrm{P}_1$ state of the pp system is forbidden by the Pauli principle! Clearly I should make more mistakes since it encourages people to think. In any case the selection rule makes my argument stronger rather than weaker.