Amplitude analysis of the reaction $pp \to d\,K^+\bar{K}^0$ near threshold

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The $pp \to d\,K^+\bar{K}^0$ reaction has been recently measured at the ANKE–COSY [1, 2]. In the analysis of this reaction only the lowest allowed partial waves were included to describe the available data on the differential cross sections [1, 2, 3]. The first permitted final states are Sp and Ps, where the first label denotes the orbital angular momentum between the $K^+\bar{K}^0$ pair and the second that of the kaon-antikaon pair relative to the deuteron. In both cases the initial pp pair has spin one and is in the P or F partial wave. These properties of the $pp \to d\,K^+\bar{K}^0$ reaction amplitude follow from angular momentum and parity conservation laws combined with the Pauli principle for the initial proton-proton state.

Each of the reaction amplitudes $M_{Sp}(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{k},\boldsymbol{\epsilon})$ or $M_{Ps}(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{q},\boldsymbol{\epsilon})$ could be presented as the superposition of four spin structures

$$\begin{aligned} &A_1\,(\boldsymbol{S};\boldsymbol{\epsilon}) = \frac{\sqrt{2}}{\sqrt{3}}\left(\boldsymbol{S}\cdot\ \boldsymbol{\epsilon}^\dagger\right), \\ &A_2\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa},\boldsymbol{\epsilon}) = \left\{(\hat{\boldsymbol{p}}\cdot\boldsymbol{S})\left(\boldsymbol{\kappa}\cdot\ \boldsymbol{\epsilon}^\dagger\right) - (\boldsymbol{\kappa}\cdot\boldsymbol{S})\left(\hat{\boldsymbol{p}}\cdot\ \boldsymbol{\epsilon}^\dagger\right)\right\}, \\ &A_3(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\epsilon}) = \sqrt{3}\left\{(\hat{\boldsymbol{p}}\cdot\boldsymbol{S})\left(\hat{\boldsymbol{p}}\cdot\ \boldsymbol{\epsilon}^\dagger\right) - \frac{1}{3}\left(\boldsymbol{S}\cdot\ \boldsymbol{\epsilon}^\dagger\right)\right\}, \\ &A_4\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa},\boldsymbol{\epsilon}) = \left\{(\hat{\boldsymbol{p}}\cdot\boldsymbol{S})\left(\boldsymbol{\kappa}\cdot\ \boldsymbol{\epsilon}^\dagger\right) + \right. \\ &\left. + (\boldsymbol{\kappa}\cdot\boldsymbol{S})\left(\hat{\boldsymbol{p}}\cdot\ \boldsymbol{\epsilon}^\dagger\right) - 2\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{S})\left(\hat{\boldsymbol{p}}\cdot\ \boldsymbol{\epsilon}^\dagger\right)(\hat{\boldsymbol{p}}\cdot\boldsymbol{\kappa})\right\}, \qquad (1) \end{aligned}$$

where $A_2\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa},\boldsymbol{\epsilon})$ and $A_4\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa},\boldsymbol{\epsilon})$ should be taken at corresponding value of the momentum $\boldsymbol{\kappa} = \boldsymbol{k}$ or $\boldsymbol{q}$. Here $\boldsymbol{q}$ is the relative momentum between the two kaons, $\boldsymbol{k}$ the deuteron cms momentum, and $\hat{\boldsymbol{p}}$ is the unit vector parallel to the beam direction. The polarisation vectors of the intial pp pair and the final deuteron are denoted by $\boldsymbol{S}$ and $\boldsymbol{\epsilon}$, respectively. The contributions to the matrix element from the Sp and Ps final states are

$$\begin{aligned} &M_{Sp} = a^1_{Sp}\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{k})\,A_1\,(\boldsymbol{S};\boldsymbol{\epsilon}) + a^2_{Sp}\,A_2\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{k},\boldsymbol{\epsilon}) + \\ &a^3_{Sp}\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{k})\,A_3\,(\boldsymbol{S};\boldsymbol{\epsilon}) + a^4_{Sp}\,A_4\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{k},\boldsymbol{\epsilon})\ ; \qquad (2) \\ &M_{Ps} = a^1_{Ps}\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{q})\,A_1\,(\boldsymbol{S};\boldsymbol{\epsilon}) + a^2_{Sp}\,A_2\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{q},\boldsymbol{\epsilon}) + \\ &a^3_{Sp}\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{q})\,A_3\,(\boldsymbol{S};\boldsymbol{\epsilon}) + a^4_{Sp}\,A_4\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{q},\boldsymbol{\epsilon})\ . \qquad (3) \end{aligned}$$

Here the two sets of coefficients a^i_{Sp} and a^j_{Ps} are independent complex amplitudes which in general case depend upon the scalar kinematic quantities q^2, k^2, and $\boldsymbol{k}\cdot\boldsymbol{q}$. The structures $A_1(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\epsilon})$ and $A_3(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\epsilon})$ in Eq. (2) or Eq. (3) are multiplied by the corresponding kinematic factor $(\hat{\boldsymbol{p}}\cdot\boldsymbol{k})$ or $(\hat{\boldsymbol{p}}\cdot\boldsymbol{q})$.

The basic structures introduced in Eqs. (1) satisfy to the condition of orthogonality, i.e. for non-diagonal interference terms arising in the matrix element squared after the procedure of averaging over the initial spins of the protons and summation over the spin states of the final deuteron we have

$$\overline{A^\dagger_i\,A_j} = 0 \quad \text{for} \quad i \neq j\ . \qquad (4)$$

Furthermore, $\overline{A^\dagger_2\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa},\boldsymbol{\epsilon})\,A_4(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa'},\boldsymbol{\epsilon})} = 0$ also at $\boldsymbol{\kappa'} \neq \boldsymbol{\kappa}$. The non-zero diagonal terms are

$$\begin{aligned} &\overline{A^\dagger_1\,(\boldsymbol{S};\boldsymbol{\epsilon})\,A_1(\boldsymbol{S};\boldsymbol{\epsilon})} = \overline{A^\dagger_3\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\epsilon})\,A_3(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\epsilon})} = 1, \\ &\overline{A^\dagger_2\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa},\boldsymbol{\epsilon})\,A_2(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa'},\boldsymbol{\epsilon})} = \\ &\overline{A^\dagger_4\,(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa},\boldsymbol{\epsilon})\,A_4(\hat{\boldsymbol{p}},\boldsymbol{S};\boldsymbol{\kappa'},\boldsymbol{\epsilon})} = \\ &= \left[(\boldsymbol{\kappa}\cdot\boldsymbol{\kappa'}) - (\hat{\boldsymbol{p}}\cdot\boldsymbol{\kappa})\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{\kappa'})\right]. \end{aligned}$$

The basic amplitudes of four types from Eqs (2-3) related to the parameters a^i_{Sp} or a^i_{Ps} are the result of the linear transformation applied to the set of spin structures $(\boldsymbol{S}\cdot\boldsymbol{\epsilon}^\dagger)\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{\kappa})$, $(\hat{\boldsymbol{p}}\cdot\boldsymbol{S})(\boldsymbol{\kappa}\cdot\ \boldsymbol{\epsilon}^\dagger)$, $(\boldsymbol{\kappa}\cdot\boldsymbol{S})(\hat{\boldsymbol{p}}\cdot\ \boldsymbol{\epsilon}^\dagger)$ and $(\hat{\boldsymbol{p}}\cdot\boldsymbol{S})(\hat{\boldsymbol{p}}\cdot\ \boldsymbol{\epsilon}^\dagger)(\hat{\boldsymbol{p}}\cdot\boldsymbol{\kappa})$ taken at $\boldsymbol{\kappa} = \boldsymbol{k}$ or $\boldsymbol{q}$ and previously used for the amplitudes M_{Sp} and M_{Ps} in Ref. [3].

For analysis of the unpolarized measurements of the $pp \to d\,K^+\bar{K}^0$ reaction performed close to its threshold the square of the matrix element with the lowest allowed parial waves taken into account should be averaged over the initial and summed over the final spins. This leads to the following expression

$$\begin{aligned} &\overline{|M(\boldsymbol{k},\boldsymbol{q})|^2} = \overline{|M_{Sp}(\boldsymbol{k}) + M_{Ps}(\boldsymbol{q})|^2} \\ &C^k\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{k})^2 + S^k\left[k^2 - (\hat{\boldsymbol{p}}\cdot\boldsymbol{k})^2\right] + \\ &+C^q\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{q})^2 + S^q\left[q^2 - (\hat{\boldsymbol{p}}\cdot\boldsymbol{q})^2\right] + \\ &+C^{kq}\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{k})\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{q}) + S^{kq}\left[(\boldsymbol{k}\cdot\boldsymbol{q}) - (\hat{\boldsymbol{p}}\cdot\boldsymbol{k})\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{q})\right]. (5) \end{aligned}$$

The C- and S-coefficients are bilinear combinations of the amplitudes a^i_{Sp} and a^j_{Ps} of Eqs. (2) and (3):

$$\begin{aligned} &C^k = \left|a^1_{Sp}\right|^2 + \left|a^3_{Sp}\right|^2, \quad S^k = \left|a^2_{Sp}\right|^2 + \left|a^4_{Sp}\right|^2, \\ &C^q = \left|a^1_{Ps}\right|^2 + \left|a^3_{Ps}\right|^2, \quad S^q = \left|a^2_{Ps}\right|^2 + \left|a^4_{Ps}\right|^2, \\ &C^{kq} = 2\,\mathrm{Re}\left(a^{1*}_{Sp}\,a^1_{Ps}\right) + 2\,\mathrm{Re}\left(a^{3*}_{Sp}\,a^3_{Ps}\right), \\ &S^{kq} = 2\,\mathrm{Re}\left(a^{2*}_{Sp}\,a^2_{Ps}\right) + 2\,\mathrm{Re}\left(a^{4*}_{Sp}\,a^4_{Ps}\right). \qquad (6) \end{aligned}$$

It is seen that there is no dependence on the relative phases between the a^i and a^j amplitudes at $i \neq j$ in the square of the matrix element $\overline{|M(\boldsymbol{k},\boldsymbol{q})|^2}$ (5) summed over the spin states of the final deuteron and averaged over the spins of the initial protons. The above mentioned relative phases could be determined only from the polarization observables. Hence, the general form used for the contributions from the lowest allowed partial waves M_{Sp} (2) and M_{Ps} (3) to the $pp \to d\,K^+\bar{K}^0$ reaction amplitude near the threshold gives minimum number of the interference terms in the matrix element squared summed over the final and averaged over the initial spins. Moreover, *the parameters that could not be measured in the unpolarized experiment are eliminated.*

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References:

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